RADIO TELEGRAPHY and TELEPHONY

ROBISON'S MANUAL

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1927

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AND

TELEPHONY

FOR USE OF

NAVAL RADIOMEN

BY

COMMANDER (NOW REAR ADMIRAL) S. S. ROBISON, U. S. NAVY

REVISED BY

COMMANDER S. C. HOOPER, U. S. NAVY

AND

LIEUTENANT COMMANDER T. A. M. CRAVEN, U. S. NAVY

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ROBISON'S MANUAL
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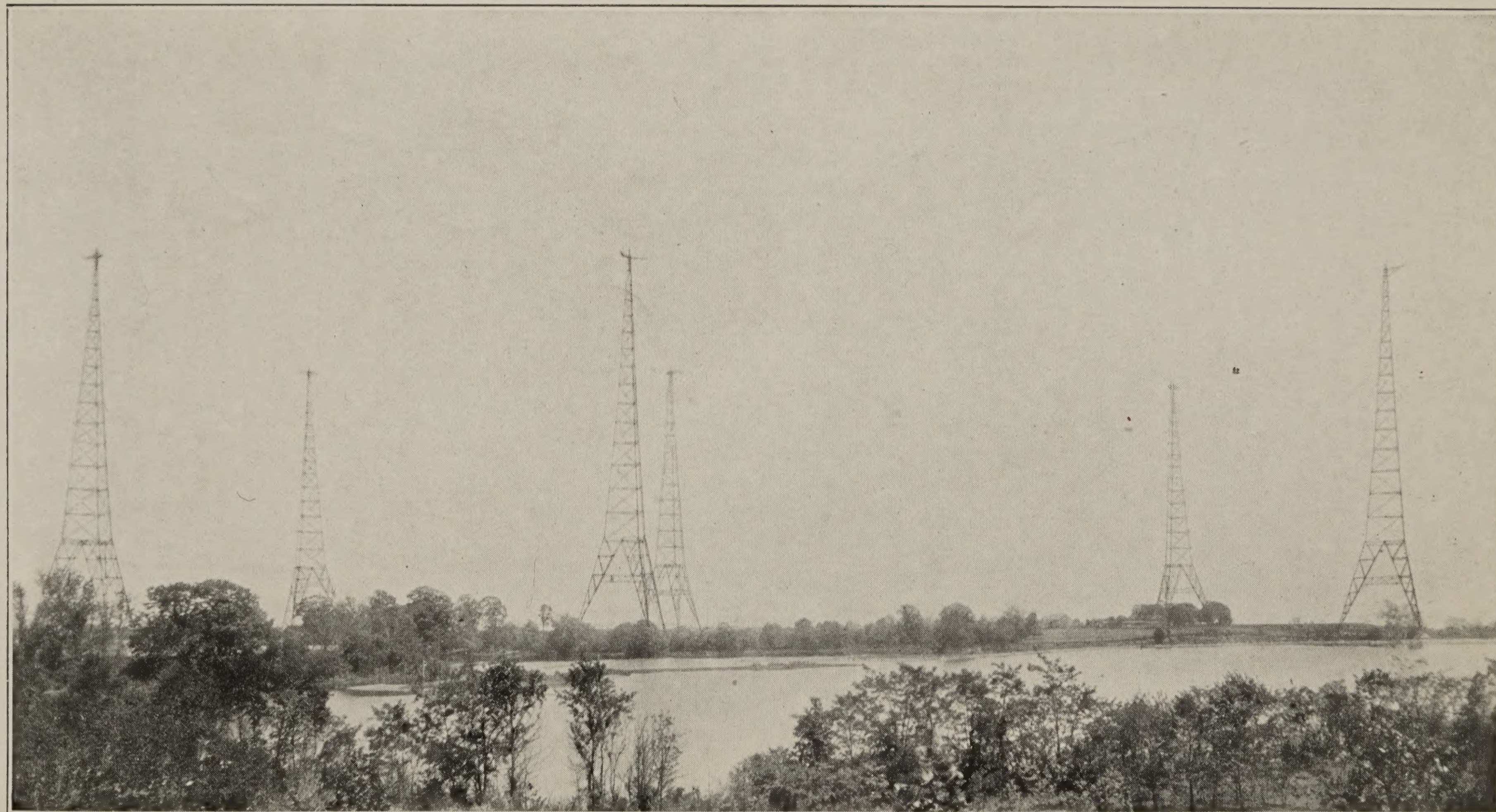


Annapolis—Showing Six 600-Foot Towers Viewed From West

PREFACE

The Radio Manual presents, for the use of student operators and others, the elementary principles of the art of radio telegraphy and telephony, together with descriptions of apparatus commonly used, also the elements of electricity necessary as a basis for understanding the radio principles. This volume is the seventh revision of the volume originally written by Lieutenant (now Rear Admiral) S. S. Robison, U. S. Navy, in 1907.

In the old days, many centuries ago, those who performed the greatest feats for the benefit of civilization were the leaders who fought for religious freedom. Then came the great benefactors of mankind whose battles were for political equality and liberty of thought and action. More recently the champions of human freedom and international peace and happiness have been the world's statesmen and the representatives of the people—the politicians. But today and tomorrow the individuals, collectively and singly, who are performing and will further accomplish miracles for the advancement of human contentment and equality of enjoyment are the scientists. The telephone has drawn all the people closer together. Through mechanical advances the poor can already travel with nearly the same speed and comfort as the rich, and through moving pictures can witness the happenings of the world and its great dramas. Radio has provided means for every one, at practicable expense, to hear the same music, to obtain the news, to enjoy the best sermons and addresses, and ultimately to see a picture of transpiring great events in his own home.

The champions of human happiness in the scientific arena are still unknown to the masses, but their day will dawn. Many of those who are assisting in the major advances are in the Government, giving their lives to service, the silent heroes of their day.

ROBISON'S MANUAL OF RADIO TELEGRAPHY AND TELEPHONY

CONTENTS

SECTION I

Theory of Radio Communication

SECTION II

Application

SECTION III

Radio Measurements and Precision Instruments

SECTION IV

Useful Information

Robison's Manual
of
Radio Telegraphy and Telephony

SECTION I.

THEORY OF RADIO COMMUNICATION.

PART 1. GENERAL THEORY.

CHAPTER I. CONSTITUTION OF MATTER.

Matter is anything that is acted upon by gravity. The force of gravity gives matter weight. Matter occupies space. Although it is always physical and concrete, matter can be in an invisible form, for example, hydrogen or oxygen.

States of matter. Matter exists in three states: **solid, liquid** and **gaseous.**

Solid matter tends to hold its shape when acted upon by some force. For example, if a weight of one pound is suspended from one end of a 10 inch length of No. 20 B&S gauge steel piano wire it will be found by careful measurement that the wire has stretched. Another measurement of the length of the wire after the weight has been removed will show that the wire has returned to its former length.

Liquid matter, in general, does not oppose a change in its shape when acted upon by a force. Liquid mercury will run about when spilled on a flat surface or, if it is poured into a bottle, it will take the shape of the bottle.

Gaseous matter tends continually to occupy a greater volume, in other words, to diffuse. Liquid ammonia is a solution of a gas called ammonia in water. If some liquid ammonia is spilled, the ammonia gas is liberated and fills the whole room with its distinctive odor.

Many substances can occur in the three states. The state in which a substance is found is dependent upon the temperature and pressure. For example, air can be liquefied by subjecting it to a very low temperature and a very high pressure. In turn, this liquefied air can be frozen by a further reduction in the temperature.

Structure of matter. Molecules. All matter is composed of extremely small particles called molecules. There are as many kinds of molecules as there are substances.

The **molecule** is the smallest integral part of any substance. Thus, a water molecule is the smallest water particle and is totally different from a mercury molecule, a wood molecule, or a molecule of any other substance. In other words, the molecule has an individuality. The nature of a given substance is dependent solely upon the molecules of which it is composed.

Each molecule is separate and distinct from all other molecules. It is free to move about and is believed to be in a continual and very rapid state of vibration, or oscillation, at ordinary temperatures. It is thought that one molecule collides with other molecules many millions of times per second, even in air. The rate of vibration and the number of collisions vary with the temperature and the density of the substance.

The molecules are relatively very closely packed together in solids, yet each molecule is able to oscillate about a mean position and is never in permanent contact with other molecules. They cannot, however, travel far. When heat is applied to one end of a metal rod, the molecules at that end vibrate more rapidly and violently and drive the neighboring molecules away, thus increasing the space around themselves. This action is transmitted through the length of the rod and heat is finally felt at the far end. The length of the rod is also increased. This expansion is caused by the space between the molecules having been increased.

The difference between solids, liquids and gases is in the degree of separation of the molecules, the molecules themselves remaining unchanged. There is very little attraction among the molecules in liquids, and still less in gases. This is evidenced by the fact that liquids flow and that gases will become diffused, or mix with air.

Atoms. The molecules are composed of small particles called atoms. The **atom** is the smallest particle of matter obtainable by chemical action.

Some molecules are made up of only one atom, such as the monatomic gas molecules, while others are more complex and contain many atoms of different kinds. For example, two atoms of hydrogen (H) combined form hydrogen gas (H_2), and the combination of twelve atoms of carbon (C), twenty-two atoms of hydrogen (H) and eleven atoms of oxygen form cane sugar, $C_{12}\ H_{22}\ O_{11}$.

It is thought that there are about 90 different kinds of atoms and that the infinite number of combinations of atoms possible constitute the endless variety of molecules and, therefore, of substances in the world.

Atoms vary in weight and size. The hydrogen atom is the lightest known. Its weight is taken as unity and the **atomic weight** of all other atoms is given relatively to that of the hydrogen atom. Thus, the atomic weight of lead is 206.4.

The following constants of hydrogen will give some idea of the extremely small dimensions within which the molecule and atom exist.

The hydrogen molecules have a diameter of about $2 \cdot 10^{-8}$ cm. and, at atmospheric pressure and 32° F., are separated by an average distance of $3 \cdot 10^{-6}$ cm., may travel at a velocity of about $1.7 \cdot 10^{5}$ cm./sec. for a distance of about $1.6 \cdot 10^{-5}$ cm. before colliding with another molecule. The hydrogen atom has a mass of $1.662 \cdot 10^{-24}$ gram.

Electrons. According to the electron theory, atoms have a structure. This structure is, essentially, of two parts, one of which is called the **nucleus,** while the other part consists of one or more **electrons.** Scientists differ in their conceptions of the structure of the atom but agree that the electron is a minute particle of **negative electricity,** that it shows none of the properties of matter, and that the electron of any atomic system is similar to that of any other atomic system. It appears, therefore, that the electron is a charge of negative electricity only, and is not material.

The electron has a diameter of $4 \cdot 10^{-13}$ cm., a mass of $1.59 \cdot 10^{-28}$ gram, when it is moving at a velocity much less than that of light, and a charge of negative electricity equal to $1.59 \cdot 10^{-19}$ coulomb. The mass of the electron is about $\frac{1}{1,850}$ of that of the hydrogen atom.

The atom is often, as a whole, electrostatically neutral and sometimes positive. Hence, there must also be positive electricity in the atom. According to one hypothesis, the positive electricity in each atom is concentrated in a small massive nucleus, having a diameter of about $1 \cdot 10^{-13}$ cm., or $\frac{1}{100,000}$ of the diameter of the atom. This nucleus may be made up of minute particles of positive electricity held together so compactly by electrons that it may be considered a **point charge of positive electricity.** Although practically the entire mass of the atom is contained in the nucleus, it occupies a very minute part of the volume of the atom. The large outer part of the atom is where the electrons are which render the atom neutral.

The electrons themselves probably revolve about the nucleus in a series of circular non-radiating orbits at inconceivably great speeds, the distribution and number of the electrons being such as to neutralize the positive charge of the nucleus. Thus, the atom can exert no external attractive force so long as its internal attractive forces are neutralized. The **normal atom,** therefore, exhibits no electrical properties. Figure 1 is a conventional representation of a normal atom.

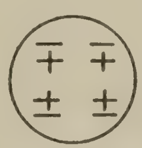


FIG 1.—A Normal Atom.

The **number of electrons** that belong to a normal atom, varies with the kind of atom. It is apparent that the greater the number of electrons

in a normal atom, the greater must be the nucleus. Every atom tends to take or lose enough electrons to acquire the structure of a neutral atom of an inert gas, unless prevented by electrostatic forces. The number of electrons in the normal atom of any element is approximately equal to one-half the atomic weight of that element. For example, hydrogen, the lightest gas, has one electron, while neon has ten electrons.

CHAPTER II. ELECTRON THEORY OF ELECTRICITY

Ions and ionization. The normal atom, to whatever element it may belong, has been shown to be electrostatically neutral, that is, incapable of exerting an attraction for other atoms or electrons, because its internal forces are in exact balance. The atom, however, is a part of the molecule which has a continual and extremely rapid haphazard movement about a mean position, even in the most dense material. The electrons are disturbed by this movement of the molecules and set up disturbances in the ether. These disturbances cause heat.

If a normal atom is struck sufficiently hard by another atom, or in any other way sufficiently disturbed, it will lose an electron. The atom is thereafter no longer neutral, because its internal balance has been destroyed, and it will continue to exert an attraction for electrons until it has picked up one, whereupon it again becomes neutral.

Figure 2 shows in a conventional manner an atom that has lost an electron. The electron is **free,** but only in the sense that it is for an instant not a part of the structure of some atom. It is not perma-

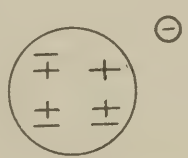


Fig. 2—A Positive Ion.

nently free, but will be drawn into the structure of an atom that has lost an electron. It can be said, therefore, that **electrons do not normally exist apart from atoms,** but, when free, are merely in transit from atom to atom.

An atom that has a deficit of electrons is called a **positive ion,** and has a strong attraction for electrons.

An atom that has been overbalanced by the addition of an electron too many is called a **negative ion,** and will repel electrons in its vicinity and readily part with the extra electron it has acquired. Figure 3 is a conventional drawing of a negative ion. Hence, it appears that atoms tend to remain normal.

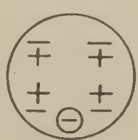


Fig. 3.—A Negative Ion.

Ionization is the process of changing a normal atom to a **positive** or **negative ion.** The **blue glow** sometimes seen in a vacuum tube is

due to the ionization of the residual gas. The glow is caused by the bombardment of the gas molecules by the electrons.

A body is said to be **charged positively** when the atoms of which it is composed have a deficit of electrons. The amount of the charge depends upon how many of its atoms have lost an electron rather than upon how many electrons a few of its atoms have lost. This is based on the belief that an atom can hardly lose more than one electron.

A **negatively charged** body would, therefore, be one, the atoms of which have acquired an electron in excess of their normal complement.

The electric field. If a stick of sealing wax is rubbed briskly against the clothing on a dry, cold day it will acquire the property of picking up lint or small pieces of paper. The friction of the sealing wax against the clothing has made the wax a negatively charged body, and the clothing positively charged. The two charges are equal and opposite because the electrons removed from the clothing have become attached to the sealing wax.

Charges generated in this manner are called **frictional electricity.** The portion of electrical theory which deals with electrical charges that are at rest is called **static electricity.** Table 1, Section IV, gives the Tribo-Electric Series of various materials.

Charged bodies can exert forces of attraction or repulsion upon charges. The force, or strain, is called **electric force** and may be one of attraction or repulsion depending upon whether the body acted upon has, respectively, the opposite or the same kind of charge as the original body. The electric force extends in all directions from the charged body. The space in which the charged body exerts an electric force is called the **electric field.**

It has been found experimentally that the electric force between two small charged bodies varies inversely as the square of the distance between them and is proportional to the product of the charges. Further the force between two charges depends upon the medium through which the force is acting. The **unit charge,** called the electrostatic unit of quantity, has, therefore, been defined as that charge which, when acting upon a similar charge at a distance of one centimeter, will repel it with unit force, or one **dyne,** when the medium is a vacuum or, for all practical purposes, when the medium is air.

On the basis of experiment and definition, therefore, the general law expressing the force between charges is given by the formula:

$$F = \frac{QQ'}{Kr^2}$$

where F = force in dynes,

Q and Q' = charges expressed in terms of the unit charge,

r = distance in cms.

K is a constant called the **dielectric constant,** and

depends upon the medium It is **unity for air or vacuum.** It will be dealt with more fully later. For bodies negatively charged, Q will be negative, and vice versa. Hence, when F is positive, it denotes repulsion; when negative, it denotes attraction.

The electric field of a charged body, or of a number of charged bodies is conveniently characterized at all points by means of the **electric field intensity.** This is numerically equal at any point of the field to the force which would be exerted on a unit positive charge located at the point in question.

Electrical potential. Another important and related quantity, which characterizes the field of charged bodies as well as the condition of any charged or uncharged bodies in the electric field, is called the **electric potential,** or simply **potential.** The difference in potential between two points in the electric field is a measure of the tendency of charges to move or flow from one point to another. Similarly, the difference in the potential of two bodies is a measure of the tendency of charges to flow from one to the other, in case such flow is permitted by providing a conducting path between the bodies. If two bodies are at the same potential there will be no flow and, hence, no change in the charge on either body if they are connected together by a wire. If the two bodies are at different potentials and are connected together by a wire, electrons will flow from the body at the lower potential to the body at the higher potential, until the potentials of the two bodies become equal, and thus the body at the higher potential will have a lower positive charge. In terms of the older theory it would be said that a positive charge has flowed from the body at the higher potential to that at the lower potential.

Since the difference in potential between two bodies is a measure of the tendency of charges to move from one body to the other, it can be determined in terms of the work which must be done in order to transport charges in the opposite direction. Hence, the **difference in potential** between two points is defined as the work required to move a unit positive charge from one point to the other. If work is required to move the unit charge from point A to point B, then point A is at a higher potential than point B, and if the work required is three **ergs,** then the potential of A is three units greater than that of B. If the unit positive charge is allowed to move from A to B, then the electrical force will perform three ergs of work upon it.

The foregoing has been concerned primarily with difference in potential. In order to assign definite values of potential to all points or bodies, it is necessary to fix arbitrarily upon something which will be called zero potential, then differences in potential as measured from the zero basis will give the potential itself. The earth is chosen as this zero of potential, and the difference in potential between any body or point and the earth is the potential of the body or point, and can be positive or negative. The **potential** of any body or point can then be defined as

equal to the work required to move a unit positive charge from the surface of the earth to the body or point. If, in performing the motion, the electrical forces impel the unit charge, and hence, do work upon it, the work required is considered to be negative and the unit charge has been moved to a point where the potential is negative.

The connection between electric field intensity and potential can now be readily seen. The work required in moving a unit positive charge from one point to another is equal to the force which is overcome, times the distance through which the charge is moved. Now the force acting on a unit positive charge is, as has been previously shown, the electric field intensity.

Take two points which are close together so that the electric field intensity can be assumed to be practically the same at the two points. Also, assume the points to be so located that, when moving a unit positive charge from one to the other, the full force of the electric field opposes or aids the motion. Then the difference in potential between the two points will be equal to the work done, and the latter will be equal to the product of the electric field intensity and the distance between the points. Or, conversely, the electric field intensity is the difference in potential between the two points divided by the distance between them. The difference in potential between two such points divided by the distance between them is called the **potential gradient** and gives the rapidity of the most rapid rate of change of the potential in that region of the electric field. From the above, it is seen that the electric field intensity is equal to the potential gradient.

Induced charges. Since difference in potential determines the motion of charges, it is evident that the potential of a conductor must assume the same value at all points after the charges on it have had sufficient time to distribute themselves and have come to rest. If a conductor without a charge is brought into the field of a charged body, it will assume the potential of the field at the point to which it is brought. Thus, an uncharged conductor can have a potential different from zero. If the conductor is then connected to earth, charges will be transferred between it and the earth until its potential becomes zero. The conductor is now charged, but its potential is zero. If a large conductor is brought into the field of a charged body, various parts of it will be in regions of the field where the potentials are different. Since the conductor as a whole must have the same potential throughout, there will be a transfer of charges which will bring this about and the potential of the conductor as a whole will assume some intermediate value of potential. If the charged body has a positive charge, the portions of the large conductor which are nearer the charged body will be in a field of higher potential. Hence, electrons will flow from the farther portions to the nearer portions to equalize the potentials; the nearer portions will become negatively charged and the farther portions positively

charged. These charges, which are produced in the conductor, by reason of its proximity to another charged conductor, are called **induced charges.**

Electric currents. An **electric current** is the transfer of charges, or the motion of electrons, which occurs when a conducting path is provided between two bodies or points which are at different potentials.

If a wire connected to ground is touched against a sphere that has a positive potential, electrons will flow from ground (zero potential) to the sphere. The charge on the sphere will leak off to ground and the sphere will finally have the same potential as the ground, for the reason that its potential has been expended in moving the charge to ground.

This leaking off of the charge on the sphere through the wire is called an electric current. The direction of current flow is from the higher to the lower potential and, in this case, is from sphere to ground.

The flow of current ceases as soon as the potential difference becomes zero. Hence, in order to maintain a current between two points, it is necessary to maintain a difference of potential between them.

Conduction currents. It has been shown that an electric current will flow in a conductor connecting two points having a difference of potential. The current flowing in the conductor is called a **conduction current.**

Let figure 4 represent a longitudinal section of a conductor in which no current is flowing. Each circle represents the mean position about which some molecule is rapidly vibrating due to thermal agitation. As stated before, the molecules collide at great frequency even at ordinary temperatures and, naturally, the probability of collision is greatly

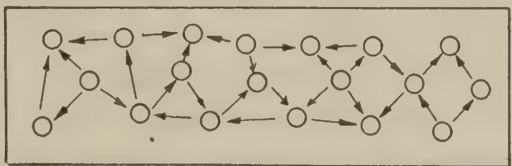


FIG. 4.—Haphazard Motion of Electrons.

increased as the density of the substance is increased. Due to these molecular collisions, electrons are detached from some atoms only to be drawn into other atomic structures that may be deficient in electrons. This interchange of electrons goes on continually but without any definite movement of the electrons in any given direction.

The movement of electrons in some substances is facilitated for the reason that their atomic structure is relatively loose and the hold any atom has on its electrons is weak. Atoms having this characteristic readily part with an electron and find it easy to pick up another in its place. The ease with which the atoms of a given substance part with an electron is believed to be a measure of the suitability of that substance as a **conductor** of electricity.

When a difference of potential is applied to the ends of a conductor, the electrons, which before were moving about at random, take a definite direction in their movement, urged by the electric force. This direction is always from the negative to the positive end of the conductor. The arrows in figure 5 show the trend of the electrons from atom to atom but always in the general direction of the positive terminal. The

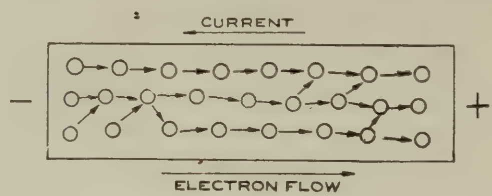


Fig. 5.—Drift of Electrons Under Influence of Electric Forces.

thermal agitation of the molecules, combined with the loose structure of the atom in a conducting material, is responsible for the major movement of the electrons. It is only when the electrons are temporarily free that they travel in a definite direction toward the positive terminal, although impeded by innumerable collisions.

This progressive movement of the electrons, sometimes called **drift,** is extremely slow, while the average velocity imparted to the electrons due to the thermal agitation of the molecules is in the neighborhood of 35 miles per second. The average velocity of the drift is not over a few hundredths of a centimeter for currents such as are carried safely by the conductors used in commercial practice.

The slow drift of the electrons should not be confused with the velocity of propagation of the electric charge, which is about $3 \cdot 10^8$ meters per second. The charge is conveyed through the conductor by a relaying action of the electrons. Each electron travels an inconceivably short distance after leaving an atom before it is incorporated into the structure of an atom beyond it in the direction of the positive terminal. This atom instantly loses an electron to another atom. A similar action occurs throughout the conductor and in the same direction so long as there is a difference of potential.

It should be remembered that only the carriers of electricity, the electrons, travel in conductors. The atoms or molecules do not travel. If this were not so, the metals of which the various parts of an electric circuit are composed would mingle.

Displacement currents. Some substances do not act as conductors, that is, when they are acted upon by a difference in potential there is practically no drift of electrons and, hence, no current flow. Such substances are classed as **insulators** or **dielectrics** and serve a very useful purpose.

According to the electron theory, these substances have an atomic structure that holds the electron very tenaciously within its influence, that is, it is extremely difficult to separate an electron from the atom

even when the electric force applied to the substance is very strong. Hence, there is practically no interchange of electrons by the atoms.

The atomic structure of insulating and dielectric substances is believed to be of such a nature that the electrons are imprisoned in cells within the limits of which they can move. Under the influence of an electric force every electron is impelled in one direction out of its normal position in its cell and held out of equilibrium only so long as the electric force is acting. With every increase or decrease of the electric force the electron is supposed to be, respectively, more or less displaced and the tension increased or decreased. As soon as the electric force is removed, the electrons return to their normal positions.

The thermal agitation of the molecules of the substance plays a very important part in the action of the electrons. As the temperature of the substance increases, the agitation of the molecules increases and the atomic structure is apparently loosened sufficiently to permit the electrons to move more readily from atom to atom. This explains why a good insulating substance becomes a fair conductor when raised to a high temperature. For example, glass at ordinary temperatures is a very excellent insulator and dielectric. When red hot it becomes conducting to such a degree that it will allow enough current to flow through it to melt it.

A **perfect insulator** or **dielectric** is one in which the atoms never lose an electron and in which the electrons also retain their elasticity unimpaired, that is, the electrons are able to move instantly and to follow the variations in the electric force without lag and to return to their normal positions as soon as the electric force is removed. Not all insulating substances come under this category. Air and various other gases, various oils and mica are considered to be extremely good dielectrics. A **rupture** of the insulator occurs if the electrons are strained beyond the elastic limit of the atomic structure. The atoms then lose an electron and the insulator becomes a conductor.

When a difference of potential is applied across an insulating material, an electron is forced out of its normal position in the atom, that is, all the electrons are driven in the same general direction, but each remains within the influence of the particular atom of which it is a part. This displacement of the electrons is a movement of charges and, hence, an electric current. The current flow is only **momentary** because the electrons can not move from atom to atom as in a conductor and ceases as soon as the strain on the electrons equals the electric force producing the strain. This momentary current in dielectrics is called the **displacement current.** The direction in which the electrons are displaced is from negative to positive in the dielectric. The electrical displacement, according to the older theory, which assumes the movement of positive charges, is from positive to negative through the dielectric and is thus shown throughout the Manual. The displacement

current depends upon the rapidity with which the electrons are moving either towards their displaced positions or back towards their normal positions. It is evident, therefore, that when the displacement itself is maximum the motion of the electrons and, hence, the displacement current will be zero.

Thus, it is seen that a displacement current will flow whenever there is a **variation in the electric force,** and will cease when the electric force reaches a steady value. The strain in the insulator will continue, however, so long as the difference of potential is maintained.

Convection currents. In addition to conduction and displacement currents, there is the current that flows through electrolytes, across spark and arc gaps, and from the filament to the plate of vacuum tubes. This type of current is called **convection current.**

In order that a convection current may flow, it is necessary that the electric force impel the electron not only out of the atomic structure of a substance, but also out of the substance itself. This is accompanied in every case with ionization, a process which has already been described.

Convection currents through the electrolytes. If a bar of copper (Cu) is immersed in a solution of copper sulphate ($CuSO_4$) together with a piece of iron (Fe) and a difference of potential is applied across the two metals, making the copper bar positive (**anode**) and the piece of iron negative (**cathode**), a current will pass through the electrolyte from anode to cathode. This a convection current and is caused by the movement of electrons and ions through the electrolyte under the action of the electric force.

The electrolyte consists of two parts—the metallic part (Cu) and the acid part (SO_4). The molecular structure of the solution is easily broken up under the influence of the electric force. Electrons leave some of the atoms of the solution and travel toward the anode to make up the deficit of electrons there. The electrons detached from the atoms attract neutral atoms and, combining with them, make negative ions (acid part SO_4) of them. These travel to the anode and, combining with the copper, form more copper sulphate. The positive ions (metallic part, Cu), left after the electrons leave the atoms, travel to the cathode and are deposited there.

The result of this ionization of the electrolyte is that the piece of iron is plated with pure copper, conveyed through the electrolyte from the copper bar.

Convection currents across spark gaps. When a difference of potential is applied across the electrodes of a spark gap, a displacement of the electrons towards the positive electrode takes place in the atoms of the gaseous medium (usually air) between the electrodes. This displacement of the electrons constitutes a strain of the air dielectric of the gap. When the potential gradient becomes too great for the

dielectric, it is disrupted and a spark passes. The air dielectric is ionized and becomes conducting, aided somewhat by the vaporization of the metal of the electrodes, which are heated by the bombardment of the electrons and positive ions.

A discharge of short duration with a heavy current flow is typical of a spark gap when operating properly. The **spark** has an intense bluish-white appearance and is accompanied with a crackling sound, both phenomena occurring because the conductive bridge between electrodes does not have time to develop fully and then collapses rapidly when the difference of potential drops below the spark-over value. Under certain conditions—such as improper degree of coupling, too short a gap—the difference of potential across the gap remains sufficiently high to maintain the spark for a longer period than normal operation demands. In such an event, the electrodes become very hot in spots, more metal is vaporized and the discharge changes from a **spark** to an **arc** with its characteristic hissing sound and flame-like appearance. The arc discharge will vary in degree with the amount of current, its duration, and with the metal of which the electrodes are made. It is difficult to say at which point sparking ceases and arcing commences, even though each is clearly defined.

FIG. 6.—Arc Between Horizontal Electrodes.

Convection currents across arc gaps. The Poulsen arc has a water-cooled copper electrode for the anode and a carbon electrode for the cathode. The flow of convection current is explained in the following manner.

When the arc is struck, the two electrodes, across which the difference of potential is applied, are momentarily held together and a conduction current flows across the area of contact between the electrodes. Electrons are then passing from carbon to copper. As the carbon is withdrawn from contact with the copper, the area of contact is reduced until, just before the two electrodes are separated, only a point of the carbon is in contact with the copper. The flow of electrons is restricted to so small an area of contact that the carbon is heated to incandescence and gives off an extremely hot vapor. The electrons pass through this vapor and ionize it, and at about the same time, the point of carbon leaves the copper, or is vaporized. In the gap just formed is a vapor made conducting by ionization. The electrons from the cathode have ionized the vapor by forcing electrons out of the atoms. The resulting positive ions travel toward the cathode where they free more electrons and positive ions, the whole process being assisted by the high temperature of the arc.

The arc will persist as long as the difference of potential between the electrodes is sufficient for the length of gap. The path through the ionized vapor will become more and more conducting as the current increases because there will be more ionized vapor. The reason that the arc tends to bulge upwards is because the hot conducting vapor rises. Figure 6 shows an arc between two horizontal electrodes. The arc bulges upward because the hot vapor rises.

Thermal effect. It was stated previously that the atoms of many substances at ordinary temperatures are in continual vibration or agitation, and that the electrons participate in this agitation by travelling from atom to atom. This condition, called **thermal agitation,** is supposed to exist except at extremely low temperatures. As the temperature is increased the agitation becomes more violent, the effect being very pronounced in conducting materials, the atoms of which seem to have a relatively weak hold on their electrons. The movement of the electrons is very much more rapid than that of the atoms of the metal, because of their lack of mass.

Metals can be vaporized at very high temperatures. During the vaporization, atoms are driven through the surface of the metal. The metal must, however, be in vacuum in order that vaporization occur, as otherwise oxidization of the surface will increase the **surface tension** of the metal to such an extent that the atoms will not be able to break through.

If it is possible for the relatively slow moving and heavy atoms to break through the surface of a metal, it is apparent that the electrons should be able to do so much more easily and at a lower temperature of the metal. The following will make this clear.

Electron emission. When a metal, such as tungsten, is heated to a high temperature in an extremely high vacuum, electrons are emitted from its surface. Each electron is emitted with what is called an **initial velocity,** which is dependent upon the temperature of the heated metal. It should be mentioned here that the emission of electrons is an effect caused by the temperature of the metal and **not** by the current that is passed through the metal to heat it.

The emitted electrons do not all have the same initial velocity. Some electrons acquire just enough velocity to break through the surface, others are able to travel a slightly greater distance and only one in a billion, perhaps, can travel any considerable distance from the surface. The **average velocity** of the emitted electrons is very high—many miles per second—but the effect is almost negligible on account of the very large charge associated with the very small mass of each electron.

The number of electrons emitted from a given cathode in a high vacuum depends greatly upon the nature of the cathode material, the

condition of its surface and upon its temperature. The effect of the electron emission from the metal is as follows.

Each electron emitted from the cathode renders the cathode more positive. The positive charge on the cathode is equal and opposite to the sum of the charges on the emitted electrons, so that no matter to what distance any given electron may travel it will finally be pulled back into the cathode.

Figure 7 shows a straight cathode made of tungsten in a highly evacuated glass container. Suppose that this cathode is brought to a high temperature. The emitted electrons will travel to varying distances from the cathode. The small dots in the figure represent the electrons. The great majority of electrons form a dense cloud just above

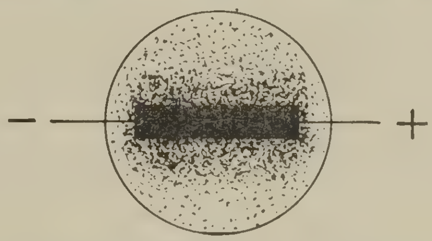


FIG. 7.—Emission of Electrons From Hot Cathode.

the surface of the cathode. This cloud is practically stationary as a whole, but is being constantly renewed by electrons flying out from the cathode to take the place of those that fall back into it. The cloud becomes less and less dense the greater the distance from the cathode, because only a very few electrons are emitted with an initial velocity sufficient to carry them to the glass wall of the container.

To sum up: The purpose of the hot cathode is to render available great numbers of electrons by keeping them in a free condition in its immediate vicinity.

Thermionic currents. If a plate of metal, P, figure 8 is placed close to a hot cathode in a highly evacuated container and a difference of potential ($P.D.$) maintained between the cathode and the plate, making the latter positive (anode) with respect to the cathode, a current

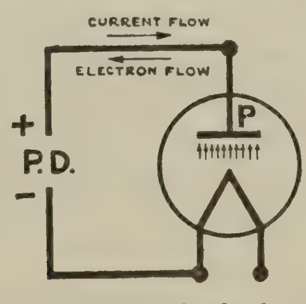


FIG. 8.—Electron Flow From Hot Cathode to Anode in a Vacuum.

will flow from the anode across the space to the cathode and back to the anode from the cathode through the external conductor joining them. The current flowing across the space between the two electrodes

is a convection current and is caused by the flow of electrons from cathode to anode **inside** the container.

The positive charge on plate *P* exerts an attraction on the electrons emitted by the cathode and many of them are drawn to the plate, but not all, when the difference of potential is small. This is because there is a **region of minimum potential** just above the cathode surface where the electric force from the plate is zero, because all the lines of force (imaginary) from the plate end on sufficient electrons to equalize the charge of the plate. Many electrons, however, have no lines of force ending on them tending to draw them to the plate. These are the ones which fall back to the cathode. Therefore, only those electrons that have an initial velocity sufficient to carry them over this **dead-line** are drawn into the plate; the others fall back into the cathode.

As the plate is raised to a higher potential, the region of minimum potential draws in closer and closer to the cathode surface, more and more electrons come into the attraction of the plate, until finally all the electrons emitted are drawn to the plate. This occurs when the electric field at the surface of the cathode is positive or zero. In ordinary practice with the tungsten cathode heated to an operating temperature of 2400° Kelvin, the region of minimum potential is very much less than one-thousandth of an inch from the cathode surface. The minimum potential is believed to be about 0.3 volt. Hence, an electron has to travel against this **retarding potential** before it comes into the attraction of the plate.

The convection current across the space from anode to cathode, due to the flow of electrons from cathode to anode, is called a **thermionic current.** Strictly speaking, all thermionic currents are due not only to the electron flow between the electrodes but also in some measure to the ionization of the residual gas. Practically, it may be considered to be due solely to a pure electron flow when a tungsten cathode and anode are used in a container which has been carefully pumped to a pressure of $1 \cdot 10^{-8}$ mm. of mercury, following modern practices of removing any gases held by the cathode, anode and the walls of the container, and when the plate potential is normal.

On the other hand, some vacuum tubes are first carefully pumped to a low pressure and then certain gases admitted to bring the pressure up to a predetermined value. Under such conditions, the convection current is carried by both electrons and ions.

Further remarks on the vacuum tube appear in Part 6 of this Section.

PART 2.

ELEMENTARY THEORY OF ELECTRICITY.

CHAPTER I. ELECTROMOTIVE FORCE.

Potential. It was explained in Part 1 how a current is caused to flow in a conductor joining two points that differ in potential and that this difference of potential is measured by the work required to move a unit charge from one body to another. It does not matter by what means a body or bodies acquire a charge, the effect produced when a conductor connects them is a flow of current tending to equalize the potentials. The current flow may be momentary or continuous.

If the unit charge is an electrostatic unit as previously defined, and the work is measured in ergs, the potential difference will be measured in **electrostatic units.** The electrostatic unit of potential difference together with the other electrostatic units do not make a system of units which is convenient for practical purposes; consequently, another consistent system of units which is convenient for practical purposes has been adopted for practical usage. This system of units is called the **practical system.**

The unit of potential difference in the practical system is called the **volt,** and it is made equal to $\frac{1}{300}$ of the electrostatic unit. In terms of powers of 10

$$1 \textbf{ volt} = 3.33 \cdot 10^{-3} \text{ esu.}$$

When dealing with decimal parts of one volt, the two following units are convenient:

$$1 \textbf{ millivolt (mv)} = 0.001 \text{ volt} = 1 \cdot 10^{-3} \text{ volt.}$$

$$1 \textbf{ microvolt} (\mu\text{v}) = 0.000001 \text{ volt} = 1 \cdot 10^{-6} \text{ volt.}$$

Also, for convenience,

$$1 \textbf{ kilovolt (kv)} = 1{,}000 \text{ volts} = 1 \cdot 10^{3} \text{ volts}$$

is used.

The **voltmeter** is used to measure the difference of potential between two points in an electric circuit. The instrument may be designed to read *P.D.* in volts, in millivolts or in microvolts, and gives the **average** value of the voltage.

Figure 9 shows a closed circuit ABR, between points A and B of which a **difference of potential** of 10 volts is maintained, point A being positive and point B negative. Now point A can be made positive or negative with respect to ground G by as many volts as may be desired. According to the figure, point A is at a positive **potential** of 100 volts.

The potential of point B is $+100$ volts -10 volts, or $+90$ volts. The closed circuit ABR is not affected internally by the fact that it is at a potential of $+100$ volts. The **flow of current** in circuit ABR is determined by the potential difference between points A and B, and its direction is from the higher to the lower potential as shown by the arrows. This potential difference is called **electromotive force.**

Electromotive force is established by a difference of potential, and causes a current to flow whenever a conducting path is provided, whereas **potential** is merely a level.

Electromotive force (**emf**) is the propelling force which drives current through a closed circuit. It is developed in the following ways:

(a) By friction,
(b) By thermal means,
(c) By chemical action,
(d) By induction.

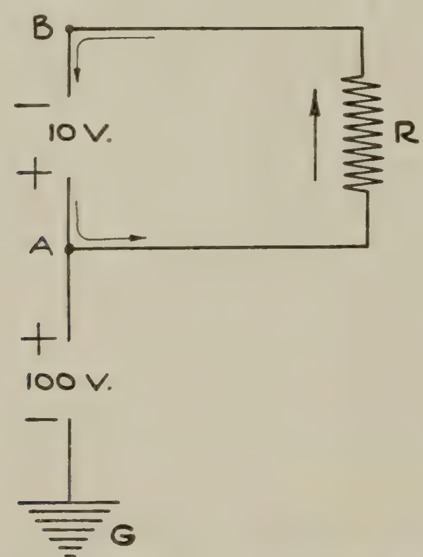


FIG. 9.—Potential and Potential Difference.

Emf by friction. High voltages can be produced by various **static** or **frictional machines.** The disadvantages connected with this method of producing an emf so far outweigh the advantages that it may be considered an impractical method.

Some of these disadvantages are: insulation difficulties; machine must be kept free from dampness; non-uniformity of performance; impossibility of maintaining a current of commercial value due to great drop in emf when current flows. The last-mentioned disadvantage is the most serious from the practical viewpoint.

High voltage and very small current are characteristic of electricity produced by friction.

Thermo-electromotive force. Electricity produced by heating the junction of two dissimilar metals is called **thermoelectricity.** Such a junction is called a **thermocouple** or **thermoelement.**

Figure 10 shows a thermocouple. AJ is a short length of thin steel wire and BJ a similar piece of constantan, which are held in contact at junction J by a small globule of solder.

When heat is applied to point J, an emf is produced which will cause current to flow in the circuit $JARB$, ARB being the external circuit. The steel and constantan junction may not be the only junction of dissimilar metals in a given circuit. Thus, there may be a junction of copper-steel at point A, german silver-copper on either side of R and copper-constantan at point B. If any one of these junctions is heated, a thermal emf will be produced and current will flow in the circuit. The junction J is, however, considered to be the true thermocouple and is the seat of the main emf.

The **thermoelectric power** of a circuit of two metals is the emf produced by 1° C. difference in temperature between the junctions.

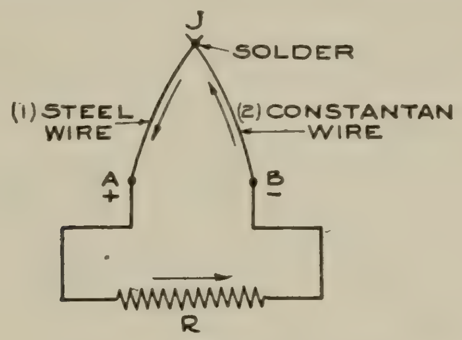


Fig. 10.—The Thermocouple.

Normally, heat is applied to junction J, figure 10, and all the other junctions are maintained at room temperature, about 20° C. Heat energy is transformed by the junction into electrical energy. The emf developed depends solely upon the temperatures to which any two metals are raised, and ordinarily varies directly as the temperature.

The presence of any intermediate metal, such as the solder, does not affect the value of the emf; but where current is employed to heat the junction, more current will be required to heat the junction to a given temperature, and the sensitivity of the thermocouple is thereby lowered.

Table 2, Section III, gives the thermoelectric power of the metals most commonly used for thermocouples. The emf developed at junction J can be predetermined in the following manner.

Example:

What will be the emf in μv of the junction shown in figure 10?

Solution.

From Table 2, the thermoelectric power in μv of

(1) Steel $= +10.62$

(2) Constantan $= -22.0$

Subtracting (2) from (1) algebraically $= +32.62\ \mu$v

And, since the difference is positive, the current flows in (1) steel from J. Hence, A is the positive terminal of the junction.

The effect of any intermediate metal, such as copper, at a junction on the emf can be shown to be zero.

Thus, from the Table, the thermoelectric power in μv of

(1) Steel	$= +10.62$	and (1) Constantan	$= -22.0$
(2) Copper	$= +\ 0.10$	(2) Copper	$= +\ 0.10$
(1a) Algebraic diff.	$= +10.52$	and (2a)	$= -22.10$

By combining (1a) and (2a)

$$
\begin{array}{lr}
 & (1a) +10.52 \\
 & (2a) -22.10 \\
\hline
\text{Algebraic difference} & +32.62 \quad \text{Proof.}
\end{array}
$$

Thermoelectricity is characterized by low values of emf. As a rule the currents are small, although fairly large currents are possible. The practical application of thermoelectricity is confined to temperature and electric current measuring devices. The thermocouple is used for measuring temperatures from the highest to the lowest, where the ordinary thermometer cannot be used. Such an instrument is the **pyrometer,** which is employed in the measurement of very high temperatures. In radio, the thermocouple is used to measure the current flowing in a circuit, for which purpose it is particularly excellent. This use of thermoelectricity will be explained later.

Emf developed by chemical action. When two different substances, such as zinc and copper or zinc and carbon, are placed a little distance apart in certain chemical solutions, a difference of potential will be found to exist between them. Due to this emf, a current will flow from the copper or carbon to the zinc when they are connected externally by a wire. The direction of current flow internally is from the zinc to the copper or carbon.

Chemical energy is transformed directly into electrical energy by the combination just described and, as the most direct method of energy transformation is generally the most efficient, the chemical method has the possibility of a higher efficiency than any of the other methods of transformation. The materials used, however, are very expensive and the cost of production of electricity on a commercial scale by the chemical method is therefore prohibitive.

Zinc in contact with copper in air becomes positively charged. Also, the internal current flows from the zinc to the copper and, in addition, the zinc dissolves. For these reasons, the **zinc** may be considered as the chemical source of the current and is, therefore, the **electro-positive** element. The exposed part of the electro-positive element, to which the external circuit is connected, is the **negative pole** or terminal.

The **copper** or **carbon** electrode is the **electro-negative** element, and its terminal end is the **positive pole.**

The combination of the two plates, electrolyte and container is called a **primary cell.**

Figure 11 shows the essential parts of a primary cell, all parts of which are named. The current flows by convection through the electrolyte from the electro-positive to the electro-negative element, and in the external circuit by conduction from the positive pole to the negative pole. The action that takes place in the electrolyte is quite similar to that described in Part 1. The zinc is gradually dissolved in the electrolyte, while the carbon remains unchanged chemically, but becomes covered with a film of hydrogen. This accumulation of hydrogen on the carbon is called **polarization** and gives rise to an emf which opposes that of the cell proper. Consequently, the emf of the cell decreases as soon as current flows in the external circuit, and the larger the current the more rapid is the fall in emf available from the cell. Polarization results in only a temporary reduction of the cell emf. As soon as this altered condition of the plates or electrolyte is removed, the emf again rises.

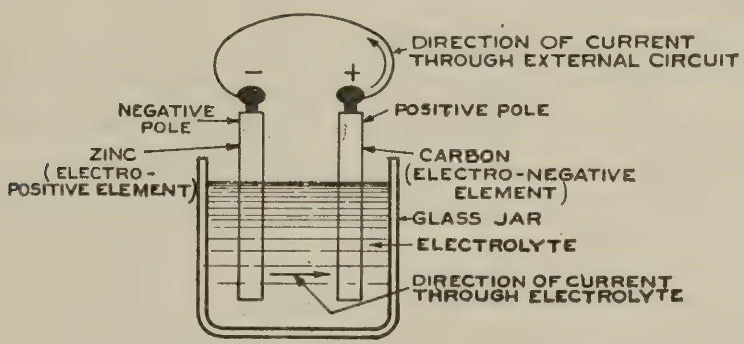


Fig. 11.—The Primary Cell.

The removal of the polarization is called **depolarization** and may be mechanical or chemical, usually the latter. The chemical depolarizer, such as manganese dioxide, is placed in contact with the carbon and absorbs the hydrogen, thus keeping the carbon free of film. Depolarization is also aided by allowing the cell to rest.

When the emf of a cell becomes permanently lowered as current is produced, the cell is said to be **exhausted.** The zinc is usually found to be nearly all dissolved and the acid part of the electrolyte neutralized. It is then necessary to renew the elements and the electrolyte of the cell.

The emf developed by a primary cell is dependent solely upon the nature of the elements and electrolyte, and not in the least on the size of the cell.

The quantity of current that can be drawn from a primary cell depends upon the mass of the elements. Table 3 in Section IV gives the electro-chemical equivalents of some metals.

The rate of flow of current through a given external circuit depends upon the area and material of the elements, their distance apart and also on the conductivity of the electrolyte.

Which element will be electro-positive and which electro-negative of the various pairs of elements is dependent upon its position in the electro-chemical series of elements as given in Table 4, Section IV.

Types of primary cells. Primary cells are divided into two types,

(a) Double-fluid,
(b) Single-fluid.

The emf of the average double-fluid cell is somewhat higher than that of the single-fluid cell. Single-fluid cells are used to a greater extent, however. The so-called dry cell is a single-fluid cell. It has an emf of 1.5 to 1.6 volts when fresh, and has a distinct advantage over all other kinds of cells in that it is portable, compact and can be made in various shapes and sizes as circumstances may require.

The Weston Normal Cell. The Weston normal cell is the standard used to maintain the international volt. It has an emf of 1.0183 international volts at 20°C. The constancy of the emf of this cell is 1 part in 100,000 over a period of one year. These cells are also reproducible within 1 part in 100,000

Storage cells. It was said in the preceding paragraphs that primary cells convert chemical energy into electrical energy. By so doing, they become exhausted and are discarded, as in the case of dry cells. If the primary cell is of the wet type, the elements are usually renewed. Primary cells can not be charged by an electric current because some of the chemical reactions which occur in them are irreversible.

Storage or secondary cells, on the other hand, convert chemical energy into electrical energy by chemical reactions which are essentially **reversible,** that is, they may be charged by an electric current passing through them in a direction opposite to that of their discharge. During the charging process, electrical energy is transformed into chemical energy to be made available at a later time in the form of electrical energy. For this reason, they are called storage cells or accumulators because they store energy and so, potentially, electricity, but do not store electricity as such.

Types of storage cells. There are two general types of storage cells in commercial use, the **lead-acid type** and the **nickel-iron type.** The first type uses lead plates immersed in an electrolyte of sulphuric acid, the elements and electrolyte being contained in a jar of glass or of hard rubber compound. The second type employs steel plates with pockets for holding the active material of nickel and iron oxides; the plates are immersed in a solution of potassium hydroxide in water, the container being made of steel. A brief description of the two types of storage cells and of the reactions occurring in each follows.

The lead-acid type. There are two types of plates in general use, the Planté and the Faure, or pasted plate. The Planté plate is made of pure lead and its surface increased by cutting, scoring or other means. A layer of lead peroxide (PbO_2) is formed on the plates electro-chemically

from the lead plate itself. These are the positive plates. The negative plates are formed from the positive plates by reducing the lead peroxide to sponge lead (Pb). The entire process of forming the plates must be carefully carried out in order to produce successful results.

The Faure, or pasted plate, consists of a frame or grid made of an alloy of lead and antimony into which the active materials, in paste form and made from lead oxides, are pressed. The construction of the grid is such that the paste is held very securely in place. The construction of the plates varies with the different manufacturers. Plates made in this manner are then **formed.** This process consists of passing direct current, or rectified alternating current, through the cell for a prolonged period. During this process, the plates connected to the positive terminal of the supply are oxidized and the active material is transformed into lead peroxide (PbO_2), while the active material of the plates connected to the negative terminal of the supply is reduced from oxides of lead to sponge lead (Pb).

After the forming process has been completed, the **positive plate** of the storage cell, using either type of plate, has a **dark brown** color, while the **negative plate** is of a **dull gray** color.

The storage cell is generally constructed so as to be portable. The compact form of cell usually has several positive and negative plates. The elements are assembled with the negative plates **always** on the outside. Consequently, there is one more negative than positive plate in each cell. The positive plates are joined together by a connecting strap which is equipped with a suitable terminal to project outside the jar. The negative plates are connected together in the same manner. Metallic conduction between plates, which would cause short circuits in the cell, is prevented by perforated thin wooden or hard rubber separators. These separators, however, permit electrolytic conduction. The entire assembly of plates is supported above the bottom of the hard rubber jar by deep insulating strips to prevent short circuits caused by sediment touching the plates. The jar is also equipped with a hard rubber top through which the terminals project. The top is also supplied with a filling hole and vent cap, and made leak-proof by pouring pitch into the joints. The positive terminal is marked by **red** paint or stamped **POS** or + to facilitate the making of correct connections when charging the cell. The electrolyte is chemically pure sulphuric acid (H_2SO_4) diluted with distilled water (H_2O) and varies in its **specific gravity,** depending upon the design of the cell. For high capacity, compact types, the range is from 1.275 to 1.290 when charged and from 1.115 to 1.130 discharged, at 25° C. The specific gravity is measured by an instrument called the **hydrometer.**

The emf per cell varies between 2.06 and 2.14 volts on open circuit, and is independent of the size of the cell, but depends on the strength of the electrolyte, the temperature and the age of the cell. The average

emf during discharge at the normal rate is very nearly 2 volts, falling to 1.75 volt at the end of discharge, at which point the cell should be charged.

The chemical reactions occurring in the storage cell during one cycle, beginning with the cell in a fully charged condition, then discharging through an external circuit and next being charged to its initial condition, are not thoroughly understood. The following, however, is about what occurs.

As the cell is discharging, the sulphuric acid (H_2SO_4) in the electrolyte combines with the sponge lead (Pb) of the negative plate, forming lead sulphate ($PbSO_4$) and with the lead peroxide (PbO_2) of the positive plate, forming lead sulphate; that is, the sulphuric acid burns the active materials of both plates into lead sulphate, and by this process transforms chemical energy into electrical energy. During the discharge, the electrolyte becomes weaker by the amount of the sulphuric acid that is used in the plates and also by the amount of the water (H_2O) formed. The amount of lead sulphate formed also increases, filling the pores of the plates and, as the plates become clogged, the free circulation of the acid is retarded and the normal action of the acid cannot be maintained. As a consequence, the cell becomes less active, this condition being shown by the drop in voltage. The cell then requires charging.

During the charging, direct current is passed through the cell in a direction opposite to that of discharge, and the chemical action is the reverse of that just given. The sulphate, formed in the plates during discharge, by the sulphuric acid combining with the sponge lead and the lead peroxide, is removed. The plates are returned to their original condition and the acid is returned to the electrolyte, which becomes stronger as the charging process continues. In the fully charged condition no more sulphate remains, and all the acid is in the electrolyte. The cycle has been completed and the fully charged cell is again able to supply electrical energy as before. There is no transfer of material as in electroplating, because the lead, lead peroxide and lead sulphate are practically insoluble in the electrolyte.

The reactions given above are represented by the following equations:

Discharge:

$$Pb+SO_4 = PbSO_4, \text{ negative plate,}$$
$$PbO_2+H_2+H_2SO_4 = PbSO_4+2H_2O, \text{ positive plate.}$$

Charge:

$$PbSO_4+H_2 = Pb+H_2SO_4, \text{ negative plate,}$$
$$PbSO_4+SO_4+2H_2O = PbO_2+2H_2SO_4, \text{ positive plate.}$$

The following reversible equation is obtained by combining the above four equations:

$$PbO_2+Pb+2H_2SO \rightleftarrows 2PbSO_4+2H_2O.$$

The reactions are shown graphically in figure 12.

The nickel-iron type. The positive plates are made up of perforated sheet metal tubes into which is tamped the active material consisting of alternate layers of nickel hydroxide [$Ni(OH)_3$] and flake nickel (Ni), the tubes being mounted in steel frames. The negative plates are steel frames with thin rectangular pockets of perforated sheet steel which hold the active material, iron oxide (Fe_2O_3) and metallic iron (Fe). The frames, tubes and pockets are nickel-plated. Both of the outside plates are negative, as in the case of the lead-acid type of cell. The container is a can of cold-rolled sheet steel, nickel plated, to which is welded a cover of the same material supplied with a filling aperture and gas vent. Tapered steel terminals project through the gas and water-tight bushed holes in the cover. Hard rubber insulation is used throughout the cell. The alkaline electrolyte is a solution of potassium hydroxide (KOH) and water (H_2O) with a little lithium hydroxide [$Li(OH)_3$] and other substances added. Its chemical composition and specific gravity do

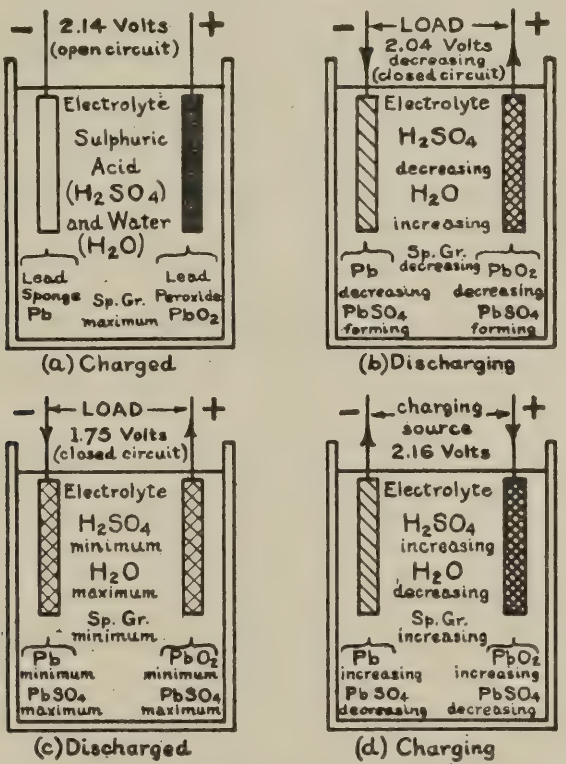


FIG. 12.—Reactions Occurring in a Lead-Acid Storage Cell.

not change during the charge and discharge, because the oxidation and reduction of the active material in the plates occur in equivalent amounts. Fresh electrolyte has a specific gravity of about 1.220, and should be renewed when it has fallen to 1.160.

The exact chemical changes that take place in the cell are not positively known. The combined reaction at both plates as given by Allmand, Applied Electrochemistry page 234, 1912, is:

$$2\,Ni(OH)_3 + Fe \rightleftarrows 2\,Ni(OH)_2 + Fe(OH)_2$$

This equation, read from left to right, represents discharge, and read from right to left represents charge.

The open-circuit voltage of the nickel-iron type of cell varies from 1.45 to 1.52 volts. On discharge, the voltage falls gradually from about the open-circuit value to 0.9 volt at end of discharge made at the normal rate, the average voltage during normal discharge being approximately 1.14 volts.

The capacity of a cell is usually expressed in **ampere-hours**, which is the product of the normal rate of continuous discharge in amperes and

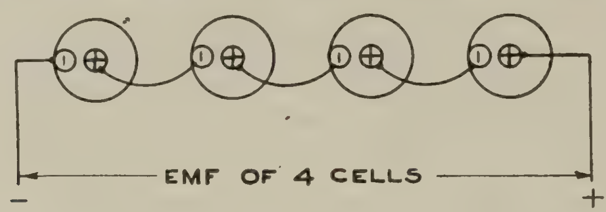


Fig. 13.—Series Connection of Cells.

the number of hours during which this discharge takes place. The ampere-hour capacity of a single cell is dependent upon the number, thickness, and area of the plates which, in turn, is dependent upon the character and the amount of the active material of the total quantity in the cell that is actually used in producing current. Both types of cells lose capacity by lying idle, and show temporary losses of capacity due to long periods of idleness and low temperatures.

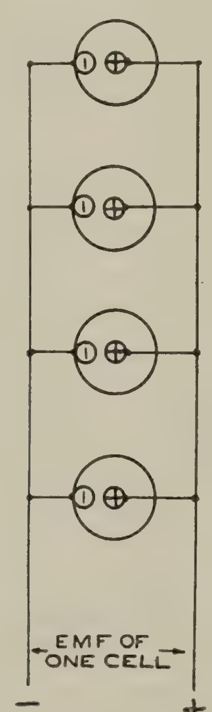


Fig. 14.—Parallel Connection of Cells.

A **battery** is a combination of cells so arranged that either the emf or the amount of electricity available may be increased over that for one cell. The combination of cells can also be arranged so that both the emf and amount of electricity available are increased.

The **series connection** of cells is employed when high emfs are desired. In this case, the positive terminal of one cell is connected to the negative terminal of the next, and the positive terminal of this cell to the negative of the next, etc. This arrangement is shown in figure 13. The emf of the battery equals the emf of 1 cell times the number of cells. Thus, if the emf of each cell is 1.5 volts, then the battery shown in the figure has an emf of 4×1.5 volts or 6.0 volts. The amount of electricity that is available is the same as that for one cell.

The **parallel connection** of cells is used when a greater amount of electricity than is available from one cell is desired. The positive terminals of all the cells are connected together and the negative terminals are similarly connected. The parallel connection is shown in figure 14.

The emf of the battery equals the emf of one cell. The amount of electricity available is equal to that of one cell times the number of cells in the battery.

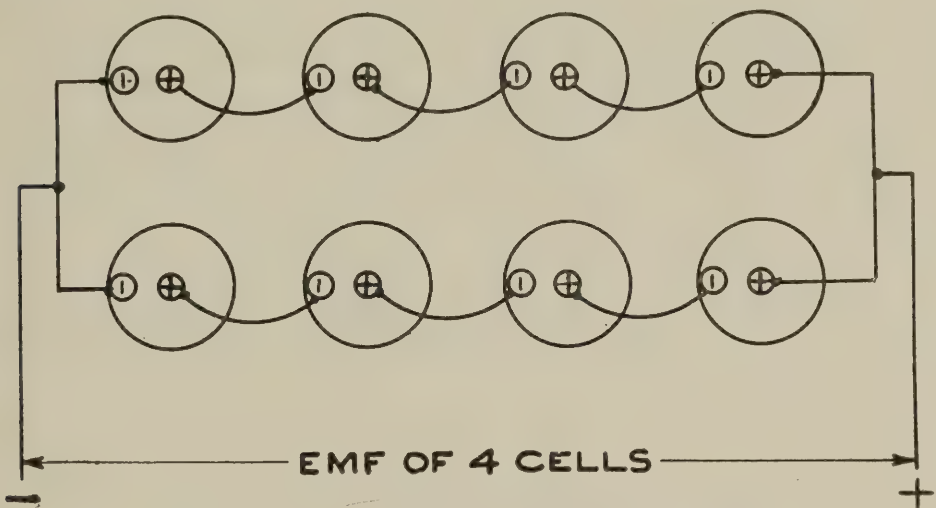


FIG. 15.—Series-Parallel Connection of Cells.

The **series-parallel** or **parallel-series** combination of cells consists of the combination of cells in series and cells in parallel. The number of cells in series determines the emf of the battery, and the number in parallel increases the amount of electricity available. Figure 15 shows a series-parallel combination of eight cells—four cells in series and two cells in parallel.

Induced emf and electric coupling. It has been pointed out previously that, when a conducting body is brought into the vicinity of a charged body, there will be a difference of potential in the first body between the part nearest the charged body and the part farthest away. This difference of potential in the conducting body cannot continue, however, and a transfer of charges takes place through the conductor until the whole conductor has assumed an equal potential throughout. Hence, there is a momentary emf acting in the conductor until the potentials in it become equalized. The emf developed by **electrostatic** or

electric induction is caused by variations in the electric field of the charged body.

Radio circuits may be considered to be conducting bodies, the circuit in which the emf is inserted being the charged body. The electric field about such a circuit is constantly varying, due to the varying emf in the circuit. If another radio circuit is in the vicinity of the first circuit, different and varying potentials will be developed in different parts of this circuit and current will flow between the points of different potential. The effect is due to **electric coupling,** and is frequently very troublesome and difficult to eliminate.

Electromotive force induced in a conductor cutting lines of force is discussed in Chapters V and VI of this Part.

Types of emf. The various types of emf have been treated in this chapter, but have not been named. They may be classified as follows:

(a) The steady emf,

(b) The alternating emf.

All emfs vary to some extent. For example, the emf of a fresh dry cell is about 1.5 volts, but will gradually decrease as current is drawn from it.

A steady emf is one which is maintained between two points having a difference of potential, practically unvarying in degree or in polarity. Such an emf is maintained by static machines, thermocouples, primary batteries and continuous or direct-current generators.

An **alternating emf** is the type of emf which is maintained between two points having a difference of potential which periodically varies in value of changes in polarity.

CHAPTER II. CURRENT.

The various sources of electromotive force were described in the previous Chapter, and it was shown that emf drives current through an electric circuit. The different kinds of current—conduction, displacement and convection currents—have also been described from the viewpoint of the electron theory of electricity. This Chapter will deal with the current itself and some of its physical effects.

Static electricity has been defined as electricity at rest. The charges of electricity on the electrons and positive and negative ions are not in motion, nor is the potential of a charged body varying. **Electric current,** however, is electricity in motion. It is the movement of charges of electricity from point to point in an electric circuit caused by a difference in potential between the points. That is to say, the electrons, each carrying a definite quantity of electricity as its charge, convey a total quantity of electricity past any given point in a circuit.

Quantity of electricity may be expressed in terms of the electrostatic unit of charge of quantity previously defined. The resulting unit is of inconvenient size and is not used for electricity in motion, so a unit of quantity in the practical system of units is taken equal to $3 \cdot 10^9$ electrostatic units.

The unit of quantity of electricity is the **coulomb (Q)**. It is equal to the quantity of electricity conveyed by $6.4 \cdot 10^{18}$ electrons past a given point in an electric circuit. The coulomb is, therefore, only a definite **amount of electricity** and is very rarely used. The rate of flow of electricity is, however, of practical importance.

The **rate of flow** of electricity is the quantity of electricity that passes a given point in a given length of time. If one coulomb passes the given point per second, the electric current is moving at the rate of one unit in the practical system of units. This unit is the **ampere,** and is the name of **rate of flow.**

The **ampere** is the unit of current (I) and is the rate of flow of 1 coulomb per second. Sub-multiples of the ampere are used when small currents are to be measured. They are:

1 **milliampere** (ma) = 0.001 ampere $= 1 \cdot 10^{-3}$ ampere
1 **microampere** (μa) = 0.000001 ampere $= 1 \cdot 10^{-6}$ ampere.

The instruments used to measure current are the **ammeter, milliammeter,** and **microammeter,** and indicate the **average** value of the current.

A current in amperes is defined as coulombs per second.

Then
$$Q = It$$
and
$$I = \frac{Q}{t}$$

Example:

How much electricity has passed a given point in a circuit when a current of 25 amperes has flowed for 10 seconds?

Solution:

Formula: $Q = It$

substituting $I = 25 \times 10 = 250$

whence $Q = 250$ coulombs.

When the duration of current flow is very long, that is for hours, a larger unit of quantity than the coulomb, called the **ampere-hour (amphr.)** is used.

Thus, 1 amphr. = 1 ampere flowing for 1 hour,
= 1 coulomb per second for 1 hour,
= 3,600 coulombs.

The term ampere-hour is used in connection with the capacity of storage batteries. Thus, a storage battery having a capacity of 100 amphrs. will supply a current of 10 amperes for 10 hours, or 1 ampere for 100 hours, etc.

Effects of electric current. The fact that an electric current is flowing in a circuit cannot always be ascertained by the senses. The metal and other substances in the electric circuit have the same appearance while current is flowing as when it is not, unless some part of the circuit is unduly stressed, such as when current is flowing at too high a rate, in which case heating of that part will probably result in smoke, flame or incandescence. The effect of current flow in this case is perceived directly by the senses, but may be considered to be an abnormal condition.

The flow of electricity can, however, be detected by its effects and, not only detected, but also measured both as to its rate of flow and amount. Thus, the effects produced by current are:

(a) Chemical,
(b) Heating,
(c) Magnetic.

Chemical effect. It was stated in the previous Chapter that the electro-positive element of a primary cell is consumed when current is drawn from the cells, and an example was given showing how much zinc would be consumed in a given time for a given current. The rate at which the electro-positive element, if pure, is consumed depends upon how much chemical energy is to be converted into electrical energy per unit of time, say the hour. This process is reversible. If current is forced through the cell in the opposite direction—from carbon to zinc—zinc will be recovered from the electrolyte and deposited on the zinc electrode. This process is called **electrolysis.** This chemical effect of current is employed in electrotyping, electroplating and in refining of metals. An example of electroplating was given in the explanation of Convection Currents through Electrolytes.

The chemical effect of current may, however, result in serious damage when it is caused by stray currents. Thus, the destruction of gas and water mains by electrolysis is caused by heavy currents flowing from the rails of electric car lines to the mains, and thence back to the rails or power house. This destructive electrolysis occurs only at the points where the current leaves the iron piping. At these points, all the requirements for electrolytic action are generally present. The result is that the iron pipe is eaten away and the main bursts at the weakened point.

Definition of the Ampere. The rate of flow of current can be measured very accurately by its chemical effects. Thus, a definite amount of a given metal will be deposited per hour by electrolysis when one ampere is flowing. The amount that will be deposited per amphr. is given in Table 3, Electro-chemical Equivalents of Metals. Equal quantities of electricity will deposit different amounts, depending upon the different metals, but the amount of any given metal is always the same for the same quantity of electricity. Hence,

The **ampere (international unit) is that unvarying current which, when passed through a neutral solution of silver nitrate, will deposit silver at the rate of 0.001118 gram per second.**

This measurement is made by the **Silver Voltameter.** The weight of the silver deposit is found by a very accurate measurement of the element before and after the run, which is timed by a standard chronometer.

Example:

18 grams of silver are deposited in 7 hours from a neutral solution of silver nitrate. Find the value of the steady current required.

Solution:

Silver is being deposited at a rate of 18 grams in 7 hours or $\frac{18}{7} = 2.5714$ grams per hour.

From Table 3, the electrochemical equivalent for silver is

0.2485 amphr. per gram.

but $I = \text{grams/hr.} \times \text{amphrs./gram}$

$= 2.5714 \times 0.2485$

whence $I = 0.639$ ampere.

The voltameter method of measuring current is used only for calibration purposes.

Heating effect. When a current flows through a conductor, the temperature of the conductor will be raised. As the current is increased, the temperature of the conductor will increase. This increase of temperature with increase of current will continue until the conductor is melted. The heating effect of current is utilized in incandescent lamps, where the filament is heated to such a high temperature in vacuum that it becomes incandescent and gives forth light.

The heating effect is mainly dependent upon:

(a) Current,

(b) Cross-section of conductor,

(c) Length of conductor,

(d) Material of conductor.

With a given conductor, the heat produced per second by an unvarying current is proportional to the square of the current. With the same conductor, but with varying current, the heat produced per second is proportional to the average of the squares of the current values during one second. Also, if the direction of current flow is reversed, the heating effect will remain the same.

The heating effect of current is taken up in more detail in the next Chapter and also in Part 3.

Magnetic effect. The definition of the electric field previously given stated that the space about a charged body is in a condition of electric strain and that an electric force acts through the space. The electric field is an effect of electricity at rest.

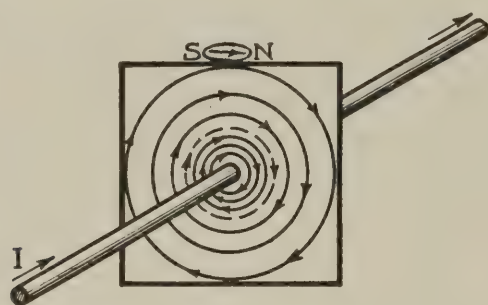


FIG. 16.—Magnetic Field about a Conductor Carrying Current.

Now, another kind of strain is set up by electricity in motion through a conductor. Each electron carries a negative charge of electricity and, as it is impelled through the conductor by the applied emf, its electric field sets up a **magnetic strain** in the surrounding space. The space in which the magnetic strain exists is called the **magnetic field.** The magnetic field is composed of imaginary **lines of force** which form closed loops or circuits about the conductor. The total number of lines in the circuit composes the **magnetic flux,** Φ. These lines of force can be combined into resultants and resolved into components in the same manner as any other kind of lines of force.

Figure 16 shows a conductor through which current is flowing from left to right, as shown by the arrows. The lines of magnetic force are concentric circles both inside and outside the conductor, as shown in the figure. The arrows indicate the direction of the magnetic force, which acts in a clockwise direction in a plane perpendicular to the axis of the conductor. Figure 17(a) shows an end view of a conductor with the current flowing away from the reader through the paper and figure 17(b) shows the same conductor with the return current coming toward the

reader from the paper. The lines of magnetic force are more dense just within and without the surface of the conductor.

The direction of the lines of magnetic force about a conductor carrying current may be found by placing a magnetic compass in the immediate vicinity of the conductor and noting how the compass needle points. The north-seeking pole of the compass needle will point in the

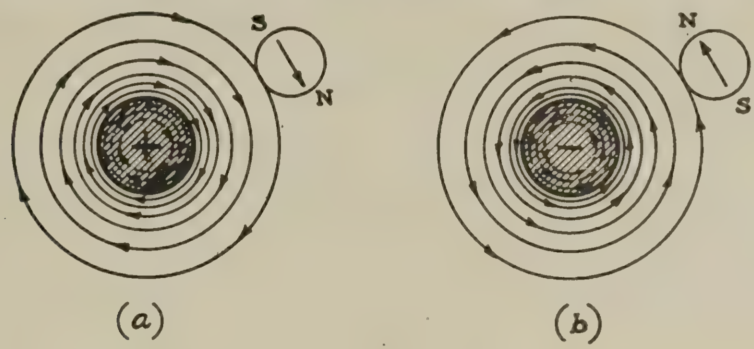


FIG. 17.—Direction of Magnetic Force about a Conductor for Both Directions of Current Flow.

direction of the magnetic force. This is shown in figures 17 (*a*) and 17 (*b*) for both directions of current flow through the conductor.

The direction of the magnetic effect may be remembered by the **Right-hand Rule,** which is—**Grasp the conductor with the right hand with the thumb extended along the conductor in the direction of the current. The fingers will then point in the direction of the magnetic effect.**

CHAPTER III. RESISTANCE, CONDUCTANCE AND OHM'S LAW.

Ohm's Law. When a difference of potential exists between two points, an emf is established, and if the two points are connected by a conductor, a current will flow. The emf is measured by the work that has to be done to transport a unit quantity of electricity from one of the points to the other against electrical forces.

A definite relation between the emf E between any two points on a wire and the current I which this emf causes to flow was found to exist by Ohm. Ohm found that, as long as the physical state of the wire was not changed, the ratio

$$\frac{E}{I}$$

was constant. This constant ratio is called the **resistance *R*** of the conductor. Hence, **Ohm's law** stated symbolically is

$$R=\frac{E}{I}$$

$$I=\frac{E}{R}$$

and

$$E=IR.$$

It is apparent, therefore, that the resistance R of the conducto- does not depend upon either the emf or the current, but upon the dimenr sions of the conductor, the material of which it is composed and also upon the physical state of the material, such as its temperature and the strain to which it is subjected. Therefore, **resistance is a measure of the opposition offered to current flow by a substance.**

Ohm's law is purely an empirical law, because it is not based on theory. It has been found to hold for all cases of **metallic** and **electrolytic conduction** of currents, but does not hold for **convection** currents.

Unit of resistance. Ohm. The resistance of a conductor is defined as the ratio $\frac{E}{I}$. Hence, a conductor has unit resistance when unit difference of potential produces unit current in it. The unit of resistance in the practical system is the **ohm,** Ω. The **International Ohm** is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area of a length of 106.3 cms. The **international ampere** defined in the previous Chapter and the international ohm just defined permit the **international volt** to be defined as the electrical pressure which, when steadily applied to a conductor having a resistance of one international ohm, will produce a current of one inter-

national ampere. These three international units are used in practical measurements.

When dealing with very large resistances, another unit of convenient size is

$$1 \textbf{ megohm } (\omega) = 1{,}000{,}000\Omega = 1 \cdot 10^6\Omega.$$

Also, for very small resistances a unit much used is

$$1 \textbf{ microhm} = 0.000001\Omega = 1 \cdot 10^{-6}\Omega.$$

Table 14 gives the multipliers to be used when changing from one system of units to another.

Volume resistivity. The resistance of a metallic conductor depends upon (a) the material of which it is composed, (b) the cubic dimensions of the conductor, (c) the temperature, and (d) the strain to which it is subjected. For a given conductor and under conditions that are constant, the **resistance *R* is directly proportional to the length and inversely proportional to the cross-section.** Hence,

$$R = \frac{kl}{S}$$

where R = resistance of conductor in microhms,
l = length of conductor in cms,
S = cross-section of conductor in cm^2,
k = constant depending upon material of conductor.

If l and S are each made equal to unity, then

$$R = k$$

and the **volume resistivity** ρ of a given material may then be defined as the resistance of a cube composed of the material the length of whose edge is 1 cm. (**a centimeter cube**), the measuring current being supposed to enter at one face and leave at the opposite, and to be uniformly distributed throughout the cube. The volume resistivity of material is expressed in microhms per cm. of a bar 1 square centimeter in cross-section. The unit of volume resistivity is the **microhm-cm.** Hence, the resistance of a given conductor can be determined, if the volume resistivity is known, by the formula

$$R = lS\rho$$

Table 5A gives the volume resistivity, at 20° C, of the metals and alloys most commonly used. It should be remembered however, that a mere trace of impurity in any of the pure metals will very largely change the resistivity.

The **volume resistivity of alloys** composed of lead, tin, zinc or cadmium can be calculated, if the proportions of the constituent metals are known, by taking the mean value of their resistivities for the resistivity of the alloy. In case of an alloy composed of any of the other metals, its resistivity will be higher than the mean of its constituent parts.

Conductance. The reciprocal of resistance $\frac{1}{R}$ is called **conductance.** The unit of conductance is the **mho (reciprocal ohm).** Thus, Ohm's law expressed in terms of conductance, instead of resistance becomes

$$g=\frac{I}{E}$$

$$I=Eg$$

$$E=\frac{I}{g}$$

Conductance is a measure of the facility with which current flows through a substance.

Conductivity. The **conductivity** γ of a material is the reciprocal of its resistivity, or $\frac{1}{\rho}$ and is expressed in mhos per cm.[3] Knowing the conductivity of the material of which a conductor is composed, its conductance can be calculated from the formula

$$g=\frac{\gamma}{lS}$$

or, from the formula

$$g=\frac{1}{lS\rho}$$

when the resistivity is known.

Effect of temperature on the resistivity of metals. The resistivity of **pure metals** always increases with increase of temperature. It has also been found that the resistivity of such metals approaches zero as the temperature is reduced. Thus, at the absolute zero of temperature ($-273°C$), the resistance of all pure metals would be zero. For comparatively small changes in temperature, the changes in resistance of pure metals is practically proportional to the change in temperature. The formula for determining the resistance of a material at a given temperature is

$$R_t=R_s\{1+a_s(t-t_s)\}$$

where R_t = resistance at given temperature,
R_s = resistance at standard temperature (usually $20°C$),
a_s = temperature coefficient,
t = given temperature ($°C$),
t_s = standard temperature (usually $20°C$).

a_s is the temperature coefficient of the material and is practically the same for all pure metals. Its value for commercial, high conductivity, annealed copper at $20°C$ is

$$a_s=+0.00393.$$

The **temperature of coefficient of alloys** is less than that of its constituents. This is of great importance, and is utilized in the manufacture

of standard resistances where large changes in resistance due to changes in temperature would be intolerable. Alloys having a zero temperature coefficient at ordinary temperatures, or even a negative one, have been prepared. Manganin is an alloy composed of copper, nickel and iron-manganese. Its volume resistivity is very high, being about 25 times that of copper, yet its temperature coefficient is negligible at ordinary temperatures.

The temperature coefficients of carbon, glass, porcelain and quartz are negative, that is, their resistivities decrease with increase of temperature. Thus, the cold resistance of a carbon filament is very much greater than its hot resistance. Glass, porcelain and quartz are very good insulators at ordinary temperatures, but become conducting at high temperatures; that is, their resistance becomes low enough to permit current to pass through them.

Table 5A gives the temperature coefficients of various metals and alloys.

Circular-mil measure. Since most wire has a circular cross-section, the circular measure is most convenient to use. The unit of circular area is called the **circular-mil, C.M.** or **cir. mil.,** and is the area of a circle having a diameter of 0.001 inch, or 1 mil. Thus, in order to find the area in cir. mils. of a circle whose diameter is given in mils, it is only necessary to square the diameter.

Example:

A circle has a diameter of 0.125 inch. What is the area in cir. mils?

Solution:

Diameter $d = 0.125$ inch $= 125$ mils.

area $S = (1.25 \cdot 10^2)^2 = 1.5625 \cdot 10^4$

whence $S = 15{,}625$ cir. mils.

Another unit frequently used for volume resistivity is the **mil-foot,** which is a wire having a length of one foot and a diameter of one mil, hence a cross-sectional area of one circular mil. Table 5 also gives wire sizes in circular mils.

Application of Ohm's law to electric circuits. The foregoing part of this Chapter has been devoted to the statement of formulas showing the interrelationship of current I, emf E, resistance R, and conductance g. It was also shown that resistance and conductance are properties inherent in a conductor and not in the least dependent upon the current and emf except so far as they may be changed due to the heating effect of the current.

It is very important that the application of Ohm's law to electric circuits be clearly understood, because this law is used in some form or other in nearly all calculations. Thus, **Ohm's law applies as well to each part of an electric circuit as to the complete circuit** and, for this reason, is of prime importance.

Ohm's law applied to a part of a circuit. Figure 18 shows a part of a complete circuit. The part of the circuit under consideration is from a to b and has a resistance R through which a steady current I is flowing as indicated by the reading of the ammeter A. For purposes of illustration, let $R = 5\Omega$ and be constant and let $I = 10$ amperes; then, by Ohm's law

$$E = IR$$
$$= 10 \times 5 = 50$$

whence $E = 50$ volts.

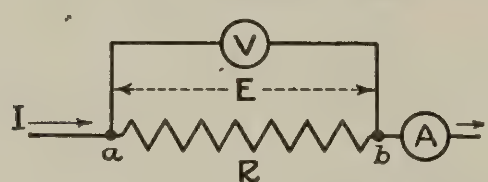


Fig. 18.—Part of an Electric Circuit.

This is the **difference of potential** of points a and b and will be indicated on the voltmeter V connected between the two points. Stated in other words, this E is the **IR drop across the resistance,** or is the **fall in potential** or the **voltage drop,** and represents the emf required to produce the given current in the resistance. It can now be stated that, for a

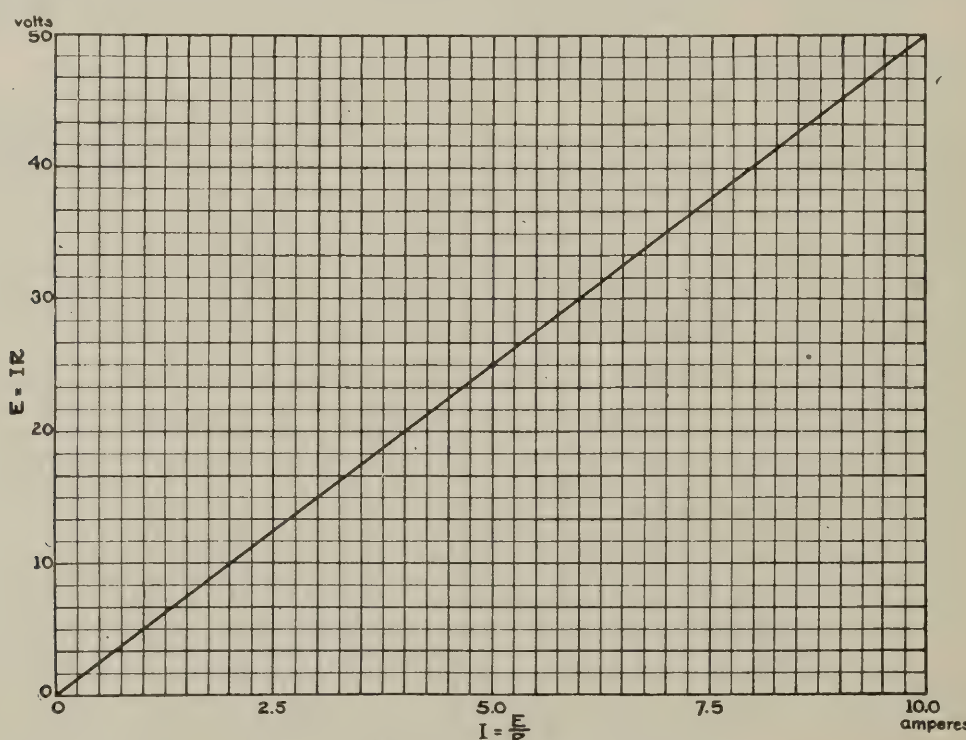


Fig. 19.—Current—Voltage Curve.

constant resistance and a constant current, the IR drop across the resistance remains constant.

Assume now that the resistance R remains constant at 5Ω and that the current I flowing through it is varied from 0 to 10 amperes in steps of 1 ampere each. The variations in emf across this resistance with

the variation in current have been plotted in figure 19. The values of E can be calculated by Ohm's law

$$E = IR$$

or read from the voltmeter. It will be seen that the IR drop across a given resistance can be predicted when the current is known. Under these conditions, it is unnecessary to use a voltmeter to measure the IR drop, because **the emf across a constant resistance varies directly as the current,** or conversely, **the current varies directly as the emf across a constant resistance.** Thus, figure 19 shows that $E=25$ volts when $I=5$ amperes and, when $I=10$ amperes then $E=50$ volts, R being constant at 5Ω

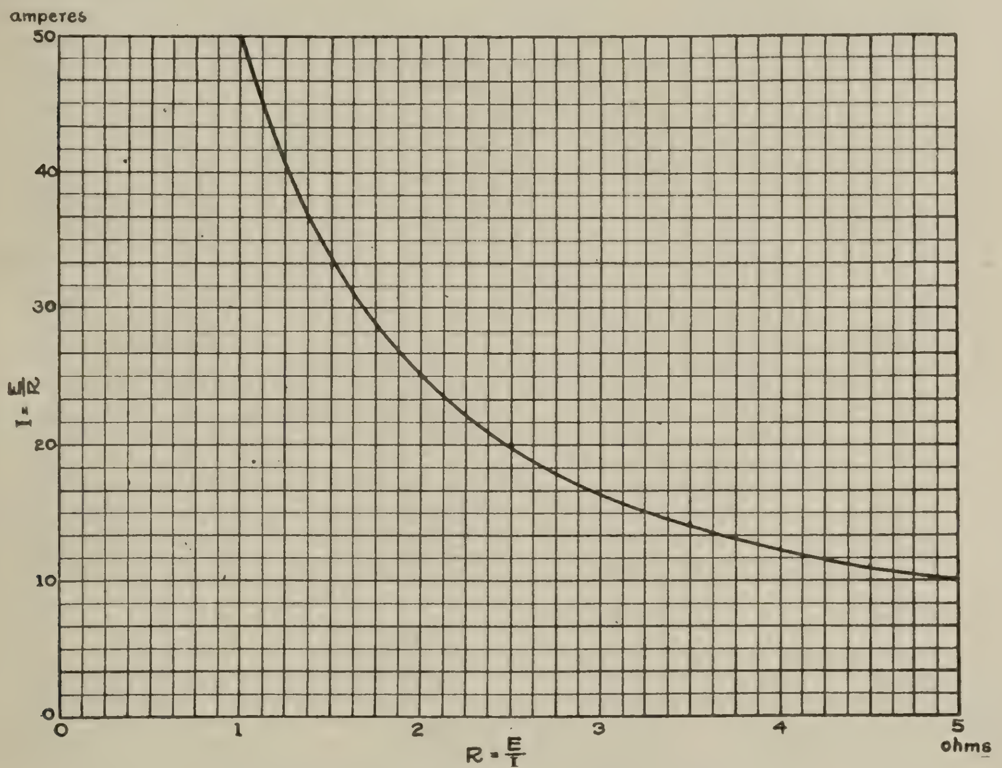


Fig. 20.—Resistance—Current Curve.

Next, let it be assumed that the emf is constant at 50 volts and that the resistance is varied. The effect on the current is shown in figure 20. The current I can be calculated by Ohm's law,

$$I = \frac{E}{R}$$

or read directly from the ammeter A, figure 18. This curve shows that, **for a constant emf across a varying resistance, the current is inversely proportional to the resistance.**

If, instead of plotting the change in current against change in resistance for a constant emf, as in figure 20, the change in current is plotted against change in conductance, the curve will be as given in figure 21. A constant emf of 50 volts across the varying resistance R is again assumed. It is seen that **the current is directly proportional**

to the conductance when the emf across the resistance is constant and, conversely, the conductance is directly proportional to the current when the IR drop across the resistance is constant.

To summarize: Ohm's law applied to any part of an electric circuit permits any one of the three quantities—I, E or R—to be determined if the other two are known and, similarly, any one of the following three quantities—I, E and g—can be calculated if the other

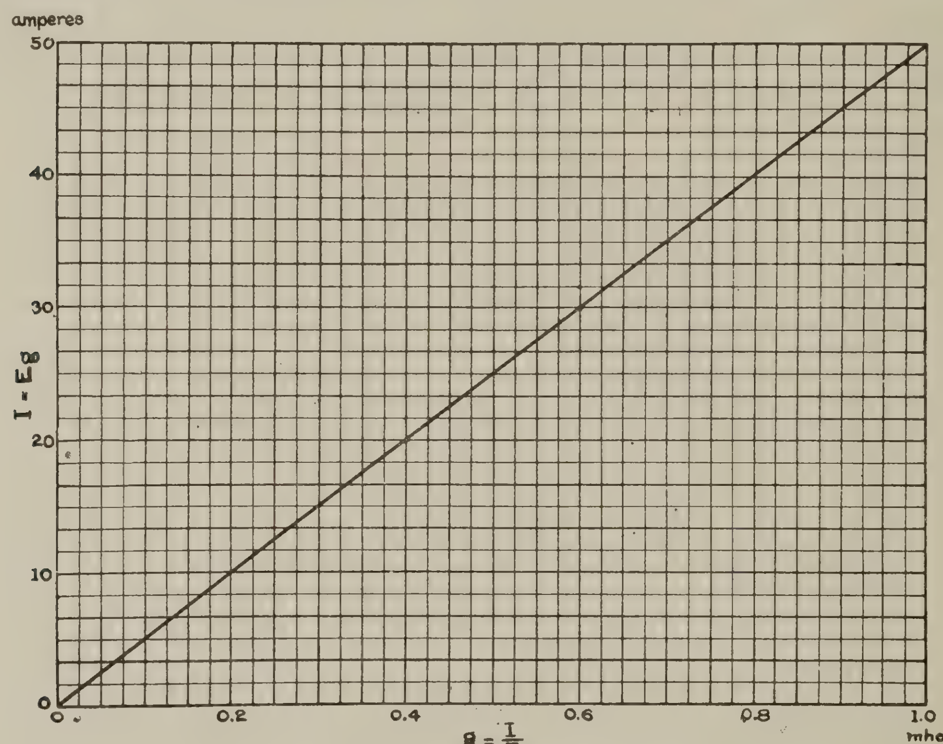


FIG. 21.—Conductance Current Curve.

two are known. In practice, the value of the current flowing in the part of the complete circuit under consideration is measured by means of an ammeter, the voltage drop measured by a suitable voltmeter connected across the part and the resistance calculated by Ohm's law or measured by means of a Wheatstone bridge.

In the following examples, showing the application of Ohm's law to a part of a circuit, it is assumed that there is no source of emf inserted in the part under test and that the temperature and current are constant.

Example:

An incandescent lamp burning at normal brilliancy on 125 volts requires 0.4 ampere. What is its resistance? Its conductance?

Solution:

Formula $$R=\frac{E}{I}$$

substituting $$=\frac{125}{0.4}=312.5$$

whence $R = 312.5\Omega$

Formula $g = \frac{1}{R}$

substituting $= \frac{1}{312.5} = 0.0032$

whence $g = 0.0032$ mho.

Example:

The voltmeter connected across a resistance of 50Ω reads 125 volts. What should be the reading of the ammeter connected as shown in figure 18?

Solution:

Formula $I = \frac{E}{R}$

substituting $= \frac{125}{50} = 2.5$

whence $I = 2.5$ amperes.

Example:

What is the voltage drop across a resistance of 5Ω through which a current of 7 amperes is passing?

Solution:

Formula $E = IR$

substituting $= 7 \times 5 = 35$

whence $E = 35$ volts. (See figure 19.)

Application of Ohm's law to a complete circuit. A complete circuit can be considered to be composed of parts connected together, each part having a certain resistance. The current, resistance and voltage drop of every part can be found as just shown. Hence, the total current, resistance and voltage drop for the entire circuit can also be determined. This will be explained in the following paragraphs.

Electric circuits. The continuous conducting path followed by an electric current is called an **electric circuit.** All electric circuits must be completed, or be closed, in order that a current may flow. A complete electric circuit consists essentially of two parts, (a) the part inside the source of emf and (b) the part external to the source of emf. As long as the conducting path is unbroken, the circuit is said to be **closed.** A break anywhere in the circuit will stop the current flow and is called an **open,** or the circuit is said to be **broken** or **opened.** Ohm's law shows that, for a given voltage, the lower the resistance the larger will be the current, and the higher the resistance the smaller will be the current. Thus, an open circuit may be considered as having infinite resistance, because no steady current can flow in it.

Electric circuits are divided into three general classes, which are (a) **series circuits** (b) **parallel circuits** and (c) **series-parallel circuits.**

Any circuit can be reduced to an **equivalent simple series circuit** containing a resistance and a source of emf. This type of circuit will now be considered.

Simple series circuit. Figure 22 shows a simple series circuit containing a source of steady emf *B*, leads *ac* and *bd*, and a resistance *R*. The circuit is called a **series circuit** because the current follows **only one continuous path with respect to the source.** The cell *B* causes a difference of potential between points *a* and *b* and, as a result, an emf is available to produce a current throughout the complete circuit. This current should not be thought of as starting at *a*, then flowing to *c*, then through *R* to *d* and thence to *b* and stopping there, but rather to be passing through every point in the complete circuit, including the interior of the cell. Further, **this current has the same value at all points in the series circuit.** It does not start out with a given initial strength and gradually decrease as it travels the circuit. An ammeter placed in series at any point in the circuit would give the same reading. The arrows show the direction of the current in the circuit.

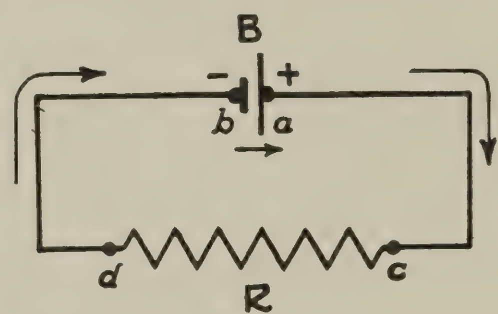


FIG. 22.—The Simple Series Circuit.

Referring again to the figure, it will be seen that the circuit consists essentially of two parts, (a) the source of emf *B*, which is the **internal circuit,** and (b) the **external circuit** *acRdb*. The external circuit contains (a) the apparatus, such as *R*, and (b) the leads connecting it to the source of emf. Hence, every complete series circuit can be considered to be composed of three parts, as follow:

(a) The part inside the source of emf,—**internal circuit,**

(b) The leads connecting the apparatus to the source—**line,**

(c) The **apparatus** itself, such as incandescent lamps, motors, etc.

Now, each of these parts has resistance, and as the same current passes through them, each has an *IR* drop which is directly proportional to its resistance. The *IR* drop in the cell is called the **internal drop** and that in the line is called the **line drop.** The *IR* drop in the apparatus is the only one that can be utilized, the other two being considered as wasted. Since each *IR* drop represents an expenditure of emf, the total expenditure of emf in the circuit is the sum of the emfs expended in each part, and the difference of potential produced by the source must equal the total emf expended. It is evident, therefore, that the source

of emf must always supply a higher voltage than is required at the terminals for the apparatus alone, in order to compensate for the internal and line drops.

Figure 23 shows a method of determining the amount of the internal drop. A voltmeter is connected to the terminals of the cell, and with switch S open, the **open-circuit voltage** E of the cell is read. The switch is then closed and the resistance R is varied from a maximum

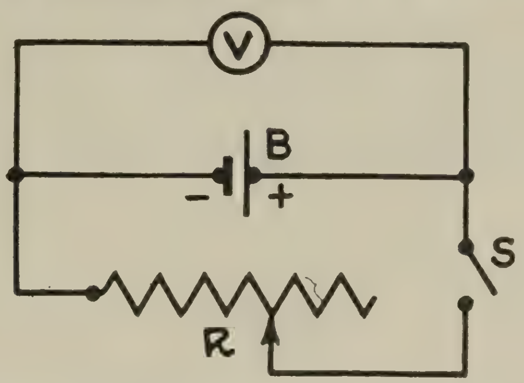


FIG. 23.—Circuit for Measuring the Internal Resistance of a Cell.

value to zero. The voltmeter reading will drop from E to a value E' As the resistance is further decreased, the voltmeter reading E' will decrease and finally become zero when the cell is short-circuited, that is, when $R=0\Omega$. The current at any instant is

$$I=\frac{E}{R+r}$$

where r = internal resistance of cell, and

therefore, $$E=IR+Ir$$

where IR = voltage drop in external circuit,

Ir = voltage drop in cell.

but $$E'=IR$$

hence $$E'=E-Ir$$

where E' = **terminal voltage** of cell.

Thus, the terminal voltage of a cell is always less than the open-circuit voltage by the amount Ir. The maximum current that a cell can deliver is

$$I=\frac{E}{r}$$

when $R=0$ and the cell is short-circuited.

The internal resistance r is a small quantity in the case of a fresh dry cell, and also for lead-acid storage cells. It has a relatively high value in the nickel-iron storage cell. The internal resistance varies with the rate of discharge, temperature, age of cell, etc., and is a more complex quantity than would appear from the above discussion. It should be remembered, however, that every source of emf has an internal resistance that must be considered.

Example:

A cell has an open-circuit voltage $E = 1.5$ volts. When discharging through a resistance $R = 14\Omega$, the current is 0.1 ampere. What is the internal resistance of the cell?

Solution:

Formula $E' = IR$

substituting $= 0.1 \times 14 = 1.4$

whence $E' = 1.4$ volts (terminal voltage)

Internal drop $= E - E'$

$= 1.5 - 1.4 = 0.1$ volt.

Since the same current flows through the cell as through the external circuit

$$r = \frac{E - E'}{I}$$

$$= \frac{0.1}{0.1} = 1$$

whence $r = 1\Omega$ (internal resistance)

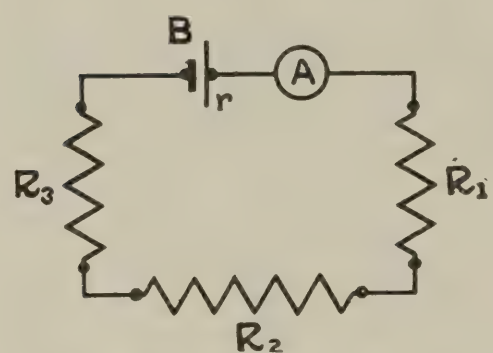


FIG. 24.—The Series Circuit.

It is seen, therefore, that a cell, or any other source of emf, having a high internal resistance will not permit a large current to flow in the external circuit, because even a small current will cause an internal expenditure of voltage equal to the total voltage of the source, and hence no voltage will be available at the terminals for external use. The reason that the value of current calculated from the open circuit voltage and the external resistance is never obtained is also apparent.

Application of Ohm's law to a series circuit. Figure 24 shows a series circuit containing the source of emf B and three resistances connected in series. As stated before, the same current passes through every part of the circuit, including the source. Then **the total emf *E* will be the sum of all the *IR* drops,** or

$$E = Ir + IR_1 + IR_2 + IR_3 + IR_{\text{leads}+A}.$$

whence $E = I(r + R_1 + R_2 + R_3 + R_{\text{leads}+A})$.

Thus, **the equivalent resistance *R* of a series circuit is equal to the sum of the resistances of the individual parts,** and in this case

$$R = (r + R_1 + R_2 + R_3 + R_{\text{leads}+A}).$$

and the current I is equal to the total emf E divided by the total resistance R of the series circuit, or

$$I=\frac{E}{R}.$$

If the source of emf consists of several cells connected in series, the internal resistance of the combination is equal to the sum of the internal resistances of the cells.

Example:

In figure 24 assume the following quantities: $r=0.05\Omega$; $R_1=15\Omega$; $R_2=10\Omega$; $R_3=25\Omega$; $R_{\text{leads}+A}=0.1\Omega$; $I=0.5$ ampere. Calculate the open-circuit voltage E and the terminal voltage E' of the source.

Solution:

Formula	$R=r+R_1+R_2+R_3+R_{\text{leads}+A}$
substituting	$=0.05+15+10+25+0.1$
whence	$R=50.15\Omega$
Formula	$E=IR$
substituting	$=0.5\times50.15=25.075$
whence	$E=25.075$ volts (open-circuit voltage).
Formula	$E'=E-Ir$
substituting	$=25.075-(0.05\times0.5)=25.05$
whence	$E'=25.05$ volts (terminal voltage).

The sum of all the IR drops should equal E. Thus,

$$E=Ir+IR_1+IR_2+IR_3+IR_{\text{leads}+A}$$
$$=0.025+7.5+5.0+12.5+0.05=25.075$$

whence $E=25.075$ volts.

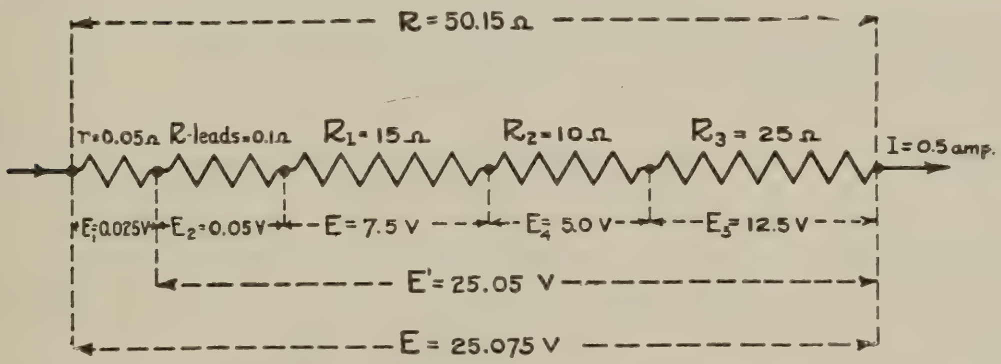


Fig. 25.—Resistances Connected in Series.

The diagram, figure 25, shows the same circuit and gives the relations of I, E, R and the IR drop for each part of the circuit and the entire circuit.

An important point to be remembered concerning series circuits is that, for a given voltage, the current at every point in the circuit is inversely proportional to the total resistance of the circuit. Hence, the insertion of apparatus having a relatively high resistance in an otherwise low resistance circuit will cause the expenditure of a large percentage of the total emf and prevent the proper operation of any

of the other apparatus requiring more current than can now be produced in the circuit due to the insertion of the high resistance. Thus, a motor requiring a current of 5 amperes and a terminal voltage of 125 volts would not operate satisfactorily if connected in series with an incandescent lamp requiring 0.5 ampere at 125 volts, if the source of emf were not changed from 125 volts, because the high resistance of the lamp would limit the current in the circuit. In this case, Ohm's law shows that the resistance of the motor is 25 ohms and that of the lamp 10 times as great, or 250 ohms, and the current available in the circuit only 0.455 ampere. The lamp, however, would light to almost full brilliancy.

This current limiting characteristic of resistance in series circuits is utilized when it is desired to operate a device normally from a source of emf that is higher than the normal operating voltage of the device. The resistance, called **dead resistance,** used for this purpose must have sufficient resistance to produce the required IR drop and also have ample current carrying capacity. This is shown in the following. Example:

A searchlight arc requires a current of 50 amperes when operating normally. Its resistance is 1Ω. The supply voltage available is 125 volts. Calculate the voltage across the searchlight arc and the amount of dead resistance necessary to limit the current to 50 amperes.

Solution:

The voltage across the arc is

$$E = IR$$

substituting $$= 50 \times 1 = 50$$

whence $$E = 50 \text{ volts (voltage across arc)}$$

Since the supply voltage is 125 volts, the dead-resistance must expend 75 volts and be able to carry 50 amperes.

Hence $$R = \frac{E}{I}$$

substituting $$= \frac{75}{50} = 1.5$$

whence $$R = 1.5\Omega \text{(dead resistance)}.$$

It should be mentioned here that the dead resistance serves a useful purpose in that it prevents an excessive current when the arc is struck and fixes the maximum current that can be drawn by the arc at any time. This is necessary on account of the negative temperature coefficient of the carbon electrodes. Without the steadying effect of the dead resistance it would be necessary to vary the terminal voltage of the source to suit the requirements of the arc.

Resistances used for current control are called **rheostats** and may be fixed or variable, the latter type being either continuously variable or variable step by step.

The **voltage divider,** or potentiometer as it is frequently misnamed, is a device which permits a lower voltage than that of the source to be conveniently obtained. The voltage divider is simply a resistance of suitable size, having ample current carrying capacity, which is connected across the source of emf. A sliding, or otherwise variable contact, is an essential part of the device.

Figure 26 shows a voltage divider. B is the source of emf having a terminal voltage E' which is impressed across the fixed terminals, a and b, of the resistance R. The current in the resistance R is usually determined by the reading of the ammeter A. The current I flowing through R can be adjusted to the desired value by the choice of the resistance, R. Frequently a rheostat is placed in series with the voltage divider to make this current adjustment. The voltage drop between points a and b is

$$E' = IR$$

That is, the voltage drop in R is directly proportional to R so long as the current remains constant. Hence, the voltage drop E between

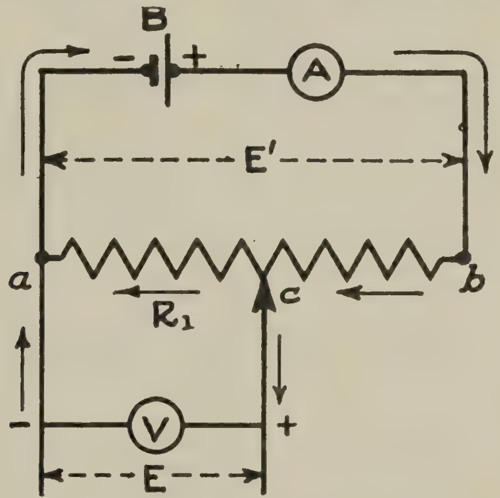


Fig. 26.—The Voltage Divider.

any two points on R, such as a and c, is equal to the product of the voltage drop E' across the whole resistance R and the ratio between the resistance R_1 of the part and that of the whole, or

$$E = E'\frac{R_1}{R}$$

The resistance R should be large as compared to the internal resistance of the source. It is also important that the resistance of the device connected across any part of the voltage divider be large, relatively to the part, as otherwise the value of R will be lowered and thus change the amount of current.

The **potentiometer** is an arrangement of circuits for measuring voltage, and is similar to the circuit shown in figure 27. Such a circuit is used for measuring voltages not much greater than that of the Weston standard cell represented by B_1 in the figure.

Opposing emfs in series circuits. It happens frequently that there are two opposing emfs in a series circuit. Such is the case when a storage battery is being charged. The direction of the current in the complete circuit is controlled by the source that has the higher emf. The difference in the two emfs and the resistance of the circuit determines the amount of current that will flow. Thus

$$E = E_1 - E_2$$

where E = emf acting in circuit,
E_1 = higher emf from one source,
E_2 = lower emf from other source.

If E_1 and E_2 are equal, then $E = 0$ and no current will flow in the circuit.

This characteristic of opposing emfs is advantageous in charging storage batteries. The charging emf may be given a constant value of 2.3 volts per cell for lead-acid storage batteries and, as the battery becomes charged, its terminal voltage will rise and gradually reduce the charging current. This is called the **constant potential** or **tapering**

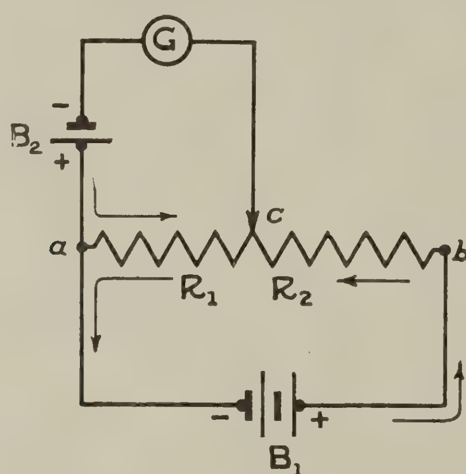


FIG. 27.—Opposing Emfs.

charge method. The initial charging rate is high but at the end of the charging period the rate will have fallen below the normal. When charging storage batteries from a relatively high voltage source it is necessary to insert a rheostat in series with the circuit to expend the excessive voltage.

Figure 27 shows the effect of opposing emfs. R is a voltage divider through which a certain current from source B_1 is flowing. Let $E_1 = IR$. Then the voltage drop E_2 across the part R_1 is

$$E_2 = E_1 \frac{R_1}{R}$$

B_2 is another source of emf connected to oppose the current flow from B_1 through any part of R such as R_1. G is a high-resistance galvanometer in series with source B_2 and R_1. A sliding contact c on R completes this second circuit, which is B_2acG, with the part ac or R_1 common to the other circuit. Due to source B_2, there is another IR drop E_3 in R_1.

Thus, there are two opposing emfs, E_2 and E_3 acting in R_1. The difference of these two emfs E determines the intensity of the current through the galvanometer G, and the direction of the current is determined by which is the higher emf, E_2 or E_3. For example, if contact c is moved to a the galvanometer deflection would be due only to source B_2 and be a maximum in one direction; then, as c is moved towards b, the deflection would be reduced until at some point it would become zero, showing that the emfs E_2 and E_3 are equal and opposite. If the contact c is moved still farther towards b, the deflection would increase, but in the opposite direction, showing that E_2 is greater than E_3.

Parallel circuits. A parallel circuit is one which has two or more parts connected between two points in a circuit. Figure 28 shows a parallel circuit consisting of two parts, R_1 and R_2, connected between points a and b of a circuit. Let E be the emf acting between points

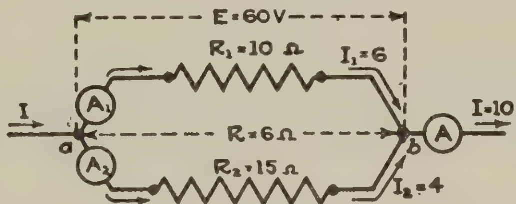


FIG. 28.—The Simple Parallel Circuit.

a and b. Then, by Ohm's law for a series circuit, the current I_1 flowing through R_1 is

$$I_1=\frac{E}{R_1}$$

and the current I_2 flowing through R_2 is

$$I_2=\frac{E}{R_2}$$

This is so because either R_1 or R_2 alone is in series between points a and b. The following important law for parallel circuits is obtained.

The current flowing through any branch of a parallel circuit is equal to the emf acting between its terminals divided by its resistance.

Now, if R_2 were removed, the current I_1 read on ammeter A would be the same as that read on A_1. Likewise, were R_1 removed and R_2 kept in circuit, the current I_2 read on ammeter A would be the same as that read on ammeter A_2. When both R_1 and R_2 are in circuit, the current I read on A will be equal to the sum of I_1 and I_2 read on A_1 and A_2, respectively. Hence, the total current I flowing through the combined parts R_1 and R_2 is

$$I=I_1+I_2$$

Therefore, **the total current flowing through any parallel combination inserted between two points in a circuit is equal to the sum of the currents through the branches.**

It is seen that the total current I is greater than can be obtained through any one branch, provided that the emf acting between its

terminals is kept constant. Therefore, it is apparent that **the joint resistance of the parallel combination is less than the resistance of any one of the branches.** In other words, the combination can be replaced by a single resistance R equivalent to that of the combination. Thus, the equivalent resistance R of R_1 and R_2, figure 28, can be expressed in terms of R_1 and R_2. Since E is the same for both resistances,

$$I=\frac{E}{R}$$

and, since

$$I=I_1+I_2$$

then

$$\frac{E}{R}=\frac{E}{R_1}+\frac{E}{R_2}$$

whence

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

It follows that:

The reciprocal of the joint resistance R of a parallel combination is equal to the sum of the reciprocals of the resistances of the branches,

and, as

$$\frac{1}{R}=g,$$

The joint conductance g of a parallel combination is equal to the sum of the conductances of the branches, that is

$$g=g_1+g_2$$

The general formula

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

becomes

$$R=\frac{R_1R_2}{R_1+R_2}$$

for two resistances in parallel. Hence,

The joint resistance of two resistances in parallel is equal to their product divided by their sum.

Also, the joint resistance of any number n of equal resistances in parallel is equal to the resistance R_1 of one unit divided by the number, or

$$R=\frac{R_1}{n}.$$

It is evident that, since the emf across the parallel combination is constant, **the IR drop across any one of the branches must equal the impressed emf.** Hence, any device connected in parallel with any other device will receive the full emf acting across the other. It is very important, therefore, to remember that the device must have a sufficiently high resistance to keep the current within the proper limits. Thus, a piece of apparatus designed for 80 volts should not be con-

nected across a 125 volt line, because its resistance remaining constant would permit a current $\frac{125}{80}$ times greater to pass through it than would be able to pass were it connected across an 80 volt circuit.

Example:

Two resistances $R_1=15\Omega$ and $R_2=25\Omega$ are connected in parallel across a 125 volt circuit. Calculate I_1, I_2, I and R.

Solution:

Formulas	$I_1=\frac{E}{R_1}$	$I_2=\frac{E}{R_2}$
substituting	$I_2=\frac{125}{15}=8.33$	$=\frac{125}{25}=5$
whence	$I_1=8.33$ amperes	$I_2=5$ amperes.
Formula	$I=I_1+I_2$	
substituting	$=8.33+5=13.33$	
whence	$I=13.33$ amperes (total current).	
Formula	$R=\frac{R_1R_2}{R_1+R_2}$	
substituting	$=\frac{15\times 25}{15+25}=\frac{375}{40}=9.375$	
whence	$R=9.375\Omega$ (joint resistance)	

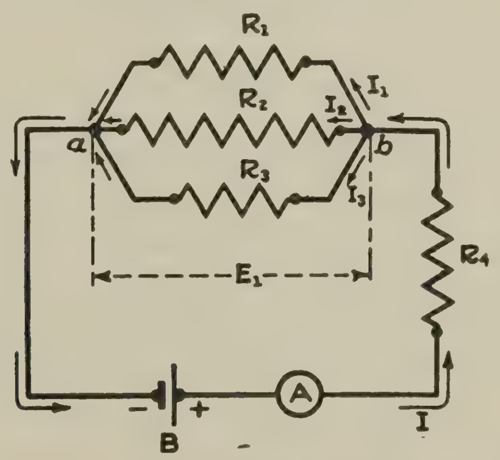


Fig. 29.—The Series-Parallel Circuit.

If the source of emf consists of several cells in parallel, the internal resistance of the battery may be calculated by the law of resistances in parallel. Thus, if the cells are of the same kind and of the same age, the internal resistance r of the battery is equal to that of one cell r divided by the number n of cells, or

$$r=\frac{r}{n}$$

Series-parallel circuits. A series-parallel circuit is a combination of series and parallel parts. Thus, there may be several parallel branches in series with other parallel branches. Figure 29 shows a series-parallel

circuit. The complete series circuit includes the battery B, the resistance R_4 and the joint resistance R_p of the parallel part included between points a and b. Complex circuits can be reduced to an equivalent simple series circuit having a definite emf E, and an effective resistance R. Ohm's law applies to every part of the circuit. In reducing a complicated circuit, it is necessary first to reduce each parallel combination to its equivalent series resistance or conductance before combining it with the remainder of the circuit.

Example:

In figure 29, the voltage drop E_1 across R_1 is 50 volts; $R_1=10\Omega$; $R_2=20\Omega$; $R_3=5\Omega$; $R_4=4.28\Omega$. Neglecting resistance of ammeter and leads and internal resistance of B, calculate I, E and R of the complete circuit.

Solution:

Parallel combination. Let R_p=joint resistance of parallel combination.

Formula $$\frac{1}{R_p}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

substituting $$=\frac{1}{10}+\frac{1}{20}+\frac{1}{5}=\frac{7}{20}$$

whence $$R_p=\frac{20}{7}=2.86\Omega$$

Formula $$I=\frac{E_1}{R_p}$$

substituting $$=\frac{50}{2.86}=17.5$$

whence $I=17.5$ amperes.

Formula $R=R_p+R_4$ (Series circuit)

substituting $=2.86+4.28=7.14\Omega$

whence $R=7.14\Omega$

Formula $E=IR$

substituting $=17.5\times7.14=125$

whence $E=125$ volts

This example may be calculated by using conductance.

The shunt law. The two resistances R_1 and R_2, figure 28 are in parallel. They are also said to be in **shunt,** one with the other; that is, R_1 is in shunt with R_2, and vice versa. As the voltage drop is the same across each resistance, the currents in the two branches are inversely proportional to the resistances of the branches, the larger current passing through the branch having the lower resistance. This relation is called the **law of shunts.**

Let R_a = resistance of instrument,
R_s = resistance of shunt,
I_a = current through instrument,
I_s = current through shunt,

and $I = I_a + I_s$ = total current

Then
$$\frac{I_s}{I_a} = \frac{R_a}{R_s}$$

and
$$1 + \frac{I_s}{I_a} = 1 + \frac{R_a}{R_s}$$

whence
$$\frac{I_a + I_s}{I_a} = \frac{R_s + R_a}{R_s}$$

But
$$I_a + I_s = I$$

hence
$$I_a = I\left(\frac{R_s}{R_s + R_a}\right)$$

The **multiplying factor** is $\left(\frac{R_s + R_a}{R_s}\right)$

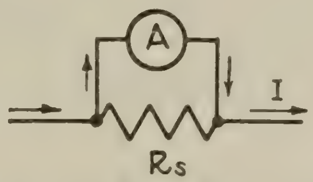


FIG. 30.—The Shunted Ammeter.

The shunt resistance is usually the one that carries the main current, while the other branch usually has a fixed resistance. Thus, practically all ammeters used for measuring large currents (above 5 amperes) are shunted because it is much easier to construct an ammeter to carry small currents, and the instrument is more sensitive to changes in current and can be mounted on a switchboard without regard to the position of the bus bars.

Example:

Figure 30 shows an ammeter A that has a full scale reading of 1 ampere. Let $R_a = 0.05\Omega$ resistance of ammeter. It is desired to provide the instrument with a shunt such that when a current of 10 amperes is flowing in the main circuit, the ammeter will read 1 ampere. Determine the resistance of the shunt and the multiplying factor.

Solution:

If $I_a = 1$ ampere

then $I_s = I - I_a = 9$ amperes.

Formula
$$R_s = \frac{I_a}{I_s} R_a$$

substituting $=\frac{1}{9}0.05=0.0055+$

whence $=0.0055+$

Formula Multiplying factor $=\left(\frac{R_s+R_a}{R_s}\right)$

substituting $=\left(\frac{0.0055+0.05}{0.0055}\right)=10$

whence Multiplying factor $=10$;
that is, the shunted ammeter readings should be multiplied by 10 to give the value of the current in the main circuit.

The Wheatstone bridge. A **Wheatstone bridge** is an arrangement of resistances to facilitate the measurement of resistances. Figure 31 shows the essential circuit of a Wheatstone bridge. A source of emf B is

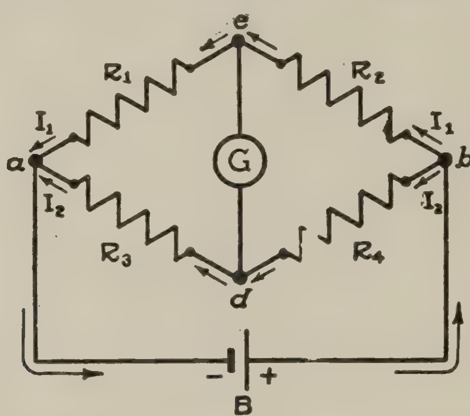


FIG. 31.—Wheatstone Bridge Circuit.

connected across points a and b of a parallel circuit composed of the two branches aeb and adb. A galvanometer G is connected from branch aeb to branch adb at points e and d, respectively. Let I_1 be the current flowing through aeb, and I_2 the current in adb. Then

$$I_1R_1+I_1R_2=I_2R_3+I_2R_4$$

It is evident also, that there is no difference of potential between the two branches at either point a or point b. Therefore, for any point e on branch aeb, there is a corresponding point d on branch adb between which there is no difference of potential and, hence, no current flowing through the galvanometer G. This adjustment is made by varying the three resistances R_1, R_2 and R_3, R_4 being the unknown quantity under measurement.

When this adjustment of the resistances has been made, then

$$\frac{R_1}{R_2}=\frac{R_3}{R_4}$$

and, as there is no current flowing in the galvanometer, the voltage

drop across R_1 equals that across R_3, and the voltage drop across R_2 equals that across R_4; that is,

$$I_1R_1 = I_2R_3 \text{ and } I_1R_2 = I_2R_4$$

Dividing equations
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

from which
$$R_4 = R_3\frac{R_2}{R_1}$$

The practical Wheatstone bridge has a rheostat R_3, which is variable in small steps, while the **ratio arms** R_2 and R_1 are arranged so that the ratio can be read directly in multiples and submultiples of 10. The resistance R_4 to be measured is connected to terminals b and d, and when a balance has been obtained, as indicated by zero reading of the galvanometer G, the reading of the rheostat multiplied by the ratio equals the resistance of the unknown resistance.

When measuring the resistance of thermocouples, small incandescent lamps, etc., care must be exercised. Enough current can be passed through the thermocouple being measured to destroy it. It is usual to place a high resistance in series with the battery B to reduce the current to safe limits.

CHAPTER IV. ENERGY AND POWER.

Work. When a force acts upon a body and the point of application of the force travels in the direction in which the force is acting, the force is doing **work** on the body. The amount of **work** W done by the force F is measured by the product of the force into the distance s along the path which the point of application of the force travels, or

$$W = Fs.$$

If the distance s moved through by a body has a direction opposite to that in which the force is acting, the work is done **against** the force. If the point of application of the force does not travel in the same direction as that in which the force acts, but is inclined to it, then the work done is equal to the product of the actual displacement of the point of application along the direction of the displacement and the component of the force, in which case it is assumed that, where the force is resolved along the direction of the displacement, the other component is at right angles to the direction.

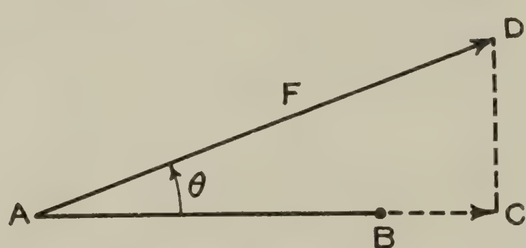


FIG. 32.—Work done by a Force.

Suppose that the line AD, figure 32, represents a force F, and that the displacement takes place from A to B. The component of the force in the direction of AB is given by the projection of AD on AB, or AB produced. In figure 32 this component is

$$AC = AD \cos\theta = F \cos\theta$$

The work done will be

$$W = AB \cdot F \cos\theta$$

F can be any kind of a force—either mechanical or electrical.

Units of work. If the force is measured in dynes, and the displacement is measured in centimeters, the work done will be in ergs. The cgs unit of work, called **the erg, is** defined as **the work done on a body when a force of one dyne moves the body through one centimeter.**

The erg is an extremely small unit. It represents approximately the work done in raising a milligram through one centimeter against the force of gravity.

The difference in potential between two points is measured by the work done in ergs in moving a unit positive charge from one point

to the other. In this case, the force is the force on a unit positive charge, or the electric field intensity. Therefore, the potential difference between the two points is equal to the product of the distance between them and the component of the electric field intensity in that direction. The work done on any other than unit charge is the product of the difference in potential and the amount of the charge. In practical units, the unit difference in potential is the volt, which equals $1 \cdot 10^8$ electrostatic units. The unit charge is the coulomb, which is 10^{-1} times the electrostatic unit. Hence, the work done in transporting a coulomb between two points having a potential difference of one volt is

$$W = 1 \cdot 10^8 \times 1 \cdot 10^{-1} = 1 \cdot 10^7 \text{ ergs.}$$

This unit of work is called the **joule**. It can also be defined as follows, which is the equivalent of the above:

The joule is the work done in one second by a current of one ampere flowing through a resistance of one ohm.

Energy. Energy is the capacity for doing work. When a weight is raised, work is done in raising the weight to the higher level. The weight now posses **potential energy** due to its new position, and will deliver this energy when it is allowed to return to the level from which it was raised. While returning to its original level, it is capable of doing work, due to its motion. Energy due to motion is called **kinetic energy.** Energy can also exist in both of these forms at the same time. Thus, during the return of the weight, there is a gradual change of potential energy into kinetic, but the total energy remains constant.

Energy also exists in many other forms such as—chemical, heat, light, sound, magnetic, mechanical and electrical, etc. Energy can be converted from one form to another, but during each conversion some of the energy being converted is transformed into heat energy and is **wasted, but is neither lost nor destroyed.** The **principle of the conservation of energy** is that, in any system, the sum total of the energy remains constant no matter what energy transformations take place, provided that no energy enters or leaves the system. If energy does enter or leave the system, the energy in the system at any instant plus that which has left the system and minus that which has entered the system is a constant quantity.

Units of energy. The units of energy are the same as those for work, namely, the erg and the joule. When mechanical or electrical energy is transformed into heat energy, there is a definite relationship between the energy utilized and the heat developed. Thus, 1 joule of energy will raise the temperature of 1 gram of water 0.24°C. The heat developed is measured in **calories,** where 1 calorie is the heat required to raise the temperature of 1 gram of water 1°C. Hence,

$$1 \text{ joule} = 0.24 \text{ calorie}$$

and

$$1 \text{ calorie} = 4.2 \text{ joules.}$$

Another heat unit which is frequently used is the **British thermal unit, btu,** and is the energy required to raise the temperature of 1 pound of water 1° F. In terms of this unit

$$1 \text{ joule} = 9.5 \cdot 10^{-4} \text{btu}$$

and

$$1 \text{ btu} = 1{,}050 \text{ joules.}$$

In general, the heat H is determined by the formula

$$H = \frac{W}{J}$$

where W = energy in joules,

$J = 4.2$ when H is to be expressed in calories,

$J = 1{,}050$ when H is to be expressed in btu.

Thus, $H = 0.24W$ (calories)

$H = 9.5 \cdot 10^{-4} W$ (btu)

The unit of mechanical energy is the **foot-pound, ft. lb.,** and is the work done in raising one pound to a height of one foot.

$$1 \text{ foot-pound} = 1.356 \text{ joules}$$

$$1 \text{ joule} = 0.737 \text{ foot-pound.}$$

Electrical energy. Electricity is a form of energy. It results from the conversion of energy by friction, chemical reactions, heat and magnetic induction. For example, an electric generator converts mechanical energy into electrical energy. This electrical energy may be converted by an electric motor into mechanical energy, or the current may be passed through a resistance and produce heat or light, or both. It may also be changed into chemical energy, as when a storage battery is being charged.

Electrostatic electricity is potential electrical energy. A definite amount of work must be done to charge a body. This produces the potential. Such energy is available and, when a conductor is provided, a current will flow from the higher to the lower potential. This current is electricity in motion and possesses kinetic energy. During this transformation, some of the electrical energy may perform useful work, while some of it is converted into heat and is wasted.

The charge Q, which is transported in an electric circuit by a current I in the time t, is equal to the charge transported in 1 second multiplied by the time. This is

$$Q = It$$

The work done in transporting this charge between two points having a difference of potential E requires an expenditure of energy

$$W = QE$$

If this E is the counter electro-motive force (**cemf**) of a motor, then the energy is being transformed into mechanical energy.

If the difference in potential is caused by the IR drop in a resistance R, then this energy is transformed into heat in the resistance.

By Ohm's law

$$E = IR$$

Substituting for E in the previous formula

$$W = I^2Rt \qquad \text{(Joule's law)}$$

By Ohm's law

$$I = \frac{E}{R}$$

Substituting for I in the formula

$$W = EIt$$

and

$$W = \frac{E^2}{R}t$$

Thus,

$$W = EIt = I^2Rt = \frac{E^2}{R}t$$

where W = energy in joules,
E = voltage in volts,
I = current in amperes,
R = resistance in ohms,
t = time in seconds.

To find the energy in calories, multiply the second member of the equation by 0.24. Multiply by $9.5 \cdot 10^{-4}$ when the answer is to be expressed in btu, and by 0.737 when foot-pounds are required. Thus,

$$W = 0.24\ EIt \qquad \text{(calories)}$$
$$W = 9.5 \cdot 10^{-4} EIt \qquad \text{(btu)}$$

and

$$W = 0.737\ EIt \qquad \text{(foot-pounds)}$$

Example:

A current of 100 amperes flows for 1 hour through a resistance of 2 ohms. What is the energy in joules? in calories? in btu?

Solution:

Formula $W = I^2Rt$

substituting $= (1 \cdot 10^2)^2 2 \times 3.6 \cdot 10^3 = 7.2 \cdot 10^7$

whence $W = 72{,}000{,}000$ joules.

$= 0.24 \times 7.2 \cdot 10^7 = 1.728 \cdot 10^7$

whence $W = 17{,}280{,}000$ calories

$= 9.5 \cdot 10^{-4} \times 7.2 \cdot 10^7 = 6.84 \cdot 10^4$

whence $W = 68{,}400$ btu.

Joule's law shows that the quantity of electrical energy that is converted into heat, when a given current flows through a given resistance is **independent of the direction of the current flow.** The heat developed is proportional to the **square of the current irrespective of the direction in which the current may be flowing,** and the conversion into heat is an **irreversible process**; that is, if the direction of current flow is reversed, the heating effect continues in the same proportion as before.

Since the heat produced in accordance with Joule's law is, in most cases, a waste of energy, it is very important to reduce the resistance of the electric circuit to the lowest practical value in order to reduce this waste. On the other hand, the heat developed by the passage of current is utilized in the incandescent lamp, arc light, electric furnace, heating

units, etc. In the first three, both heat and light are produced, the heat being a by-product, so to speak, while in the electric furnace the light produced is unimportant relatively to the heat developed. Some heating units also become luminous, but their main function is to produce heat

Power. Power P is the rate at which work is done; that is, it is the **activity** with which one form of energy is being converted into another form. It is evident that a definite quantity of energy can do a definite amount of work, and also that the work can be done in a very short time or be prolonged indefinitely. In the first case, the energy would be doing work at a high rate, while in the second case, the rate of doing work would be low. In either case, the rate of doing work (power) would be equal to the work done (energy converted) divided by the time taken in the operation; that is, the power

$$P=\frac{W}{t}$$

When a current of sufficient intensity flows through a conductor, the conductor is heated. For example, a definite quantity of energy is required to heat the filament of an incandescent lamp to incandescence. The filament radiates both light and heat energy when incandescent; that is, energy is leaving the system continually. It is necessary, therefore, that more electrical energy be continually supplied the lamp in order to maintain the filament at the necessary temperature; that is, heat must be developed at a definite rate to supply that which is being dissipated. When the rate of supply of electrical energy is greater than the rate of dissipation of heat by conduction or radiation, the temperature will continue to rise. When the rate of supply is equal to the rate of dissipation, then the temperature becomes constant, the final temperature of the resistance being dependent upon its surroundings. Under these conditions, the resistance assumes a constant value.

Units of power. It is apparent from the foregoing that power must be continually supplied to compensate for the power utilized and lost. Since power is the rate at which work is done, the rate can be expressed as the energy converted per second. Thus, the units of power derived from the units of energy given above are: **ergs per second, joules per second,** and **calories per second.**

The unit most frequently used in power measurements is called the **watt,** w.

1 watt = 1 joule per second

and, by definition of the joule,

1 watt = 1 volt $\times$ 1 ampere

Also **1 kilowatt (kw)** = 1,000 watts = $1 \cdot 10^3$ watts

1 kilovolt-ampere (kva) = 1,000 volt-amperes = $1 \cdot 10^3$ volt-amperes (ac unit)

1 microwatt (μw) = 0.000001 watt = $1 \cdot 10^{-6}$ watt.

The units of mechanical power are:

1 **foot-pound per second** (**ft. lb./sec**) $=4.2 \cdot 10^{-2}$ watt

1 **horsepower** (**hp**) $=550$ ft. lb/sec $=746$ watts $=0.746$ kw.

Hence 1 watt $=23.8$ ft. lb/sec. $=1.34 \cdot 10^{-3}$ hp.

1 kilowatt $=1.34$ hp.

Electrical power is measured by means of an instrument called the **wattmeter,** which is a combination of a voltmeter and ammeter and a special coil in series with the movable coil which compensates for the power expended in the instrument itself. The wattmeter gives the **average** value of the power.

Work and energy can now be defined in terms of power.

Thus,

$$W=Pt$$

Other units of work and energy are expressed in terms of the watt and various periods of time. They are the **watt-second,** the **watt-hour** and the **kilowatt-hour, kw-hr.**

1 watt-second $=1$ joule $=1$ volt-coulomb.

1 watt-hour $=3{,}600$ watt-seconds.

1 kilowatt-hour $=1{,}000$ watt-hours

The kilowatt-hour is the commercial unit of electrical energy, and is the basis on which electrical energy is bought and sold. The electrical energy is measured by an instrument called the **watt-hour meter.** Such an instrument is the **integrating wattmeter,** which automatically adds up the work done, although the power may be continually varying.

The equations for power are formed by dividing the equations for energy W, by t.

Hence,

$$P=EI$$

$$P=I^2R$$

$$P=\frac{E^2}{R}$$

where P = power in watts (joules per second).

E = voltage in volts,

I = current in amperes,

R = resistance in ohms.

To find the power in other units, multiply the second number of the equation by

$1 \cdot 10^7$	for ergs per second,
0.24	for calories per second,
23.8	for foot-pounds per second,
$1.34 \cdot 10^{-3}$	for horsepower.

The equations given above are very important. They are used for calculating the power utilized or lost in electric circuits, and for finding the resistance and the efficiency of electrical apparatus.

Example:

An incandescent lamp requires 0.5 ampere at 125 volts. Find the power required.

Solution:

Formula $P = EI$

substituting $= 125 \times 0.5 = 62.5$

whence $P = 62.5$ watts.

Example:

100 incandescent lamps, each having a hot resistance of 250Ω and requiring 125 volts, are connected in parallel. The total line resistance is 0.4Ω. Neglecting the resistance of the leads to each lamp, find (a) the power required for each lamp, (b) the power required for all the lamps, (c) the power lost in the line, (d) the total cost for 3 hours at 12c per kw-hr.

Solution:

(a) Formula $P = \dfrac{E^2}{R}$

substituting $= \dfrac{(125)^2}{250} = \dfrac{1.5625 \cdot 10^4}{2.5 \cdot 10^2} = 62.5$

whence $P = 62.5$ watts per lamp.

(b) $P = 62.5 \times 100 = 6{,}250$

whence $P = 6{,}250$ watts $= 6.25$ kw for 100 lamps.

(c) The current in the line can be found by dividing the power used by the lamps by the voltage.

Formula $I = \dfrac{P}{E}$

substituting $= \dfrac{6.25 \cdot 10^3}{1.25 \cdot 10^2} = 50$

whence $I = 50$ amperes

Formula $P = I^2R$

substituting $= (50)^2 \times 0.4 = 1{,}000$

whence $P = 1{,}000$ watts $= 1$ kw (power lost in line)

(d) The total power is the sum of the power used in the lamps and the power lost in the line, or

total $P = 7.25$ kw.

To find the kilowatt-hours, multiply the power in kw by the number of hours

Formula $W = Pt$

substituting $= 7.25 \times 3 = 21.75$

whence $W = 21.75$ kw-hrs.

At 12c per kw-hr

cost $= 21.75 \times 0.12 = 2.61$

whence cost \$2.61.

Efficiency. The conversion of energy of one form into another form is always attended with the conversion into heat of part of the

energy. This is a waste of energy unless the apparatus is to be used for heating purposes. In mechanics, the heating is caused by friction in the bearings, gears, belts, etc. In electric circuits, the heating is caused by resistance. Since all electric circuits have resistance, it is apparent that power must be continually expended to supply the losses. Therefore, more power must be supplied the circuit than can be drawn from it; that is, the **input power** is always greater than the **output power.** The input power is equal, therefore, to the output power plus the power lost. The **efficiency** of an electric circuit is measured by the ratio of the output to the input. Thus,

$$\eta=\frac{\text{output}}{\text{input}}=\frac{\text{output}}{\text{output}+\text{losses}}$$

where η = efficiency in per cent.

The output is, of course, the power that is useful. Whenever the input cannot be measured directly, it can be calculated from the output and the losses.

The **efficiency of transmission** is never perfect; that is, some energy is lost in the lines connecting the apparatus to the source of power. The waste of energy is due to the resistance of the leads. It is very important, therefore, to keep the line resistance as low as is practical. Power must be expended to supply the loss of power in the line. Efficiency of transmission is defined as the ratio of the power delivered to the apparatus to the power supplied the line at the source. Thus, a wattmeter at the source would read higher than the wattmeter at the end of the transmission line to the apparatus. The difference between the readings of the two wattmeters would give the power in watts lost in the line.

Example:

While a certain motor is delivering 30 hp, the voltmeter and ammeter connected in at the motor terminals read 220 volts and 107 amperes, respectively. The wattmeter on the generator switchboard reads 24 kw. Calculate (a) efficiency of motor, (b) efficiency of transmission and (c) overall efficiency.

Solution:

(a) Formula $P=1.34\cdot10^{-3}EI$ (hp)

substituting $=1.34\cdot10^{-3}\times2.2\cdot10^{2}\times1.07\cdot10^{2}=31.5$

whence $P=31.5$ hp (input)

Formula $\eta=\frac{\text{output}}{\text{input}}$

substituting $=\frac{30}{31.5}=0.952$

whence $\eta=95.2\%$ (efficiency of motor)

(b) Efficiency of transmission equals power delivered to motor divided by power delivered to transmission line.

Formula $P = EI$

substituting $= 2.2 \cdot 10^2 \times 1.07 \cdot 10^2 = 2.354 \cdot 10^4$

whence $P = 23.54$ kw. (output to motor)

Formula $\eta = \dfrac{\text{output}}{\text{input}}$

substituting $= \dfrac{23.54}{24} = 0.981$

whence $\eta = 98.1\%$ (efficiency of transmission)

(c) The overall efficiency is found by dividing the output of the motor by the input to the transmission line. It is necessary to change the kw input to hp input.

Formula $\eta = \dfrac{\text{output}}{\text{input}}$

substituting $= \dfrac{30}{32.16} = 0.933$

whence $\eta = 93.3\%$ (overall efficiency).

Thus input $= 24\text{kw} = 32.16$ hp.

CHAPTER V. MAGNETISM AND ELECTROMAGNETIC INDUCTION.

The various effects produced by an electric current were mentioned in Chapter II of this Part. Among these effects was that of magnetism. It has been found that electricity and magnetism are very closely related. The object of this chapter is to show the various relationships existing between the electric current and magnetism.

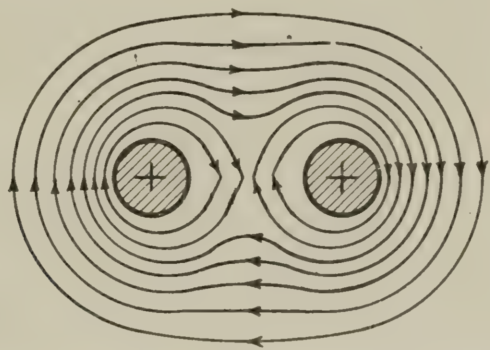


Fig. 33.—Resultant Magnetic Field about two Parallel Wires in which Current is Flowing in same Direction.

Resultant magnetic fields. The magnetic field about a single straight wire carrying current is circular. Now, if two wires, each carrying current, are placed parallel and close together, the resulting magnetic field is a combination of the two fields, which are thus super-

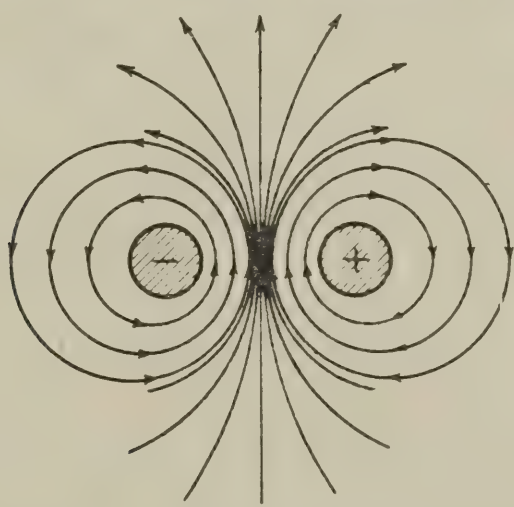


Fig. 34.—Resultant Magnetic Field about two Parallel Wires in which Current is Flowing in Opposite Directions.

imposed. Figure 33 shows two wires close together, each carrying current in the same direction. The lines of force about each wire have the same direction and tend to combine as shown, making one large field. As a result, the wires tend to come together.

If the current is flowing in opposite directions in the same two wires, the resultant magnetic field is as shown in figure 34. In this case the wires tend to be forced apart.

The solenoid. If the wire carrying current is wound in a loop, figure 35, it will be found that all the lines of force about the wire combine and pass through the loop in the same direction. A magnetic field then exists both inside and outside the loop itself. The field inside the loop has a tendency to draw a magnetic substance into it.

When a length of wire is formed into several loops which are wound side by side, the resulting coil is called a solenoid. In this case, most

Fig. 35.—Magnetic Field about a Loop Carrying Current.

of the lines of force about one loop will combine with those of the next loop, etc., somewhat as shown in figure 34 for two parallel wires carrying current in opposite directions. The result is that lines of force thread the coil and form closed loops by returning outside the coil to the other end. The path followed by the lines of force is called the **magnetic circuit.**

If the loops are placed close together, practically one-half of the lines will continue through the coil, leave by one end and return outside the coil to the other end. This is shown in figure 36. The lines of

Fig. 36.—Magnetic Field of a Solenoid Carrying Current.

force between closely wound loops are neutralized to a greater extent than shown in figure 36, and are not able to encircle each wire with a separate field, but continue through the coil as represented in the figure.

The **north pole** of the coil is the end from which the lines of force leave, and the **south pole** is the end into which the lines of force return.

The **polarity** of a coil can be found by the **End Rule,** which is—Look at one end of the coil; if the current is flowing into or out of the end turn in a **counter-clockwise** direction, that end is the **north pole** of the coil; if in a **clockwise** direction, it is the **south pole.**

If a magnetic substance is placed near either end of the coil while it is carrying current, the coil will tend to draw in the substance. This property of the coil is utilized in the tripping mechanism of some forms of circuit-breakers and in some types of arc lamps for automatically striking the arc.

Magnetic substances and induced magnetism. If a bar of soft iron is brought close to the north pole of a solenoid carrying current, it will be strongly attracted by the solenoid and, in addition, will exhibit magnetic properties. The attraction exerted is really an attraction between the north pole of the solenoid and south pole that is induced in the bar of iron. The farther end of the bar becomes a north pole. If the bar is removed from the field of the solenoid, it will be found that the bar has practically lost its magnetic property. Soft iron may, therefore, be called a **temporary magnet.**

Iron, steel, nickel and cobalt are called **ferromagnetic** substances, as they show magnetic properties most strongly when subjected to a magnetic force. Many other substances show magnetic properties by induction, but to a very much lesser degree. These are called **paramagnetic** substances. Still other substances are repelled by a magnetic field. These are the **diamagnetic** substances. Iron and steel alone will be considered in this Manual.

If a bar of hard steel is substituted for the bar of iron, it will be found that, in addition to its display of induced magnetism while in the field of the solenoid, it retains some magnetization after removal from the field. The bar of steel has become a **permanent magnet,** by virtue of its power of retaining part of the induced magnetism. This property of steel is called **magnetic retentivity.**

Magnets. The bar of steel may be broken into any number of pieces, and each piece, no matter how minute, will be a magnet. Thus, each molecule of iron is an **elementary magnet** having equal and opposite poles. The electron theory of magnetism explains this phenomenon.

Electron theory of magnetism. The magnetization of the elementary magnet is due to the magnetic action of electrons revolving about the nucleus of the atom. If the number of electrons revolving in one direction is equal to the number revolving in the opposite direction, then a nonmagnetic atom is the result. However, when this type of atom is in a magnetic field, magnetic lines are set up in opposition to those of the inducing field and a repelling action occurs. Thus, the atom is diamagnetic. When a greater number of electrons are revolving in one direction than in the opposite direction about the nucleus of the atom, the atom is naturally paramagnetic. In the case of such atoms, it is thought that the velocities of the electrons and the planes of the electron orbits are changed, with the possibility that the atoms themselves are rotated by the inducing field.

The elementary magnets in a bar of iron or steel which exhibits no magnetism are supposed to be turned in all directions or to form closed magnetic loops among themselves. Their individual external magnetic effects are thus neutralized, and the bar exhibits no external magnetism. This is shown in figure 37 (*a*). When the iron or steel is partially magnetized, some of the elementary magnets are turned in the same direction, as illustrated in figure 37 (*b*), and their external fields add. When the bar is magnetized to saturation, all the elementary magnets are turned in the same direction. Figure 37 (*c*) illustrates this condition.

The more elementary magnets that turn in the same direction under the influence of the magnetic induction, the more strongly is the bar of iron or steel magnetized.

Magnetic attraction and repulsion. If two long and slender needles are magnetized and one is hung on a silk fiber, it will be found that when the north pole of the other needle is brought close to the north pole

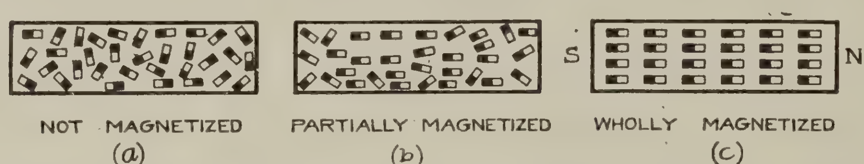


Fig. 37.—Effect of a Magnetizing Force on the Elementary Magnets of a Magnetic Substance.

of the suspended one, it will be **repelled,** and if the south pole is brought near the north pole of the suspended needle, it will be **attracted.** Hence, **unlike** magnetic poles **attract** and **like** magnetic poles **repel.**

Definition of unit magnetic pole. On the assumption that the two poles of the long slender needle are far enough apart so that the pole of the farther end has a negligible effect on a similar magnet held vertically with its north pole near the north pole of the suspended magnet, the law of magnetic force may be studied. Coulomb's law of magnetic force states that **the force between two poles varies inversely as the square of the distance between them.** Also,

A **unit magnetic pole** is one which, when placed at a distance of 1 cm. in a vacuum from an equal and like pole, repels it with a force of one dyne. The formula is:

$$F=\frac{m_1m_2}{r^2}$$

where F = force in dynes,
m_1 = strength of one pole,
m_2 = strength of other pole,
r = distance in cms.

This unit magnetic pole is the basis of the electromagnetic system of units.

If any medium other than a vacuum intervenes, a factor for that medium must be introduced. This factor is:

$$\frac{1}{\mu}$$

where μ = **permeability.**

The formula for any medium then becomes:

$$F = \frac{m_1 m_2}{\mu r^2}$$

The permeability of air is taken as unity.

Example:

An N pole having a strength of 20 units is placed 5 cms. in air from another N pole of 50 units. Find force of repulsion acting between them, neglecting effect of S poles.

Solution:

Formula $$F = \frac{m_1 m_2}{\mu r^2}$$

substituting $$= \frac{20 \times 50}{1 \times 5^2} = \frac{1{,}000}{25} = 40$$

whence $$F = 40 \text{ dynes.}$$

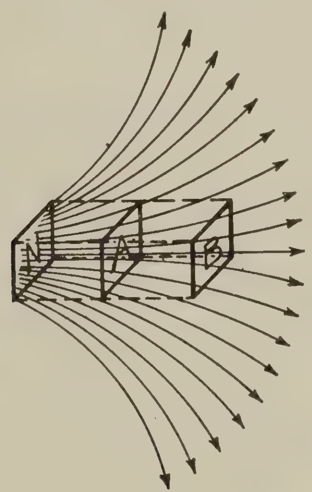


Fig. 38.—Graphical Representation of Variation of Magnetic Field Intensity due to Distance from Pole.

The force exerted by a magnetic field on a unit pole in a medium such as air whose permeability is equal to one, is numerically equal to the field intensity H or number of lines of force per cm^2. in a plane perpendicular to the lines of force.

In figure 38 let N represent a north pole. Let the area of the pole face be 1 cm^2. Let the squares marked A and B each have an area of 1 cm^2. and be parallel with an opposite to the pole face N. Also, let A be 5 cms. from N and let B be twice as far from N, or 10 cms. distant. It is evident that fewer lines pass through A than through N, and that still fewer pass through B. Although A is at one-half the

distance of B from N, more than twice the number of lines passing through B pass through A. In this case, **four** times the number pass through A. If A were in closer to N by one-half its present distance, or at one-fourth the distance of B from N, **sixteen** times as many lines would pass through it as through B. Hence,

The magnetic field intensity varies **inversely** as the square of the distance from the pole.

That is,

$$H \propto \frac{m}{r^2}$$

where m = strength of isolated magnetic pole,
r = distance in cms.

Example:

If the magnetic field intensity at a point 5 cms. from a magnetic pole is 1,000 lines per cm²., what will be the intensity at 10 cms.?

Solution:

Formula $$H \propto \frac{1}{r^2}$$

Hence, at twice the distance, H will be one-fourth as great. Therefore,

$$H = \frac{1.000}{4} = 250 \text{ lines per cm}^2.$$

Electromagnets. When current flows through a coil of insulated wire wound on an iron core, the resulting magnet is called an **electromagnet.** The iron core takes its polarity from the coil, because the elementary magnets in iron tend to turn in the direction of magnetization.

Electromagnets are used where very strong magnetic fields or poles are required, and when it is desired to vary either the intensity of the field or the polarity. Thus electromagnets are used in dc and ac generators, lifting magnets, relays, etc. Hence, a study of the magnetic properties of the iron core together with a calculation of the magnetic field of the coil and the effect of air-gaps in the magnetic circuit are required in the design of an electromagnet.

Magnetic induction. As explained before, whenever a piece of iron is subjected to the influence of the magnetic field of a solenoid, it becomes magnetized by induction. The force which induces the magnetism in the iron is called **magnetomotive force,** $\mathfrak{F}$. The unit of mmf is called the **gilbert.** The mmf of a solenoid is represented by the formula:

$$\mathfrak{F} = 0.4\Pi NI$$

where N = number of turns,
I = current in amperes.

Example:

A current of 10 amperes is flowing through a solenoid having 100 turns. What is the value of $\mathfrak{F}$?

Solution:

Formula $\mathfrak{F}=0.4\Sigma NI$

substituting $=0.4\times3.1416\times100\times10=1.257\cdot10^3$

whence $\mathfrak{F}=1{,}257$ gilberts.

In each unit length of the magnetic circuit, a certain part of the mmf is consumed in maintaining the magnetic lines of force or flux. This is, then, the magnetic potential gradient, or drop, and is called **magnetizing force, *H*.** Consequently, the unit is the **gilbert per cm.**

The formula for magnetizing force is:

$$H=\frac{0{,}4\pi NI}{l}$$

where H = gilberts per cm.,
N = number of turns,
I = current in amperes,
l = length of magnetic path in cms.

The total number of lines of magnetic force induced in the iron is called the **magnetic flux Φ.** The unit of magnetic flux is called the **maxwell.**

The number of lines of force, or maxwells, passing at right angles through a square centimeter of the cross section of the iron is the **flux density,** or **induction, *B*.** The unit of flux density, or induction B, is called the **gauss.**

Thus $$B=\frac{\Phi}{S}$$

where Φ = total flux in maxwells,
S = total area in cms².

Example:

Find the average flux density for a surface of 35 cms². when the total flux is 300 kilomaxwells.

Solution:

$S=35$ cms². $\Phi=300$ kilomaxwells,

Formula: $$B=\frac{\Phi}{S}$$

substituting $$=\frac{3\cdot10^5}{3.5\cdot10^1}=0.857\cdot10^4$$

whence $B=8.57$ kilogausses.

The relation between flux density and magnetizing force is a measure of the magnetizability of the iron and is called the **magnetic permeability, μ.**

Thus, $$\mu=\frac{B}{H}$$

where B = flux density in gausses,
H = magnetizing force in gilberts/cm.

The magnetic circuit. The magnetic circuit is the path which the magnetic flux Φ travels. It is always a complete circuit, as has been shown in the figures. The flux is produced by a magnetizing field. Now this field is set up by the magnetomotive force, $\mathfrak{F}$, which is analogous to the emf in an electric circuit.

Also, just as in an electric circuit, resistance to current flow, I, has to be overcome, so in the magnetic circuit, resistance to the flux Φ, has to be overcome. This magnetic resistance is called **reluctance,** $\mathcal{R}$. The unit of reluctance is the **oersted.**

The reluctance of a substance is dependent upon its (a) length, (b) cross-section and (c) permeability. Expressed as a formula:

$$\mathcal{R}=\frac{l}{\mu S}$$

where l = length in cms.
μ = permeability,
S = cross-section in cms^2.

Expressed in words:

The reluctance of a given magnetic circuit varies **directly** as the length and **inversely** as the cross-section and the permeability.

The quantity $\frac{1}{\mu}$ is a measure of the difficulty with which a substance can be magnetized, and is called **reluctivity.**

Magnetic leakage. Since there is no known insulation for lines of magnetic force, they cannot be entirely confined to the desired path. The magnetic circuit consists, therefore, of parallel branches, one of which is the path through the iron, while the other is through the surrounding medium whatever it may be. That part of the flux that flows through the surrounding medium is called **magnetic leakage.**

Magnetic shielding. Although there is no known insulator for magnetism, apparatus can be protected from stray magnetic fields to a considerable extent by making use of the high permeability of iron. The apparatus to be protected is enclosed as much as possible in an iron box having thick sides. Very sensitive astatic galvanometers are magnetically shielded by a series of concentric hemispheres or coaxial cylinders of soft iron. Apparatus protected in this manner is termed **iron-clad.**

Stray magnetic fields can be greatly reduced by placing the shield about the source of magnetic disturbance. For example, placing current-carrying wires in iron conduit will practically restrict the magnetic field to the conduit.

Magnetomotive force, reluctance and flux are interrelated in the magnetic circuit similarly to electromotive force, resistance and current in the electric circuit.

Thus
$$\Phi=\frac{\mathfrak{F}}{\mathcal{R}}$$

and substituting $\frac{l}{\mu S}$ for $\mathcal{R}$

$$\Phi=\frac{\mu S \mathfrak{F}}{l}$$

but $\mathfrak{F}=0,4\pi NI$

hence $\Phi=\frac{0.4\pi NI\mu S}{l}=\frac{1.26NI\mu S}{l}$ for the usual type of electromagnet.

The simple formula for ampere-turns is derived from the above formula. It is:

$$NI=\frac{0,8\Phi l}{\mu S} \text{ approximately.}$$

Example:

A closed ring of Norway iron having a circular cross-section of 8 cms². and a mean radius of 20 cms., figure 39, has a uniform winding consisting of 200 turns. Calculate the current required to produce a total flux of 112 kilomaxwells.

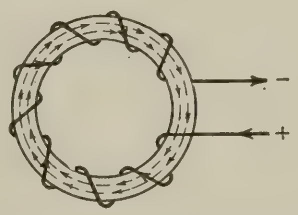


FIG. 39.—Closed Iron Ring with Toroidal Winding.

Solution:

$S=8$ cms².; $r=20$ cms.; $\Phi=112$ kilomaxwells.

Then $l=2\pi r$

substituting $=6.28\times 20=125.7$

whence $l=125.7$ cms.

Formula: $B=\frac{\Phi}{S}$

substituting $=\frac{112}{8}$

whence $B=14$ kilogausses.

Referring to the B-H curve, figure 40, it will be seen that the magnetizing force H for 14 kilogausses is 6.8 gilberts per cm. The permeability for $H=6.8$ may be read from the permeability curve also given in the figure, or calculated from the formula:

$$\mu=\frac{B}{H}$$

substituting $=\frac{14,000}{6.8}$

whence $\mu=2,060.$

The current necessary to produce a total flux of 112 kilomaxwells may now be calculated.

Formula: $$I=\frac{0.8\Phi l}{N\mu S}$$

substituting $$=\frac{8\cdot 10^{-1}\times 1.12\cdot 10^{5}\times 1.257\cdot 10^{2}}{2\cdot 10^{2}\times 2.06\cdot 10^{3}\times 8}=\frac{1.126\cdot 10^{7}}{3.30\cdot 10^{6}}=3.4$$

whence $I=3.4$ amperes.

Example:

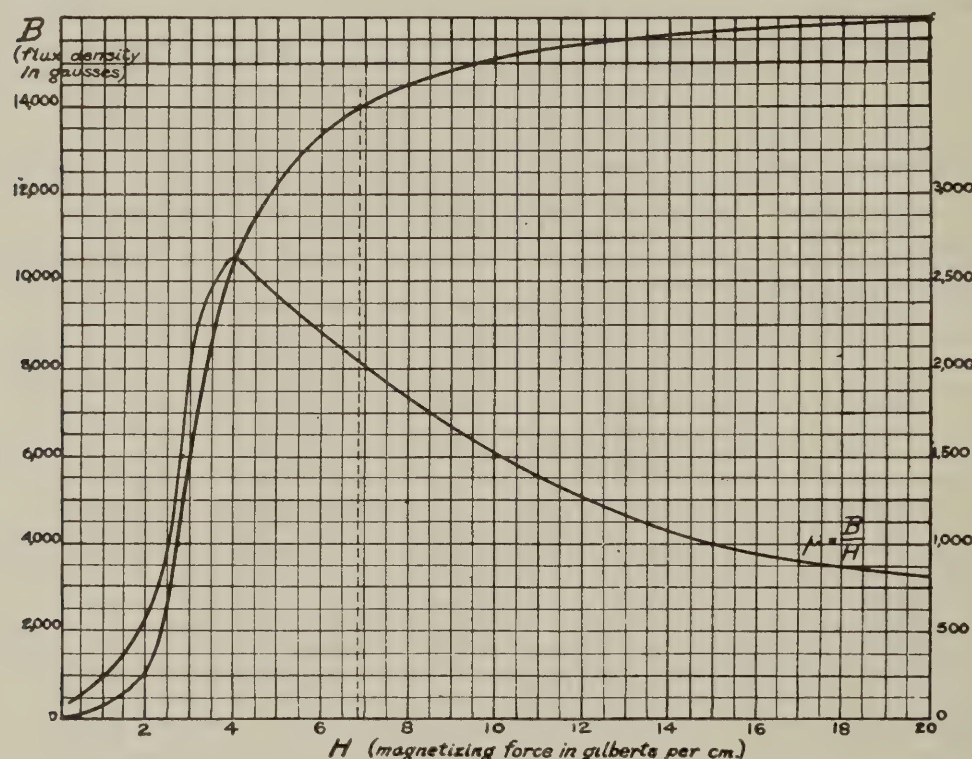


FIG. 40.—*B–H* Curve for Norway Iron.

A gap of 5.4 cms. is cut in the closed iron ring used in the previous example and an armature of Norway iron having a diameter of 5 cms. is placed in the gap, figure 41. Find the value of current required to maintain the flux, neglecting all leakage other than that across the clearance.

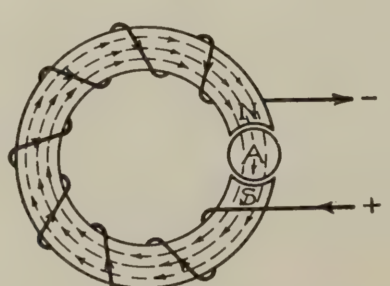


FIG. 41.—Iron Ring with Toroidal Winding, Air-Gaps and Armature.

Solution:

Let l = length of path through iron core,

l_1 = length of path across air-gaps.

Then $l = 125.3$ cms.

$l_1 = 5.4 - 5.0 = 0.4$ cm.

As before $S = 8$ cms². (approx.)

$N = 200$

$\Phi = 112$ kilomaxwells

$\mu = 2{,}060$ for iron

and $\mu = 1$ for air

$$\frac{l}{\mu S} = \mathcal{R} \text{ of iron path}$$

$$\frac{l_1}{\mu S} = \mathcal{R}_1 \text{ of air-gaps.}$$

The entire flux through the iron must cross the air-gaps because the lines of force form closed loops. Hence, the formula becomes:

$$I = \frac{0.8\Phi}{N}(\mathcal{R} + \mathcal{R}_1)$$

whence
$$I = \frac{0.8\Phi}{N}\left(\frac{l}{\mu S} + \frac{l_1}{\mu S}\right)$$

substituting
$$= \frac{8 \cdot 10^{-1} \times 1.12 \cdot 10^5}{2 \cdot 10^2}\left(\frac{1.253 \cdot 10^2}{2.06 \cdot 10^3 \times 8.0} + \frac{4 \cdot 10^{-1}}{1 \times 8.0}\right)$$

$$= 4.48 \cdot 10^2(7.61 \cdot 10^{-3} + 5 \cdot 10^{-2})$$

$$= 3.4 + 22.4$$

whence $I = 25.8$ amperes.

The answer indicates that an increase in current of 22.4 amperes is necessary to overcome the reluctance of the air-gaps, while only 3.4 amperes are required for the closed iron ring.

Hysteresis. When steel has once been subjected to a magnetizing force, it tends to retain some magnetism even after the force is removed. This remaining magnetism is called **residual magnetism.** Iron possesses some **retentive power,** but not in great degree. The residual magnetism is caused by the elementary magnets remaining partly lined up, and it will require a magnetizing force of a certain value to reduce the magnetization to zero. This value of the reversed magnetizing force required to reduce the induction to zero is called the **coercive force,** H_c. If the reversed magnetizing force is increased to the same value as the original magnetizing force, the resulting induction will have the same numerical value as the original induction, but of opposite sign (direction).

This phenomenon, by virtue of which the magnetization lags behind the magnetizing force, is called **hysteresis.** It requires a certain

definite amount of electrical energy to carry the material through a complete cycle of magnetization from positive to negative and back again. This energy is always expended in heating the iron, and is called **hysteresis loss.** This loss occurs in all apparatus containing iron in which the magnetization is varied as a result of variations in the current, and forms an appreciable part of the total losses.

Electromagnetic induction. The production of magnetic fields by current flowing through conductors and coils, and the influence of magnetic substances and air-gaps in the magnetic circuit have been discussed up to this point. It has been shown that a magnetic field is one of the effects of current flowing in a conductor, and exists so long as there is a current. The effect of a magnetic field on a conductor will be discussed in the following.

Induced emf. Figure 42 shows a conductor *ab* of length *l* placed in the air gap between the *N* and *S* poles of a magnet. It is immaterial whether the magnet is a permanent magnet or an electromagnet. The

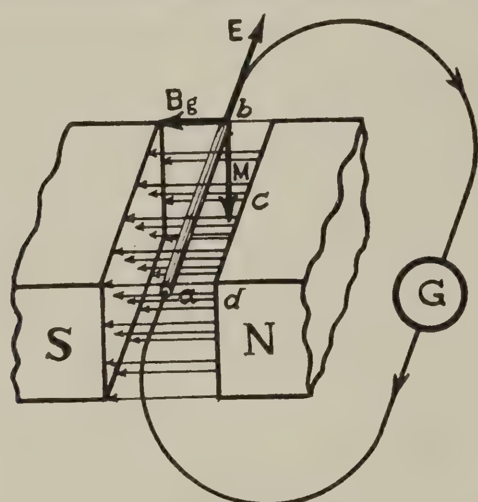


FIG. 42.—Conductor in a Magnetic Field.

direction of the magnetic flux Φ in the air-gap is shown by the arrows, and is from the *N* to the *S* pole. It is also assumed that the flux is uniformly distributed in the air-gap and is unvarying or steady. A lead *bGa*, outside of the magnetic field, is connected to the ends of the conductor. *G* is a galvanometer to indicate both the value and the direction of the current in the complete circuit *abGa*.

So long as the conductor remains immovable in any position the galvanometer will show no deflection, indicating that nothing is happening. Next, if the conductor is moved from the *N* to *S* pole, or vice versa, and parallel to the direction of the flux, the galvanometer will not deflect. Similarly, if the conductor is oscillated parallel to the horizontal edge *c*, or is turned through 90° and then oscillated parallel to the vertical edge *d* of the pole, but in both cases remains entirely within the uniform field, the galvanometer will not deflect. However, if the conductor is moved downwards between the pole faces in the direction

indicated in the figure by the heavy arrow M, the galvanometer will show a deflection. Similarly, if the conductor is turned through 90° and then moved from front to back, or vice versa, across and at right angles to the flux, the galvanometer will deflect. In both these cases, the direction of the motion of the conductor is at right angles to the direction of the magnetic flux, and the maximum effect will be produced. If the conductor, after having reached the bottom of the air gap, is moved upwards to its original position, the galvanometer will show a deflection opposite in direction to that produced while the conductor was being displaced downwards.

It will be noted that a galvanometer deflection is obtained only **during the movement of the conductor across the flux.** It is apparent, therefore, that this movement of the conductor across the flux produces a difference of potential between the ends a and b of the conductor, or an emf. This is the **emf produced by electromagnetic induction,** and the current acting in the circuit, as indicated by the galvanometer deflection, is due to this emf. The same effect would be produced if the conductor were fixed in the position shown in the figure, and the magnetic flux were moved across it by moving the magnet up and down. It is thus seen that emf is induced in a conductor whenever there is a **relative motion between the conductor and the magnetic flux.**

Direction of induced emf. The direction of the induced emf is related to the direction of motion of the conductor and the direction of the magnetic flux in the following manner. Referring to figure 42, the conductor is assumed to be moving **downwards** across the magnetic flux in the direction indicated by the arrow M. The magnetic flux is acting at right angles to this direction of motion of the conductor, and from right to left. Under these conditions, the emf induced in the conductor is acting in the direction indicated by the arrow E, that is, from a to b, and this direction is at right angles both to the direction of the motion of the conductor and the direction of the magnetic flux. When the direction of the motion of the conductor is upwards, the direction of the induced emf is opposite to that just given and shown in the figure.

The direction of the induced emf is thus seen to be dependent upon the direction of the flux and the direction of the motion of the conductor. The heavy line arrows B_g, M and E show, respectively, the directions of the magnetic flux, the motion of the conductor, and the induced emf. The direction of the resulting current in the conductor is, of course, the same as the direction of the induced emf. These directions can be remembered by **Fleming's right hand rule.**

Extend the **thumb, forefinger** and **middle finger** of the **right hand** until they are mutually at right angles. When the forefinger indicates the direction of the magnetic **flux** and the thumb the direction of motion of the conductor, the middle finger will point the direction of the

induced emf. It should be remembered that the direction of the magnetic flux is **always from the *N* pole to the *S* pole across the air-gap.** The right hand rule is shown by the hand in figure 43.

Fig. 43.—Right Hand Showing Directions of Flux, Motion, and Induced Emf for Moving Conductor and Stationary Magnetic Field.

Value of induced emf. Assume that the conductor of length l figure 42, is moving at a velocity v through the air gap and at right angles to the direction of the magnetic flux. Then the value of the induced emf at any instant will be

$$e = lvB_g$$

where e = instantaneous emf in emu,
l = length of conductor in cms,
v = velocity in cms per sec,
B_g = magnetic flux density in the air gap.

If both the velocity and the flux density are uniform, the emf will be constant and the **average emf,** E_{ave}, will be the same in value as the instantaneous emf.

It is evident, since velocity is equal to the distance which the conductor moves per second, or

$$v = \frac{d}{t}$$

that
$$lv = \frac{ld}{t}$$

but
$$ld = S$$

where S is the area swept over in a given time t. Then $\frac{S}{t}$ is the area swept over by the conductor per sec.

Since
$$lv = \frac{S}{t}$$

$$e = \frac{SB_g}{t}$$

where S = area in cms²

Since
$$\Phi = SB_g$$

$$e = \frac{\Phi}{t}$$

where Φ = flux cut in a given time t.

As above, if v and B_g are uniform

$$e = E_{ave}$$

Whether v and B_g are uniform, or not

$$E_{ave}=\frac{\Phi}{t}$$

The magnitude of the emf is seen to be dependent upon:

(a) **Velocity at which the conductor moves across the magnetic flux.** The greater the relative motion of conductor and flux, the greater will be the induced emf.

(b) **Flux density in the air-gap.** The induced emf is directly proportional to the flux density.

(c) **Relative angle of direction of motion of conductor and direction of magnetic flux.** The maximum emf that can be induced in a given conductor moving at a uniform velocity through a given magnetic field, is when the direction of motion of the conductor is at right angles to the direction of the flux. For any other relative angle of motion, the induced emf will be less.

The **electromagnetic unit of emf** is defined as the emf induced in a conductor cutting 1 line of magnetic force per second. This unit is exceedingly small, and the **practical unit** of emf is called the **volt.** In terms of the rate of cutting of lines of magnetic force, an emf of 1 volt is induced in a conductor which is cutting lines of magnetic force at the rate of $1 \cdot 10^8$ lines per second. Hence

$$1 \text{ volt} = 1 \cdot 10^8 \text{ emu.}$$

When the emf is expressed in volts, the formulas become:

$$e = lvB_g \cdot 10^{-8}$$

and

$$e = \frac{\Phi}{t} \cdot 10^{-8}$$

$$e = lvB_g \cdot 10^{-8}$$

Example:

A conductor 25 cms. in length crosses at right angles the flux threading an air-gap. The flux density B_g is 8.57 kilogausses, and the velocity of the conductor is 100 ft. per sec. Calculate the instantaneous value of the induced emf.

Solution:

$$100 \text{ ft.} = 3.048 \cdot 10^3 \text{ cms.} \qquad \text{(Table 15).}$$

Formula $e = lvB_g \cdot 10^{-8}$

substituting $= 2.5 \cdot 10^1 \times 3.048 \cdot 10^3 \times 8.57 \cdot 10^3 \cdot 10^{-8}$

$= 6.53$

whence $e = 6.53$ volts.

Example:

A conductor cuts $5 \cdot 10^9$ lines in 2.5 secs. Calculate the average induced emf.

Solution:

$$1 \text{ volt} = 1 \cdot 10^8 \text{ lines per sec.}$$

Formula $E_{ave} = \dfrac{\Phi}{t}$

substituting $$= \frac{5 \cdot 10^9 \cdot 10^{-8}}{2.5} = 2 \cdot 10^1 = 20$$

whence $E_{ave} = 20$ volts.

Emf induced in a loop. Figure 44 is a plane view of one of the pole faces of an electromagnet. Assume that it is the N pole and that the direction of the flux is toward the reader. Let it also be assumed that the flux density in the air-gap is uniform and unvarying. A rectangular loop of wire *abcd*, with its plane parallel to the plane of the pole face, is located in the air-gap. This loop can be considered to consist of two pairs of parallel conductors—*ab* and *dc*, *ad* and *bc*—connected together. If the loop is moved back and forth between the pole face and the reader and parallel to the lines of magnetic flux, no emf will be induced in any side of the loop. If the loop is moved up and down, an emf will be induced in each of the sides *ab* and *dc*, but none in either *ad* or *bc*. Since the sides *ab* and *dc* are equal in length and both move with equal velocities and in the same direction in a uniform field, the direction and magnitude of the emfs induced in these two sides are the same. The direction of the induced emfs can be determined by the right hand rule given above. These two emfs are in opposition around the loop. As a result, there can be no emf acting around the loop. If

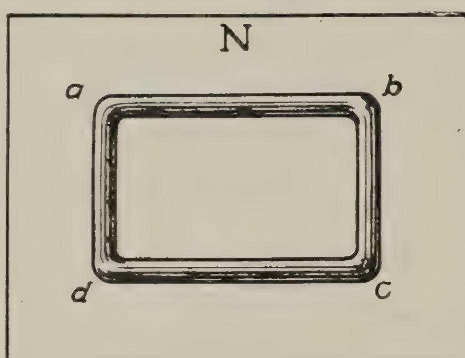


Fig. 44.—Loop in a Uniform and Unvarying Magnetic Field.

the loop is moved from right to left, or vice versa, then the sides *ad* and *bc* have emfs of the same magnitude and direction induced in them, while sides *ab* and *dc* are inactive. Again, no emf can act around the loop. The loop may also be rotated like a pinwheel with its plane **always parallel** to the pole face and no emf will act around the loop. However, if the loop is rotated on either its horizontal or perpendicular axis the effect is very different.

Figure 45 shows a conducting loop arranged in the air gap of an electromagnet so that rotation on its horizontal axis can be effected. The loop may again be considered to consist of four conductors joined together. The loop, as shown in the figure is supposed to be rotating about its horizontal axis in a clockwise direction. It is apparent that the conductors *ab* and *dc* are moving in opposite directions, that is, *ab* is being displaced downwards while *dc* is being displaced upwards.

The emfs induced in these two conductors will, therefore, have the directions shown by the arrows in the figure. As a result, the two induced emfs will add and an emf will act around the loop, thereby causing a current to flow around the loop. Sides *ad* and *bc* are inactive because they are moving in planes parallel to the direction of the flux. They serve only to complete the electric circuit. It should be remembered, however, that an emf is induced in the loop only during the

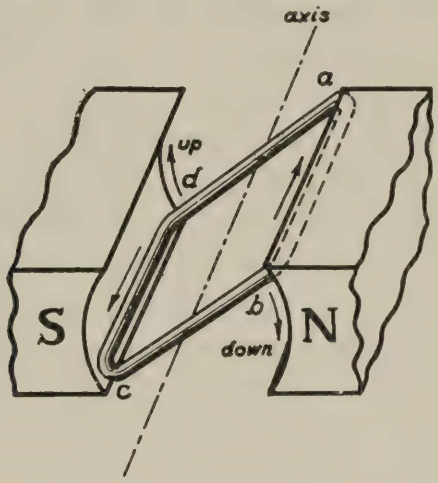


FIG. 45.—Conducting Loop Rotating on its Horizontal Axis in a Magnetic Field.

times that there is a relative motion between the conductors and the magnetic flux.

Law of induced emfs. The foregoing description of how an emf is induced in a conductor which is cutting lines of flux and the formulas for calculating the magnitude of the induced emf, although of great practical value, can be applied only in special cases such as in the design of electric generators. The fundamental relations existing between lines of magnetic flux and electric circuits will now be considered.

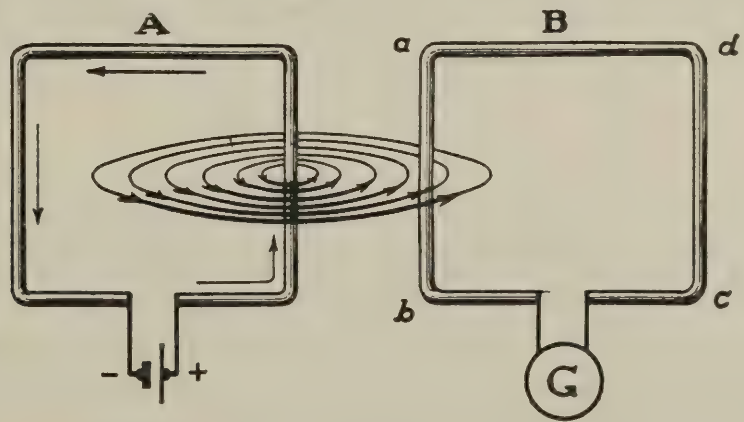


FIG. 46.—Linkage of Flux Lines with a Circuit.

A magnetic field always exists about a circuit in which current is flowing. The lines of magnetic flux are closed curves. The electric circuit also is closed. Figure 46 shows two circuits, *A* and *B*. A current is flowing in circuit *B* and, as a result, lines of magnetic flux are set up throughout all the space about this circuit. Circuit *B* is in the vicinity

of circuit A. Some of the flux lines of circuit A thread through circuit B, as shown in the figure. Each of these lines of flux **link together** the two circuits. The linkage is not great for the relative position of the circuits given in the figure. As circuit B is moved in closer to circuit A, more and more flux lines will link with B. The total number of flux lines which are linked with any circuit is spoken of as the **flux through the circuit.**

The number of lines of flux linking with B, or the flux through B, will vary whenever either circuit is moved, or when the current in A is varying. No change in the flux through B will occur if neither coil is moved and if the current in A remains steady. Faraday discovered experimentally that whenever, from any cause whatever, the flux through any conducting circuit is altered, an emf is induced in the circuit during the change in the number of linkages, and that this emf is directly proportional to the rapidity with which the flux is changing. Thus, if in the time t, the flux through a circuit changes uniformly in value from Φ_1 to Φ_2, the change in flux will be $\Phi_2-\Phi_1$ and the rapidity of the change will be

$$\frac{\Phi_2-\Phi_1}{t}$$

The average induced emf will be

$$E_{\text{ave}}=\frac{\Phi_1-\Phi}{t}\cdot 10^{-8}$$

where E_{ave} = average induced emf in volts,
Φ_1 and Φ_2 = flux in maxwells,
t = time in seconds.

If the flux is not changing at a uniform rate, then the induced emf will be different at different instants of time. If, at any particular instant, the flux changes by an amount $d\Phi$ in an extremely short time dt, the **induced emf at the instant** is given in volts by

$$e=\frac{d\Phi}{dt}\cdot 10^{-8}$$

This law states the fundamental principle on which electric generators produce current. The galvanometer G in circuit B will not deflect so long as there is no change in the flux through that circuit. However, during the time that the relative positions of A and B are changing or when the current in A is varying, a deflection will occur, showing that a current caused by induced emf is flowing in B.

In the first explanation of how an emf was induced in a conductor cutting magnetic flux, the complete circuit was ignored. It is now apparent that an emf is induced in a circuit due to changes in flux through the circuit. Referring to figure 42, the conductor ab is a part of the circuit $abGa$. The flux should be considered as threading this circuit. For the position of the conductor shown in the figure,

the flux through the circuit is a minimum. As the conductor is displaced downwards, more and more flux threads the circuit until, when the conductor is at the bottom, the flux through the circuit is a maximum. In other words the flux enclosed within the area of the circuit is varied from a minimum to a maximum value as the position of the conductor is changed. An emf is induced in the circuit during the alteration in amount of the enclosed flux, the magnitude of the induced emf depending upon the time rate of change in the flux through the circuit.

The loop in figure 44, during any of the movements described, always enclosed the same amount of flux and, since the linkages between the flux and loop did not change, no emf was induced. Such was not the case with the rotating loop in figure 45. The flux enclosed within the area of the loop changed continually as the loop was rotated, and an emf was induced in the loop. The similarity between the first and the last examples can now be seen.

Referring again to figure 46, it will be seen that the flux lines due to the current flowing in circuit A not only link with B but also with A. Whenever the current in circuit A varies, the linkage of the flux lines of circuit A with the **circuit itself** also varies and an emf is **self-induced** in that circuit. This is called the **emf of self-induction,** and will be described more fully in Chapter VII of this Part.

The coils or loops, shown in figure 46, are single-turn coils, and the formula given above determines the induced emf in such coils. When the coil has a number of turns and the flux threads all of the turns, then when any change in the flux takes place, an emf will be induced in each turn of the coil. The total emf induced in the coil will be given by the sum of the emfs induced in the separate turns. In case the change in flux is the same in each of the turns, then the same emf will be induced in each turn. The total emf will then be the product of the emf induced in one turn and the number of turns. Thus,

$$E_{\text{ave}}=\frac{n(\Phi_2-\Phi_1)}{t}\cdot 10^{-8}$$

where the various quantities are as previously given.

It is seen that, with a given time rate of change in flux through a coil, the **induced emf is directly proportional to the number of turns** in the coil. It is also apparent that high voltages can be induced in a coil having a large number of turns. The same effect can be produced by increasing the time rate of change of flux and by using fewer turns.
Example:

Let the flux through a 200-turn coil at a certain instant be $3 \cdot 10^6$ lines. Assume that the flux through the coil changes to $4 \cdot 10^6$ lines in 0.02 second. What is the average value of the induced emf?
Solution:

Formula $$E_{\text{ave}}=\frac{n(\Phi_2-\Phi_1)}{t}\cdot 10^{-8}$$

substituting $$=\frac{2 \cdot 10^2(4 \cdot 10^6-3 \cdot 10^6)}{2 \cdot 10^{-2}} \cdot 10^{-8}=100$$

whence $E_{\text{ave}}=100$ volts.

The method of increasing the emf induced in a coil with a given time rate of change of flux by increasing the number of turns in the coil is utilized in **step-up transformers** and also in **generators** where a high peripheral velocity of the rotating element is not desirable.

Lenz's law. The direction of the emf induced in a conductor moving across flux lines can be determined by the right hand rule, and is shown in figures 42, 43 and 45. Now, the direction of the current flow in the circuit is the same as the direction in which the emf acts. The **law of Lenz** states that when there is a relative motion between a conductor and a magnetic field, the current induced in the conductor is in such a direction as to oppose the motion. A more general statement of this important law is that, **whenever a current is induced in a circuit by a change in the flux through the circuit, the magnetic field produced by the induced current acts in such a direction as to oppose the change in flux.** That this law follows the principle of the conservation of energy will be apparent from the following.

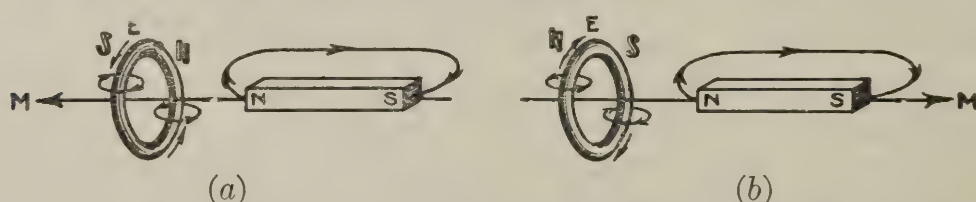


FIG. 47.—Opposition Offered to a Change in the Relative Positions of Loop and Magnet.

Figure 47(*a*) shows a closed conducting loop and a permanent magnet of the bar type. Assume that the magnet is moving toward the loop, as shown by the arrow M. An emf will be induced in the loop in the direction shown by the arrow E. The resulting induced current will set up a magnetic field acting in the direction shown by the circles around the loop. It will be seen that the N polar face of the loop is toward the N pole of the bar magnet and, therefore, that a repulsion exists between the loop and the magnet, and their relative motion is thus opposed. If the direction of motion of the bar magnet is reversed, all the effects are also reversed. The face of the loop toward the retreating magnet then becomes S, and an attraction between the two exists which opposes their separation. This is shown in figure 47(*b*).

A force is necessary to move the magnet against the opposition of the magnetic field produced by the induced current. Therefore, mechanical work is done in changing the relative positions of the loop and magnet. Electrical energy is produced in the loop by the conversion of mechanical energy, and the total electrical energy thus produced is equal to the work done in changing the relative positions of the loop

and magnet. Consequently, it is necessary to supply mechanical energy continuously to a generator in order to obtain electrical energy continuously.

It is customary in writing the formulas for induced emf to introduce a **minus sign** before the second member; for example

$$e = -\frac{d\Phi}{dt} \cdot 10^{-8}$$

This minus sign is used to indicate the direction of the induced emf as determined by Lenz's law. If, in figure 48(a), the flux passes through the circuit in the direction shown by the arrow, then the direction of the induced emf E in the circuit will be **counter-clockwise** when the **flux** is **increasing** and **clockwise** when the **flux** is **decreasing,** figure 48(b).

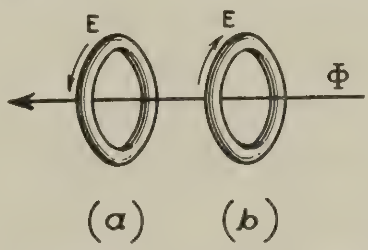


FIG. 48.—Direction of Emf Induced in Loop with: (a) Flux Increasing and (b) Flux Decreasing.

Thus, when the change in Φ——$d\Phi$——is positive, then the induced emf is counter-clockwise, or negative. This accounts for the presence of the minus sign in formulas for induced emf. Should difficulty arise in any case, the direction of the induced emf can always be determined by Lenz's law.

Another important point to remember concerning the mutually perpendicular directions of flux, motion and induced emf given by the right hand rule, is that the **motion** is that of a **conductor moving** in a **stationary magnetic field.** In cases where the **conductor** is **stationary** and the **magnetic field** is **moving,** such as in **revolving field alternators** and in **radio reception,** the direction of the induced emf in the conductor can be found by using the **left hand** instead of the right hand to point the directions. If the thumb is used to indicate the direction of **motion of the magnetic field** and the forefinger the direction of the **flux,** then the middle finger shows the direction of the **induced emf.** The left hand, figure 49, shows these directions. If the *abdc* coil, figure 45, is assumed

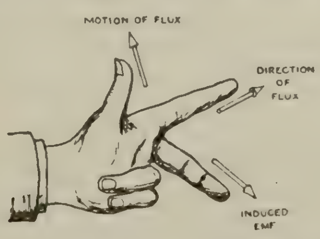


FIG. 49.—Left hand showing Directions of Flux, Motion, and Induced Emf for Stationary Conductor and Moving Field.

to be stationary and the N and S poles are rotated in a counter-clockwise direction about the coil, the relative motion will be the same as if the field were stationary and the coil was moved in the direction shown by the arrows. On applying the left hand rule just given, it will be seen that the direction of the induced emf is the same as shown in the figure.

Emf induced in a conductor by an advancing electromagnetic wave. It has been shown that there must be a relative motion of conductor and magnetic field in order that an emf may be induced in the conductor. The magnitude of the induced emf is maximum for any velocity of the conductor when the conductor is moving at right angles across the flux. The instantaneous induced emf in emu in this case is

$$e = lvB_g$$

where l = length of conductor in cms.,
v = velocity of conductor in cms. per sec.,
B_g = flux density in gausses.

Now, assume that the conductor is stationary and the flux is uniform and unvarying but moving at a velocity v, and that the conductor

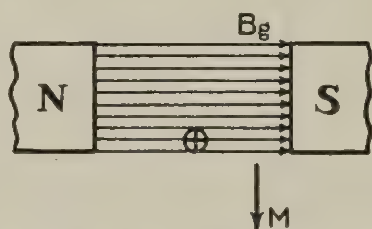


FIG. 50.—Emf Induced in a Stationary Conductor by a Moving Magnetic Field.

has unit length, that is, is 1 cm. long. Then the emf in emu induced in this stationary conductor is

$$e = E_{ave} = vB_g.$$

Figure 50 shows an end view of a conductor in which an emf is being induced by an advancing uniform and unvarying magnetic field which is carried along in the air gap between the N and S poles of a magnet in the direction shown by the heavy arrow M. The magnet is supposed to be sliding on the page towards the bottom, while the conductor is perpendicular to the plane of the page. By the left hand rule, the direction of the emf induced in the conductor is downwards into the page. If the conductor were not present, there would still be an **emf acting in space** in the same direction and having the same magnitude, or

$$e = vB_g \cdot 10^{-8} \qquad \text{(volts per cm.)}$$

This emf, or difference in potential per unit length in space, is the **electric field intensity,** $\mathcal{E}$. Hence

$$\mathcal{E} = vB_g \cdot 10^{-8} \qquad \text{(volts per cm.)}$$

The directions of the magnetic flux and the electric field are mutually perpendicular in space; that is, they are in **space quadrature.** The **left hand rule shows that each is at right angles to the direction in which the magnetic field is travelling.**

The magnetic field of an electromagnetic wave is a **moving sinusoidal field.** An attempt to picture this is made in figure 51, which shows the magnetic field at a given instant. The field is supposed to be travelling in the direction shown by the heavy arrow M, that is, towards the bottom of the page. Each of the lines represents the **front** of the **wave,** which may also be considered to be **plane** at a considerable distance from the source. These lines, therefore, represent planes in which the magnetic field intensity H is uniform and acting in one direction, the planes themselves being perpendicular to the page. For example, the line drawn through point b represents a plane at all points in which the same field intensity exists, the intensity in this case being a maximum and the direction of the force being from right to left. The sinusoidal

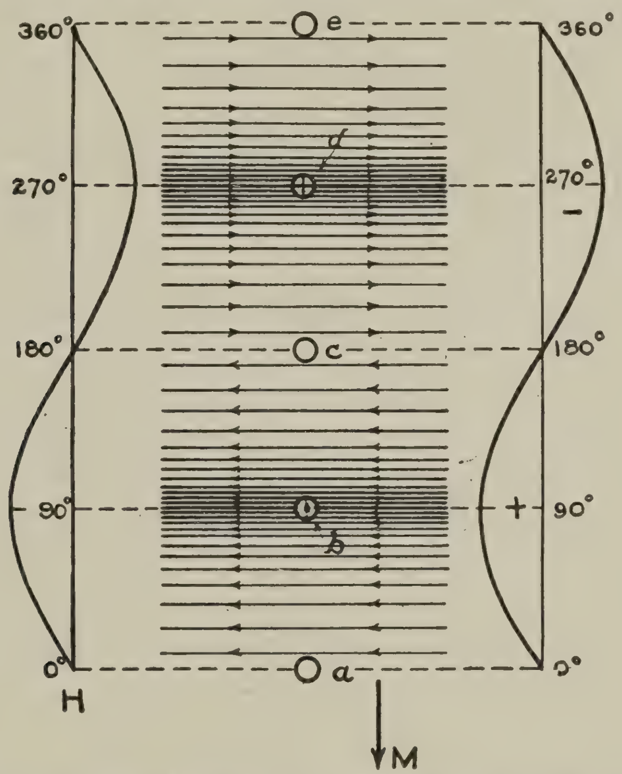


FIG. 51.—Manner in which an Electric Field Intensity is produced by an Electro-Magnetic Wave.

variation in the field intensity is shown by the shading and separation of the lines, while the change in direction of the magnetic field is given by the arrowheads on the lines. The magnetic force acts throughout all the space, however, between the lines drawn. Thus, at 0° no line is drawn meaning that the intensity is zero, while in the vicinity of 90°, the lines are drawn most closely together and most heavily shaded in an effort to represent a maximum field intensity. The sine curve also reaches its maximum displacement from the axis at this point and indicates maximum field intensity. From 90° to 180° the intensity is decreasing until it becomes zero at 180°.

Between 0° and 180° the direction of the field is the same, namely, from **right to left.** At 180°, the field changes in direction and acts from

left to right. At this instant the intensity is zero and immediately begins to increase until, at 270°, it is again a maximum. The intensity then decreases to zero at 360°. This completes **one cycle,** which is followed by another cycle, etc. It is evident from the foregoing that the magnetic field of an electromagnetic wave **varies in intensity** H**,** and **changes periodically in direction** and **travels continuously in the same direction** with a velocity v equal to that of light, the velocity of light being

$$c=3\cdot10^8 \text{m. per sec.}$$

Now, as this sinusoidal magnetic field travels past point a, the field at the point varies in intensity and changes in direction as above described. Thus, at successive instants, the intensity may be considered to be uniform at the point, but to have a different value, depending upon the part of the wave which is at that instant at the point. This may be seen by placing a finger on point a and moving the page in the direction of the arrow M.

Assume that a conductor perpendicular to the page is located at point a. Then this advancing field will induce an emf in it. As the intensity of the field varies, so will the magnitude of the induced emf vary. When the direction of the field changes, the direction of the induced emf will also change. These variations and changes in the emf take place simultaneously with those of the magnetic field; hence, the **induced emf is in time phase with the electromagnetic wave.**

The sine curve on the right side of the figure shows the variations in magnitude and the changes in direction of the induced emf. The left hand rule gives the direction of the emf. Thus, at point a, there is no emf; at point b the emf is acting upwards and is maximum; at c the emf is zero again, while at d it is acting downwards and is maximum; at point e the emf is zero. The **direction upwards** is called **positive** and the **direction downwards negative.**

As in the case of the moving unvarying field, there will be an emf induced in space at right angles to the direction of the magnetic field. This **electric field intensity** $\mathcal{E}$ **is** now seen to be **in time phase and space quadrature with the magnetic field and** also **at right angles to the direction of propagation,** and has a value

$$\mathcal{E}=3\ H\cdot10^{10} \qquad (\mu\text{v per m.})$$

It is important to remember that a moving magnetic field generates an electric field that travels with the same velocity, varies instantaneously with every variation of the magnetic field, and also changes in direction when the magnetic field changes. It should not be assumed, however, that the two fields can be separated and only one used. A moving electric field also generates a magnetic field. One field cannot exist without the other, and the energy in the electromagnetic wave is always equally divided between the two fields. This is shown in Part 6, where radio transmission and reception are treated from the viewpoint of the electric field intensity.

CHAPTER VI. ELECTRIC GENERATORS AND MOTORS.

The electric generator. The **electric generator,** commonly called **generator,** is a machine that converts mechanical energy into electrical energy. The principle on which it operates is electromagnetic induction. The subject of magnetism and electromagnetic induction was treated in the previous Chapter. The practical application of the laws and principles given there will be taken up in the following.

It was shown that work must be done in changing the relative positions of a closed conducting loop and a magnetic field. The power necessary to perform this work may amount to several thousand horsepower in the case of large generators. The prime mover may be a steam engine, steam turbine, water turbine, gas or oil engine, or an electric motor. The mechanical power is applied to the shaft of the rotating element of the generator by any of the methods used in transmitting mechanical energy, such as by direct coupling, gears or belts.

Generators are divided into two broad classes, namely, **alternators** and **continuous-current generators.** The latter are frequently called **direct-current generators.** Both classes of generators operate on the same principle—electromagnetic induction. Since the dc generator is essentially an alternator, it is necessary that the principle of the alternator be understood before studying the theory of the former.

The alternator. It was shown in the previous Chapter that, whenever the flux through a coil is varying, an emf is induced in the coil. The laws governing the production of induced emf as well as rules and formulas for determining the direction and magnitude, respectively, of induced emf were also given. The essential thing to remember is that there must be a relative motion of coil and flux in order to produce an emf by induction; that is, a change must occur in the number of flux lines threading the coil.

One of the simplest methods of varying the flux through a coil is to rotate the coil on its horizontal axis in a stationary magnetic field. This method is used in the **revolving armature** type of alternator. Another method is to have the coil fixed in position and to revolve a magnet within it. This type of alternator is called the **revolving field, stationary armature alternator,** and appears in two forms—the one in which the electromagnet with its coils is revolved, and the other in which the core of the electromagnet alone is revolved while its magnetizing coil and the armature coils are both stationary. The last type is called the **inductor alternator.** In each of the types of alternators mentioned, the flux through the armature coil is varied and an induced emf results. An understanding of all these types will be gained by the following description of the simple alternator consisting of a single-turn

coil revolving on its horizontal axis in the air gap between the poles of a magnet.

The simple alternator. Figure 52 shows a conducting loop *abcd* arranged so that it can be rotated on its horizontal axis in the air gap between the N and S poles of a magnet. The loop is cut on side *bc* and insulated leads are brought out along the shaft (not shown) to two **collector rings** r_1 and r_2 which are insulated from the shaft as well as from each other. Conducting **brushes** b_1 and b_2 make electrical contact with the collector rings r_1 and r_2, respectively, and serve to connect the external circuit, represented by the resistance R, to the rotating loop. The complete circuit necessary to a flow of current is $b_1\ R\ b_2\ r_2\ b\ a\ d\ c\ r_1$.

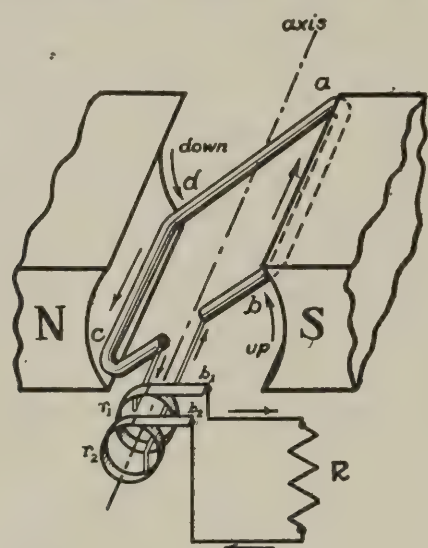


FIG. 52.—Simple Alternator.

Assume that the loop is rotating in a counter-clockwise direction, as shown by the arrows. The direction of the emfs induced in sides *ab* and *cd* at the instant when the loop is in the position shown in the figure is indicated by arrows. It will be seen that the induced current flows **out** from conductor *cd* at end *c* to the collector ring r_1, through brush b_1, through the external circuit to brush b_2, through collector ring r_2, and **into** conductor *ab* at end *b*.

As the loop continues to revolve in the same direction, conductors *ab* and *cd* will, for an instant, be moving parallel to the flux lines, and the flux through the loop will not vary. At that instant there will be no emf induced in the loop; the plane of the loop will then be perpendicular to the flux lines, and the maximum number of flux lines will thread the loop. This position, where the change in Φ through the loop is zero, is sometimes called the **neutral position.** As soon as this position is passed, conductor *ab* will be moving upwards and *cd* downwards. The emf induced in each conductor will have a direction in accordance with the right hand rule. The direction of the current in the conductors will now be **out** of *ab* at *b*, and **in** at *c* for *cd*. It will be noticed that the

direction of the emf in *ab* is the same as it was in *cd* for the same position and direction of motion of the conductor; that is, there is merely an exchange of the conductors. If the out direction is called **positive,** then collector ring r_1 was positive and r_2 negative in the first case and then reversed in polarity as the loop turned through one-half revolution, or 180°. As a result of this change in polarity, the emf acting in the entire circuit will **periodically alternate in direction** and is, therefore, called an **alternating emf.** The time elapsing between two successive reversals in directions is called an **alternation,** while the time between two successive changes in the **same** direction is a **cycle.** A cycle is, therefore, composed of two alternations—one positive and one negative. The number of times per second that a cycle occurs is called the **frequency, *f*.**

The sine curve. Although there is a periodical change in the direction of the induced emf, it should not be inferred that the change is abrupt, nor that the emf instantly attains its maximum value and is

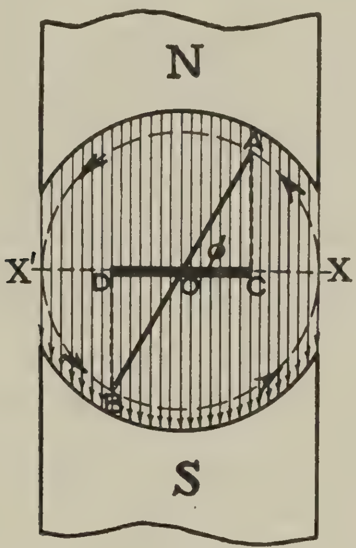


FIG. 53.—Showing Cosine Law of Variation in Flux Through a Revolving Loop.

then maintained at this value during the entire alternation, after which it abruptly reverses in direction and goes through the same operation. Such a condition cannot be obtained with a revolving loop.

The variation in the flux through the loop as it revolves at a uniform speed in an unvarying and uniform field depends upon the variation in the area of the loop made available to the passage of the flux lines. The law of the variation in the flux through the revolving loop will now be considered.

Figure 53 shows an end view of a rectangular loop as viewed from end of the shaft. The loop is supposed to be revolving in a counterclockwise direction at a uniform speed in a uniform field, starting from the position XOX'. At a later instant, its position is AOB. As the loop revolves it sweeps through an angle ϕ at the center O with an **angular velocity,** ω. Now, ω is the angle measured in radians that is

swept through in one second. The magnitude of the angle ϕ which is swept through in t seconds will be t times ω. Hence

$$\phi=\omega t \qquad \text{(radians)}$$

Since there are 2π radians in one revolution, the **periodic time, T of one** revolution in terms of ω is

$$T=\frac{2\pi}{\omega} \qquad \text{(seconds)}$$

and the **frequency,** f, or number of revolutions per second, is equal to one second divided by the periodic time of one revolution, or

$$f=\frac{1}{T}$$

Substituting in this equation the value of T in the previous equation

$$f=\frac{\omega}{2\pi}$$

from which

$$\omega=2\pi f$$

Now, when the loop has turned through an angle ϕ, the flux through the loop has varied. It will be seen from figure 53 that as many flux lines pass through the heavy line COD as though the line AOB, which represents the loop itself. Line COD lies in line XOX', which is the initial position of AOB, and also makes an angle ϕ with the latter. Thus in the triangle COA

$$OC=OA\ \cos\phi$$

and, since OA is equal to one-half of the breadth b of the loop, then

$$OC=\frac{b}{2}\cos\phi$$

and

$$CD=b\ \cos\phi$$

The area of the rectangular loop is

$$S=bl$$

and it is apparent that, as this area revolves, there is an area S' at right angles to the flux through which as many lines pass as through the area of the loop. This area remains constant in length, but varies in breadth as the cosine of the angle ϕ between it and the plane of the loop. Hence,

$$S'=bl\ \cos\phi$$

The flux through this area is

$$\Phi=blB_g\ \cos\phi$$

Hence, the flux through the loop at any instant is

$$\Phi=blB_g\ \cos\omega t.$$

The cosine curve, figure 54, shows this variation in flux through the loop as it rotates through one revolution in a counter-clockwise direction, starting from the position XOX', where the number of flux lines through the loop is a maximum.

The emf induced in a loop is determined by the time rate of change of flux. If, in a very short time dt, the flux through the loop changes by an amount $d\Phi$ the time rate of change of Φ is

$$\frac{d\Phi}{dt}$$

As seen above, the flux Φ at any time is given by

$$\Phi = blB_g \cos \omega t$$

If, in a very short time dt, the flux increases by an amount $d\Phi$, the total flux will be given by $\Phi + d\Phi$, and the time by $t+dt$.

Hence, $$\Phi + d\Phi = blB_g \cos \omega(t+dt)$$
$$= blB_g \cos \omega t \sin \omega dt - \sin \omega t \cos \omega dt.$$

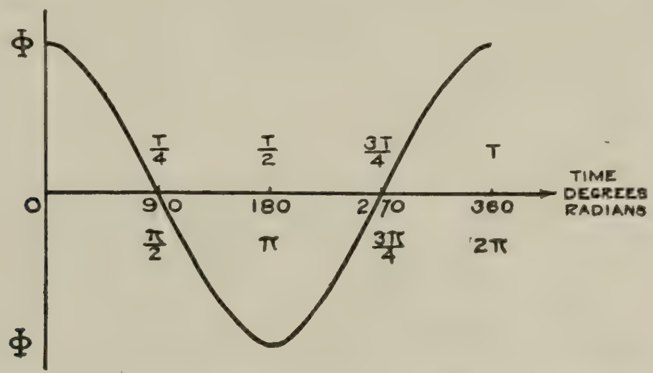


FIG. 54.—Cosine Curve Showing Variation in Flux Passing Through a Rotating Loop.

If dt is very small, the cosine of the angle is unity, while its sine is equal to the angle itself.

Hence, $$\cos \omega \, dt = 1$$
$$\sin \omega \, dt = \omega \, dt$$

Hence, $$\Phi + d\Phi = blB_g \cos \omega t - blB_g \, \omega \sin \omega t \, dt$$

subtracting $$\Phi = blB_g \cos \omega t$$

$$d\Phi = -blB_g \, \omega \sin \omega t \, dt$$

whence $$\frac{d\Phi}{dt} = -\omega blB_g \sin \omega t$$

But the instantaneous value of the induced emf is

$$e = -\frac{d\Phi}{dt}$$

Therefore, $$e = \omega blB_g \sin \omega t$$

and $$e = \omega blB_g \sin \omega t \cdot 10^{-8} \qquad \text{(volts)}$$

where ω = angular velocity in seconds of time, degrees or radians, and the other quantities are as previously given.

Since $$\phi = \omega t$$

then $$e = \omega blB_g \sin \phi \cdot 10^{-8} \qquad \text{(volts)}$$

When angle ϕ is 90° or 270°, then $\sin \phi + 1$ numerically, and the induced emf is a maximum. Its value is

$$E_0 = \omega blB_g \cdot 10^{-8} \qquad \text{(volts)}$$

In terms of this maximum value, the instantaneous emf becomes

$$e = E_0 \sin \omega t \qquad \text{(volts)}$$

or

$$e = E_0 \sin \phi \qquad \text{(volts)}$$

If the loop has n turns, the emf is increased in direct proportion to the number of turns, as has already been described in the previous Chapter, or

$$e = \omega b l n B_g \sin \phi \cdot 10^{-8} \qquad \text{(volts)}$$

and

$$E_0 = \omega b l n B_g \cdot 10^{-8} \qquad \text{(volts)}$$

The foregoing mathematical proof shows that the instantaneous value of emf induced in a single-turn loop of given dimensions, revolving at a constant angular velocity, is directly proportional to the **sine** of the angle it is making with the initial position at that instant. The variation in the instantaneous value of the emf, is therefore, **sinusoidal.**

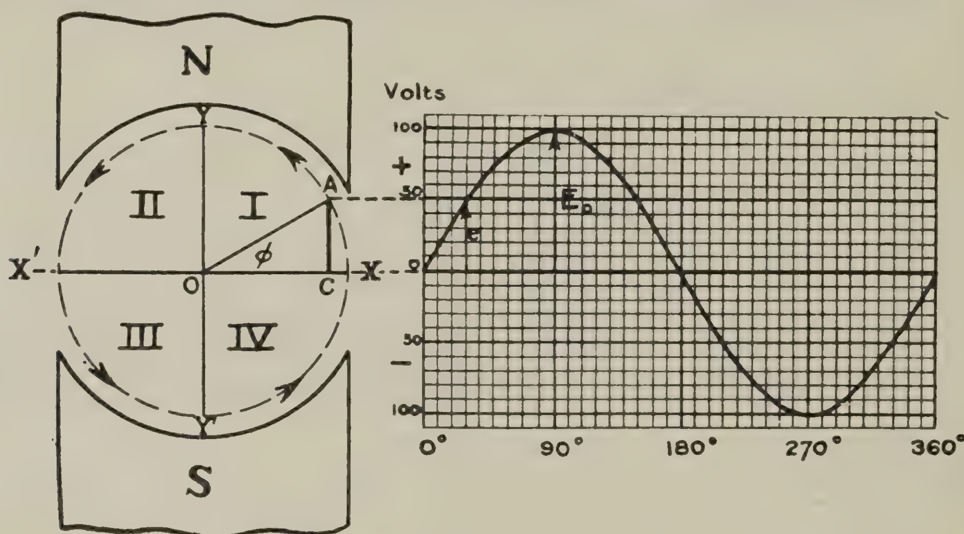


FIG. 55.—Production of a Sine Wave of Emf in a Loop Revolving in a Magnetic Field.

Figure 55 shows how a sine curve is developed. For the sake of clearness, only one-half of the breadth of the loop is shown. The initial position of the loop is XOX'. The change in the sine of the angle ϕ as the terminal line OA rotates in a counter-clockwise direction through four quadrants, or one complete revolution, is described in Section IV, Trigonometry, while Table 17 gives the changes in the natural sine of an angle as it varies through 90°, or $\frac{\pi}{2}$ radians, In quadrant I, sin . varies between 0 and +1. Therefore, when the loop is in position XOX' no emf is induced. As the loop revolves, the emf increases to its maximum positive value E_0 when the loop is in position YOY', as shown in the figure. In quadrant II, sin ϕ varies between +1 and 0, that is, is decreasing, and the emf is decreasing from its maximum positive value to zero. One hundred and eighty degrees of revolution, or one alternation, or one-half cycle, is now completed. As the loop revolves through quadrants III and IV, sin ϕ is negative but varies in magni-

tude in exactly the same manner as in quadrants I and II, respectively. This completes one cycle. So long as the loop continues to revolve at a uniform speed, the cycle will be reproduced at regularly recurring intervals.

The relation existing between the time rate of change in flux through the loop and the induced emf can now be seen, figure 56. The curves show the variation in flux and induced emf as the loop rotates in a counter-clockwise direction starting from the XOX' line, figure 55. An inspection of Table 17 shows that the natural cosine of an angle changes least rapidly at 0°, but has its largest numerical value. The induced emf at this instant is zero. At 90°, the cosine is changing most rapidly, and passing through its smallest numerical value. The induced emf is, therefore, a maximum. In other words, as angle ϕ changes from 0° to 90°, the natural cosine varies from +1 to 0 and the natural sine varies from 0 to +1.

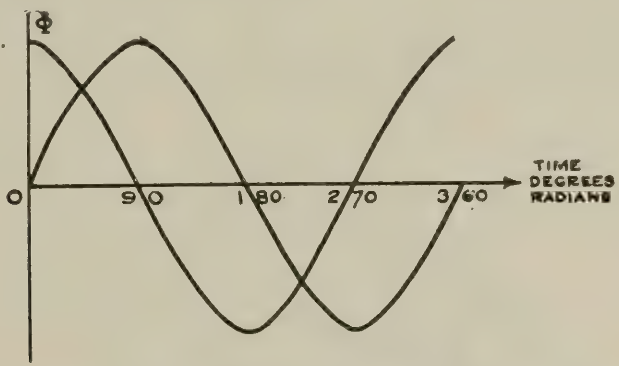


FIG. 56.—Relation Between Variation in Flux and Induced Emf.

If, in a sine wave, either the maximum emf or any instantaneous value of emf, together with the angle ϕ at which it occurs, is known, the variation in the emf for any quadrant can be calculated by the use of Table 17. As the variation in emf in every quadrant is numerically the same, it is necessary to calculate the instantaneous values only for quadrant I. This has been done in plotting the sine wave of emf shown in figure 55. The values of e are given below. The instantaneous emf for angle $\phi = 30°$ was assumed to be 50 volts. The maximum emf was, therefore, 100 volts, and the instantaneous values used were then calculated using the formula:

$$e = E_0 \sin \phi$$

Degrees	e	Degrees	e
0	0.0	50	76.6
10	17.4	60	86.6
20	34.2	70	94.0
30	50.0	80	98.5
40	64.3	90	100.0

Multipolar alternators. The simple alternator described in the foregoing consisted of a single-turn loop revolving in the field between **two poles.** A two pole alternator is frequently called a **bipolar alternator.** It was shown that one cycle occurred for each complete revolution of the loop, that is, one **mechanical revolution** caused one **electrical revolution.** Angular velocity is properly a velocity of revolution in **space** and may be expressed in degrees, in time, or in radians. To differentiate between a cycle in mechanics and in electricity, the term **electrical degree** is used for the latter. Thus, one electrical cycle equals 360 electrical degrees. Frequency is always expressed in cycles per second. Hence, in the case of a bipolar alternator

$$f = \text{revs. per sec.}$$

Example:

A loop revolves 3,600 rpm in a bipolar alternator. What is the frequency?

Solution:

The loop revolves $\frac{3,600}{60} = 60$ times per sec.

Hence $$f = 60\sim$$

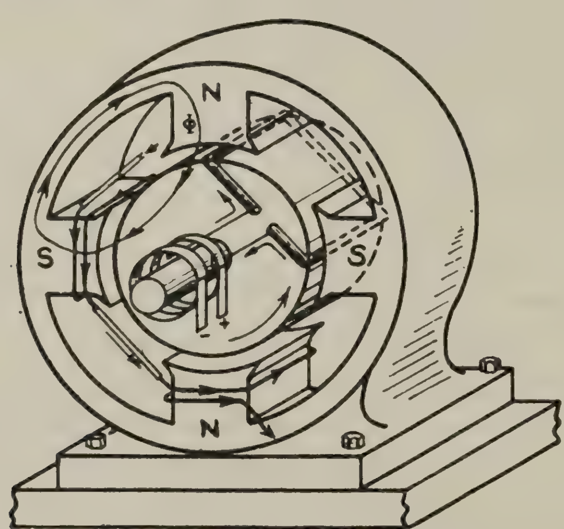


FIG. 57.—Multipolar Alternator Having Four Poles.

A **multipolar alternator** is one having two or more **pairs of poles.** The main purpose in employing several pairs of poles is to reduce the velocity of rotation of the loop, and yet maintain the desired frequency. Figure 57 shows a four-pole alternator. The loop is now arranged so that the flux from the N to S pole passes through it as in the case of the bipolar alternator, but on account of the arrangement of the poles of the multipolar alternator, the loop is secured to the surface of a cylinder which revolves on a shaft. It will be noticed that one side of the loop lies under the center line of the N pole while the other side lies under the center line of the S pole. The flux passes through the air gap from each N pole and through the loop to the adjacent S poles.

As the cylinder revolves with the loop, the loop passes under two pairs of poles per revolution and, since one cycle occurs as the loop passes under each pair of poles, two cycles occur per mechanical revolution. Hence, the loop revolves through 720 electrical degrees during a mechanical revolutions of 360°. The difference in the two terms is now apparent. The general formulas for determining the frequency are:

$$f=\frac{\text{rpm}\times n}{120}$$

and

$$f=\frac{\text{rps}\times n}{2}$$

where rpm = revolutions per minute,
rps = revolutions per second,
n = number of poles.

Example:

A 24-pole alternator runs at 2,500 rpm. What is the frequency?

Solution:

Formula $$f=\frac{\text{rpm}\times n}{120}$$

substituting $$=\frac{2.5\cdot 10^3\times 24}{1.2\cdot 10^2}=\frac{6.0\cdot 10^4}{1.2\cdot 10^2}=5\cdot 10^2=500$$

whence $f=500\sim$

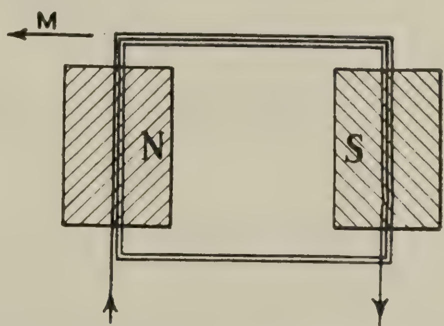


FIG. 58.—Concentrated Winding (one-Slot-per-Pole) with 100 Per Cent Pitch, i. e., Winding Pitch Equals Pole Pitch.

A bipolar alternator would have to run at 12 times this speed, or at 500 rps, to give 500 cycles. Such a speed is not practical. A speed of even 60 rps is excessive for large machines. The advantage gained by the use of multipolar alternators is, therefore, apparent.

Concentrated and distributed windings. In order to obtain high emfs, multiple-turn loops, or windings, are used. The turns may be **concentrated** into one loop or coil, in appearance like a single-turn loop, or **distributed** in overlapping loops. Thus, the loop in figure 57 is a concentrated winding, which can be wound in one pair of large slots in the rotating element. It is also called the **one-slot-per-pole** winding. Figure 58 shows this type. The view is looking from the shaft toward the pole faces. The turns forming the coil are insulated from each other, and the whole coil bound together and insulated.

The **distributed winding** is also composed of many turns which, instead of being confined to two large slots—one under the center line of each pole face—occupy **several small slots per pole,** as is shown in figure 59. The concentrated type of winding gives a peaked wave form, while the distributed type gives a more gradual increase in emf, hence, a more rounded wave form, which more nearly approaches a sine wave. If, in addition, the pole-tips are shaped so that the distribution of flux in the air gap is sinusoidal, the induced emf will be practically a sine wave. This form of wave is highly desirable.

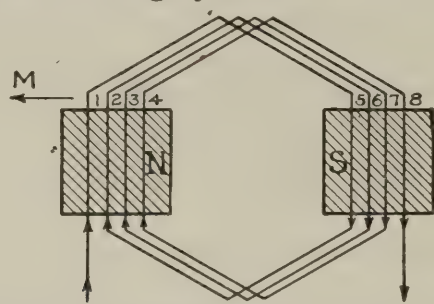


FIG. 59.—Distributed Form of Winding.

Equations for emf. The formulas given previously for the instantaneous and maximum values of the emf induced in a multi-turn loop, revolving in a bipolar field, can be made more general. Thus, in the formula

$$e=\omega blnB_g \sin \phi \cdot 10^{-8} \qquad \text{(volts)}$$

$$blB_g= \cdot\phi$$

whence Φ = flux per pole in maxwells, or in flux lines, as frequently used.

Hence, $$e=\omega n\Phi \sin \phi \cdot 10^{-8} \qquad \text{(volts)}$$

The formula $$E_0=\omega blnB_g \cdot 10^{-8} \qquad \text{(volts)}$$

also becomes $$E_0=\omega n\Phi \cdot 10^{-8} \qquad \text{(volts)}$$

Although the maximum value of induced emf is of some interest, another quantity, called the **effective emf,** E, is used in practice. The effective value of an alternating emf having any wave form is equal to the **square root of the mean of the squares of all the instantaneous values**; otherwise named the **root mean square** (rms) value of emf, or the **virtual** value. The **effective value of a sine wave** can be found by dividing the maximum value by $\sqrt{2}$. Hence,

$$E=\frac{E_0}{\sqrt{2}}=0.707\ E_0$$

and $$E_0=\sqrt{2}\,E=1.414\ E$$

Since $$E_0=\omega n\Phi \cdot 10^{-8}$$

$$=2\pi fn\Phi \cdot 10^{-8}$$

then $$E=\frac{2\pi fn\Phi \cdot 10^{-8}}{\sqrt{2}}=\frac{6.2834}{1.414}fn\Phi \cdot 10^{-8}$$

whence $$E=4.44fn\Phi \cdot 10^{-8} \qquad \text{(volts)}$$

in which all the quantities have previously been given. This formula is somewhat modified for mechanical reasons, which it is beyond the scope of this Manual to explain. These modifications can be expressed by inserting a factor k in the formula, which then becomes

$$E=4.44\,fkn\Phi\cdot 10^{-8} \qquad \text{(volts)}$$

Types of fields. The N and S magnets which produce the flux that threads the loop or coil, are called the **field magnets,** or simply the **field.** In all alternators, excepting magnetos, the field magnets are electromagnets which are energized by direct current passing through the winding on the core of each pole in such a direction as to make the poles alternately N and S around the circumference. The path of the flux is out from the N pole, across the air gap, through the loop and armature, back acorss the air gap to the S pole, and thence through the frame to the N pole, which completes the circuit. The magnetic path includes as much magnetic material as is possible, and the air gaps are reduced to a minimum in order to reduce the reluctance to the lowest possible value compatible with good engineering practice. Laminated steel is used whenever necessary to reduce losses.

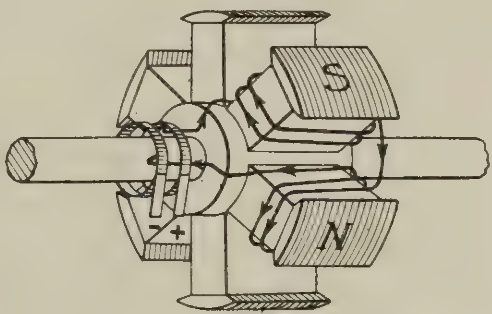


FIG. 60.—Type of Rotor Field for Moderately High Speed of Rotation.

There are three types of fields, which are: (*a*) **stationary,** (*b*) **revolving** and (*c*) **inductor revolving.** In the **stationary** type of field, the poles are connected to a frame which, in turn, is bolted to a bed-plate. This type is shown in figure 57, and is called the **stator field** type. Field coils are shown on two of the poles, the other two coils (not shown) being connected in series in the proper sense to give the proper magnetic polarity. Its greatest application is in dc generators.

The **revolving field,** or **rotor field,** is generally used in large alternators when high voltages are generated. In one form, the rotor consists of a hub which is keyed to the shaft. The poles are secured radially to this hub. The direct current for energizing the magnets is supplied to the magnetizing coils from an external source through brushes which make contact with two collector rings secured to, and insulated from, the shaft and each other. Insulated leads run from the rings to the ends of the windings. This arrangement is similar to that shown in figure 52. A view of this type of rotor field is given in figure 60.

Another type of rotor field is constructed more on the order of a flywheel with the poles inserted in the face. It is used in alternators

driven by prime-movers having a low speed, such as reciprocating steam engines and Diesel oil engines. This type is shown in figure 61.

The **inductor revolving** type of field is essentially a rotor field, but is different in that the magnetizing coil **does not rotate.** Only the core revolves. This type of field is of special interest to radio engineers because it is practically the only form used in the 500~ alternators used in radiotelegraphy. It has the additional advantage that collector rings and brushes are not needed to connect the magnetizing coils to the source.

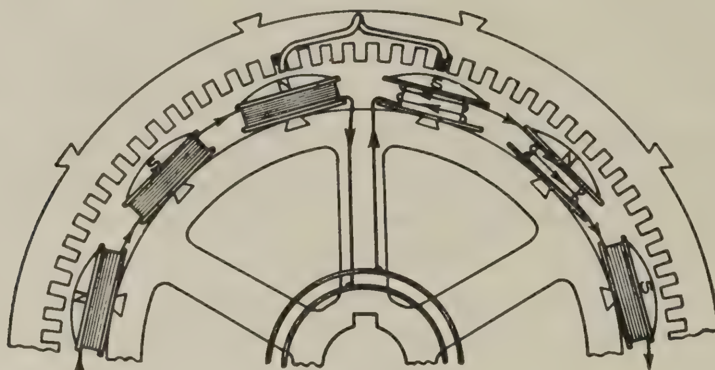


FIG. 61.—Flywheel Type of Revolving Field with its Stator Armature.

The construction is simple and rugged, and the characteristics of this type of alternator are well adapted to the work. Figure 62 shows the complete rotor and magnetizing coil. The coil is secured to the frame of the alternator concentrically with the shaft and the rotor. The rotor core consists of a solid cylindrical body which is keyed to the shaft. At each end of the core is a flange made up of laminations. The flanges are toothed radially, one size having 12 teeth in each flange and, there-

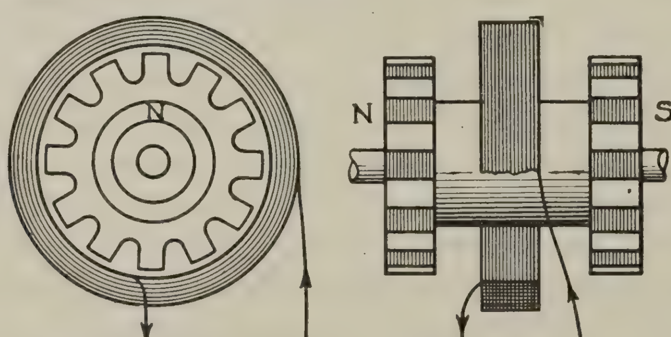


FIG. 62.—Inductor Type of Revolving Field.

fore, 24 poles in all. The single field coil magnetizes the core, making one end of the core N and the other end S. The path of the flux is out of the core through the rotor teeth at the N end, through the air gap into the adjacent stator teeth through the frame, and then back into the S end rotor teeth through the stator teeth. As the magnetized rotor revolves, the path of the flux is alternately increased and decreased in length, thereby producing the necessary variation in flux through

the fixed coils, called **inductors.** This alternator gives 500 cycles at 2,500 rpm.

Types of armatures. An **armature** is essentially a body made of a magnetic material used to reduce the reluctance of any magnetic path which would otherwise be completed through air or other substance. The familiar block of iron kept across the poles of a permanent horseshoe magnet when not in use, and called a **keeper**, is an armature. The term armature, as applied to electric machinery, refers to the **iron** or **steel form** inserted in the air gap between the poles of the field. On this form are disposed the **armature coils** or **windings,** called **inductors,** in which the emf is induced by the variation in the flux from the field passing through them.

The armature may be the revolving element of the electric machine, in which case it is termed the **rotor armature**; or it may be stationary, and is then called the **stator armature.** Both types of armatures are used in alternators, while only the rotor type is employed in dc generators on account of the practical necessity for rotating the commutator, which

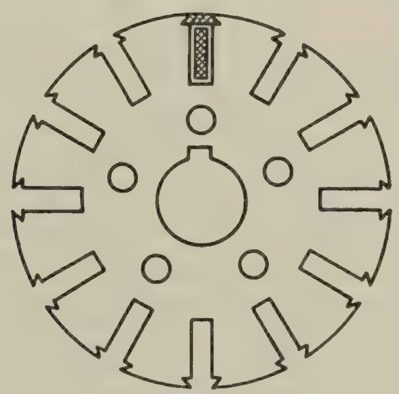


Fig. 63.—Lamination for Rotor Armature Showing Cross-Section of an Inductor and Method of Securing Coil in Slot.

will be explained later. In both types of armatures, the part through which the main flux from the field passes is made of laminated iron or steel in order to reduce losses. Such a lamination for a small generator is shown in figure 63. The thickness of the lamination varies with the type of machine. Each lamination is usually given a thin coat of enamel to insulate it electrically from the adjacent laminations. The armature coils are wound in slots in the periphery. The holes around the center are for ventilation, and connect with openings along the length of the armature formed by spacers between groups of laminations. In the rotor type, these laminations are held securely together by end plates and through bolts.

The rotor armature. Figure 64 shows one form of rotor armature. The reluctance of the air gap between the N and S poles of the field is reduced to a minimum by placing as much of the armature as possible in the path of the flux through each armature coil. It will be seen that the coil itself is practically surrounded by iron. The air gap between

the poles and the armature is then made as narrow as is mechanically safe.

Rotor armature windings. The form of armature winding in general use is called the **drum winding.** A number of loops, or inductance coils, are connected in series additively for high emfs and small currents, or in series-parallel for moderate emfs and larger currents, or in parallel

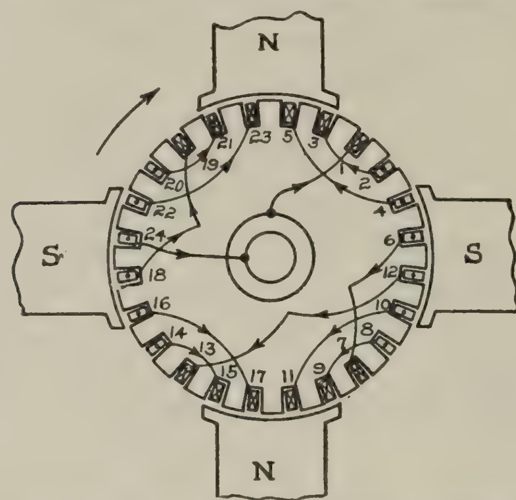


Fig. 64.—Showing one Form of Rotor Armature and Distributed form of Winding for Single Phase.

alone for low emfs and very large currents. The coils themselves consist of one or more turns of insulated wire wound in **former** in order to give them the required shape to fit the armature for which they are intended. They are then taped and impregnated with an insulating compound and retaped, varnished and otherwise treated so that they will not break down under operating conditions. Terminal leads are

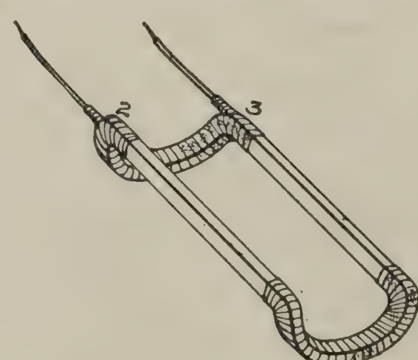


Fig. 65.—Form of Armature Coil to be Placed in Slots 2 and 3 of Armature in Figure 64. Three Different Coil Forms will be Required.

brought out from each coil for making the coil to coil connection. Figure 65 shows a formed armature coil ready for placing in armature slots 2 and 3 of figure 64.

The method of placing the armature coils in the armature is shown in figure 64. The coils are connected in series additively around the armature, and the lead to the first is then connected to one collector

ring, and the lead from the end of the last coil to the other ring. The figure has only enough coils (end view) to show the method of connection. The similarity between this figure and figure 57 is apparent. The pitch of each of the armature coils, as shown, is not the same as that of the poles. The winding is of the **throughly distributed type.**

The coils are protected from abrasion in the slots by the taping, and also by insulating strips placed between the coil and the armature; see figure 63. Wooden wedges are driven into the space provided at the peripheral opening of the slots to hold the coils securely in place against centrifugal force. They are additionally held in by steel wire wound in bands around the armature.

The stator armature. The stator armature is essentially the same as the rotor armature, and the description of the latter applies equally well to the stator form. A section of a stator armature is shown in figure 61 with one coil in place. The coils are necessarily concave in shape to conform with the curve of the armature. The distributed form of winding shown in figure 59 would be used in this armature.

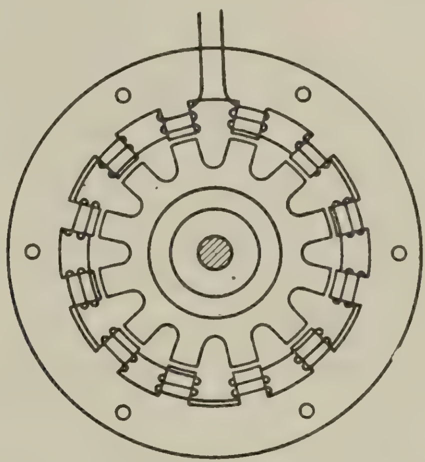


FIG. 66.—Showing One Set of Armature Coils in the Inductor Type of Alternator.

The inductor alternator used in radiotelegraphy, the field of which was described above, has a slightly different form of armature winding. The individual coils are wound around the projecting stator armature teeth in the same manner as field magnet coils are wound. The width of the armature teeth is approximately the same as that of the rotor teeth. There are two similar sets of inductor teeth and coils, corresponding to the two sets of teeth on the rotor. The coils and armature of each set are frequently assembled in such a manner that they can be slipped into place over the rotor teeth and secured to the frame of the generator. The two sets of armature coils can be connected in series, in which case the resulting emf will be twice that obtainable from one set, or they may be connected in parallel, when the emf will be that of one set and the current will be double that obtainable with one set of coils. Figure 66 shows one set of the inductor teeth with their armature coils. Note that the rotor and stator teeth have the same width. The reason

for the great variation in the reluctance of the magnetic circuit and, therefore, in the flux through the inductors, as the rotor teeth are rotated past the inductor teeth may be seen.

Polyphase or **multiphase alternators.** The alternators of all the types described up to this point are of the **single-phase** type, that is, only one alternating emf is available at any instant, this emf varying periodically in magnitude in more or less close agreement with the sine wave form. Only two terminals are necessary for connecting this simple alternating emf so that it may act in an external circuit.

A **polyphase alternator** is one which produces two or more alternating emfs that pass through their maxima and minima at different instants; that is, each of the emfs reach corresponding values throughout their respective cycles at different instants of time. This difference in time is called **difference in phase,** and is usually expressed in degrees. The practical polyphase alternator is so designed that each of the

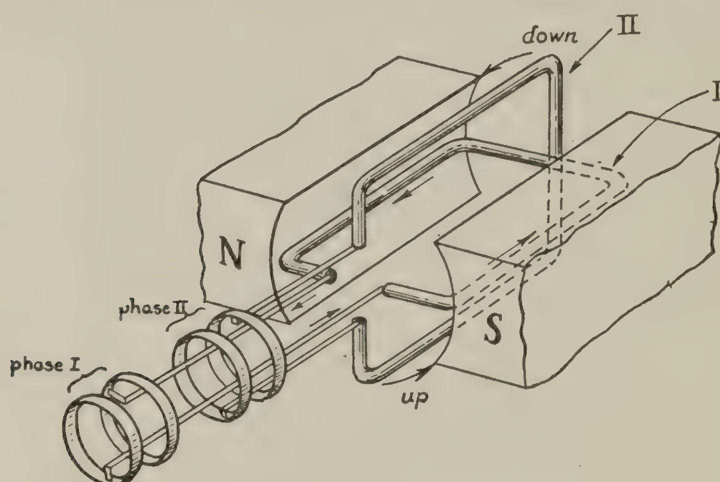


FIG. 67.—Simple Two-Phase Alternator.

alternating emfs induced in the inductors passes through cycles which are exactly the same in every respect for all phases, but displaced by a constant and equal number of degrees. For example, a **two-phase** alternator produces two identical alternating emfs which differ in phase by 90°, or one-quarter of a cycle. One of the emfs is said to **lag behind,** or to **lead,** the other by 90°, or one-quarter of a cycle. These two emfs are also said to be **quadrature.** The inductor winding for each phase is called a **phase winding,** and is numbered I or II as the case requires.

The **three-phase** alternator is the type generally used for power purposes. It is rarely encountered in radio engineering, single-phase alternators being used exculsively nowadays. In this type, the phases differ by 120°, and are numbered I, II and III.

A polyphase alternator in its simplest form consists of two or more sets of inductors, each set producing a single-phase emf in the manner just described for the simple alternator. Each phase winding is the same as that of the other phases, in pitch, number of coils compos-

ing the winding, and in the number of turns per coil, but is displaced 90 or 120 electrical degrees, as the case may be, on the armature. Each phase winding can have its own set of collector rings. This is generally the case with two-phase alternators, but the usual number of collector rings and, therefore, the number of wires in the transmission line from a three-phase alternator is three.

Polyphase alternators are usually built multipolar with rotor field and stator armature. The alternator shown in figure 61 is typical of polyphase alternator design. Figure 67 shows the arrangement of the

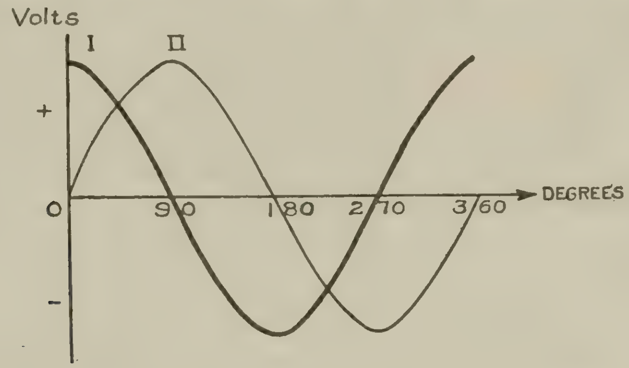


Fig. 68.—Emfs in Quadrature Induced in a Two-Phase Alternator.

armature coils for a two-phase alternator. It is seen that the two coils form independent windings, which are at right angles to each other. The pitch is 180° for each coil. As the loops revolve, coil I is having a maximum emf induced in it in one direction at the instant that the coil II has zero induced emf. A little later the emf in coil I is decreasing while that in coil II is increasing. The phase relation between the two emfs is shown in figure 68. The heavy-line curve represents the emf being induced in coil I, while the light-line curve shows the same for coil II.

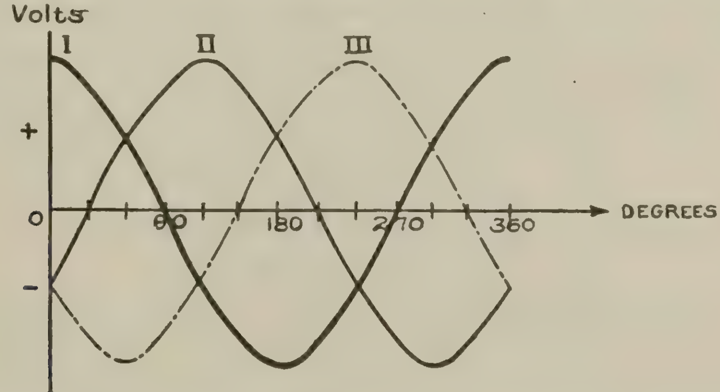


Fig. 69.—Phase Relation of Emfs in a Three-Phase Alternator.

In the practical two-phase alternator, either the concentrated or the distributed form of winding can be used, the latter being the more usual.

The phase windings of the three-phase alternator are spaced 120° apart and, when properly connected, three collector rings are sufficient. The phase relation of the emfs induced in the three coils is shown in

figure 69. The usual methods of connecting the coils together and to the collector rings are given in figure 70.

Eddy currents. The armature in which the inductors are placed serves primarily to reduce the reluctance of the magnetic path and, hence, to increase the efficiency of the alternator. In addition to this important function, the armature should also be considered as a metallic and conducting body threaded by a varying magnetic flux in the same

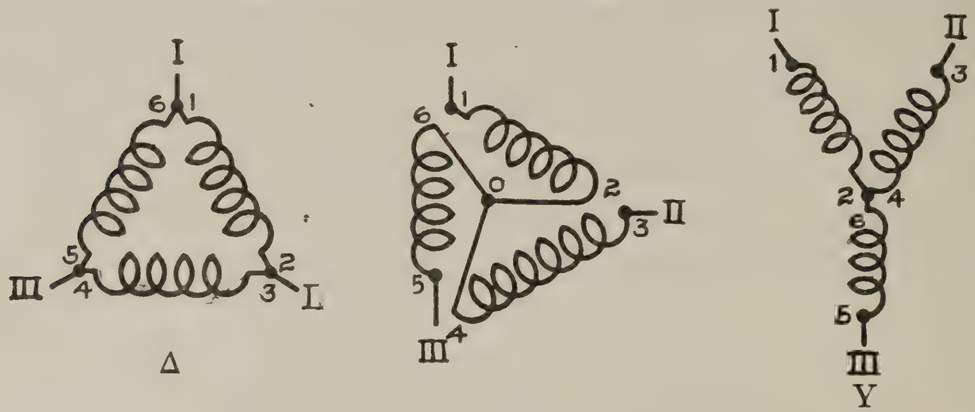


FIG. 70.—Three Methods of Connecting the Phase Windings of a Three-Phase Alternator.

manner as are the inductors that are imbedded in it and, therefore, to have emfs induced in it. These emfs cause currents to circulate in the armature itself, which are called **eddy currents.** Since the eddy currents cannot be utilized, they represent a loss of power which is expended in heating the armature and, in turn, the inductors. This causes an increase in the resistance of the armature winding, and brings about further losses which produce more heat. Unless this condition is remedied, the efficiency of the machine will be seriously impaired.

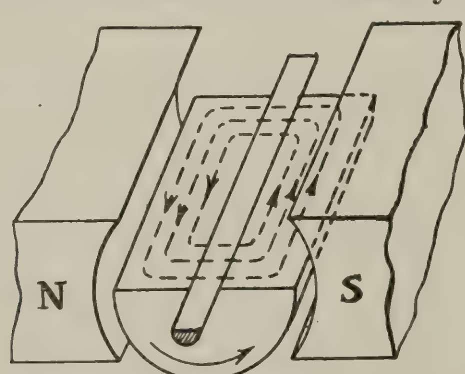


FIG. 71.—Direction of Eddy Currents Induced in a Rotor Armature.

Figure 71 shows the direction of the eddy currents in an armature. The direction in which the emfs act is found in the same manner as for an armature winding. The armature shown in the figure is made of one piece of iron or steel. The conducting path is therefore continuous, and the resistance has a minimum value which is dependent upon the resistivity of the iron. Under these conditions, the eddy currents

attain a maximum value. Anything that can be done to increase the resistance of the circuit will aid in reducing the eddy currents. This is accomplished by constructing the armature from a great number of thin discs, called **laminations.** The laminations are placed with their **planes parallel to the flux lines,** and are insulated electrically from each other. The emfs induced in the individual laminations are not able to combine, and the losses are thereby greatly reduced. The laminated structure and the necessary insulation between the laminations have, practically, no effect on the reluctance of the magnetic circuit except that the cross-sectional area of the magnetic material in the armature is slightly reduced. This is remedied by making a corresponding increase in the cross-sectional area of the iron.

Armature reaction. The emf induced in the armature windings causes a current to flow when the circuit is closed. The circuit flowing in the armature winding causes a reacting emf, which is the emf of self-induction, in the armature winding. In addition, the magnetic effect of the current in the armature winding produces great changes in the distribution and strength of the flux from the field. These changes in the main field flux are caused by **armature reaction.** The change in the distribution of the flux through the armature is a distortion of the field, and results in a change in the wave form of the induced emf, the change tending either towards, or away from, a sine wave form. The change in the strength of the main field flux either increases or decreases the magnitude of the emf induced in the armature windings. More will be said about armature reaction under direct current generators.

Losses. Not all the mechanical energy supplied to the alternator by the prime-mover is transformed into available electrical energy. Some is transformed by friction into heat energy, and some into electrical energy, and then into heat energy. This transformation of part of the mechanical energy into unavailable energy represents a complete loss and is spoken of as an energy or power loss, usually the latter. These losses are divided into three classes, as follow:

(a) **Mechanical losses,**
(b) **Copper losses,**
(c) **Core losses.**

The **mechanical losses** are the same as those that occur in all similar types of rotating machinery. They include friction in bearings, friction between the air and the rotor, called **windage,** and, in addition, friction between the brushes and the collector rings. The mechanical losses remain practically constant irrespective of the load on the alternator, because alternators are operated at a nearly constant speed. If the normal speed of rotation is high, the mechanical losses are correspondingly high, the windage loss being very important at high speeds. These losses are reduced to a minimum in a properly designed and carefully operated machine.

The **copper losses** are due to the heating effect of the current in both the field and armature windings, and are proportional to the square of the current in the windings. The copper losses in the field windings, frequently called **excitation losses,** are

$$W_f = I_f^2 R_f$$

where W_f = watts lost in field windings,
I_f = current in amperes in field,
R_f = resistance in ohms in field.

The copper losses in the armature winding are calculated in the same manner as for the field, but the current is generally very much greater than in the field. Hence, the resistance of the armature winding should be very low. It is a very small fraction of an ohm in the case of large machines.

Core losses include those caused by eddy currents and by hysteresis, both of which have previously been described.

All the losses result in heat, which must be dissipated in order to prevent a cumulative effect. These losses produce the rise in temperature which is noticeable in the vicinity of electric machines. The temperature of a properly designed alternator, operating at rated full load rises to a certain value, called the **normal operating temperature,** which is well within the limits of safety. Momentary overloads are not very injurious, but a continuous overload is likely to ruin the insulation, and cause a complete breakdown of the alternator. Thus, the output of a machine is wholly dependent upon the allowable operating temperature. The power lost in heat varies with the size, design, normal speed, etc., of the alternator, amounting to as much as 20 per cent in small machines and to 5 per cent in very large machines.

The **efficiency** of an alternator is the ratio of the output to the input. Thus, the efficiency of a small alternator with 20 per cent losses would be 80 per cent, while that of a very large alternator with 5 per cent losses would be 95 per cent. The efficiency increases with the size of the machine and, for any given machine, will vary with the load, being low at small loads and usually reaching its maximum value at the rated full load.

Regulation and voltage control. An alternator is designed to run at a constant speed. This is necessary, as otherwise the frequency and magnitude of the induced emf would be continually varying. Such a varying condition is not desirable, as it would cause a continual change in the brilliancy of incandescent lamps connected to such a circuit and would cause certain types of motors to vary in speed.

The frequency variation can be eliminated by a proper governor control of the speed of the prime-mover. There is usually, however, a change between the **no-load voltage** and **load voltage,** even when the speed and field excitation are kept constant. As the load is increased,

the armature reaction becomes greater, and there is a drop in voltage. The characteristics of the external circuit play an important part in the change of terminal voltage with change in load, but will not be discussed here. The automatic manner in which an alternator acts in this respect, when the load is reduced from rated full load to no load, with the speed and field excitation kept constant, is called its **voltage regulation** and is expressed in percentage, the formula being

$$\text{Regulation} = \left(\frac{E_1 - E_2}{E_2}\right) \times 100 \qquad \text{(per cent)}$$

where E_1 = voltage at zero load,
E_2 = voltage at full load.

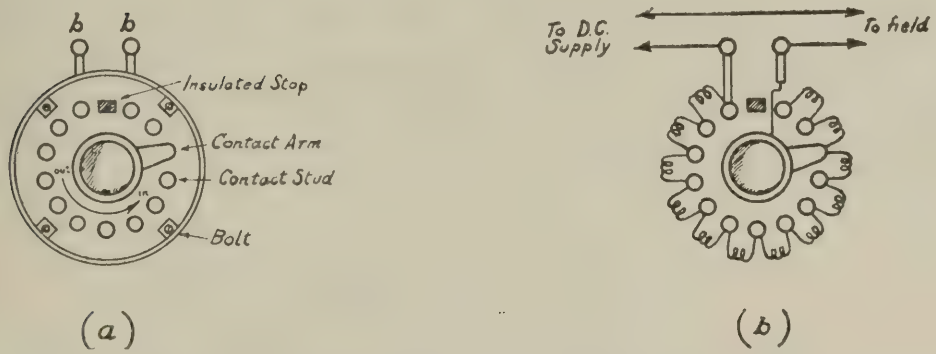


FIG. 72.—Field Rheostat used for Controlling Terminal Voltage of Shunt and Compound Wound Generators.

If the voltage regulation of an alternator is inherently good, variations in voltage due to very rapid fluctuations in load will be automatically limited. Such sudden variations are most objectionable. Slow variations in voltage can be cared for by automatic voltage regulators, or by manual control of the field excitation. In the last two cases, the current flowing in the field magnetizing coils is changed by varying the amount of **regulating resistance** in series with the field winding, some resistance being cut out of circuit when the voltage drops, and more being inserted when the voltage rises. The apparatus used for this purpose is called a **field rheostat.** Figure 72(a) gives an external view of such a rheostat. It consists of a number of contact studs, mounted in a circle on an insulating base, which are swept over by a revolving contact arm supplied with an insulating knob. The resistance coils are connected between the studs, as shown in figure 72(b). The connections are such that, as the contact arm is revolved in a clockwise direction over the studs, the amount of resistance inserted in the circuit is reduced. An insulating stop between the first and last studs prevents the contact arm from passing suddenly from the **all resistance out** to the **all resistance in** position. The method of connecting the field rheostat R into circuit is shown in figure 72(b).

The direct-current generator. The direct-current generator is simply an alternator which is provided with an automatic reversing

device instead of the usual collector rings. What has been said concerning the alternator, therefore, applies equally well to the dc generator. The main difference between the two lies almost entirely in the manner in which the alternating emfs induced in the armature windings are permitted to act in the external circuit. In the case of the alternator, the emf is allowed to act first in one direction and then in the reverse direction, while **the dc generator is arranged in such a way that the emf can act only in one direction in the external circuit,** that is, the emf acts continuously in the same direction.

The simple dc generator. Figure 73 shows a dc generator in its simplest form. The similarity between this figure and figure 52 should be noted. The description of how an emf is induced in the revolving loop given there applies to the case now under consideration. A reversing device, called the **commutator,** is substituted for the collector rings, and in this case is simply one of the collector rings cut into halves, s_1 and s_2, which are insulated from each other and from

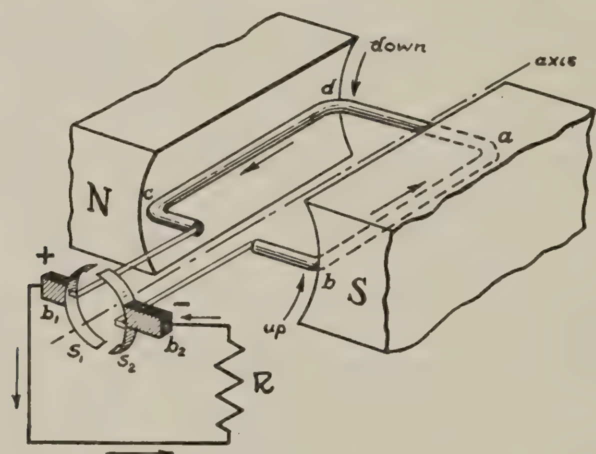


FIG. 73.—Simple DC Generator Showing how Commutation is Effected.

the shaft, but securely connected to the latter. The halves are arranged on the shaft so that the line drawn between them is at right angles to the plane of the loop, which is connected to the two semicircular **segments,** as shown in the figure. Brushes b_1 and b_2, which are thick enough to bridge the gaps in the commutator, are secured to, and insulated from, the frame of the generator. They serve to make the electrical connection between the revolving loop and the external circuit R.

Commutation. The loop is supposed to be rotating in a counterclockwise direction. At the instant shown in the figure, the emfs induced in the loop are acting in the directions shown by the arrows; for example, the direction in side dc is towards the reader, and **out** at segment s_1 to brush b_1. This is called the positive direction, and brush b_1 is therefore the **positive terminal** of the generator. The direction in side ab is away from the reader, that is, **in** at brush b_2 and through segment s_2 to side ab. Brush b_2 is the **negative terminal.**

As the loop revolves, whichever side of the loop is moving down past the N pole has an emf induced in it which always has an outward direction. Contact with this side is made with brush b_1 through the segment of the commutator which is connected to that side and which rotates in synchronism with it. Hence, the polarity of brush b_1 is always positive, and that of b_2 always negative, excepting at the instants that no emf is being induced in the loop. This is when the loop is moving parallel to the flux and, consequently, the flux through the loop is not being varied. This position is called the **neutral position** and, theoretically, is midway between the two poles. At these instants the loop is short-circuited by the brushes which bridge the gaps between the segments.

This operation of periodically reversing the connections from the revolving loop to the external circuit, so that the emf induced in the loop always acts in the same direction in the external circuit, is called **commutation.** Although the emf at the terminals of the generator does not change its direction, it varies sinusoidally in the positive direction;

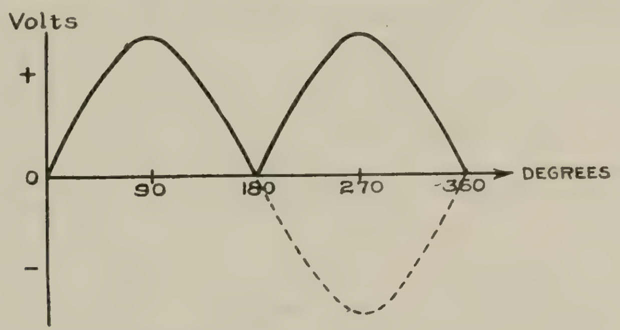


Fig. 4.—Pulsating Emf Produced by a Loop with Commutator During One Revolution in a Bipolar Field.

due to the manner in which the flux through the loop varies. Consequently, it changes periodically from zero to a maximum value and then decreases to zero, performing this operation twice per cycle, or electrical revolution of 360°. Figure 74 shows the effect of commutating, or completely **rectifying**, a sine wave of emf. The result is a **unidirectional** and **pulsating emf.** The dash-line curve shows the negative alternation, which is reversed in direction by the commutator. The necessity for a rotating armature is now apparent. All dc generators are constructed with a rotor armature and a stator field.

The magnitude of the induced emf can be increased by using the same methods as were described for the alternator, but so long as only one coil is used, the pulses of emf in the external circuit are practically as shown in figure 74. An increase in speed will only result in an increase in the frequency and the amplitude of the pulses. This pulsating emf produces objectionable effects in the external circuit, such as a flickering in the light from incandescent lamps.

In order to obtain a more steady emf, resort is made to multi-coil, multipolar generators having a great number of commutator segments.

For example, if the simple two-phase alternator, figure 67, is equipped with a commutator having four segments, the negative halves of each phase will be rectified. This is called the **open-coil** type of armature winding. The position of the commutator segments relatively to the two loops is given in figure 75, which is an end view of figure 67. The brushes b_1 and b_2 are shown inside the commutator, for convenience. Each commutator segment subtends an angle of slightly less than 90° at the center.

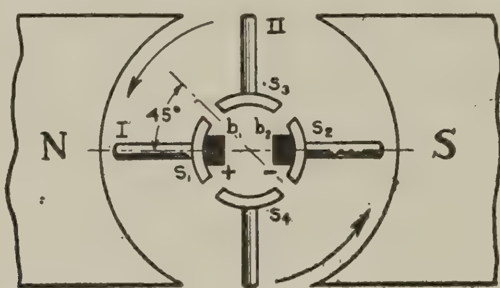


FIG. 75.—Two Coil, Open-Coil Type of Armature Winding for Bipolar Field.

With the brushes fixed in the position shown, and the armature revolving in a counter-clockwise direction, loop I is having a maximum emf induced in it; also, brush b_1 is positive and brush b_2 is negative. As the armature continues to revolve, segments s_1 and s_2 leave the brushes and, at the same instant, segments s_3 and s_4 touch the brushes, the gaps being closed by the brushes during this interval. Both loops are, therefore, momentarily connected together. This occurs when both loops are in the same angular position relatively to their neutral positions, that is, when they are displaced 45°, and the emfs induced

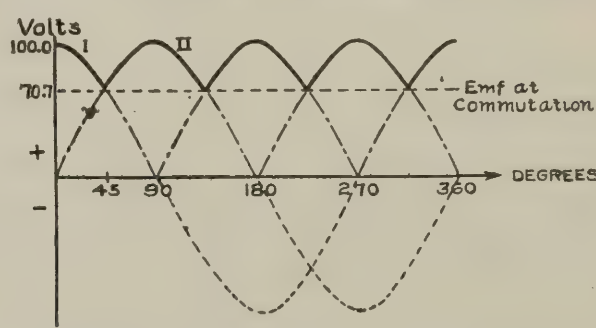


FIG. 76.—Resultant Pulsating Emf Produced by Properly Commutating a Two Phase Alternating Emf.

in each loop are equal in magnitude. The resultant emf for this condition is plotted in figure 76. The dash-line curves represent the negative alternations which are rectified. On the assumption that $E_0 = 100$ volts, commutation would occur at an emf of 70.7 volts, corresponding to an angular position of each loop of 45°. The heavy-line wave is the resultant emf, which varies between a maximum and 70.7 percent of the maximum eight times per mechanical revolution of the armature.

Position of brushes. The position of the brushes is very important. The form of the resultant emf in figure 76 was obtained with the brushes in

the proper position. If the brushes are kept diametrically opposite (for a bipolar generator), but moved 45° **against** the direction of rotation, figure 77, they will leave the segments connected to phase winding I at the instant that maximum emf is being induced and, simultaneously, connect phase winding II, in which no emf is being induced, to the external circuit. Commutation will, therefore, occur as shown in figure 77.

If the brushes are moved **with** the direction of motion of the armature through an angle of 45° from the first position, the emf in phase winding I will be allowed to drop sinusoidally to zero, at which instant phase winding II will be connected into circuit, while its emf is a maximum. This condition is obtained with the brushes in the position shown by the light brushes in figure 77.

Violent sparking will occur at the brushes when they are in either of the last-mentioned positions. This sparking becomes serious when the current flow is heavy, and should therefore be remedied by bringing the point of commutation where the emfs are equal.

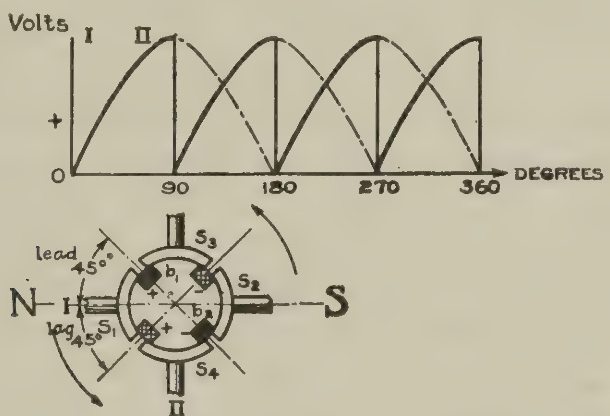


Fig. 77.—Effect Produced by Moving Position of Brushes 45° against Direction of Rotation.

Practical dc generators. The foregoing description of a simple dc generator will give the reader an insight into the operation of all dc generators. In the practical generator, the armature is **drum-wound** with many coils. The two sides of each coil (the part in the slot) are so separated that they will pass under adjacent poles at the same instant. The commutator is connected to the windings in such a way that the coil in which the direction of the induced emf is on the point of being reversed (position of zero induced emf) has its terminals reversed relatively to the external circuit at the same instant. The terminals of each coil are connected to **adjacent** commutator segments, while the next coil has its **in** terminal joined to the same segment as the **out** terminal of the preceding coil. There are usually as many pairs of brushes as there are pairs of poles. All the positive brushes are connected to a common positive terminal. The same is done with the negative brushes.

Figure 78 shows in developed form one of the simplest types of armature winding. It is called the simplex, singly reentrant drum winding. There are eight coils which form two layers of conductors in the eight slots. The winding can be easily traced by starting at segment 1, passing through slot 1 to slot 5, and back to segment 2. This is continued until all the slots are filled with the first layer. The second layer, or set of 4 coils, is then wound, again beginning at slot 1. The beginning of the first coil of the second set is secured to segment 6 and

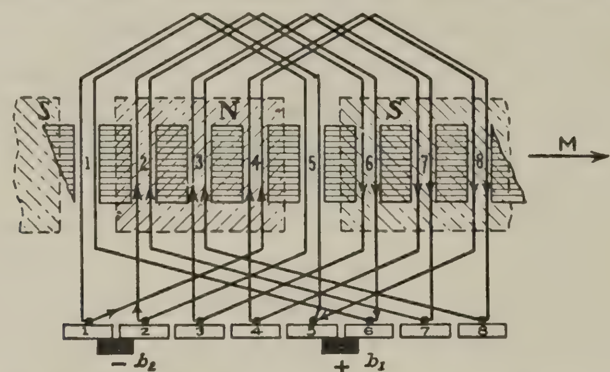


FIG. 78.—Simplex, Singly Reentrant Drum Armature Winding for a Bipolar Field; Shown in Developed Form.

its end to segment 5, etc. The view is from **above** the poles, through them, and into the armature which is revolving to the right and beneath the poles. A study of the diagram will show that coils 1 and 5 are being short-circuited at the instant shown, with the emf acting **into** the winding from segments 1 and 2, and **outward** from the winding at segments 5 and 6, making brushes b_2 and b_1 negative and positive, respectively. The resultant emf at the brushes of this generator would be very much more steady than in the case of the rectified two-phase

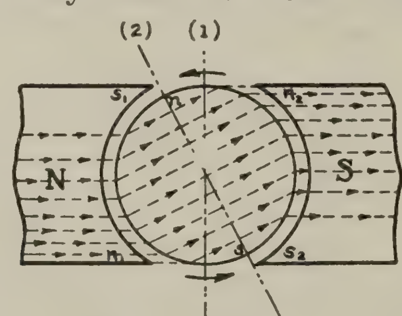


FIG. 79.—Distortion of the Magnetic Field Caused by Armature Reaction.

alternating emf. Dc generators having over a hundred commutator segments give a practically uniform emf, the pulsations being reduced to a mere **ripple.**

Armature reaction. The armature is under the magnetic influence of the field magnets, and acquires magnetic properties by induction from them. Magnetism of opposite polarity to that of the inducing field magnets is induced in those parts of the armature that are nearest to the field magnets. This is shown in figure 79 for a bipolar field. As the

armature is rotated, these induced poles do not rotate with the armature, but **remain opposite** the field poles; also, the flux lines from the N field pole thread the air gap and armature to the S field pole without being distorted. Now, the neutral axis is **always** at right angles to the direction of the flux. The brushes should be placed on this axis in **every instance,** because commutation should take place when the coil is moving parallel to the flux and, hence, is having no emf induced in it.

However, a displacement of this neutral axis occurs when current is flowing in the armature winding, that is, when the generator is supplying current to the external circuit. The current flowing in the armature winding makes the armature an electromagnet. For the direction of rotation shown, the magnetic polarity of the armature due to the current flow will be as indicated by the letters n and s. These two poles induce opposite poles s_1 and n_1 in the N field pole; also, n_2 and s_2 in the S field pole. The result is that the magnetism of both the field poles is weakened at the **entering** edge and strengthened at the **leaving** edge. The flux lines between them are no longer uniformly distributed, but are crowded together near the leaving edge and are practically nonexistent at the entering edge. Maximum induction, therefore, is along a line more nearly connecting n_1 and n_2, which are located at the leaving edges. Since the neutral axis is at right angles to the flux, it will be displaced **in the direction of rotation** from the ideal position (1) to a new position (2) figure 79. The brushes must, therefore, be shifted to this position in order that commutation may occur at the correct instant. Unless this is done, violent sparking will occur. The neutral position shifts with variation in the load on the generator. Modern generators are particularly free from sparking at the brushes. This is desired condition is obtained by the use of high-resistance brushes (carbon), and by **interpoles** which reduce the distortion by counteracting the magnetic effect of the current in the armature winding.

Field excitation. After the alternating emf induced in the armature of a dc generator has been rectified, it is suitable for use in energizing the field magnets. Consequently, there is seldom any reason to use current supplied from any other source, the dc generator itself being able to supply the proper kind of current for this purpose. Thus, practically all dc generators are of the **self-excited** type. The only time it is absoultely necessary to use current from an external source is when the generator is run for the first time. This is because the field magnets have no **residual magnetism** and, therefore, the armature winding is not threaded by any flux. After the generator has once been operated, the field magnets will retain sufficient magnetism to start a current in the armature winding, and the field will gradually be brought up to full operating strength. Various methods of connecting the field coils to the armature are employed, depending upon the operating characteristics desired. These methods are outlined in the following.

The series field. When all the current flowing in the armature windings also passes through the field winding, the latter must be connected in series with the armature terminals and the load. This type of field is called the **series field.** Since the armature current is usually large, the field winding consists of only a few turns of heavy wire. The number of ampere-turns to produce the required flux is thus obtained by passing a large current through a few turns. Figure 80 shows the series field method of self-excitation.

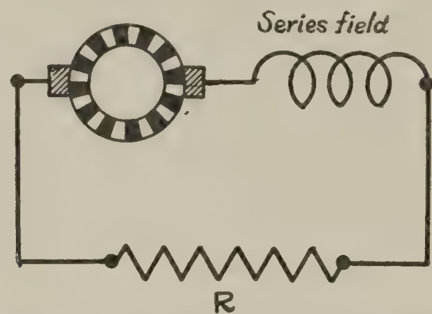


FIG. 80.—Circuit Diagram of a Series Wound Generator.

The **characteristics of the series generator at constant speed** are such that the emf generated varies with every change in the load. At zero load, the terminal voltage is only that which is due to the residual magnetism. As the current drawn from the generator increases, the terminal voltage continues to increase, because all of this current flowing through the series field increases the flux through which the armature winding is rotated. This type of generator is employed with series arc lights where, with proper automatic regulation, it becomes a

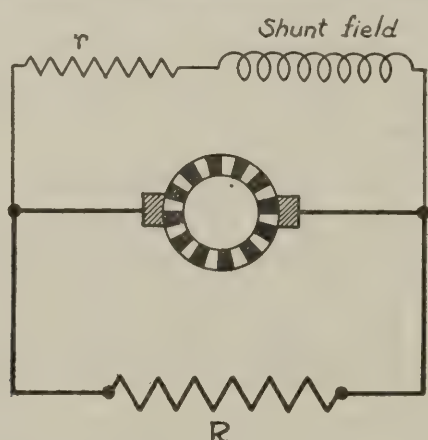


FIG. 81.—Circuit Diagram of a Shunt Wound Generator.

constant current machine, its terminal voltage increasing with th addition of each arc light cut into circuit.

The shunt field. When only a part of the current in the armature winding is used to energize the field magnets by connecting them in parallel with the brushes, the **shunt field** method of excitation is employed. Since the field is connected across the armature windings, it receives the full emf induced in the armature and, consequently, must

have a sufficiently high resistance in order to limit the current drawn to a reasonable value. The required flux is obtained by passing a small current through a large number of turns. Figure 81 shows diagrammatically the connections of a shunt wound generator with rheostat r for regulating the terminal voltage.

The **characteristics of a shunt wound generator at constant speed** are such that, at zero load, the terminal voltage is a maximum. As the load is increased, the terminal voltage is decreased by both the IR drop in the armature windings and reaction and, consequently, the current in the field winding is also decreased, which operates to lower the voltage still more. Thus, the terminal voltage decreases as the current drawn from the armature increases. This characteristic is the opposite of that of the series wound generator. The terminal voltage may be kept constant by varying the regulating resistance in series with the field winding, resistance being cut out as the load is increased, and vice versa. The shunt generator operates best on a constant load, in which case it will maintain a constant terminal voltage.

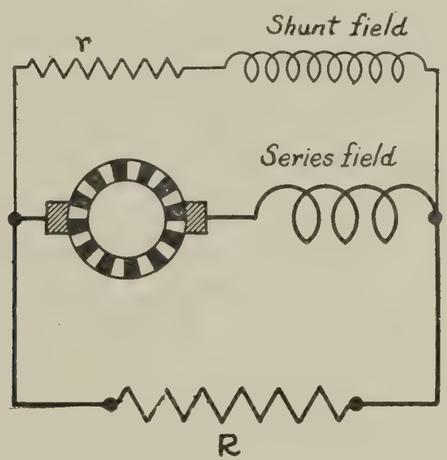


FIG. 82.—Circuit Diagram of a Long Shunt, Compound Wound Generator with Terminal Voltage Control.

The compound field. The simple series and shunt field methods of excitation with their opposing characteristics are usually combined in large generators where a practically constant terminal voltage is required irrespectively of any variations in load. The contribution of each of the field windings to the total flux can be so proportioned that the terminal voltage will remain constant from no-load to rated full-load. Such a generator is said to be **flat-compounded.** The proportions of the two fields may also be such that the terminal voltage will rise slightly with increase in load. This is the **over-compounded** condition. Figure 82 gives a diagram of the connections of a **long shunt,** compound wound generator. It will be seen that the shunt field is connected across both the armature and the series field. In the **short shunt** compound wound generator, the shunt field is across only the armature.

Motors. A **motor** is an electrical machine used to convert electrical energy into mechanical energy in the form of rotary motion, the amount of electrical energy converted into available mechanical energy being dependent upon the efficiency of the motor. So similar are the generator and motor in general appearance that a casual inspection would not reveal the differences; in fact, most generators generally can act interchangeably as motors, with only slight modifications being necessary. Much of the theory of the generator, such as the generation of emf, armature reaction, sparking, commutations, field connections, etc., applies to the motor.

Theory of the motor. It was shown in Chapter V of this Part that a force of attraction or repulsion is developed when different magnetic fields are acting in parallel directions in the same space, depending, respectively, upon whether the flux lines are acting in opposite directions or in the same direction. Now, when a wire carrying current is placed at right angles to the direction of a magnetic field, a force will also be developed which will act on the wire to move it. If a very flexible wire in loop form is placed in a strong magnetic field, and a current then passed through it, the loop will form itself into a circle, in which case it would include the greatest number of flux lines possible. This gives the rule that **a circuit always tends to move in such a direction so as to include the maximum number of flux lines.**

If the direction of the magnetic field is reversed, the direction in which the wire carrying current tends to move will also be reversed. Likewise, if the direction of the current through the wire is reversed, the direction in which the wire tends to move will also be reversed.

The left hand rule. The mutual relations of the direction of the magnetic field of the current in the conductor and of the motion of the conductor may be remembered by the **left hand rule,** frequently called the **motor rule.** It is: Extend the **thumb, forefinger** and **middle** finger of the left hand in mutually perpendicular directions; then, when the forefinger points the direction of magnetic **flux** and the middle finger the direction of the current I in the conductor, the thumb will indicate the direction of **motion** of the conductor.

The force developed, for a given current and magnetic field, is a maximum when the conductor carrying current is at right angles to the direction of the magnetic field, and is zero when the current flowing in the conductor has the same direction as the magnetic field. For all other relative positions of the same conductor and magnetic field, the force has a value between zero and a maximum. Thus,

$$F = HIl \sin \phi$$

where F = force in dynes,
H = magnetic field intensity in gausses,
I = current in emu,
l = length of conductor in cms,

ϕ = angle the conductor makes with direction of flux, which is 90° in the case of a motor and may, therefore, be omitted.

It is apparent that this force can be made very great by the same methods as are employed in generators to increase the magnitude of induced emf.

The dc motor. The simplest form of a dc motor consists of a single loop with a two-segment commutator, arranged to revolve in a bipolar field. Figure 83 shows this arrangement. Note the similarity between this and figure 73. Current is supplied to the loop by the battery B when the switch S is closed. The switch is supposed to have just been closed. The current flows from the battery B to brush b_2 through segment s_2 **into** side ab at end b and **out** of side cd at c into segment s_1, through brush b_1 and back to the battery. The position of the loop at the instant of closing the switch is such that it includes the minimum

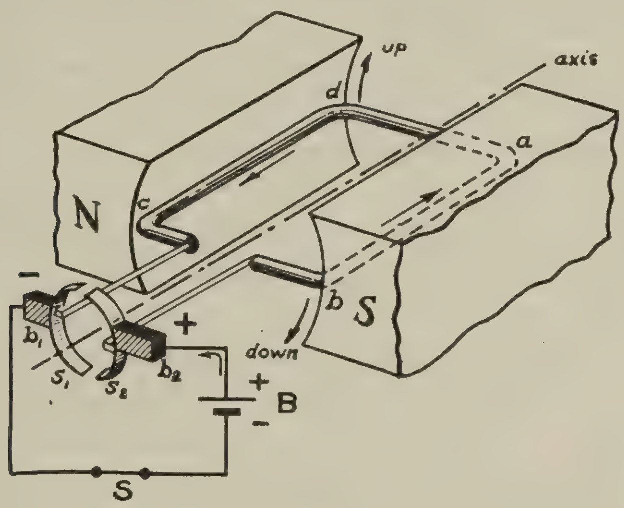


FIG. 83.—The Simple DC Motor.

number of flux lines. By applying the left hand rule, it will be seen that the side cd tends to move upwards, and the side ab downwards. This direction of motion is the **reverse** of that necessary to induce a current in the conductors having the same direction as that supplied by the battery. Hence, the direction of rotation of the armature of a generator, when operating as a motor, will be **reversed.**

As the loop continues to revolve, it reaches a position at right angles to that shown in the figure, and at that instant embraces the maximum number of flux lines. In the case of this simple machine, the loop will stop in this position unless it has acquired sufficient momentum to carry it past this **dead center,** in which case the commutator operates to cause current to flow in the conductors always in the direction indicated in the figure, and the rotary motion of the loop continues so long as current is supplied.

The practical dc motor. The form of armature winding shown in figure 78 is also typical of small motors. If current were supplied to

this armature winding, so that on leaving the brushes it had the directions shown by the arrows in the slots, the armature would move under the field poles towards the **left**, and the action would be similar to that described above for the simple motor. The force acting on the armature would, however, be more uniformly distributed. The motor would, therefore, start automatically from any position of rest.

The direction of the current flowing through the armature coils as the armature rotates is always maintained constant by the action of the commutator. The result is that the magnetic polarity of the armature always remains the same, and since it is opposite to that of the field magnets, there is a mutual attraction between the two, which acts to keep the armature revolving. This turning force developed between the armature and the field magnets is called **torque**, which, when applied through a distance r, in this case the radius of the armature, will do mechanical work when the armature is revolving. The mechanical work done is proportional to the product of the speed and torque. If the radius is expressed in feet and the pull on armature coils in pounds, the torque will be expressed in **pound-feet.** One method of measuring the mechanical power is with a **prony brake.** Another method uses a **dynamometer.**

Counter-electromotive force. As the motor armature revolves in a magnetic field, propelled by the magnetic effect of the current flowing in its coils, these coils are also being threaded by a varying amount of flux. Therefore, a generator action occurs, and an emf is induced in the motor armature winding. The magnitude of this emf can be calculated by the use of the formulas given for the determination of the emf induced in a generator armature, while its direction, which is readily found by the **right hand rule,** is **opposite,** or **counter,** to that supplied to the armature from the external source. This emf generated in a revolving motor armature is called **counter-electromotive force,** (**cemf,**) and plays a very important part in the operation of motors, because it limits the amount of current that can flow through the armature. This is shown in the following.

The armature of a motor is connected across the supply mains and therefore receives the full difference of potential. Assume that the supply voltage is 125 volts and that the motor armature is revolving at a speed sufficiently high to generate a cemf of 123 volts. Let the resistance of the armature winding be 0.05Ω at operating temperature. Now, since the applied emf and cemf act in opposition, the emf $\mathbf{E}_1$, available for forcing current through the armature winding, is equal to the difference between the two, or

$$E_1 = E - \text{cemf}$$

substituting $\qquad = 125 - 123 = 2$

whence $\qquad E_1 = 2$ volts.

The current drawn can be found by Ohm's law.

$$I = \frac{E_1}{R}$$

substituting $$= \frac{2}{0.05} = 40$$

whence $I = 40$ amperes.

This current will be registered on an ammeter connected in the supply mains.

Next, assume that, due to a sudden load, the speed of revolution of the armature drops until the cemf equals 120 volts. The current becomes 100 amperes. If this current is sufficient to carry the load, the speed of the motor will remain constant. If it is not enough, the speed of the motor will continue to drop, and the current will continue to increase. Unless the motor armature winding is protected by circuit-breakers, or fuses, in the supply mains, the current will become destructively great and burn out the armature coils. In this particular case, if the armature were stopped, the current would rise to 2,500 amperes, provided that the source could deliver such a current. This would also be the current drawn by the armature were it connected across the supply mains without a suitable means of reducing the current taken on starting the motor.

On the other hand, it might be thought that a motor armature on zero load would continue to speed up until the cemf equalled the applied emf. There is this tendency in the case of the series motor but not in a shunt motor unless the field is open. This characteristic will be treated under each type of motor.

The starting box. It has just been said that the current drawn by an armature when standing still, and suddenly connected across the supply mains, is destructively large. A rheostat capable of carrying a momentary heavy current is temporarily connected in series with the armature winding and varied from the **all resistance in** position to **all resistance out,** or **running,** position. The current flow is thereby limited to a safe value on starting. An **overload release** is also usually provided to open the circuit in case the rheostat is operated too rapidly, or when a dangerous overload is placed on the motor. Such a complete device is called a **starting box.** It takes many forms depending upon the control desired, some types being designed to operate for short periods on any one of the steps of resistance. The starting box is **always** to be connected in series with the armature circuit, and is **not** to be used for controlling the speed of the motor, except in the case of **series motors.**

Figure 84 shows the circuits included in one type of starting box used for shunt and compound wound motors, together with the connections to a compound motor and supply mains. Care must be taken to make the proper connections. The lead from the terminal F on the starting box must run to the F terminal on the terminal board of the motor which is **not** connected to the armature by a jumper.

The lever is shown in the **off** position. It will be noted that the shunt field circuit is not closed until the lever is on the first contact, and also that the **retaining magnet** winding, which, in series with a limiting resistance r, is also across the supply mains. The purpose of the retaining magnet is to hold the lever in the running position as long as the supply mains are **alive.** If, for any reason, the supply mains become **dead**, the retaining magnet is no longer energized and allows the lever to be returned to the off position by the action of a

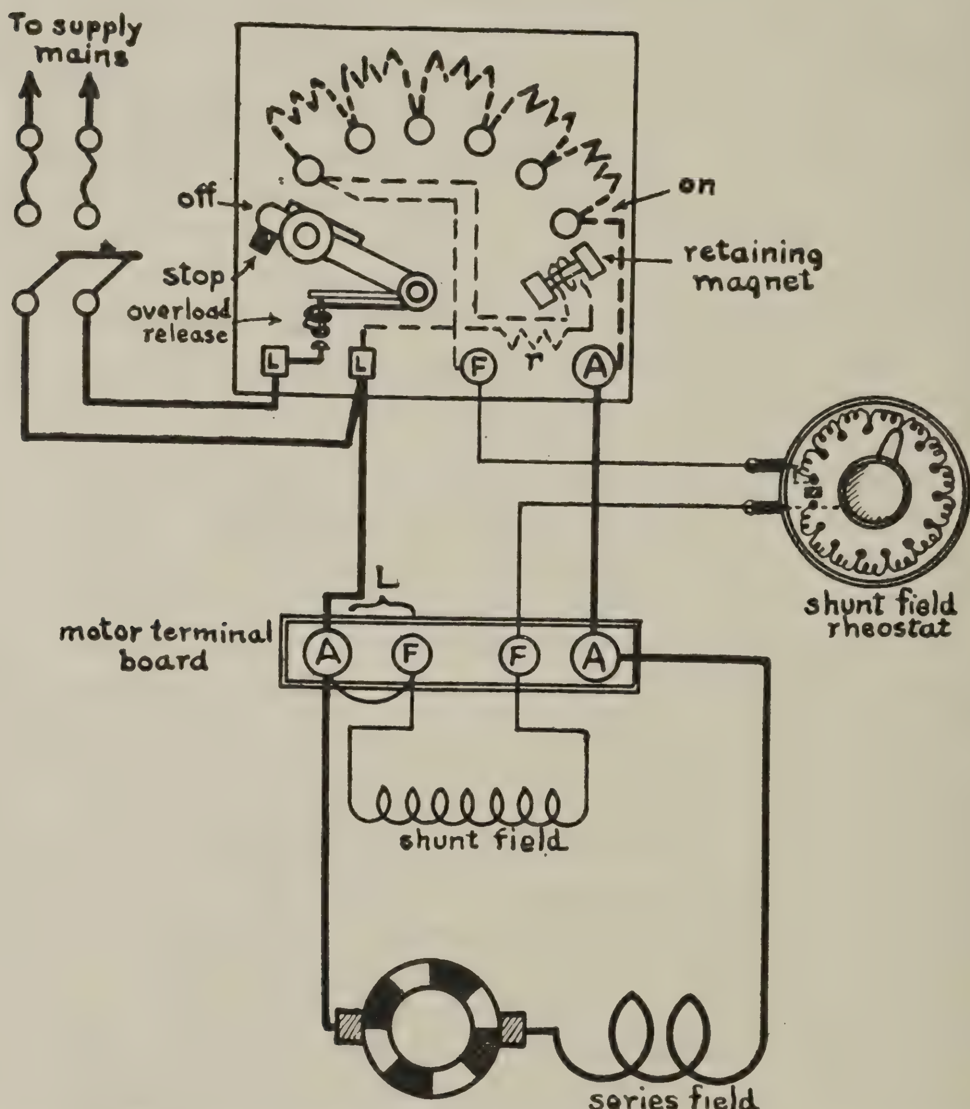


Fig. 84.—Circuit Diagram of a Starting Box and its Connections to Line and Motor.

spring. The overload release controls magnetically the amount of current that can be drawn. It is connected in series with the armature circuit. In case of an overload, it trips the starting lever free from the retaining magnet and permits it to return to the off position, or independently opens the line.

Classes of dc motors. Dc motors are classified in the same manner as are dc generators, that is, according to the method of field excitation.

Separate excitation of the field magnets is seldom employed; neither are permanent field magnets used except in toy and other very small motors.

The **series motor** is used where a large starting torque is required, such as for street cars, cranking automobile engines, and deck winches. Such duty requires that the motor start up under varying loads and, therefore, exert a high turning effort while accelerating. The series connection of the field coils makes this possible. On starting, the strength of both the armature field and the main field is limited only by the current that the starting box permits to flow. Therefore, the torque can attain a very high value. After having started, if the load is suddenly increased, the armature speed decreases very rapidly, and the reduced cemf allows a larger current to flow, the torque again increasing. **The series motor will develop a constant torque at a variable speed,** or **a variable torque at a variable speed.**

If the load is suddenly removed from a series motor, and it is allowed to run **freely**, it will increase in speed until the cemf plus the *IR* drop in the windings equals the impressed emf, neglecting bearing and windage losses. The speed attained is likely to become disastrously high unless the motor is provided with either a mechanical or electrical governor to control the speed. On account of this tendency of the series motor to race, it is generally connected directly, or by gears, to its load rather than by means of a belt, which is likely to break.

One method of controlling the speed of a series motor is by means of a special starting box that is capable of withstanding heavy currents continuous running. This method is not satisfactory, both because of the great waste of power and on account of the *IR* drop in the regulating resistance varying with every change in load and thereby affecting the speed of the motor. Another method employed to regulate the speed of a series motor is to vary the field strength by actually varying the number of turns in the field coils, or by shunting a few turns. In street car installations, two motors are connected in series when starting, thus reducing the emf impressed across each motor to one-half the line voltage.

The **shunt motor** is wound in exactly the same manner as the shunt generator and the field is, therefore, across the supply mains. For this reason the field remains constant in strength irrespectively of the load, provided that the line voltage does not drop and the field rheostat is not varied. After a shunt motor has been started by means of the starting box, and the speed has been adjusted to the desired value by means of the field rheostat, the motor will continue to run at the same speed and, whether loaded or unloaded, will vary only slightly in speed. If an overload is placed on it, the motor will slow down and probably stop. Another characteristic it has is that it will not start up on full load; in fact, it is usually the practice to bring the motor up to normal speed before putting on the load. These characteristics show that the shunt motor depends upon high speed and low torque to pull the load. Slight

variations in speed are taken care of by employing the IR drop across a variable resistance in series with the field to vary the emf impressed across the field winding.

Shunt motors are used to drive line-shafting, lathes, grinding machines, etc., which should run at a constant speed even though the load varies. **The shunt motor develops a variable torque at a constant speed.** Care must be exercised **never** to start a shunt motor with its field **open,** as it will race and probably wreck itself. This is caused by the armature increasing in speed to make the cemf practically equal to the impressed emf. This is an impossibility because the residual magnetism is not sufficient to produce the necessary amount of generator action. Note the positive method of connecting the shunt field into circuit when the motor is started by means of the starting box shown in figure 84.

The **compound motor** combines the characteristics of both the series and shunt motor, and is the type most frequently encountered in radio engineering. It is used in most **motor-generators** for spark transmitters where the load is constantly varying from zero to full load, caused by telegraphing with the key. A shunt motor used for this purpose must be oversize in order to operate as satisfactorily as the compound motor. Even slight variations in the speed of the 500-cycle alternator driven by the motor would cause an objectionable change in the group frequency, and prevent satisfactory operation.

The **differential compound-wound motor** is especially adapted to this kind of duty because its speed is but very little affected by wide variations in load, due to the manner in which its series and shunt fields operate. The current flows in opposite directions in the two fields, and they tend to neutralize each other. Such a motor normally operates as a shunt motor until a heavy load comes on. The motor slows down and more current passes through the armature and series field. The latter becomes stronger and, by reducing the total field, permits the armature to accelerate. In this manner the speed is kept constant. The shunt field is much the stronger. The starting torque is abnormally low, being even lower than that of a shunt motor. This characteristic is not objectionable because the motor-generator is not loaded until after it is up to normal speed.

Interpoles are narrow poles set between the main field poles. Their magnetizing coils are connected in series with the armature winding and the series field (if one is used), and the current passes through them in the proper direction to give each the same polarity as that of the main field pole next **behind,** relatively to the direction of armature rotation. Interpoles serve the same purpose in motors as in generators, that is, they counteract the distortion of the main field caused by armature reaction and thereby obviate the necessity of shifting the position of the brushes to obtain sparkless commutation.

Reversing a motor. The direction of rotation of a motor armature can be reversed at will by reversing the direction of the current through **either the armature winding or the field winding, but not through both.** This is evident if the left hand rule is applied. Care should be taken to see that the brushes are properly set for the reversal of motion of the commutator.

Ac-dc motors. It was stated in the previous paragraph that the direction of rotation of a motor is not reversed by a reversal in the direction of the current through both the armature and field windings. Now, alternating current can be considered to be direct current that periodically reverses in direction in addition to varying sinusoidally in strength. Therefore, a dc motor will operate on alternating current if the voltage is correct. The field magnet cores must, however, be laminated, because they are subjected to an alternating flux. The reason for this is the same as for laminating the armature of a dc motor—to reduce eddy current losses. If the field core is not laminated, the motor will become very hot in a short time and, also, be very inefficient. Such combination ac-dc motors are used mainly for running fans, sewing machines, and for performing other light work.

Alternating-current motors. The subject of alternating-current motors will be only very briefly discussed, because they take so many forms and are so little used in radio, excepting at shore radio stations where local commercial power lines are available. All ac motors, excepting the smallest sizes mentioned in the previous paragraph, can be operated only on alternating current of the proper frequency. The frequency generally employed for power purposes only is 25 cycles. When both light and power loads are carried, the standard frequency is 60 cycles. This is mainly for the reason that an objectionable flickering of the lights is noticeable when 25 cycles is employed.

The **synchronous motor.** If two identical alternators are operating in parallel, and the prime-mover of one is shut down, the alternator will continue to revolve at the same speed as when it was able to deliver power. This speed, which had to be the same for both, so that both would supply current of the same frequency, is called the **synchronous speed.** Hence, the alternator whose prime-mover has been shut down is operating as a motor running in **synchronism,** or **in step,** with the other alternator, and is drawing power from it.

A synchronous motor will continue to run at synchronous speed until greatly overloaded. The normal load, however, is about 20 percent of the maximum load, because operation at maximum load is unstable; that is, the rotor falls back as the load is increased, and then tries to get in step. This results in an oscillating motion of the rotor across the synchronous position, called **hunting.** If this condition persists, the motor may be pulled out of step, and stop. Thus, this motor is quite similar in its characteristics to the shunt motor. Special starting devices

must be used, because this type of motor will not start of itself. The synchronous speed in rps can be found by dividing the frequency (in cycles per second) of the supply by the number of pairs of poles in the motor field. Thus, a synchronous motor having 12 poles and operating on 60 cycles will have a synchronous speed of 600 rpm.

The **polyphase induction motor.** When polyphase currents flow in the stator phase windings of a polyphase motor in the proper direction, the magnetic polarity at any given point on the stator surface becomes more and more *N*; for example, it reaches a maximum intensity and then decreases, first becoming neutral, and then increasing in *S* polarity, and again returning to a neutral condition. This occurs during each cycle of each phase. The curves of emf shown in figure 69 can also be used to represent a three-phase alternating current. If the axis of abscissas is taken to represent a plane view of the inside circumference of the stator, a progressive motion to the right is seen to occur in the current amplitudes. Assume that each amplitude of current in the positive direction produces a *N* pole of maximum strength in the stator, and each amplitude in the negative direction, an *S* pole of maximum strength. It will be seen that the **equally spaced** *N* and *S* polar regions also travel to the right, that is, **around the circumference of the stator.** This progressive motion of the magnetic poles produces a continuously and rapidly rotating magnetic field in the stator.

Induction motors. The induction motor is the most widely used type of ac motor. It resembles the shunt motor in its operating characteristics; that is, it maintains a practically constant speed from no load to full load. In addition to this desirable feature, it is simple and rugged in construction.

The induction motor has two members—the stator and the rotor, which are laminated and provided with slots to take the windings. The windings are called **primary** and **secondary.** The former carries the inducing current which is supplied by the mains, while the latter is short-circuited upon itself and is entirely independent of the primary winding, current being induced in it by the magnetic action of the current in the primary winding, which is usually on the stator. Induction motors are built single-phase, two-phase and three-phase. The frequency employed is usually 25 or 60 cycles. Polyphase induction motors can be operated on single-phase current at decreased efficiency.

The rotating field. The operation of any induction motor will be more easily understood if the principle of the polyphase motor is made clear. For this purpose a three-phase, four-poles-per-phase motor will be used. This is shown in figure 85, which departs radically from the usual construction, and is used only to illustrate the principle of operation.

It will be seen from the figure that the three phase windings are symmetrical both with one another and with the core. Phase winding

I energizes only the poles *a*, *d*, *g* and *j*, which are spaced every 90° around the circumference of the stator core. Similarly, phase winding II energizes only the poles *b*, *e*, *h* and *k*, while the poles *c*, *f*, *i* and *l* are due to the magnetic effect of current flowing in phase winding III. Whenever current is flowing in a phase winding, the poles of that winding, which are spaced 90° apart, produce equal fields of opposite polarity, as will be seen by applying the rule for finding the magnetic polarity of a solenoid. Thus, in phase I, *a* and *d*, and *g* and *j* form two pairs of poles whose separation is 180 electrical degrees. A pole for phase II and another for phase III must be equally spaced between every pair of poles of phase I, etc. The magnetic effect of the alternating current flowing in any one of the phase windings, say I, will next be considered.

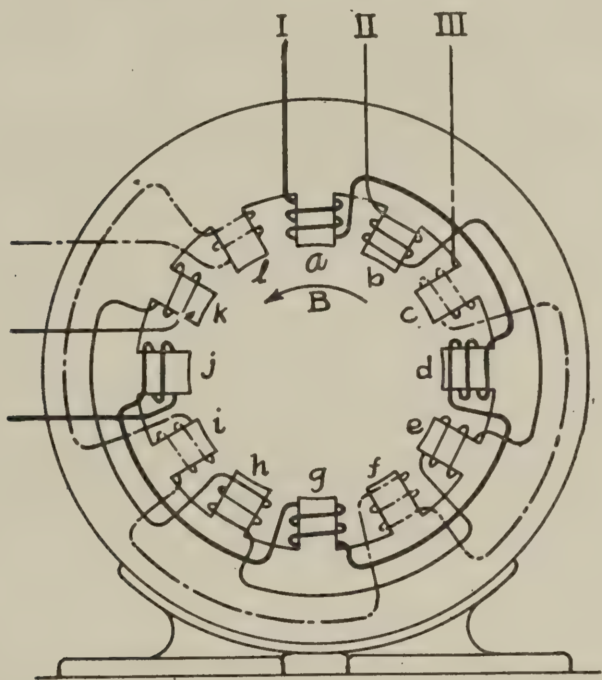


Fig. 85.—Stator of a Three-Phase, Four-Poles-per-Phase Induction Motor (illustrative only).

The sinusoidally varying current can be represented by the emf curve shown in figure 55. The direction of the winding on pole *a* is such that it will be *N* when the direction of the current is positive, that is, flowing from the source into the coil. At the beginning of the cycle, no current is flowing and, hence, the poles are not magnetized. As the current increases sinusoidally in the positive direction, poles *a* and *g* become *N*, while poles *d* and *j* become *S*. The strength of these poles varies with the current, reaching a maximum value and then decreasing until they have lost their magnetism. This completes one alternation. The direction of the current then reverses, and with it the polarity of the poles, *a* and *d* becoming *S*, and *g* and *j* becoming *N*. They attain a maximum strength and then lose their magnetism. This is the end of one cycle.

These same changes take place in the other phase windings, occurring a little later in II and in I and still later in III. The time interval between phases is, however, identical, since the three currents are displaced 120 electrical degrees as shown in figure 69. Consequently, it would be expected that, if pole *a* were *N* and maximum at a given instant, pole *b* would reach the same value 120 electrical degrees, or 1/3 cycle later, and pole *c* the same 2/3 cycle later. Such is not the case, however, because pole *b* is wound clockwise, thus making the pole face *S* polarity when the direction of current through it is positive. This direction of winding is the reverse of that on pole *a*. The effect of this reversal in winding is the same as would result were phase II itself reversed, that is, displaced 180°. A careful inspection of the direction of the winding on every pole will show the reversals. Due to this method of reversing, pole *b* will be in the same condition as pole *a* was 1/6 cycle earlier, and so will pole *c*, 1/3 cycle later.

Figure 86 is an attempt to show graphically what occurs in the stator. The heavy-line curves represent the natural sequence of the three phases, while the dashed curves represent the phases reversed, due to the method of winding. Thus, II means phase current II, while II_r is II in the reverse direction, etc. Beginning at the instant that the current in phase winding I is a maximum in the positive direction, pole *a* has a maximum *N* polarity. Pole *b* has phase II_r and is slightly *N*, and decreasing. Pole *c* has phase III, and is slightly *S*, and increasing. Pole *d* has phase I_r, and is a maximum *S* decreasing, etc. For convenience, this is tabulated for all the poles at successive instants, and is also marked on the time line of the figure.

Pole	Phase Current	Polarity at time *t* equals			
		0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$
a	I	N	N_d	S_i	S
b	II_r	N_d	S_i	S	S_d
c	III	S_i	S	S_d	N_i
d	I_r	S	S_d	N_i	N
e	II	S_d	N_i	N	N_d
f	III_r	N_i	N	N_d	S_i
g	I	N	N_d	S_i	S
h	II_r	N_d	S_i	S	S_d
i	III	S_i	S	S_d	N_i
j	I_r	S	S_d	N_i	N
k	II	S_d	N_i	N	N_d
l	III_r	N_i	N	N_d	S_i

where N or S = maximum intensity,
N_i or S_i = increasing intensity,
N_d or S_d = decreasing intensity.

It will be seen by following the curves from time $t=0$ to time $t=\frac{T}{2}$, that the magnetism travels progressively around the circumference of the stator. Referring to figure 85, the north polar region at the top includes three poles l, a and b at the instant $t=0$, increasing in intensity

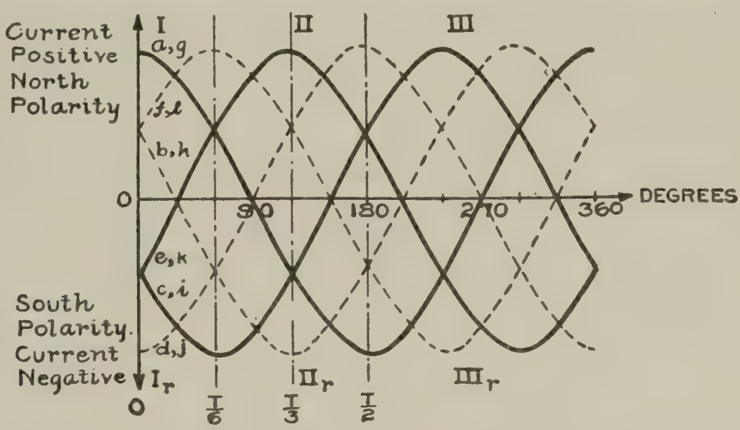


Fig. 86.—Graphical Representation of Change in Polarity and Variation in Field Strength of Poles in the Three-Phase, Four-Poles-per-Phase Induction Motor of Figure 85.

at l, being a maximum at a and decreasing at b. A little later, the north polar region includes poles k, l and a, the gradation of field intensity being the same as just described. As time passes, this north polar region travels around the stator and, simultaneously with it, the other polar regions. As a result, a rotating magnetism, or rotating field, is developed in a counter-clockwise direction around the stator.

An idea of the polarity and intensity of the magnetic field is perhaps more readily seen in figure 87. The three vectors I, II and

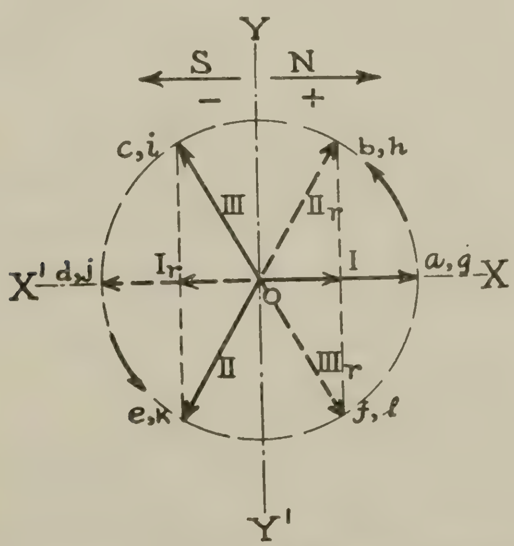


Fig. 87.—Rotating Sector Method of Showing Change in Polarity and Field Strength of Poles of Motor, Fig. 85.

III, spaced 120°, represent the three phase currents, while I_r, II_r and III_r represent the three respective opposites. The direction of rotation is counter-clockwise. The relative polarity and magnetic field intensity of any pole at any instant can be determined by the projection of the corresponding vector on the XOX' axis. The positive direction of current and N polarity are to the right of the YOY' axis, as marked. The diagram shows the relation at time $t=0$ when pole a has a maximum N polarity. Thus, poles a and g are N, b and h are N_d, c and i are S_i, etc.

The practical induction motor has the distributed form of winding. The field is therefore uniform, and revolves with uniform speed. The number of revolutions the field makes per cycle varies with the number of poles per phase. Thus, a two-pole field makes one revolution per cycle, the four-pole field one revolution in two cycles, and the six-pole field one every three cycles. The **synchronous speed** is the speed with which the rotating field revolves. Expressed in rps, and rpm, it is:

$$\text{synchronous speed} = \frac{2f}{n} \qquad \text{(rps)}$$

$$= \frac{120f}{n} \qquad \text{(rpm)}$$

where f = frequency in cycles per second,
n = number of poles per phase.

Example:

What is the synchronous speed in rpm of a three-phase, 4-poles-per-phase induction motor supplied with current at a frequency of 25 cycles?

Solution:

Formula $$\text{synchronous speed} = \frac{120f}{n}$$

substituting $$= \frac{120 \times 25}{4} = 750$$

whence synchronous speed = 750 rpm.

The rotor. The rotor carries the inductors which are imbedded in slots in the laminated core. The **squirrel cage** type of rotor has copper bars as the inductors, which are short-circuited at each end by heavy rings. The construction is simple and rugged and the resistance of the inductors is very low, which makes for high efficiency and good regulation. The starting torque, however, is low.

Another type of rotor is **phase-wound**; that is, it is wound with coils properly disposed in the slots. These windings are usually provided with adjustable resistances, which can be internal or external to the rotor. In the latter case, slip rings and brushes are employed to make the necessary connections to the windings. The purpose of the rheostat

is to increase the starting torque and to reduce the starting current. It is then cut our of circuit to give higher efficiency and better speed regulation.

The action of the rotating flux on the rotor is as follows: When the rotor is stationary, this flux induces emfs and, therefore, currents in the bars, or windings, between the points that are under polar regions of opposite polarity on the stator. A mechanical force is exerted by the flux on these conductors carrying current, and the rotor tends to turn in such a direction as to reduce the induced currents in accord with Lenz's law. This direction is the same as that of the rotating flux, and the rotor is **dragged** by it, that is, the torque produced thereby turns the motor.

A polyphase motor will start of itself, and when once started, will accelerate until its speed is nearly the same as the synchronous speed. It cannot rotate at synchronous speed, because it would cut no flux. Hence, no current would be induced and there would be no torque. The rotor would then slow down and enough torque would thereby be developed to keep up the rotation. For this reason, induction motors are frequently called **asynchronous** motors, meaning that they rotate out of step with, or lag behind, the rotating flux. The difference between the actual and synchronous speeds is called the **slip,** and is expressed in percentage of the synchronous speed. The slip increases with the load, and so also does the torque. At zero load, the slip is very small.
Example:

A three-phase, four-poles-per-phase motor operating on 25 cycles has a slip of 5 per cent. Calculate the rotor speed in rpm.
Solution:

The synchronous speed is 750 rpm, as found in the previous example. The rotor speed is 5 per cent less than 750 rpm, or

$$\text{Speed} = 0.95 \times 750 = 712.5 \text{ rpm.}$$

Single-phase induction motors. A polyphase motor operating at full speed will continue to run and deliver power if all but one of the phase windings are disconnected from the polyphase supply mains, that is, it is operating as a single-phase motor. The efficiency is, however, considerably lower, and the speed regulation is also lower than when all three phases were acting. Also, if the motor stops, it cannot be started on the single-phase supply without a special starting device to put it in motion.

A single-phase motor is quite similar in construction to the polyphase type, but has only one phase winding on the stator core. It may have as many poles as desired. The rotor may be of the squirrel cage type or the phase-wound type. When the rotor is not moving, and single-phase current is then supplied to the stator winding, the polarity of the stator varies harmonically, as was described under the polyphase motor for any one phase winding. **There is no rotation of flux when the rotor is not revolving.** This sinusoidally varying flux induces

currents in the rotor, and these currents produce N and S poles in the rotor directly under the N and S poles, respectively, of the stator. No torque can be produced between the two sets of poles, on account of their relative positions.

Now, when the rotor is turned, the conductors cut the flux from the stator. The magnitude and direction of the emfs induced in the rotor may be determined by the formulas given in the first part of this Chapter. The currents produced by these induced emfs have the proper direction to produce N and S poles in the rotor, which are midway between the stator poles, and oppose the motion that produces them. These rotor poles also produce, by magnetic induction, poles of opposite polarity directly over themselves on the surface of the stator. The main stator field, and the field produced on the stator by the rotor poles, are at right angles to each other, or **in space quadrature,** and the resultant field rotates. Explained otherwise, the rotor flux produces a cemf to balance the emf induced in it, the cemf being proportional to the time of rate change of flux. Therefore, the rotor flux reaches its maximum value 90° after the induced emf passes through maximum and, consequently, 1/4 cycle after the inducing or main stator flux. The main flux and the flux of the poles induced in the stator surface by the rotor are 90° out of phase with each other. The effect of the two is similar to that which would be produced by a two-phase alternating current. Consequently, after the rotor of an induction motor is once in motion, it is dragged around by a rotating flux in the same manner as the rotor of a polyphase motor.

When the rotor is revolving at synchronous speed, the two fluxes are equal, and a uniform (circular) magnetic field is produced. As the rotor slows down the field becomes elliptical, which flattens more and more with decrease in speed until, when the rotor stops, it becomes a straight line.

Single-phase motors must be started, because the torque is zero when the rotor is standing still. Small motors can be given sufficient momentum by hand to pick up. Larger motors use: (a) a **split-phase** starting device with or without a **starting** coil, (b) **shading coils** or (c) are started as a **repulsion** motor. These will not be considered.

CHAPTER VII. INDUCTANCE.

General. When the current flowing in a circuit is unvarying, it encounters only the resistance of the circuit, and the magnitude of such a current is determined by the resistance and applied emf in accordance with Ohm's law. In this case, the total IR drop in the circuit equals the applied emf.

Emf of self-induction. Whenever a current is flowing in a circuit, a magnetic field is produced which is proportional to the current. This magnetic flux is linked, to a greater or less extent, with the circuit in which the current producing it is flowing. When the magnetic flux linked with a circuit is varying, an induced emf results in the circuit itself, and has an instantaneous value equal to the time rate of change of the flux or

$$e=-\frac{d\Phi}{dt}$$

This is the **emf of self-induction** and, in accordance with the law of Lenz always has a direction such as to oppose the variations in current. Thus, when the current in a circuit is varying, the emf of self-induction must be considered in addition to the iR drop.

Self-inductance. The number of linkages of flux lines with any circuit for a given current is dependent upon the configuration of the circuit. Thus, if the circuit consists of s single turn of wire, the flux lines can link only once with the circuit. If, however, the current is flowing in a loop, or coil, or many turns, some of the flux due to the current flowing in one turn will thread through some or all of the other turns. The number of linkages of flux with any circuit for a given current, therefore, is dependent upon the form, shape and size of the circuit. The term self-inductance, or inductance, is used to differentiate between the varying characteristics of circuits with respect to the flux which threads them for a given current. The **self-inductance L** of a circuit is the number of linkages of flux when it is traversed by unit current. Since the flux is proportional to the current, the flux for any current I is equal to the flux for unit current multiplied by the current, or

$$\Phi=LI$$

As stated above, the emf of self-induction is given by the time rate of change of Φ, or $\frac{d\Phi}{dt}$. Hence, the emf of self-induction is also given by the time rate of change of Li, or

$$e=-\frac{d(Li)}{dt}$$

Usually, the configuration of a circuit does not change, but the current is the variable quantity. Then, the emf of self-induction is given by the product of the self-induction and the time rate of change of the current, or

$$e = -L\frac{di}{dt}$$

Thus, the self-inductance of a circuit can be defined in terms of the emf of self-induction; that is, the inductance is equal to emf of self-induction when the time rate of change of the current is unity. Hence, **inductance is that magnetic property of a circuit that opposes any change in the flux and, therefore, any change in either the magnitude or the direction of the current on the circuit.**

All circuits possess inductance to a greater or less extent, and are said to be **inductive.** Thus, a straight wire has inductance in **distributed form,** while the solenoid is an inductance in **concentrated form.** A complete circuit may have inductance in either or both forms but **always** has inductance, if only that of the leads connecting the apparatus. The value of the inductance depends upon many factors, such as the shape of the circuit, size and number of turns and the permeability of the medium. Air is the medium in practically every instance in radio circuits. Its permeability is 1. The value of the current flowing has no effect upon the value of the inductance except when iron is present, in which case the inductance is greatly increased and varies with changes in the current. The inductance of a given circuit, in which no iron is present, is a constant quantity provided that the current does not change so rapidly that the distribution of current in the conductors differs from that of a steady current; that is, so long as the current is uniformly distributed over the entire cross-section of the conductors.

Units of inductance. The emf of self-induction is a measure of inductance. Thus, the inductance of any circuit is the emf produced in it by unit rate of variation of the current through it. If the current changes **one emu in one second, one emu of emf** is produced and the circuit has **one emu of inductance,** or **1 cm of inductance.** This electromagnetic unit is very small and seldom used in this country except for very small inductances. The name **centimeter,** used as a unit of inductance, is not entirely satisfactory because it is also used as a unit of physical length.

The **practical unit** of inductance is the **henry, h,** and has such a value that, if the current flowing through it varies at the rate of **one ampere per second,** an emf of **one volt** is induced. Since 1 volt $= 1 \cdot 10^8$ emu and 1 ampere $= 1 \cdot 10^{-1}$ emu, then

$$1 \text{ henry (h)} = \frac{1 \cdot 10^8}{1 \cdot 10^{-1}} = 1 \cdot 10^9 \text{ emu}$$

The henry is too large a unit for most purposes connected with radio engineering, and the following sub-multiples of it are employed:

1 millihenry (mh) $=0.001$ henry $=11 \cdot 0^{-3}$ henry,

1 microhenry (μh) $=0.000001$ henry $=1 \cdot 10^{-6}$ henry.

Table 14 gives the multiplier to be used when changing from one system of units to another.

Derivation of general formula for inductance. In the case of n turns, if the number of linkages of flux with one turn is denoted by Φ_1; then, if the same flux is enclosed by each of the turns, the total number of linkages will be

$$\Phi = n\Phi_1$$

This is approximately the case for a long solenoid, a concentrated winding, or when the magnetic path is entirely through iron.

Hence,
$$LI = n\Phi_1$$
and
$$L = \frac{n\Phi_1}{I}$$

The formula for the flux produced by a current flowing in the winding of an electromagnet given in Chapter V is

$$\Phi_1 = \frac{1.26\ n\ I\mu S}{l}$$

This expression for the flux is the same for a long solenoid.

Substituting this value of Φ_1 in the previous equation, multiplying and changing to henries,

$$L = \frac{1.26\ n^2\ \mu S \cdot 10^{-8}}{l}$$

where
L = inductance in henries,
n = number of turns,
μ = permeability of medium, which is assumed to be constant, and is unity for air,
S = area of magnetic path in cms^2.,
l = length of magnetic path in cms., or length of solenoid.

The above formula shows that inductance is a property of the circuit as has been stated before, and that **its value varies directly as the square of the number of turns,** provided that the length l is not changed. This is true in the case of a long solenoid, or when the magnetic path is entirely through iron, and is approximately the case when the medium is air and the turns are closely wound. Certain modifications and refinements of this general formula are necessary in practical applications, especially when the current changes extremely rapidly. Methods of winding and calculating the inductance of coils are given in Chapter I, Part 7 of this Section.

Example:

Calculate the inductance in henries of the coil wound on the closed ring of Norway iron, given in figure 39, using the quantities given in that example.

Solution:

Formula $L = \dfrac{1.26\ n^2 \mu S \cdot 10^{-8}}{l}$

substituting $= \dfrac{1.26 \times (2 \cdot 10^2)^2 \times 2.06 \cdot 10^3 \times 8 \cdot 10^{-8}}{1.257 \cdot 10^2} = 0.065$

whence $L = 0.065$ henry.

Growth of current. If a circuit containing inductance, resistance and a source of steady emf is open, the current will be zero. When the circuit is closed, the current will not change abruptly from zero to its maximum steady value, but will vary **continuously** from zero to a final steady value, which is determined solely by the resistance of the circuit. It is the inductance of the circuit which prevents the current from changing abruptly. The emf of self-induction at any instant balances the excess of the impressed emf E over the iR drop. Thus, at the instant that the circuit is closed, the current is equal to zero and, therefore, the iR drop is zero. The growth of the current at this instant is such that the emf of self-induction, or the **inductive drop** is equal and opposite to the impressed emf. As the current increases, the iR drop also increases, while the inductive drop and the rate of growth of the current decreases. Finally, the iR drop becomes equal to the impressed emf, the inductive drop becomes zero and the current no longer increases. The fact that the inductive drop at any instant balances the difference between the impressed emf and the iR drop is expressed by the equation

Inductive emf = impressed emf − resistance drop

or

$$L\frac{di}{dt} = E - iR$$

The mathematical solution of this equation determines the value of the current at any instant after the circuit is closed. The solution is

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

where
i = instantaneous current in amperes,
E = impressed emf in volts,
R = resistance in ohms,
L = inductance in henries,
t = time in seconds,
$\epsilon = 2.7128$.

When the circuit is closed, $t = 0$. Then $\epsilon^{-\frac{R}{L}t} = \epsilon^0 = 1$, and $i = 0$. As t increases, the value of the exponential term decreases and finally becomes zero, when infinite time has elapsed. The current then reaches its final value

$$I_0 = \frac{E}{R}$$

Although, theoretically, it takes infinite time for the current to reach its final value, in the majority of cases the current for all practical purposes reaches its final value in a very samll fraction of a second.

Example:

An inductance coil has an inductance of 1 henry. Its resistance is 500 ohms. Calculate the percentage of the final value the current attains in 0.01 second.

Solution:

For this purpose it is necessary to use only the expression in the parentheses

$$\left(1-\epsilon^{-\frac{R}{L}t}\right)$$

The exponential term is first solved by substituting the values of R in ohms, L in henries and t in seconds, and using Table 12.

Substituting

$$\epsilon^{-\frac{5\,.\,10^2}{1}\cdot 1\cdot 10.^2} = \epsilon^{-5} = 0.00674$$

whence $$\left(1-\epsilon^{-\frac{R}{L}t}\right) = 0.993 \text{ or } 99.3 \text{ per cent.}$$

Time constant. At a time $t=\dfrac{L}{R}$

$$\epsilon^{-\frac{R}{L}t} = \epsilon^{-\frac{R}{L}\cdot\frac{L}{R}} = \epsilon^{-1}$$

Substituting this value of the exponential term in the previous formula

$$i=\frac{E}{R}\left(1-\frac{1}{\epsilon}\right)$$

and solving the parentheses

$$i=\frac{E}{R}\left(1-0.368\right)$$

whence $$i=0.632\frac{E}{R}$$

This shows that the current reaches 63.2 per cent of its final value, $\frac{E}{R}$, in a time $t=\frac{L}{R}$. This ratio $\frac{L}{R}$ is called the **time constant** of a circuit containing inductance and resistance, and is used as a measure of the rapidity of the growth of current in different circuits. The time required for the current to reach 63.2 per cent of its final value is not dependent upon the size of either the inductance or the resistance, but solely upon their **ratio.** Figure 88 shows the growth of current in the circuit having the constants given in the above example, the time constant being 0.002 second. The growth is logarithmic, and was calculated by means of Table 12.

One of the practical applications of the time constant is in the design of high-speed relays for radiotelegraphy. Thus, if the relay is to be operated at a speed of so many words per minute and on a given impressed emf, the time constant $\frac{L}{R}$ must be made very much less than the time required for a dot, because otherwise the current will not reach a value sufficiently high to operate the relay on the dots. In this connection, the action of a sluggish relay can be speeded up by inserting resistance in series with it and increasing the emf applied to the circuit.

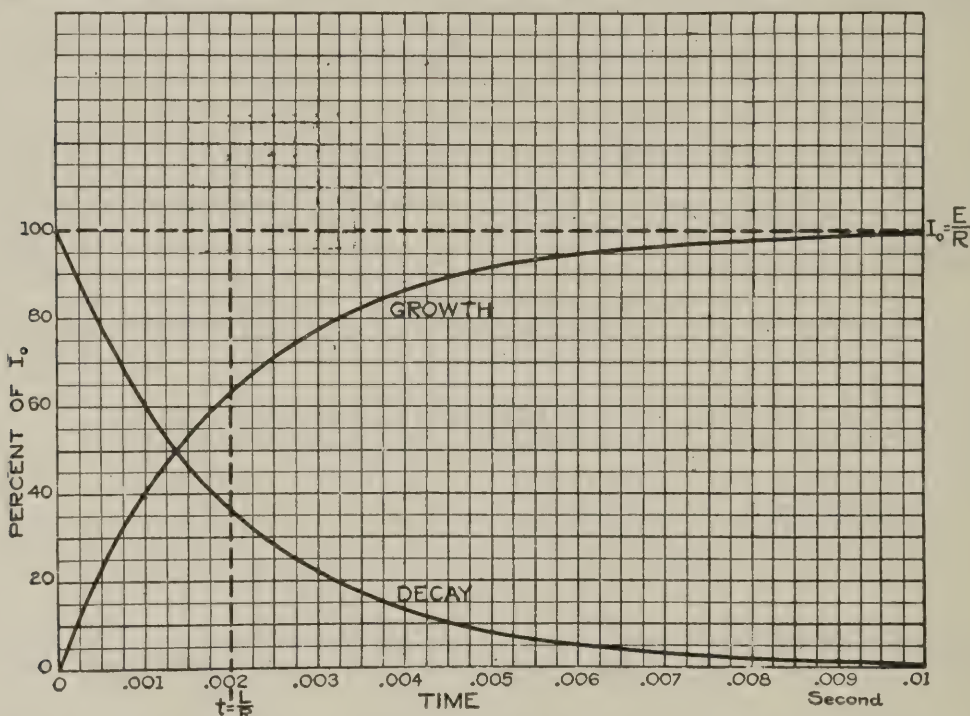


Fig. 88.—Growth and Decay of Current in a Circuit Containing only Inductance and Resistance.

Energy stored in the magnetic field. The power required to maintain a given current through a given resistance is

$$P = I^2R$$

It has just been shown that when a circuit containing inductance, resistance and a source of steady emf is closed, the IR drop increases from zero until it equals the impressed emf E, while the inductive drop $-L\frac{di}{d}$ decreases from E to zero, their sum being equal to E at every instant. The power Ei required by such a circuit at any instant is, therefore, equal to the power i^2R lost in the resistance plus that used in overcoming the inductive drop. The latter has an instantaneous value

$$p = i\left(L\frac{di}{dt}\right)$$

Thus

$$Ei = i\left(L\frac{di}{dt}\right) + i^2R$$

During the time that the current is attaining its steady value, the power Ei supplied by the source is, therefore, larger than the power required to overcome the i^2R lost in the total resistance of the circuit. This excess of power is stored in the magnetic field of the circuit, the rate of storage being greatest at the instant the switch is closed and then growing less and less until, when the current becomes steady, the energy stored in the magnetic field has also attained its final value. It can be shown that the amount of energy stored in the magnetic field about the circuit at any instant, when the current has a value i, is

$$w = \frac{1}{2} L i^2 \qquad \text{(joules)}$$

where L = inductance in henries,
i = current in amperes.

When the current has attained its final value I_0, then

$$W = \frac{1}{2} L I_0^2$$

At this time, the emf of self-induction is zero. The power now supplied to the circuit is sufficient to compensate for the I^2R losses, no more being required to maintain the magnetic field that has been produced. Were the current now increased, an emf of self-induction would be produced and more energy would be stored in the magnetic field. This process would continue until the current again became steady. A greater power would now be required to maintain the flow of current, but none of it would be expended in maintaining the magnetic field at its new value.

Decay of current. Suppose that the emf applied to the circuit be reduced instantaneously to zero. Since the sum of the inductive drop and the iR drop at every instant is equal to the applied emf, which in this case is zero, then

$$L\frac{di}{dt} + iR = 0$$

The current drops from its steady value $I_0 \times \frac{E}{R}$ to zero and, in so doing, an emf of self-induction is produced in the circuit by the varying current. The direction of this emf can be found by the law of Lenz, and is the same as that in which the applied emf was acting just before it was removed. The variation in the current, or **decay** of the current, is again dependent upon the time constant of the circuit. It can be shown that the value of the current at any instant is

$$i = \frac{E}{R} \epsilon^{-\frac{R}{L}t}$$

Example:

Using the values of inductance and resistance given in the example showing growth of current, calculate what percentage of the maximum current will be flowing 0.003 second after the emf has been removed.

Solution:

For this purpose it is necessary to use only the exponential term of the equation.

$$\epsilon^{-\frac{R}{L}t}$$

substituting $$\epsilon^{-\frac{5.10^2}{1}\cdot 3.10^{-3}} = \epsilon^{-1.5} = 0.223$$

whence $$\epsilon^{-\frac{R}{L}t} \times 22.3 \text{ per cent}$$

Figure 88 shows the logarithmic decay of current in this circuit.

During the time that the current is dropping to zero, the energy stored in the magnetic field is being returned to the circuit. The amount of this energy was previously said to be

$$W = \frac{1}{2} L I_0^2 \qquad \text{(joules)}$$

The rate at which this energy is returned to the circuit depends upon the time constant of the circuit. If the time constant is small, this rate will be high, and the emf of self-induction will fall rapidly to zero.

The above discussion has been concerned with the case where the emf was suddenly reduced to zero. Now, consider the case where the emf is kept in the circuit, but the resistance of the circuit is suddenly increased. Such a resistance increase would tend to produce an instantaneous reduction in the current. However, the emf of self-induction will prevent such a change, and will act in the direction of the applied emf; in fact, at the instant that the resistance is increased, the emf of self-induction will be such as to maintain the current at its former value. Thus, if the circuit resistance were instantly increased a hundredfold, the emf of self-induction would be ninety-nine times the applied emf; and the two emfs acting together would keep the current from changing instantly, although it would immediately start falling to a new steady value determined by the new value of the circuit resistance. Thus, sudden changes in the circuit resistance can bring into play very high values of self-induced emfs.

No circuit carrying a current can be opened instantly. If this were possible, the circuit resistance would instantly become infinite, and the emf of self-induction would be infinite. This emf would jump any gap in the circuit. The spark thus formed would make the resistance of the circuit finite. Thus, there would be a path for the induced current which would continue to flow until the energy stored in the magnetic field has been transformed into other forms, such as heat and light energy.

The value to which this self-induced emf, called the **inductive kick,** can rise is enormous, amounting to thousands of volts in some instances. This inductive kick may or may not be useful, depending upon the apparatus in the circuit. For example, the inductance of the field coils of a large generator has a value of several henries. If the field switch is opened while the field is excited, a heavy spark or arc will occur at the switch points, and the emf of self-induction may be great enough to puncture the insulation of the field coils, thereby putting the generator out of commission. In order to prevent this, a special switch having a resistance of suitable size is provided which cuts the resistance into circuit on the first movement of the switch and later opens the circuit. It is on account of this inductive kick that a Wheatstone bridge is provided with both a battery and a galvanometer switch. In measuring the dc resistance of a large inductance, it is advisable first to close the battery switch and, after allowing a sufficient length of time for the current to become steady, then to close the galvanometer switch. If for any reason it is necessary to open the galvanometer circuit, it should be done before opening the battery switch; thus, the galvanometer is not connected into circuit when the inductive kick occurs.

The choke coil. On the other hand, the energy stored in the magnetic field can be usefully employed. The **choke coil** is an example. When it is desired to protect apparatus from sudden surges of current and to keep the current within reasonable limits, or to smooth out fluctuations in current due to sudden changes in the emf impressed in the circuit, a coil having a large inductance is connected in series with the circuit, generally close to the source of emf. Its action is to prevent any sudden change in the rate of current flow by virtue of its inductance. When a sudden increase of current tends to occur, the inductance of the choke coil opposes the change and more energy is stored in the magnetic field. If the current tends to drop, the change is again opposed and some of the stored energy is returned to the circuit. In this manner, a more even flow of current is produced.

The choke coil is widely employed in dc supply mains to arc transmitters, in **filter** circuits, etc. When used in conjunction with ac circuits, it is termed a **reactance coil** and may or may not be provided with an iron core, depending upon the requirements. It is used to limit the current flow by preventing sudden rushes of current during short-circuits, and also to reduce the current that can be taken from the line. The flow of alternating current is thus regulated, but in an economical manner, because no power is lost in extra resistance as is the case when a resistance is used to regulate the flow of direct current. The reactance coil is also used in rf circuits, where it serves the same purpose as in low-frequency circuits and, in addition, is frequently employed to prevent the flow of currents of undesired frequencies in the circuit.

The make and break spark coil. Another and very well-known piece of apparatus, which operates on the principle of self-induced emf, is the **make and break spark** coil used for ignition. It consists of a large number of turns of insulated wire wound on an iron core and supplied with direct current on the **make** of a wiping contact. The spark occurs at the breaking of the contact, which is done suddenly in order that a high emf may be produced. This is a case where much of the energy stored in the magnetic field is suddenly transformed into heat energy in the spark, which ignites the explosive mixture in the cylinder of the internal combustion engine.

The buzzer. The buzzer used to excite wavemeters in radio measurements work is another example of the useful employment of the energy stored in the magnetic field. Figure 89 shows a type of circuit frequently used in buzzer excited wavemeters. The source of emf is the battery B. The action is as follows:

When the key K is closed, the current from the battery flows in the circuit $BRLvL_1B$. A magnetic field is set up about the inductance

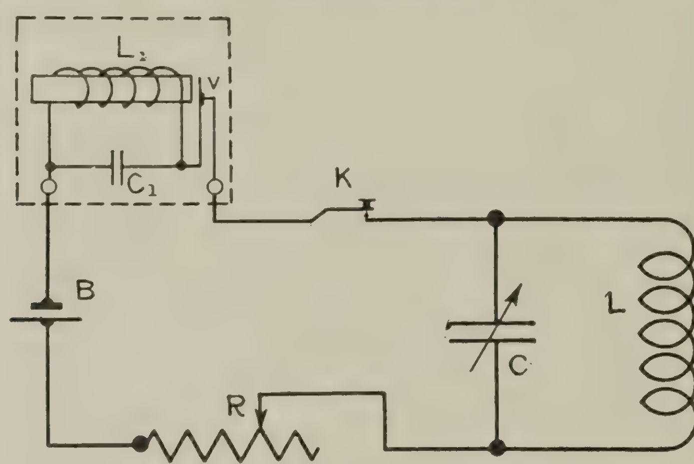


FIG. 89.—The Buzzer Excited Wavemeter.

L of the wavemeter LC during the time that the vibrator is closed. When the pull of the buzzer magnet becomes sufficiently strong, the diaphragm of the vibrator is drawn toward the magnet and opens the circuit. The energy stored in the magnetic field of the inductance L returns to the circuit. Since the circuit is open at v, this energy in the form of self-induced current flows into the condenser C and charges it. The condenser then discharges through L in a series of oscillations in the wavemeter circuit LC. This oscillating current is dealt with later.

Mutual inductance. It has just been shown that, whenever the current flowing through a circuit is varying, the flux produced by it and linked with the circuit also varies and produces an emf of self-induction in the circuit. Now, a part of this varying flux may also link with another circuit and induce an emf in it, as was explained in Chapter V. The number of linkages of flux that will occur is dependent upon the relative positions in space, the shape, size, number of turns of the

two circuits, 1 and 2, figure 90, as well as the permeability of the medium in which they lie; that is, upon the arrangement of the two circuits, relatively to each other, will depend the number of linkages of flux lines, or the flux that is **common to both circuits.** The amount of the flux which is common to both circuits per unit of current flowing in the inducing circuit is the **mutual inductance** M of the two circuits.

The definition of mutual inductance is similar to that of self-inductance. Thus, if the inducing current is changing at the rate of one emu in one second, the mutual inductance will be unity, if one emu of emf is induced in the second circuit.

Mutual inductance and self-inductance are measured in the same units. Thus, **if the current in the inducing circuit changes at the rate of one ampere in one second, the mutual inductance of the two circuits is one henry, if an emf of one volt is induced in the second circuit.**

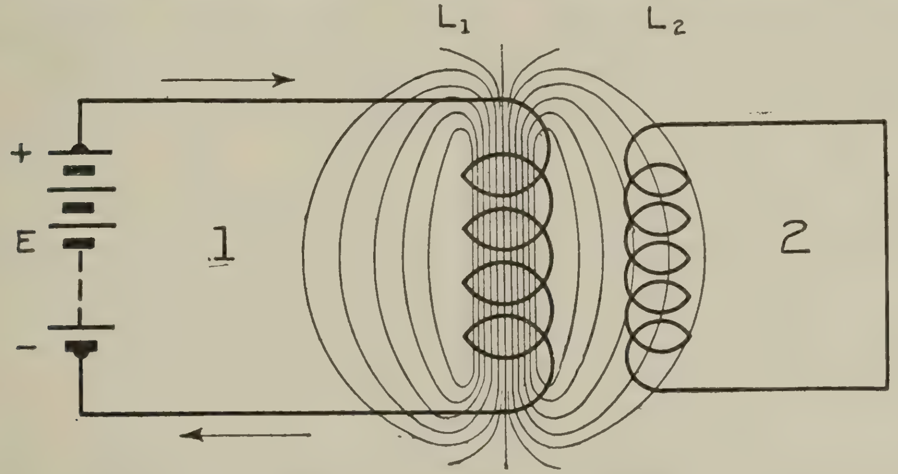


FIG. 90.—Linkage of Flux between Adjacent Circuits.

The flux Ω_{12} common to both circuits for any current I_1 flowing in circuit 1 is

$$\Phi_{12} \times MI_1$$

and

$$M_1 = \frac{\Phi_{12}}{I_1}$$

The emf of mutual induction in circuit 2 is given by the time rate of change Mi_1, or

$$e_2 = -\frac{d(Mi_1)}{dt}$$

If M is constant, that is, if no changes are made in the circuits, and if the permeability of the medium does not change, then the emf of mutual induction is given by the product of the mutual inductance and the time rate of change of current, or

$$e_2 = -M\frac{di_1}{dt}$$

The method of determining the direction of this emf of mutual induction, as well as its magnitude, has already been given. The only point to remember is that the emf of mutual induction is generated in

one circuit by a varying current in another circuit, while the emf of self-induction is generated in the same circuit in which the varying current is flowing.

It can also be shown that **for any given arrangement of two circuits, the mutual inductance has the same value whether the inducing current is flowing in one circuit or the other**; that is, if unit current is flowing in circuit 2, the flux common to both circuits is the same as if unit current were assumed to be flowing in circuit 1.

It it is assumed that there is no leakage, so that all the flux lines thread both circuits, and there are n_1 turns in circuit 1 and n_2 turns in circuit 2, then, if Φ_{12} or Φ_{21} represent that part of the flux from one turn of either circuit which passes through each turn of the other circuit,

$$M = \frac{n_1 n_2 \Phi_{12}}{I_1} . 10^{-8} \qquad \text{(henry)}$$

On the other hand, if Φ_{12} or Φ_{21} represents the total flux of either circuit threading each turn of the other circuit, then

$$M \times \frac{n_1 \Phi_{12}}{I_1} . 10^{-8} \qquad \text{(henry)}$$

The usual case is where the two circuits consist primarily of **concentrated inductances** L_1 and L_2, as in figure 90. Since the self-inductance of a coil is proportional to the square of the number of turns, the mutual inductance of two inductances will therefore vary as the square root of the product of the two inductances, or

$$M \propto \sqrt{L_1 L_2}$$

The **theoretical maximum value of mutual inductance** is given by

$$M = \sqrt{L_1 L_2}$$

This represents the condition of no leakage, that is, where all the flux due to the current flowing in either coil is linked with every turn in the other coil. In this case, the mutual inductance of two equal inductances would be equal to the self-inductance of either. In practice, the maximum value of mutual inductance is usually considerably below this value, due to leakage.

Coefficient of coupling. Two inductances are said to be **coupled** when their mutual inductance has a value greater than zero. Since the coupling is effected by the electromagnetic field, this method of coupling is called **electromagnetic** in contradistinction to electric coupling, which is effected by means of the electric field.

The type of electromagnetic coupling may be **direct** or **indirect.** If the two circuits have a part in common, that is, if some of the inductance of one circuit is also included in the other circuit, as in figure 91 (a), the coupling is called **direct inductive,** or simply **direct.** It is also known as the **conductive** type of coupling and also is termed an **auto-transformer,** and is frequently employed in radio circuits. The mutual inductance of such an arrangement equals the amount of the

total inductance common to both circuits, as shown in the figure. It has the disadvantage that the adjustment of the mutual inductance is not very flexible.

The type of coupling shown in figure 91(*b*) and (*c*) is called the **indirect inductive coupling,** or simply **inductive coupling.** This is the type usually found in radio receiving circuits and in most transmitting circuits. The mutual inductance of this arrangement is readily adjusted to the required amount by changing the relative positions of the coupling coils.

The coupling part of two coupled circuits is frequently called the **coupling transformer.** The circuit in which the inducing current is flowing is called the **primary**, while the circuit in which current is is induced is called the **secondary.**

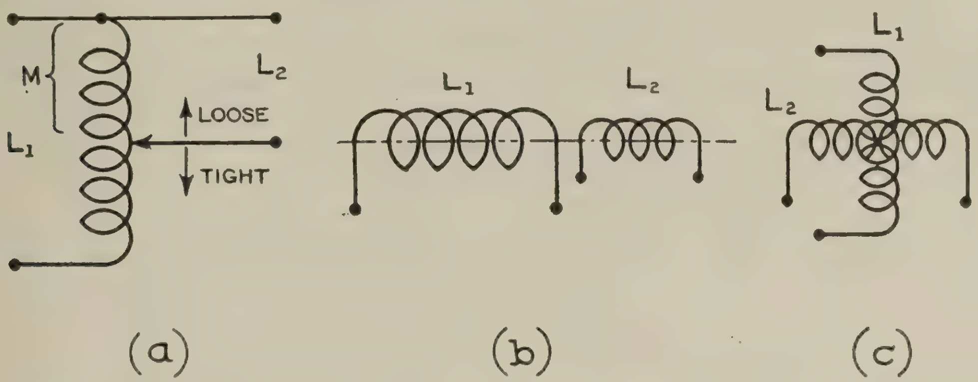


FIG. 91.—Methods of Coupling Circuits Inductively.

The coupling can be **loose** or **tight,** depending upon whether the resulting mutual inductance is, respectively, a small or large amount of the maximum possible. If the coupling is loose, the reaction between circuits is small, while with tight coupling, changes in current in one circuit will produce considerable effects in the other circuit. The **degree, or coefficient, of coupling** k is expressed as a ratio of the actual mutual inductance to the theoretical maximum value. Thus

$$k=\frac{M}{\sqrt{L_2L_1}} \qquad \text{(per cent)}$$

where M, L_1 and L_2 are expressed in the same units—henries, millihenries, etc., and k is in percent. It is evident that k is always less than 100 percent in practical apparatus.

Example:

Given: $L_1=150\mu h$; $L_2=275\mu h$; $M=160\mu h$. Calculate k.

Solution:

Formula $\quad k=\frac{M}{\sqrt{L_1L_2}}$

substituting $\quad =\frac{1.6\cdot10^2}{\sqrt{1.5\cdot10^2\times2.75\cdot10^2}}=\frac{1.6\cdot10^2}{2.03\cdot10^2}=0.788$

whence $\quad k=79$ per cent.

In this case, the theoretical maximum value of mutual inductance is 203 μh. The mutual inductance of two given coils falls off rapidly as they are separated.

Different methods of coupling air-core inductances inductively are employed, depending upon the space available. The two coils to be coupled are frequently mounted with their axes in the same straight line, and are arranged so that one coil may be moved. If one of these coils has an outside diameter slightly smaller than the inside diameter of the other, it may be moved inside the other to give a maximum mutual inductance. This method of coupling is frequently employed in radio circuits, and is shown in figure 91 (*b*). It gives the highest value of mutual inductance obtainable with a given physical separation, but has the disadvantage of requiring a great deal of space to obtain a low value of mutual inductance.

Another type of inductive coupling frequently employed is shown in figure 91 (*c*). The movable coil is arranged to rotate inside the fixed coil, either at one end or in the center. Minimum mutual inductance is obtained when the coils are at right angles.

In the case of two coupled circuits, where all the inductance of one circuit is not coupled to all that is in the other circuit, the formula

$$k=\frac{M}{\sqrt{L_1L_2}}$$

is still applicable, but the quantities are as follow:

M = mutual inductance between coupled inductances alone,
L_1 = total inductance in one circuit,
L_2 = total inductance in other circuit.

It is also assumed that there is no coupling between the two circuits other than through the coupler itself.

Example:

Using the value of L_1, L_2 and M of the previous example for the coupler, calculate k when an inductance of 200μh is added (a) in series with L_1, (b) in series with L_1 and with L_2.

Solution:

Formula $k=\dfrac{M}{\sqrt{L_1L_2}}$

(a)

substituting $=\dfrac{1.6\cdot 10^2}{\sqrt{3.5\cdot 10^2\times 2.75\cdot 10^2}}=\dfrac{1.6\cdot 10^2}{3.1\cdot 10^2}=0.516$

whence $k=51.6$ per cent.

(b)

substituting $=\dfrac{1.6\cdot 10^2}{\sqrt{3.5\cdot 10^2\times 4.75\cdot 10^2}}=\dfrac{1.6\cdot 10^2}{4.07\cdot 10^2}\times 0.393$

whence $k=39.3$ per cent.

The foregoing example shows that, with a given mutual inductance, the coefficient of coupling is reduced by the addition of uncoupled inductances to the coupled circuits.

Effect of mutual inductance on the self-inductance of a circuit. It has just been shown that mutual inductance exists between neighboring coils. Its effect on the inductance of a circuit will now be considered. The simplest case is that of two wires wound closely together, figure 92, each forming a coil, the two coils having inductances L_1 and L_2, respectively. Connect their far ends together, and their near ends to a source of varying current. The same current, therefore, flows through each coil, but in **opposite** directions. If it is assumed that the coefficient of couplings is 100 per cent, then all the flux due to one coil threads all of the other coil; and since the same current is flowing in opposite directions in the two coils, the magnetic effect of the current in each is reduced, and the total **effective inductance** L of the circuit is **reduced.** The effective inductance of each coil has been reduced by the amount of the mutual

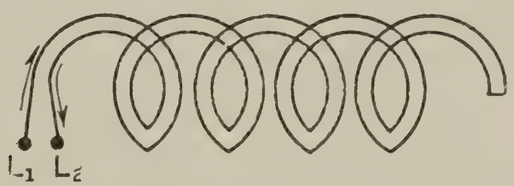


FIG. 92.—Method of Winding a Noninductive Resistance.

inductance so that the inductance of L_1 is now (L_1-M) and that of L_2 is (L_2-M) or

$$L=L_1+L_2-2M$$

and since $L_1=L_2=M$ by the method of winding, the total effective inductance of L_1 and L_2 in combination is zero, although each coil taken separately may have a very large inductance. This type of winding is called **noninductive,** and is widely used in electrical apparatus when resistance without inductance is desired.

The variometer. The **variometer** is a piece of apparatus so arranged that an increasing or decreasing amount of inductance can be inserted in a circuit in a continuously variable manner. As it is now usually constructed, the variometer consists of two inductances, one fixed and the other arranged on a pivot so that it can be rotated inside the fixed inductance. The two inductances are connected in series, and a pair of binding posts serve to connect the variometer into the circuit. The variation in the total inductance is obtained by varying the amount of the mutual inductance as well as its sign. The underlying principle of the variometer is shown in the following.

Figure 93 shows the essential parts of a variometer. Let L_1 be the fixed and L_2 the movable inductance. For the sake of clearness, the two coils are shown side by side. Let $L_1=L_2=600$ μh when measured separately. On connecting them in series and placing them at right angles and far apart, thus reducing the mutual effect, the measured

inductance was 1,193 μh. This shows that **the inductance of two coils in series is practically the sum of the separate inductances.** The two coils were then moved closer together and connected in series in such a way that the varying current would flow through them in the same direction, that is, into A through L_1 out at B into L_2 at C and out at D, figure 93(a). The effect of the mutual inductance is **additive.** Upon measurement the total inductance was

$$L_x = L_1 + L_2 + 2M = 1{,}285\ \mu\text{h} \qquad \text{(additive)}$$

The connections to L_2 were then reversed so that the current flowed into L_1 at A and out at B, then into L_2 at D and out at C, its direction

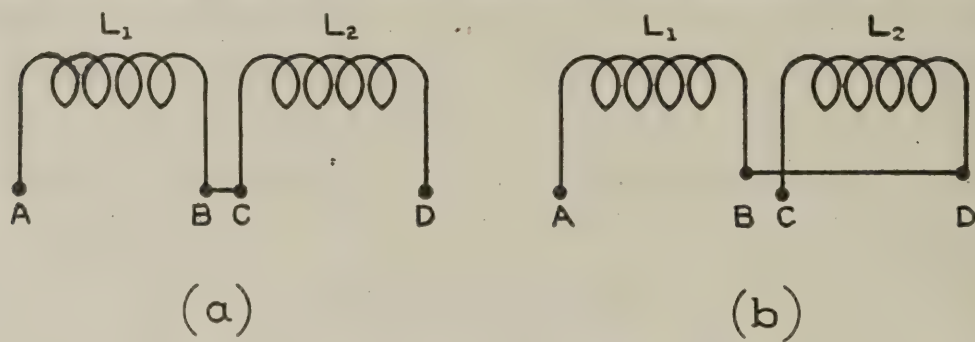


FIG. 93.—Principle of the Variometer.

through L_2 thus being reversed, figure 93(b). The effect of the mutual inductance was thereby made **subtractive,** and the total inductance

$$L_y = L_1 + L_2 - 2M = 1{,}115\ \mu\text{h} \qquad \text{(subtractive)}$$

Adding the two equations

$$L_x + L_y = 2L_1 + 2L_2 = 2{,}400\ \mu\text{h}$$

whence
$$L_1 + L_2 = \frac{L_x + L_y}{2} = 1{,}200\ \mu\text{h}$$

This value checks the one previously obtained for $L_1 + L_2$. The mutual inductance was found by subtracting the first two equations.

$$L_x - L_y = 4M = 170\ \mu\text{h}$$

whence
$$M = \frac{L_x - L_y}{4} = 42.5\ \mu\text{h}$$

This is the mutual inductance for this relative position of the two coils. The coefficient of coupling was then found by the formula

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

substituting
$$= \frac{42.5}{\sqrt{6 : 10^2 \times 6 \cdot 10^2}} = \frac{42.5}{600} = 0.071$$

whence $\quad k = 7.1$ per cent

The coupling employed in this example was very **loose.**

Applying this principle to the rotating type of variometer, when the planes of the two coils coincide, zero setting of the dial, they are tightly coupled but have subtractive inductance. The total inductance is therefore a minimum. As the movable coil is rotated, the coupling is reduced and also the mutual inductance until, at 90°, the total

inductance equals the sum of the two inductances without mutual inductance. From this point on to 180°, the mutual inductance increases in value and is additive, the maximum value being obtained at 180°, when the planes of the two coils again coincide. The theoretical limits of the self-inductance of such a variometer are:

$$L_x = L_1 + L_2 - 2M = 0$$

and

$$L_y = L_1 + L_2 + 2M = L_1 + L_2 + 2\sqrt{L_1 L_2}$$

In practice, with two spherically wound coils, one rotating inside the other and very close together, the ratio $\frac{L_y}{L_x}$ can be as great as 12.

Effect of nearby coils on the self-inductance of a circuit. The manner in which the self-inductance of a circuit is affected by mutual inductance has just been explained. Now, as is frequently the case, if a short-circuited coil, or other closed circuit, is in the immediate vicinity of a circuit in which a varying current is flowing, and there is some mutual induction between the two, the effective inductance of the inducing circuit will be decreased, the amount of the decrease depending upon the degree of coupling, the frequency and the opposition (impedance) offered to the flow of the induced current.

This effect is usefully employed in signaling by the **compensating wave system with arc transmitters.** A loop of one or more turns, coupled inductively to the loading inductance, is alternately short-circuited and open-circuited by the signaling key, and the inductance in the antenna circuit is thereby changed from its original value when the loop is open to a lower value when the loop is closed. This results in a difference in frequency when the key is open and when it is closed. This difference in frequency permits signals to be received.

On the other hand, the effect of an adjacent closed coil, or closed circuit, may result in a serious loss of power as it will not only decrease the inductance, but will also increase the resistance of the inducing circuit very materially and thereby decrease the efficiency of the circuit. Conducting bodies in the field of a coil will also seriously affect the inductance and the resistance of the coil. In this case, eddy current losses will occur which must be supplied from the inducing circuit. Binding posts, metal spindles, washers, metal indicating dials, etc., included in the field of the coil will have eddy currents induced in them. These losses are very pronounced when the current is varying rapidly. as in radio circuits.

Transfer of power by electromagnetic coupling. Use is made of the principle of mutual induction to transfer power from one circuit to another. The voltage of the inducing, or **primary,** circuit may be stepped up or down as desired. The commercial transformer, the oscillation transformer used in radio transmitters and receivers and the induction coil operate on this principle. Discussions of these appear later in the Manual, as they are all, strictly speaking, arc transformers.

CHAPTER VIII. CAPACITY.

A discussion of displacement currents has already been given (Part I, Chapter II). It was shown that when an insulator or dielectric is subjected to an electric force, a displacement of the electrons occurs in the substance which is resisted by an equal electric stress, or tension, in the substance, and which varies with every variation in the electric force. As long as the electric force is increasing, a displacement current will flow, and when the electric force becomes steady, the flow of current ceases. If the electric force is allowed to diminish, the electric stress also diminishes, and a displacement current flows in the opposite direction.

The condenser. If one metal plate is connected to the positive terminal of a dry cell, and another plate to the negative terminal, each plate will acquire the same polarity and potential by a transfer

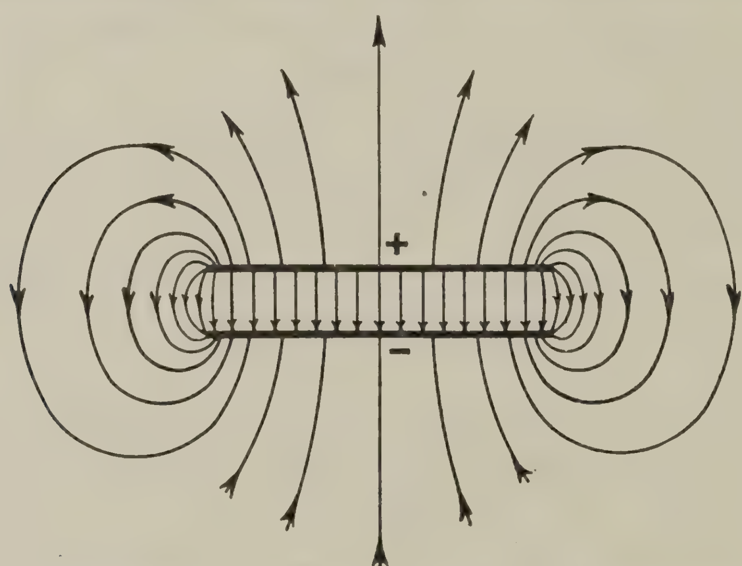


FIG. 49 —Electrostatic Field in the Dielectric of a Two-plate Condenser, showing the Direction of the Electric Displacement.

of charges as that of the pole of the dry cell to which it is connected; that is, the two plates will have the same difference of potential as the poles of the cell, and be oppositely charged. Now, if these two oppositely charged plates are brought close together, a field of electric force will be produced in the dielectric between them, and a displacement of electrons will occur therein, the direction in which the electrons are displaced being from the negative to the positive plate. The electrical displacement, according to convention, is from the positive to the negative plate through the dielectric, and is thus shown in figure 94 by the arrow-heads on the lines representing the lines of electric force. While this displacement is taking place in the dielectric, there will be a

movement of electrons in the external circuit from the negative plate to the positive plate. Hence, the charges on these two plates will be increased, the negative plate acquiring a greater negative charge, and the positive plate a greater positive charge. However, **the sum total of these two charges is always zero.** When the term **charge of a condenser** is used, it refers to the charge, or quantity of electricity, on **one** of the plates, usually the positive plate.

Assume that the difference of potential of the two plates was 1.5 volts when they were far apart. Upon bringing them close together, the positive potential of one plate is reduced by a negative potential induced in it by the negative charge on the other plate. In a similar manner, the negative potential of the other plate is reduced, and the difference of potential between the two plates is thereby made lower than that of the dry cell. As a result, a movement of charges (electrons) will occur between the cell and the plates, a conduction current will flow in the conductors and plates, and there will be a displacement current in the dielectric until the difference of potential between the plates again becomes equal to that existing between the dry cell terminals. The electric displacement in the dielectric is again equal and opposite to the electric force producing it, each of the lines of electric force representing a **line of tension** terminating on an equal and opposite charge.

If, now, the separation of the plates is reduced to one-half, they will be maintained at the same difference of potential by the cell by means of a new flow of charges, and the charges now on the plates are much greater than when the plates were far apart. The name **condenser** is given to such an arrangement of plates for the reason that a **condensing action** on the charge appears to take place; that is, for a given difference of potential between two plates of a given area, the charge that can be received by each plate is much greater when the plates are close together than when they are widely separated.

Capacity. There is a constant relation between the charge of a condenser and the difference of potential between its plates. Thus, if the charge be doubled, the difference of potential will be doubled. This constant ratio of charge Q to potential difference V is called the **capacity** $\boldsymbol{C}$ of the condenser. Thus,

$$C=\frac{Q}{V}$$

Units of capacity. If the difference of potential between the plates of the condenser is unity, the capacity is numerically equal to the charge required to cause unit difference of potential. **The capacity of a condenser may,** therefore, **be defined as the electrostatic charge which it must receive to produce unit potential difference between its plates.** Thus, a condenser has a capacity of one electrostatic unit, called the **centimeter,** when the addition of one electrostatic unit of electricity

raises its difference of potential one electrostatic unit. This unit is seldom used.

The **farad f** is the unit of capacity in the practical system, and has such a value that **one coulomb of electricity is required to raise the difference of potential of a condenser one volt.** Hence, capacity in farads is given by the number of coulombs contained in a condenser **per volt** difference of potential between the condenser plates.

$$C=\frac{Q}{V}=\frac{I}{V}t$$

Since one coulomb = one ampere-second,
where C = capacity in farads,
I = current in amperes,
V = difference of potential in volts,
Q = charge in coulombs,
t = time in seconds.

The value of one farad in esu can be found by substituting for I and V their equivalents in esu, as given in Table 14. Thus,

$$1 \text{ farad}=\frac{3\cdot 10^9}{3.33\cdot 10^{-3}}\times 1=9\cdot 10^{11} \qquad \text{(esu)}$$

The farad is much too large to use in practice. The following submultiples are, therefore, used:

1 **microfarad** (μf) = 0.000,001 f = $1\cdot 10^{-6}$ f
1 **milli-microfarad** (mμf) = 0.000,000,001 = $1\cdot 10^{-9}$ f
1 **micro-microfarad** ($\mu\mu$f) = 0.000,000,000,001 = $1\cdot 10^{-12}$ f

In radio engineering, capacity is usually expressed in decimal parts of the microfarad for all purposes except when the capacity is extremely small, when micro-microfarads are used. Thus, 0.002 μf is used instead of 2 mμf or $2\cdot 10^{-9}$ f. Table 13, giving the oscillation constant LC, is computed using C in microfarads and L in microhenries.

Interrelationship of charge, difference of potential and capacity. The interdependence of charge, condenser voltage and capacity should be thoroughly understood. The formula

$$C=\frac{Q}{V}$$

may also be written

$$Q=CV \text{ and } V=\frac{Q}{C}$$

These three equations show that

(a) the capacity equals the charge divided by the voltage,
(b) the charge equals the capacity multiplied bythe voltage,
(c) the voltage equals the charge divided by the capacity.

The first equation gives the capacity required to store a given charge at a given voltage.

Example:

What is the value of the capacity required to receive a charge of 0.002 coulomb when the available emf is 100,000 volts?

Solution:

Formula $$C=\frac{Q}{V}$$

substituting $$=\frac{2\cdot 10^{-3}}{1\cdot 10^{5}}=2\cdot 10^{-8}$$

whence $$C=2\cdot 10^{-8}\text{f}=0.02\mu\text{f}$$

The second equation shows that the charge is proportional to the capacity of condenser and the voltage to which it is charged.

Example:

A condenser of 0.002 μf capacity is charged to a voltage of 12,000 volts. Calculate the charge.

Solution:

Formula $$Q=CV$$

substituting $$=2\cdot 10^{-9}\times 1.2\cdot 10^{4}=24\cdot 10^{-5}=0.000{,}024$$

whence $$Q=0.000{,}024 \text{ coulomb.}$$

The third equation shows that, for a given charge, the voltage across the condenser is inversely proportional to the capacity.

Example:

What is the voltage of a condenser having a capacity of 0.032 μf when the charge is 0.000,384 coulomb?

Solution:

Formula $$V=\frac{Q}{C}$$

substituting $$=\frac{3.84\cdot 10^{-4}}{3.2\cdot 10^{-1}}=1.2\cdot 10^{4}=12{,}000$$

whence $$V=12{,}000 \text{ volts.}$$

Charge of condensers. When a condenser is connected to a source of steady emf, the condenser does not acquire its full charge instantaneously, but rather **accumulates** the charge. Let it be assumed that a circuit containing an uncharged condenser, a resistance and source of steady emf is open. When the circuit is closed, a momentary conduction current will flow in the circuit and a displacement current in the dielectric. During this time the electric charge is accumulating on the plates of the condenser. The charging will continue until the accumulated charge on the plates has made the condenser voltage equal to the applied emf. The only limitation put upon the value that the current can reach when the circuit is just closed is the iR drop, provided that the circuit is noninductive. Thus, if E is the applied emf, then at the first instant

$$iR=E$$

However, as the condenser accumulates its charge, its voltage acts in opposition to the applied emf so that at any instant

$$iR+\frac{q}{C}=E$$

where i = instantaneous value of charging current,
q = instantaneous value of charge on condenser.

Since the current has the same value in all parts of a series circuit, it is equal to the rate at which the charge is accumulating in the condenser, which is

$$i=\frac{dq}{dt}$$

and hence,
$$R\frac{dq}{dt}+\frac{q}{C}=E$$

The solution of this equation determines the value of charge at any instant after the circuit is closed. The solution is

$$q=CE\left(1-\epsilon^{-\frac{t}{CR}}\right)$$

where q = instantaneous charge in coulombs,
C = capacity in farads,
E = applied emf in volts,
R = total resistance of the circuit,
t = time in seconds,
ϵ = 2.7128.

When the circuit is closed, $t=0$, hence $\epsilon^{-\frac{t}{CR}}=\epsilon^0=1$ and $q=0$. The exponential term decreases as the time t increases, finally becoming zero when infinite time has elapsed, and the charge has then reached its final value

$$q=CE.$$

As in the case of growth of current in an inductive circuit, the charge reaches its final value in a very small fraction of a second.

The equation for the value of the charging current can be determined from that for the charge by using

$$i=\frac{dq}{dt}$$

Thus, the value of the charging current at any instant after the switch is closed is

$$i=\frac{E}{R}\left(\epsilon^{-\frac{t}{CR}}\right)$$

where the quantities are the same as those given above. This equation shows that, at the instant the circuit is closed, the current flow is determined solely by Ohm's law, and then decreases exponentially, reaching zero when infinite time has elapsed.

Example:

A condenser having a capacity of 0.02 μf is connected in series with a resistance of 30,000 ohms and a source of steady emf of 1,000 volts. Calculate (a) the current at the instant the switch is closed, (b) the percentage of the initial current flowing after 0.001 second and (c) the percentage of final charge accumulated after 0.001 second.

Solution:

(a) At the instant of closing the switch, $t=0$ and the exponential term is unity. Hence

$$i=\frac{E}{R}$$

substituting $$=\frac{1\cdot 10^{3}}{3\cdot 10^{4}}=0.0333$$

whence $$i=0.0333 \text{ ampere}=33.3 \text{ ma.}$$

(b) The percentage of the initial current is found by solving the exponential term

$$\left(\epsilon^{-\frac{t}{CR}}\right)$$

substituting $$\left(\epsilon^{-\frac{1\cdot 10^{-3}}{2\cdot 10^{\;8}\times 3\cdot 10^{4}}}\right)=\epsilon^{-1.67}=0.188$$

whence $$\left(\epsilon^{-\frac{t}{CR}}\right)=18.8 \text{ per cent.}$$

(c) The percentage of the final charge accumulated in 0.001 second is found by solving the exponential term

$$\left(1-\epsilon^{-\frac{t}{CR}}\right)$$

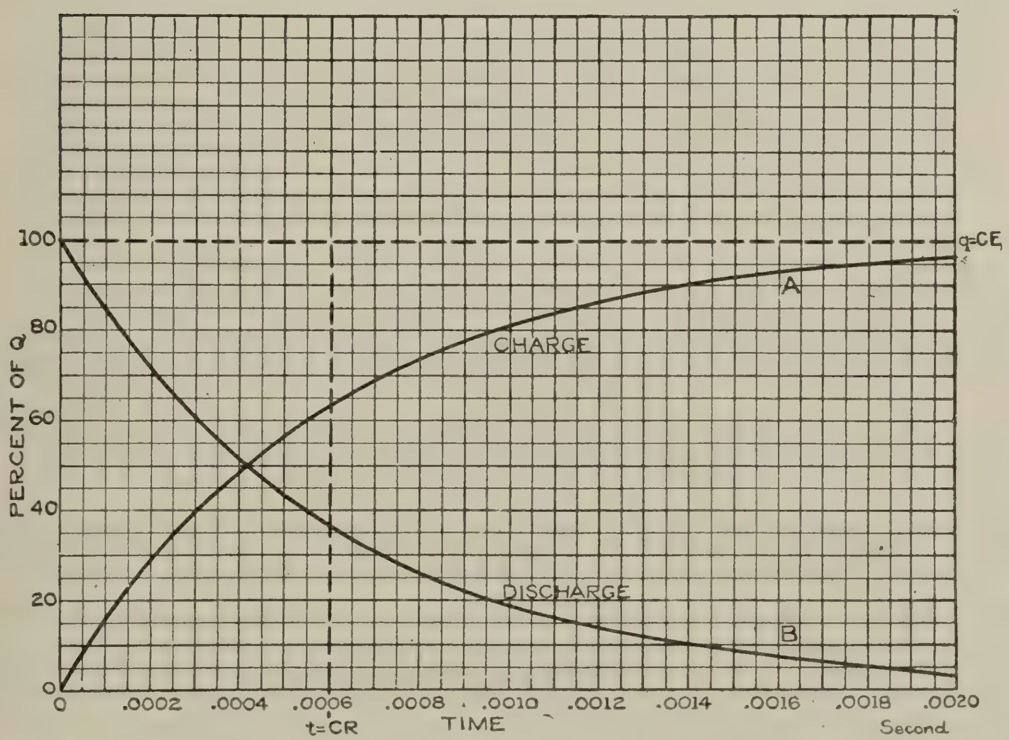


Fig. 95.—Curves Showing the Charge and Discharge of a Condenser in a Non-inductive Circuit.

substituting $(1-0.188)=0.812=81.2$ per cent.

Time constant. At a time $t=CR$

$$\epsilon^{-\frac{t}{CR}}=\epsilon^{-\frac{CR}{CR}}=\epsilon^{-1}$$

Substituting this value in the exponential terms of the formulas for the instantaneous value of the charge and the current, it is found that

$$q=0.632\ CE$$

and

$$i=0.368\ \frac{E}{R}$$

Thus, the charge reaches 63.2 per cent of its final value and the charging current drops to 36.8 per cent of its initial value in a time CR. This product is called the **time constant** of a circuit containing capacity and

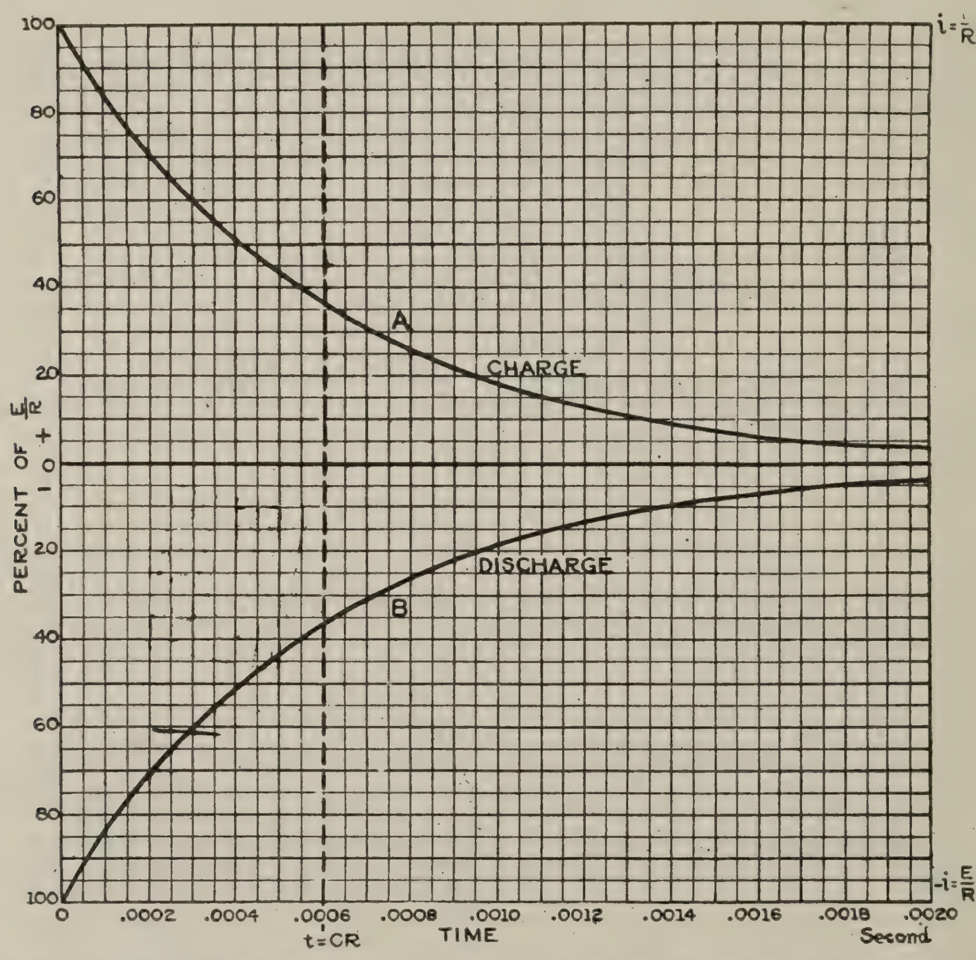


FIG. 96.—Curves Showing Current Flow During the Charge and Discharge of a Condenser in a Noninductive Circuit.

resistance, and indicates the rapidity of accumulation of charge in a condenser. Figure 95, curve A, shows the accumulation of charge in the condenser for the circuit used in the above example, the time constant being 0.0006 second. The accumulation of the charge is exponential and was calculated with the use of Table 12. Figure 96, curve A, shows the charging current for the same circuit. It is now evident that the length of time in seconds required to charge a condenser is dependent

upon the product of its capacity and the resistance in series with it, and **is independent of the value of the applied emf.**

Energy stored in the electrostatic field. Whenever charges are transported from one point to another, against an electric force, work must be done. Therefore, work must be done to charge a condenser. Now, the difference of potential, or voltage, between two points is the measure of the work done to move a unit charge from the one point to the other; in fact, the work is equal to the charge times the voltage. Thus, if a charge q is moved between two points having a $P.D.$ of v, the work w is equal to q times v. Assume that a condenser of capacity C is in an uncharged condition and is to be charged by small equal increments of charge q. The gradual increase in the difference of potential between the condenser plates and the elements of work done in moving the small charges from one plate to the other will now be shown.

At the instant that the charging emf is applied to the condenser, there is no difference of potential between the plates, so no work is done in moving in the first increment of charge. There is, however, a small potential difference after q has been transferred. Thus, after the first increment

$$v_1=\frac{q}{C} \qquad \text{and} \qquad w_1=0$$

where v_1 = the instantaneous voltage of the condenser,
w_1 = the element of work done on the first increment of charge.

After the second increment of charge

$$v_2=\frac{2q}{C} \qquad \text{and} \qquad w_2=qv_1=\frac{q^2}{C}$$

since the work done is equal to the product of the charge and the voltage.

After the third increment of charge

$$v_3=\frac{3q}{C} \qquad \text{and} \qquad w_3=qv_2=\frac{2q^2}{C}$$

It will be noticed that each succeeding element of work is greater than the one preceding it by a constant amount. This means that the transportation of the increments of each charge becomes increasingly difficult as the charging process continues. This is apparent from the charge curve in figure 96.

This process is continued until the condenser has been charged. Let it be assumed that n increments of charge are necessary to charge the condenser to its full charge Q, at which instant the potential difference

$$V=\frac{Q}{C}$$

After the nth increment has been transported

$$v_n=\frac{nq}{C}=\frac{Q}{C} \qquad \text{and} \qquad w_n=qv_{(n-1)}=\frac{(n-1)q^2}{C}$$

The total work done in transporting n increments of charge is the sum of the work done on all the individual increments of charge and **is n times the average work done on the individual increments.** The work done on each increment varies uniformly from 0 up to $\frac{(n-1)q^2}{C}$ as the condenser is charged.

The average work done on an increment of charge is

$$W_{ave} = \frac{0+(n-1)\frac{q^2}{C}}{2} = \frac{1}{2C}(n-1)q^2$$

The total work done in charging the condenser is the product of the average work for each increment times the number of increments. Hence,

$$W = nW_{ave} = \frac{1}{2C}n(n-1)q^2$$

If the increments are very small, then n is extremely large and $(n-1)$ is approximately equal to n.
Hence,

$$W = \frac{1}{2C}(nq)^2$$

But the total charge

$$Q = nq$$

Hence

$$W = \frac{Q^2}{2C}$$

This is the work done in charging the condenser. It also represents the electrical energy of the charged condenser.
Since

$$Q = CV$$

the expression can also be written

$$W = \frac{1}{2}CV^2$$

or

$$W = \frac{1}{2}QV$$

In the above equations

W = energy stored in condenser in joules,
C = capacity of condenser in farads,
Q = charge in coulombs,
V = potential difference of condenser plates in volts.

V and E can be used interchangeably whenever the condenser voltage is the same as the applied emf, as is usually the case.

The **energy is stored in the dielectric** of the condenser in the form of electric strain. At the beginning of the charge the strain is zero; and as the charging process continues, the strain increases and opposes the applied emf until no more energy can be stored unless the applied emf is increased.

Example:

Calculate the energy stored in a condenser when $C=0.02\ \mu f$ and $V=100{,}000$ volts.

Solution:

Formula $$W=\frac{1}{2}CV^2$$

substituting $$=\frac{2\cdot 10^{-8}\times(1\cdot 10^5)^2}{2}=1\cdot 10^2=100$$

whence $W=100$ joules.

Discharge of condensers. A good condenser will retain its charge for a considerable period of time after being removed from the charging circuit. Consequently, the energy stored up in the electrostatic field will be available at some later time. If the two plates of a charged condenser are connected together by a conductor, the strain in the dielectric will be relieved, and each line of electric force will shorten up by virtue of the tension which exists along it, until the two ends of the line come together and the line shrinks to nothing. During this interval the charge is transported between the plates in the conductor, a momentary displacement current occurring in the dielectric and a conduction current in the conductor. The current ceases to flow when both plates have the same potential. The condenser is said to have been **discharged,** and the stored up energy has been transformed into energy in other forms such as heat, light, etc. A perfect condenser will restore all the energy it received on charge. The work done during the discharge is, therefore, equal to the work done during the charge.

Suppose that a charged condenser is discharged through a resistance R. Then

$$R\frac{dq}{dt}+\frac{q}{C}=0$$

By solving this equation, it will be found that the value of the charge of the condenser at any instant after the discharge has begun is

$$q=CE\left(\epsilon^{-\frac{t}{CR}}\right)$$

where the quantities are the same as previously given. The time taken in discharging a condenser is determined by the time constant CR of the discharging circuit. In this case, the time constant equals the time in seconds required for the condenser to discharge 63.2 per cent of its charge. Curve B, figure 95, is the discharge curve of the condenser in the circuit used in calculating curve A, figure 95. The decrease in the charge of the condenser is exponential.

The initial value of the discharge current of a given condenser is determined by the voltage of the charge and the total resistance of the circuit in which the discharge takes place, provided that there is no inductance in the circuit. This is the assumption made in the formulas

given herewith. It is for this reason that the discharge current is shown starting from its maximum value at time $t=0$, curve B, figure 96. Since every circuit contains inductance, this assumption does not correspond exactly with the physical facts, but does show approximately what happens. Actually, the discharge current must start at zero and build up to a maximum before decreasing exponentially. The time required for the discharge current to reach its maximum is dependent upon the amount of inductance in the circuit, and can be extremely short. The effect of inductance will not be considered further at this time.

The direction of the discharge current is, of course, opposite to that of the charging current. However, its value at any corresponding instant is the same as that of the charging current, provided that the constants of the circuit are not changed. Thus, the value of the discharge current at any instant is

$$i=-\frac{E}{R}\left(\epsilon^{-\frac{t}{CR}}\right)$$

Figure 96, curve B, shows the discharge current for the condenser and circuit used in calculating the charging current.

It is evident that a given condenser can be charged slowly and then rapidly discharged in a circuit having a very low resistance and, consequently, a very small time constant. In many instances, a high rate of discharge is desirable. This is the case in **spot welding** and in the **primary oscillatory circuit of a spark transmitter.**

In spot welding, sufficient energy is stored in a condenser to perform the welding. The charging of the condenser may be done comparatively slowly. The two pieces of metal to be spot-welded are then pressed together at the welding point, and the charge of the condenser suddenly permitted to pass through the point of contact where the major part of the total resistance of the discharge circuit is located. The electrical energy stored in the condenser is almost instantly released into a circuit of extremely low resistance and inductance, with the result that the two metals are fused together by the heat developed at the point of contact. The rate of doing work is extremely high.

The primary oscillatory circuit of a 500-cycle, quenched-spark transmitter consists of a large condenser in series with a small inductance and a spark gap. When the proper adjustments have been made in the charging and discharging circuits, the condenser will discharge as soon as it has received its maximum charge, that is, the condenser will be charged and then discharged once per alternation (1,000 times per second). The duration of the charging period will always be 1/2000 second. The actual time taken for the condenser to discharge is very much less—in most instances being only a few micro-seconds. Here, again, the rate of doing work is extremely high.

Power required to charge a condenser. The element of time does not enter into expressions of energy, as was explained in Chapter IV of this Part. The same amount of energy is required to charge a condenser having a given capacity to a given voltage, whether it is done slowly or rapidly. The average power in watts required to charge a condenser can be found by dividing the total energy by the time in seconds taken to complete the charge, or

$$P=\frac{1}{2}\cdot\frac{CV^2}{t} \qquad \text{(watts)}$$

If the energy stored in a charged condenser is not allowed to return to the charging circuit, but is expended in another circuit, the result will be that power must be supplied by the charging circuit to compensate for the power lost to that circuit. In other words, a **transfer of power** occurs between the charging circuit and the discharging circuit. Hence, if the condenser is charged, and then discharged in this manner, n times per second, the power required to charge the condenser will be

$$P=\frac{1}{2}CV^2n \qquad \text{(watts)}$$

If an alternating emf of frequency f is used in charging the condenser, and the condenser is discharged once per alternation, and at the instant that the voltage equals the maximum value of the charging emf, the power equation becomes

$$P=CE_0^2f \qquad \text{(watts)}$$

where E_0 = maximum value of the alternating emf,

f = frequency in cycles per second.

It will be noted that the condenser is charged and discharged **twice per cycle.** This eliminates the $\frac{1}{2}$ from the previous equation.

This equation is used in calculating the power required to charge the bank of condensers in the primary oscillatory circuit of a 500-cycle, quenched-spark transmitter.

Example:

Calculate the power required to charge a capacity of 0.032 μf, given $E_0=17{,}000$ volts and $f=500\sim$.

Solution:

Formula $P=CE_0^2f$

substituting $=3.2\cdot 10^{-8}\times(1.7\cdot 10^4)^2\times 5\cdot 10^2=4.62\cdot 10^3$

whence $P=4{,}620$ watts $=4.62$ kw.

Dielectric Constant. The dielectric of the condensers referred to up to the present point has been assumed to be air. Now assume that a given condenser with air dielectric is charged to a given voltage, and the charge then measured. Next, suppose that some substance, such as transformer oil, is substituted for the air, and the condenser is again charged to the same voltage. It will be found that the condenser

has a greater charge. The effect is the same as if the capacity of the condenser had been increased by moving the plates closer together; that is, an additional condensing action on the charge has occurred, caused by the presence of the oil. The substitution of the oil for the air has **increased** the capacity of the condenser. If the charge accumulated, when the oil was used as the dielectric, is twice that when air was used, the condenser voltage remaining the same, the capacity has been doubled. The ratio of the capacity of the condenser with oil dielectric to that with air dielectric is called the **dielectric constant** of the oil, air being taken as the standard with a dielectric constant of 1. If some other substance, such as castor oil, is used as the dielectric, the ratio will be still higher. Thus, **the dielectric constant K of any substance may be defined as the ratio of the capacity C_x of a given condenser with this substance as the dielectric to the capacity C_a of the same condenser with only air as the dielectric.** Thus,

$$K=\frac{C_x}{C_a}$$

Example:

The measured capacity of a certain condenser with air dielectric was 0.0005μf. It was then filled with castor oil, and the capacity was increased to 0.00235μf. Calculate the dielectric constant of the castor oil.

Solution:

Formula $$K=\frac{C_x}{C_a}$$

substituting $$=\frac{0.00235}{0.0005}=4.7$$

whence $K=4.7$

The dielectric constants of various gases, liquids and solid substances frequently used in radio are given in Table 24. The values given are only approximate. An inspection of the table will show that the dielectric constant is, in most cases, given limiting values. This fact indicates that there is a wide variation in the dielectric constant. This variation is due to several causes. For example, a given material may vary in such physical properties as density, temperature coefficient, homogeneity, purity, etc. In addition, it may be hygroscopic, and the absorption of even a small amount of water will very materially increase the dielectric constant.

One of the most important factors entering into the variation of the dielectric constant of a given material is the kind of emf applied to the condenser, and also how it is applied. In the case of most dielectrics, if the charging current is supplied by a source of steady emf, the charge of the condenser and, hence, the dielectric constant of the substance will vary with the length of time that the voltage is applied. A variation in the dielectric constant is also noted when the charging

is done with alternating current, the variation occurring when the frequency of the ac supply is changed, especially at low frequencies. Consequently, the conditions under which the dielectric constant of a substance is measured should be stated in order that the results may be properly interpreted.

It is on account of this variation that condensers, the capacity of which must be accurately known at radio frequencies, should invariably be calibrated at radio frequencies by comparison with a standard condenser that shows no variation in capacity with the frequency. The air-dielectric condenser is preferred for this purpose, provided that the solid dielectric used to support the two sets of plates, and to insulate them from each other, is properly selected and located with respect to the electrostatic field and so proportioned as to introduce a negligible amount of capacity through itself from one set of plates to the other. More will be said on this subject later.

General formula for capacity. It has been shown that the capacity of a given condenser is dependent upon the separation of the plates and the dielectric constant of the medium in the electrostatic field. There is another factor which affects the capacity. This is the area of the plates. These factors are combined in the following formula for the capacity of two parallel plates close together.

$$C = \frac{KS}{4\pi\tau}$$

where C = capacity in esu,
K = dielectric constant,
S = area of one plate in cms^2. If one plate is smaller in area than the other, the smaller area should be used in calculating the capacity.
τ = thickness of dielectric between the plates in cms.
$\pi = 3.1416$.

Since $1 \text{ esu} = 1.1124 \mu\mu\text{f}$,

the above formula becomes, when C is expressed in $\mu\mu$f,

$$C = 8.85 \cdot 10^{-2} \frac{KS}{\tau} \qquad (\mu\mu\text{f})$$

where the other quantities are as given for the previous formula.

The formula shows that the capacity of a condenser can be increased in any one of the following ways:

(1) Decreasing the separation between the plates,
(2) Increasing the area of the plates,
(3) Using a dielectric having a larger dielectric constant.

Example:

Calculate the capacity in $\mu\mu$f of a condenser having two parallel plates separated 0.5 cm., the area of each plate being 150 cms^2., and the dielectric constant having a value of 2.17.

Solution:

Formula $$C=8.85\cdot 10^{-2}\frac{KS}{\tau}$$

substituting $$=8.85\cdot 10^{-2}\frac{2.17\times 1.5\cdot 10^{2}}{5\cdot 10^{-1}}=57.6$$

whence $C=57.6\ \mu\mu\text{f}.$

The formula used in the foregoing example does not take into account the curving of the lines of force at the edges of the plates. This effect is considerable even when τ is small compared with the dimensions of the plates.

The following **general formula for capacity,** which is due to Dr. L. W. Austin, was developed in order to allow for the so-called edge effect. It is

$$C=0.4\sqrt{S}+8.85\cdot 10^{-2}\frac{KS}{\tau}\qquad(\mu\mu\text{f})$$

where the quantities are as before.

The formula is the sum of the usual expression for the capacity of a disk in space

$$C=\frac{2r}{\pi}\qquad\text{(cms. and esu)}$$

and that just given for a two-plate condenser in which the edge effect was disregraded. It is rigidly true for two circular disks and any separation, within the limits of experimental error. It is approximate for other shapes of plates if the length l and breadth b are approximately equal.

When the plates are very elongated, the following formula, which includes a correction factor, should be used:

$$C=\left(0.4\sqrt{S}+8.85\cdot 10^{-2}\frac{KS}{\tau}\right)\left(1+0.015\frac{l}{b}\right)\qquad(\mu\mu\text{f})$$

Example:

Calculate the capacity of the condenser in the last example, using the general formula.

Solution:

Formula $$C=0.4\sqrt{S}+8.85\cdot 10^{-2}\frac{KS}{\tau}\qquad(\mu\mu\text{f})$$

substituting $$=0.4\sqrt{1.5\cdot 10^{2}}+8.85\cdot 10?^{2}\frac{2.17\times 1.5\cdot 10^{2}}{5\cdot 10^{-1}}$$

$$=4.9+57.6=62.5$$

whence $C=62.5\mu\mu\text{f}.$

Capacity of an antenna. It will be seen that the part of the total capacity which is calculated from the term $0.4\sqrt{S}$ remains constant for a given area of plate irrespectively of the magnitude of τ. Therefore, as τ is increased, the so-called edge effect, or **free capacity,** becomes more and more important.

Now, an antenna can be considered to be the upper plate of a two-plate condenser, the lower plate being the earth itself. The separation of the plates, or the mean height of the antenna top, is usually a large quantity. Hence, in most cases the free capacity will constitute a large, or even a major part, of the total capacity of the antenna. This is especially true when the antenna structure is a network of interconnected wires which is very high and not extremely extensive.

The value of a formula that can be used in a fairly accurate predetermination of the capacity of an antenna is apparent. The general formula can be used for this purpose and will give results that will be accurate to within about 10 percent. Adapted to this purpose, the general formula becomes

$$C=\left(4\sqrt{S}+0.89\frac{S}{h}\right)10^{-5} \qquad (\mu f)$$

where S = area of antenna top in m^2.
h = mean height of antenna in meters.

K is omitted, since air is the dielectric. The correction factor should be used when the ratio $\frac{l}{b}$ is 8 or more. The formula then becomes

$$C=\left(4\sqrt{S}+0.89\frac{S}{h}\right)\left(1+0.015\frac{l}{b}\right)10^{-5} \qquad (\mu f)$$

The capacity of the down-lead is not included in the formula, and should be estimated.

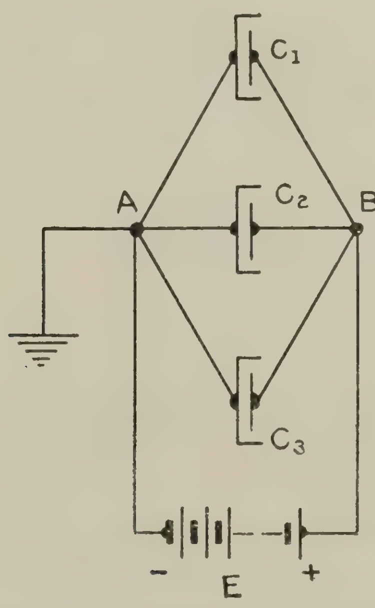


FIG. 97.—Condensers in Parallel.

Capacities in parallel. Figure 97 shows three condensers C_1, C_2 and C_3 connected in parallel between points A and B, to which is connected a source of steady emf E. The same difference of potential exists between the terminals of each condenser. Hence, if Q_1, Q_2 and Q_3 are the charges that these condensers acquire, then

$$Q_1=C_1E, \quad Q_2=C_2E \text{ and } Q_3=C_3E$$

and $$Q_1+Q_2+Q_3=(C_1+C_2+C_3)E$$
Hence

$$\text{Total capacity } C=\frac{\text{total charge}}{E}=\frac{Q_1+Q_2+Q_3}{E}$$

or $$C=C_1+C_2+C_3$$

In order that the total capacity equal the sum of the individual capacities measured separately, the condensers should be shielded and the shields connected together, as shown in the figure, in order to eliminate mutual capacities between the condensers. The shields may or may not be grounded.

Example:

Calculate the total capacity of the three following condensers connected in parallel, $C_1=0.00114\ \mu f$, $C_2=0.00218\ \mu f$, $C_3=0.00515\ \mu f$. If $E=20{,}000$ volts, what will be the voltage of each condenser?

Solution:

Formula $C=C_1+C_2+C_3$

substituting $=0.00114+0.00218+0.00515=0.00847$

whence $C=0.00847\mu f$

Since each condenser is connected across the source of emf, the voltage of each condenser will be 20,000 volts.

It is also evident that the total capacity of n condensers of equal capacity in parallel is

$$C=nC_1$$

where C_1 = capacity of any one of the condensers.

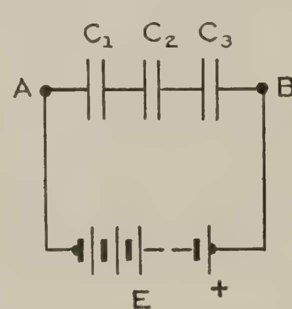


FIG. 98.—Condensers in Series.

Condensers in series. Figure 98 shows three condensers connected in series. Now, the charges on the opposite plates of a condenser are equal. When the condensers are connected to the source of emf, the right-hand plate of C_3 receives a charge $+Q$ and the left-hand plate becomes equally and oppositely charged with $-Q$. The right-hand plate of C_2, which is connected to the left-hand plate of C_3, must also have a charge $-Q$, and so on, the left-hand plate of C_1 being negative. The voltage of each condenser is equal to this charge Q divided by the capacity of the condenser, or

$$V_1=\frac{Q}{C_1},\qquad V_2=\frac{Q}{C_2}\qquad \text{and } V_3=\frac{Q}{C_3}$$

But the sum of these voltages, or the potential difference between

points A and B, is equal to E and also to $\frac{Q}{C}$, where C is the total capacity of the combination. Therefore,

$$\frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}$$

or
$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

Hence, **when condensers are connected in series, the reciprocal of the total capacity is equal to the sum of the reciprocals of the individual capacities, and the voltage across any one of the condensers is in inverse ratio to its capacity.**

Example:

Find the capacity of the following three condensers in series: $C_1=0.006\mu f$, $C_2=0.004\mu f$ and $C_3=0.002\mu f$. If $E=24{,}000$ volts, what will be the voltage of each condenser?

Solution:

Formula
$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

substituting
$$=\frac{1}{6\cdot10^{-3}}+\frac{1}{4\cdot10^{-3}}+\frac{1}{2\cdot10^{-3}}=(1.67+2.5+5.0)10^5$$
$$=9.17\cdot10^2$$

or
$$C=\frac{1}{9.17\cdot10^2}=0.001091$$

whence
$$C=0.001091\mu f.$$

It is evident that the total capacity of condensers in series is smaller than that of the smallest condenser in the combination.

The voltage of each condenser is found in the following manner.

It is evident that the voltage V of the equivalent capacity C of the combination is 24,000 volts, and that it receives a charge Q. Now,

$$Q=CV$$

Changing to farads
and substituting
$$=1.091\cdot10^{-9}\times2.4\cdot10^2=2.6184\cdot10^{-5}$$

whence
$$Q=2.6184\cdot10^{-5}\text{ coulomb.}$$

Also, since the voltage of a condenser is

$$V=\frac{Q}{C}$$

then the voltages of C_1, C_2, and C_3 are

$$V_1=\frac{2.6184\cdot10^{-5}}{6\cdot10^{-9}}=4.364\cdot10^3$$

$$V_2=\frac{2.6184\cdot10^{-5}}{4\cdot10^{-9}}=6.546\cdot10^3$$

$$V_3=\frac{2.6184\cdot10^{-5}}{2\cdot10^{-9}}=1.3092\cdot10^4$$

since each receives the charge Q.

Hence $V_1 = 4{,}364$ volts
$V_2 = 6{,}546$ volts
$V_3 = 13{,}092$ volts

Adding $V = 24{,}002$ volts.

It is also clear that the condenser having the smallest capacity receives the major part of the voltage, and therefore, is most liable to break down.

The total capacity of two unequal condensers in series is found by the following simple formula:

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

Also, the total capacity of n condensers of equal capacity in series is

$$C = \frac{C_1}{n}$$

where C_1 = capacity of any one of the condensers.

The above results obtained under the assumption of an impressed steady emf must be qualified somewhat. The distribution of voltage among the condensers will be that stated above immediately after the voltage is applied. However, all condensers show leakage and the voltage distribution will be affected by leakage; so that if one condenser has a very much higher leakage resistance than the rest, it will finally have a major fraction of the total voltage impressed upon it, and may break down. Thus, when a number of equal series condensers are used on a steady voltage higher than that which one of the condensers can stand, means must be taken to equalize the leakage of the condensers to prevent breakdown. With alternating voltages, leakage is not of importance because of the rapid reversals of the emf, and the voltage will be distributed equally among the various condensers. Or, if the condensers are of unequal capacity, the total voltage will be distributed among them in accordance with the law stated above.

Dielectric strength. Some dielectric substances can be subjected to a higher difference of potential per unit of thickness before a rupture occurs than can others. For example, if oil is substituted for air as the dielectric of a given condenser, it will be found that the condenser can now be charged to a higher voltage before sparking occurs between the plates. This is due to the greater dielectric strength of the oil. **The dielectric strength,** or **electric strength,** of a dielectric is the minimum value of the electric field intensity required to rupture it; that is, it is the maximum value of electric intensity that the dielectric can withstand without breaking down. Dielectric strength is usually measured in kilovolts per centimeter of dielectric thickness betwen the points of application of the voltage. Thus, the **disruptive voltage** is the voltage required to break down the dielectric between the points of application of the voltage. In the case of air dielectric, the terms dielectric strength and disruptive voltage are sometimes used synonymously.

The dielectric strength of a given substance is dependent upon many factors which, in the main, are similar to those governing the dielectric constant of the same substance. Some of these factors are: temperature; degree of compression, purity and dryness; condition of the surface—whether leakage will occur or not, presence of scratches, etc.; size and shape of the electrodes used in applying the voltage; the kind of voltage used and the duration of its application. It has been found that, in general, dielectrics will withstand a higher voltage without breaking down when they are subjected to a steady emf than when an alternating emf is applied and, further, that the voltage required to rupture the dielectric is less when it is of a very high frequency than when it is in the range of commercial frequencies. Thus, a given condenser, which will last indefinitely when charged and discharged infrequently using a direct-current supply, may break down after a time when used in a low-frequency circuit, and in a much shorter time if it is charged and discharged at a radio-frequency rate. It is thought that hysteresis of the dielectric is responsible for the breakdown in the cases where alternating voltages are employed, since hysteresis produces heat, which in turn gradually makes the dielectric conducting enough to allow current to pass through it, and thereby cause the breakdown.

Dielectric absorption. When a condenser that has a **perfect dielectric** is charged from a source of steady emf, the charging current will flow only during an extremely brief period, provided that the resistance and inductance in the circuit external to the condenser are negligible. The condenser, therefore, receives its full charge almost instantly. Now, if this same condenser is discharged by short-circuiting its plates, the discharge current will reach zero in an extremely short time, and the condenser will be completely discharged.

On the other hand, when a condenser having an **imperfect dielectric** is charged from a source of steady emf, a suitable current-measuring instrument will show that a charging current was flowing into the condenser long after the switch was closed. This action is similar to that which occurs when a perfect condenser is being charged through a high noninductive resistance, that is, the time constant is larger. If a condenser receives a certain charge when a given emf is applied for a short period and a greater charge when the duration of application of the same emf is increased, the dielectric is said to be an **absorbing dielectric.** This **absorption** is caused by a gradual **penetration,** or **soaking-in,** of the electric strain into the dielectric. If such a condenser is left in a charged condition and kept insulated, so that no leakage of the charge occurs, an apparent loss of the charge will take place, nevertheless, which will be indicated by a decrease in the condenser voltage.

Now, if this same condenser is short-circuited, the condenser will discharge. The charge which leaves the condenser instantaneously

is called the **free charge** and the condenser is apparently completely discharged. However, if the short-circuit is removed and replaced some time later, a second and smaller discharge will take place. This operation can be repeated until, finally, no further discharge occurs. These are called **residual discharges,** and are due to the emergence of the charge that had soaked into the dielectric.

It will be seen from the foregoing that the penetration and emergence of part of the charge is a gradual process, and is due to an imperfect elasticity of the dielectric. It is as though the dielectric had acquired a sub-permanent strain from which only a gradual recovery is possible.

Absorption is negligible in the gaseous and the most commonly used liquid dielectrics. Thus, air, carbon dioxide, castor and petroleum base oils are practically free from absorption. Of the solid dielectrics, good mica shows very little absorption, glass has some absorption, while phenolic base insulating compounds show a considerably higher absorption.

Absorption represents a loss of electrical energy, because it is always attended with a generation of heat in the dielectric. It is also responsible for the observed changes in the capacity of a condenser having a poor dielectric. Thus, the quantity of electricity that such a condenser will take, when the charging period is long, will be greater than when the duration of the charge is short. Consequently, the measured, or apparent, capacity at a low frequency will be greater than when the frequency of charge and discharge is high.

The **geometric capacity** of a condenser is the value that would be measured at a very high frequency.

The action of an absorbing dielectric in a condenser which is being charged and discharged in an alternating-current circuit is discussed in the following Part.

Leakage. All dielectrics have some conductivity, and consequently, perfect insulation is not possible; that is, an insulating substance can be thought of as being an extremely poor conductor or, to have a very small electric conductivity. Thus, if a charged condenser, supposedly well insulated from its surroundings, is allowed to stand idle, the charge will gradually leave the condenser. This is called loss of charge by **leakage,** and should not be confused with absorption of the charge, because the latter is recoverable.

Dielectrics vary greatly in their holding power. A condenser with dry air as the dielectric will hold its charge indefinitely, while certain types of paper condensers lose their charge very rapidly.

The leakage, or conduction, may be either over the surface of, or through the dielectric. Here again the advantages gained by using air dielectric and solid insulation of mica, high grade glass, porcelain or quartz in the construction of condensers are apparent. It is also evident that moisture should not be allowed to collect on the solid insulation. However, the power loss at radio frequencies which is attributable to leakage is not important if the insulation is reasonably good.

PART 3.

ALTERNATING-CURRENT THEORY

CHAPTER I. GENERAL.

The generation of an alternating emf has been treated in Chapter VI of Part 2. In the discussion of the simple alternator it was shown that the rotation of a loop in a uniform magnetic field leads to the generation of an emf which varies sinusoidally in accordance with the angular rotation of the loop. Such an emf is called a **sinusoidal emf,** or is said to have a **sinusoidal wave form.** The ordinary theory of alternating current assumes that the emfs acting in the various circuits are of this character.

When a sinusoidal emf is acting in a circuit, there will be a flow of current in the circuit. This current will also be **sinusoidal** and will have the **same frequency** as that of the emf producing it. It is true that, in practice, both low-frequency and high-frequency generators do not usually supply an emf and, hence, a current, which is exactly sinusoidal. In most cases, the deviation from the sine wave form is not important; but if it is important, the simple theory based upon a sine wave form can be extended to treat the more complicated forms.

In the theory of steady current it was shown that the current in a circuit depends upon the impressed emf and the resistance of the circuit in accordance with Ohm's law. However, when considering the effects of inductance, it was found that immediately following the closing of a circuit, the current did not correspond to Ohm's law, but while increasing from zero to its final value, the value of the current of any instant was determined also by the inductance in the circuit. The final steady value did correspond to Ohm's law. Thus, inductance plays a part in determining the flow of current when the current is varying.

Further, in studying condensers, it appeared that a change in the emf applied to a condenser resulted in a momentary flow of current into or out of the condenser. A variation in emf can, therefore, produce a current flow in a circuit containing a condenser, whereas the steady current, according to Ohm's law will be zero.

Since alternating emfs and currents are varying emfs and currents, the three quantities—resistance, inductance and capacity—are important in determining the **relation** between the emf and the current when these are alternating. This relationship is developed in the succeeding Chapters.

CHAPTER II. AVERAGE AND EFFECTIVE VALUES.

Average value of an alternating current. A dc ammeter indicates the **average** value of the current flowing through it, while the reading of a dc voltmeter corresponds to the **average** value of the emf impressed upon it. Such instruments will show a zero reading when used for alternating currents and emfs, and indicate that **the average value of an alternating current or emf is zero.** That this is true is apparent from the sine curve in which the negative alternation is a counterpart of the positive alternation. Thus, while the positive alternation will tend to deflect the pointer of the instrument by a certain amount in one direction, the negative alternation immediately following it will tend just as strongly to cause the same amount of deflection in the opposite direction. The **net result** is that there is no resultant tendency of the pointer to deflect in either direction. Although the average value over half a cycle has a definite value, the average value over a complete cycle, or number of cycles is zero.,

Effective value of an alternating current. It has already been pointed out that the heating effect of a current, that is, the rate at which electrical energy is being converted into heat, is in no way dependent upon the direction of the current. If, therefore, an alternating current is passed through a so-called hot-wire ammeter, that is, one in which the deflection depends upon the heating of a wire, both the positive and negative alternations of the current will contribute equally to the heating and, hence, to the deflection of the pointer.

Alternating currents are measured on the basis of the heating effect which they can produce as compared with direct currents. Thus, an alternating current is said to have an **effective value** of one ampere if it produces the same heating effect in a given resistance as would be produced by one ampere of direct current. But the heating effect of a steady current has previously been shown to be proportional to the square of the current, that is, the energy transformed into heat in each second is

$$P = I^2R$$

where I = average value of the direct current.

If, therefore, an alternating current produces the same heating effect, its effective value will also be I.

Figure 99 shows one cycle of a sinusoidal alternating current having a maximum value $I_0 = 2$ amperes. A steady current of 2 amperes is represented by the straight line ab in the figure. Since the heating effect of a current is proportional to the square of the current, the instantaneous values of the alternating current should be squared.

Remembering that the square of a quantity, whether positive or negative, is **positive,** the result of squaring the alternating current will be as shown by the dash-line curve, which attains a maximum value $I^2_0=4$ amperes². Likewise, squaring the steady current I of 2 amperes gives $I^2=4$ amperes². This squared steady current is represented by the line cd in the figure.

Now, the energy which is transformed into heat is

$$W=I^2Rt \qquad \text{(joules)}$$

If the abcissa OX represents the time t, then the area $OcdX=I^2t$ and will be proportional to the amount of heat developed in the time t by a steady current of 2 amperes. Also, the amount of heat produced by an alternating current is equal to the sum of the amounts of heat

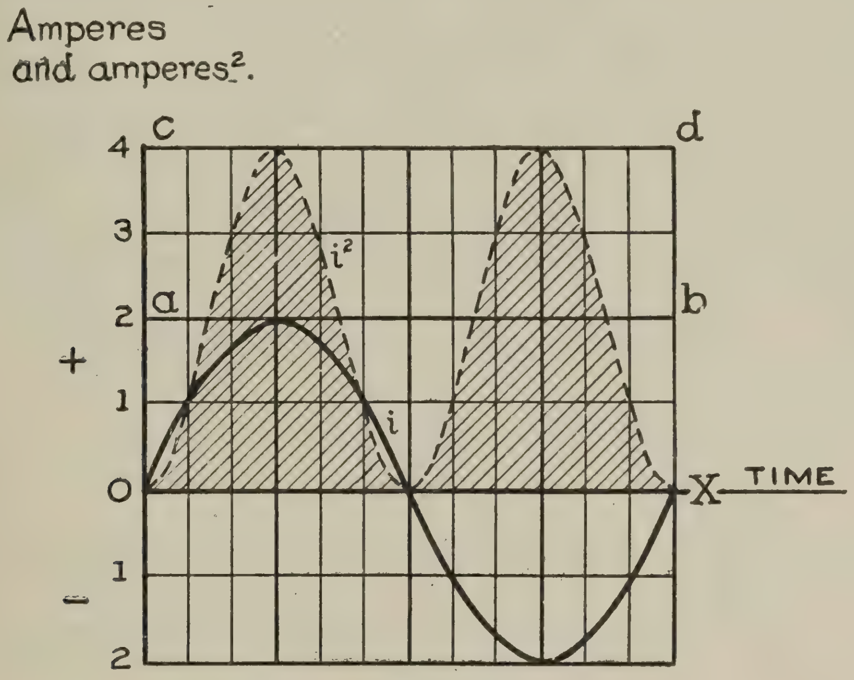


FIG. 99.

produced at every instant. Thus, the amount of heat produced by an alternating current having a **maximum** value $I_0=2$ amperes during one cycle, or in time t, can be represented by the shaded area in the figure. It will be seen that this shaded area is exactly **one-half** of the area $OcdX$. Hence, the amount of heat developed in a given resistance in the time t by an alternating current having a maximum value I_0 is equal to

$$W=\frac{I_0^2Rt}{2}$$

The direct current I which will give the same heating effect is given by

$$I^2Rt=\frac{I_0^2}{2}Rt$$

or

$$I=\frac{I_0^2}{\sqrt{2}}=\frac{I_0}{1.414}=0.707I_0$$

This formula shows that an alternating current and a steady current having an average value 0.0707 times the maximum value of the alternating current will produce the same heating effect in a given resistance. Likewise

$$I_0 = \sqrt{2}I = 1.414I$$

That is, the maximum value of an alternating current must be 1.414 times a given steady current to produce the same heating effect.

By the definition of the effective value of an alternating current given above, I is then the effective value of the sine wave alternating current having a maximum value I_0. Hence, **the effective value of a sine wave alternating current is 0.707 times the maximum value. Consequently, the maximum value of a sine wave alternating current is 1.414 times the effective value.**

Now, the **average** heating effect is meant when the expression heating effect is used. In the case of a steady current, the value of the current at any instant is the same as the average value and I^2R will, therefore, give the heating effect as mentioned before. To find the average heating effect of an alternating current, it is necessary to find the average, or mean, of the squares of all the values of the current throughout a half-cycle, or cycle. For a sine wave this mean is $\frac{I_0^2}{2}$. The heating effect is

$$\frac{I_0^2R}{2}$$

However, $\frac{I_0^2}{2}$ = mean of the squared instantaneous values. Therefore

$$\sqrt{\frac{I_0^2}{2}} = \sqrt{\text{mean of the squared instantaneous values.}}$$

which, in turn, is equal to the effective value. The second member of the last equation is called the **root-mean-square** value, or **rms** value, of the alternating current.

Another name for the effective value is the **virtual** value. Thus, the terms—effective, rms and virtual are synonymous, and are used interchangeably.

Alternating emfs are likewise measured in terms of maximum value and effective value. Thus, **the effective value E of a sine wave of emf is 0.707 times the maximum value E_0,** or

$$E = 0.707E_0$$

Conversely the maximum value is $\sqrt{2}$, or 1.414, times the effective value, or

$$E_0 = \sqrt{2}E = 1.414E$$

Whenever alternating currents or emfs are mentioned without specific reference as to whether they are instantaneous, maximum or

effective, the latter is to be assumed, and is the value measured by an ac ammeter or voltmeter.

The effective value is used in power calculations and also in determining what size a conductor should be to carry a given current. The maximum value is employed in determining the dielectric strength of insulating materials, the break-down voltage of a spark gap and the maximum antenna voltage.

Example:

An ac voltmeter connected across a part of an ac circuit reads 250 volts. What is the maximum value of the voltage across the part?

Solution:

The ac voltmeter measures the effective voltage E.

Formula $E_0 = 1.414E$

substituting $= 1.414 = 250 = 353.5$

whence $E_0 = 353.5$ volts.

CHAPTER III. AN ALTERNATING EMF IN A CIRCUIT CONTAINING RESISTANCE ONLY.

If, as stated before, an alternating current is passed through a dc ammeter or a dc voltmeter, the instrument will not give any indication, excepting perhaps that the pointer will vibrate and its position will, consequently, become blurred. Even the alternating currents ordinarily used in power and lighting circuits, which have frequencies of 25 cycles and 60 cycles, respectively, reverse in direction too rapidly for the pointer to follow the reversals.

Suppose, however, that a source of alternating emf having a frequency of one cycle in several seconds were available and that this emf were applied to a circuit containing only resistance. A dc instrument having its zero in the center of the scale could then be used to follow the variations in the current in the circuit. Similarly, a dc voltmeter with zero in the center of the scale could be used to follow the sinusoidal variation in the emf applied to the circuit. At any instant, the readings of these instruments would give the **instantaneous** values of the current and emf.

Phase relation of emf and current in a circuit containing only resistance. In such a circuit, it would be found that, at the instant the emf attained its maximum value in one direction, the current would also be a maximum in a corresponding direction. Further, both the emf and the current would pass through zero at the same instant. In such a case, when the emf and current are passing through corresponding values in the cycle at the same instant, they are said to be **in phase,** or to have the **same phase.**

It would also be found that, at any instant, the current in the circuit bears the same **constant relation** to the emf as would be the case were the current and emf both constant at their instantaneous values. In other words, Ohm's law does apply to the instantaneous values of current and emf when the circuit in which the alternating emf is acting contains nothing but resistance. Thus, the maximum value of the current I_0 corresponding to a maximum value of the emf E_0 is given by

$$I_0 = \frac{E_0}{R}$$

Since the effective current

$$I = 0.701 I_0$$

and the effective emf

$$E = 0.707 E_0$$

it is also true that

$$I = \frac{E}{R}$$

Thus, **in a circuit containing only resistance, the current and the impressed emf are in phase and are related to each other in accordance with Ohm's law.**

The in-phase relation of the current and the impressed emf in a circuit containing only resistance is shown in figure 100. The light-line sine wave represents the alternating emf acting in the circuit, while the heavy-line sine curve shows the resulting current. An alternating emf

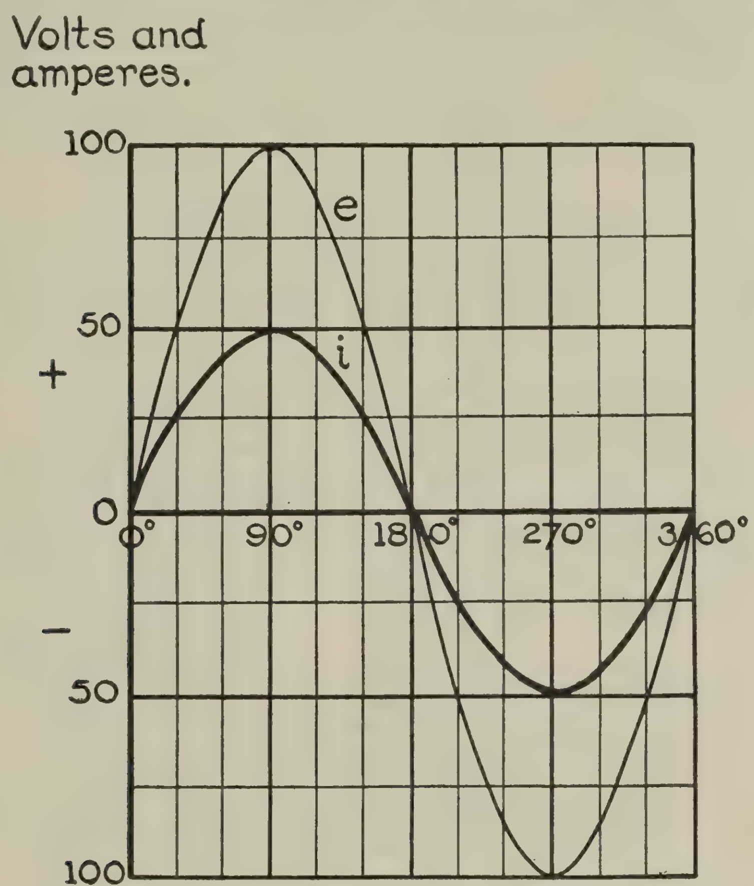


Fig. 100.—Current and Applied Emf are in Phase in a Circuit Containing only Resistance.

having a maximum value $E_0 = 100$ volts is assumed to be acting in a circuit which has a constant resistance $R = 2$ ohms. The constant relation of the current and emf throughout the cycle is apparent.

Energy and Power. From the definition of effective current, the **energy** transformed into heat in the resistance R, that is, the energy supplied to the circuit, will be

$$W = I^2Rt$$

just as in the case of steady currents. Similarly, utilizing the relation

$$I = \frac{E}{R},$$

the energy supplied to the circuit can be written in the following various forms:

$$W = I^2Rt = EIt = \frac{E^2}{R}t$$

where W = energy in joules,
E = effective emf in volts,
I = effective current in amperes.

The **power** (average) in watts is likewise given by

$$P = EI = I^2R = \frac{E^2}{R}$$

The above laws can be applied to **any part of a circuit,** provided that the part under consideration contains only resistance, regardless of what the rest of the circuit contains. Thus, figure 101 shows the instantaneous values of power p in watts, which are obtained from the equation

$$p = ei$$

using the values of e and i appearing in figure 100. Thus, when $e = E_0$ and $i = I_0$, the instantaneous value of power will be 5 kw. It is to be

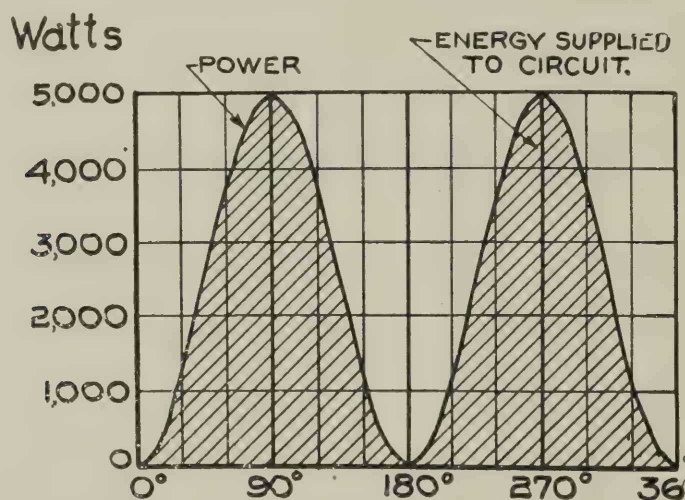


Fig. 101.—Instantaneous Power Supplied to a Circuit Containing only Resistance.

noted that the power at every instant is **positive,** by which it is meant that the source is **supplying** power to the circuit at every instant during the cycle, excepting when both e and i are zero.

Example:

Calculate the average power dissipated in a part of a circuit containing a resistance of 2 ohms, if the maximum value of the impressed emf is 100 volts.

Solution:

Formula $E = 0.707E_0$

substituting $= 0.707 \times 100 = 70.7$

whence $E = 70.7$ volts.

Formula $I = \dfrac{E}{R}$

substituting $= \dfrac{70.7}{2} = 35.35$

whence $I = 35.35$ amperes.

Formula $P = EI$

substituting $= 70.7 \times 35.35 = 2{,}499$

whence $P = 2{,}500$ watts.

Counter emf. It is desirable to distinguish between the impressed, or applied emf and the **counter emf, cemf,** offered by the resistance.

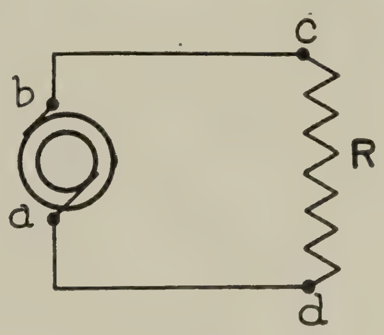


Fig. 102.

At every instant the cemf is equal to, but oppositely directed from the impressed emf. Thus, in figure 102, at any instant when impressed emf is such as to send current around the circuit in the direction *bcda*,

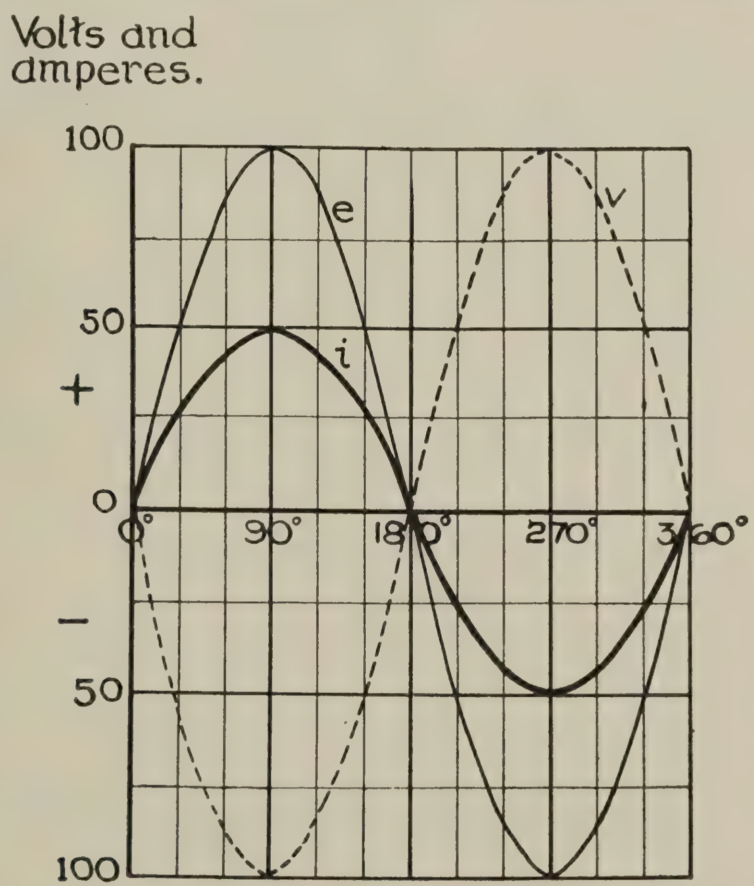


Fig. 103.—Phase Relations of Impressed Emf, Current and Cemf in a Circuit Containing only Resistance.

the positive terminal of the alternator will be b, and the negative terminal a. At that instant, the cemf across the resistance R will be such that the terminal c will be positive and the terminal d negative. The impressed emf is in the direction of the current and, hence, is in phase with it. The cemf is in direction opposite to that of the impressed emf and the resulting current.

Thus, if the current and impressed emf are represented by **sine** curves, the cemf will be a **minus sine** curve. A minus sine curve reaches its positive maximum value 180° before or after the plus sine curve reaches its positive maximum value. That is, the plus sine curve has a positive maximum when the angle is 90°, while the minus sine curve passes through this value at an angle —90°, or+270°.

This is also true for any other corresponding value of the two curves. Hence, the quantities represented by these curves are said to be 180° out of phase.

The relation between the instantaneous values of the cemf v and the impressed emf e with the resulting current i is shown in figure 103, in which the dash-line curve represents the cemf.

Thus, **in a circuit with resistance only, the impressed emf and the resulting current are in phase with each other, but are 180° out of phase with the cemf.**

CHAPTER IV. AN ALTERNATING EMF IN A CIRCUIT CONTAINING SELF-INDUCTANCE ONLY

Assume that a sine wave of emf is impressed upon a circuit which consists of an inductance coil alone and that this coil has an inappreciable resistance. Such a coil would constitute a short-circuit for a steady emf, and the resulting current would rise to an enormous value. In the case of an alternating emf, however, there would be a definite limit to the flow of alternating current, no matter how low the resistance of the coil. Thus, inductance limits the flow of alternating current in the same manner as it does a varying direct current flowing through it, as was explained in Chapter VII of Part 2. Since it is assumed that the inductance has a negligible resistance, there will be no counter emf of resistance to oppose the impressed emf. The only counter emf acting will be that which has previously been called the emf of self-induction. The current will flow in a manner so that the cemf of self-induction will at every instant be equal and opposite to the impressed emf. The cemf is given, as before, by

$$v = -L\frac{di}{dt}$$

and since the impressed emf e is always equal and opposite, it will be given by

$$e = L\frac{di}{dt}$$

Now, if the instantaneous value of a sinusoidally varying current is represented by

$$i = I_0 \sin\omega t$$

it can be shown that the quantity $\frac{di}{dt}$, which represents the time rate of variation of the current, is

$$\frac{di}{dt} = I_0\omega \cos \omega t$$

Substituting this value of $\frac{di}{dt}$ in the previous equation for e then

$$e = I_0\omega L \cos \omega t$$

The maximum value of e is, therefore,

$$E_0 = I_0\omega L$$

or

$$I_0 = \frac{E_0}{\omega L}$$

Because of the relation between maximum and effective values

$$I=\frac{E}{\omega L}$$

where I = effective current in amperes,
E = effective emf in volts,
L = self-inductance in henries,
$\omega = 2\pi f$

This equation gives the value of the resulting current when an alternating emf is applied to a circuit containing only inductance.

Inductive reactance. It will be seen that the term ωL in the equation

$$I=\frac{E}{\omega L}$$

takes the place of the R in Ohm's law. In other words, the alternating current in an inductance to which a given alternating emf is applied is limited by the value of the quantity ωL. The term ωL is called the **reactance of inductance,** or simply **inductive reactance, and is defined as the limitation placed upon the flow of alternating current by inductance.** The symbol of inductive reactance is X_L. Reactance is measured in **ohms,** like resistance. Thus,

$$I=\frac{E}{X_L}$$

Example:

An emf of 1,000 volts at a frequency of 60 cycles is applied to the primary of a transformer, which has an inductance of 10 henries and a negligible resistance (secondary open). What will be the current taken by the primary?

Solution:

Formula $X_L = \omega L$

substituting $= 2 \times 3.1416 \times 60 \times 10 = 3{,}770$

whence $X_L = 3{,}770\ \Omega$

Formula $I = \frac{E}{\omega L} = \frac{E}{X_L}$

substituting $= \frac{1{,}000}{3{,}770} = 0.265$

whence $I = 0.265$ ampere.

The above solution shows that only a small alternating current results.

Phase relation of cemf of self-induction, current and the applied emf in a circuit containing only inductance. It was pointed out in Chapter VII of Part 2 that the emf of self-induction is always in such a direction as to oppose changes in current flow, and also that the magnitude of the emf of self-induction is proportional to the rapidity with which the current is varying.

In the circuit under consideration, if the current i is represented by a sine wave, figure 104, the cemf of self-induction v will be a minus cosine curve, as shown by the dash-line curve in the figure. This agrees with previous considerations, for during the interval that the current is becoming more negative, that is, going from 90° to 270°, the cemf of self-induction v is positive. Also, the cemf of self-induction is a maximum when the current is passing through zero. The current reaches its maximum value at 90°, while the cemf of self-induction passes through a maximum at 180°, or **90° later** in the cycle. Thus, there is a **difference in phase of 90°.** The current may be said to **lead,** or to **be in advance** of the cemf of self-induction by 90°, or the cemf of self-induction **lags behind** the current by 90°.

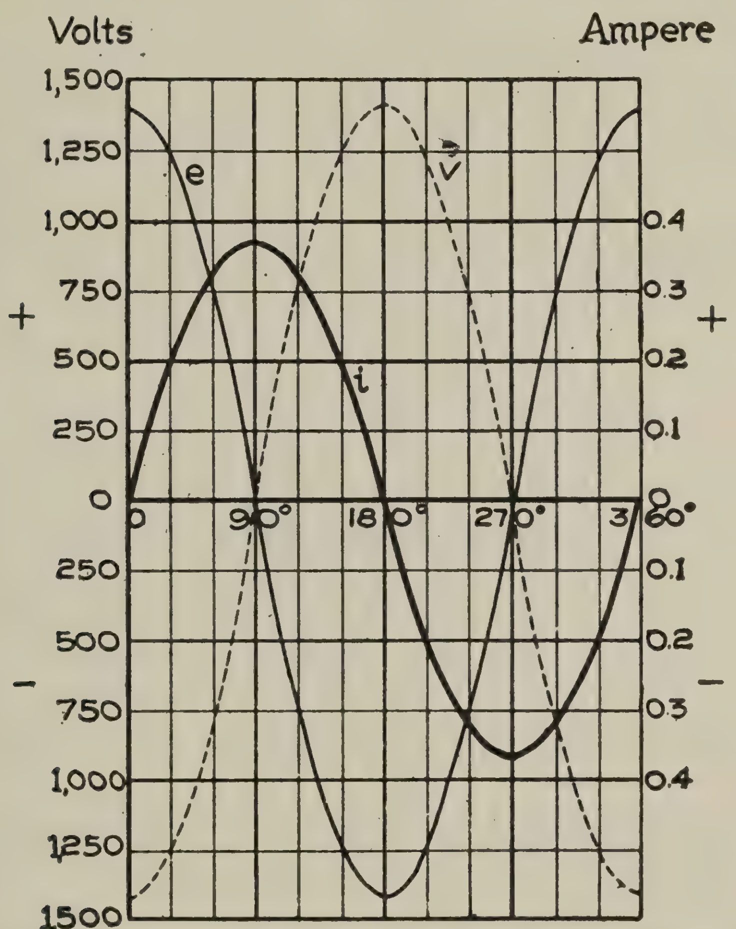


FIG. 104.—Phase Relation of Impressed Emf, Current and Emf of Self-Induction.

Just as in the case of the circuit with resistance only, the **impressed emf** is balanced at every instant by the cemf of self-induction, if only inductance is present in the circuit. In fact, it is a law for **all** circuits that the cemf, if there is but one, or the resultant of the cemfs, if there are a number, will balance the impressed emf or impressed emfs.

In the circuit being considered, the **impressed emf** e will be a plus cosine curve as shown by the light-line curve in figure 104. At every instant, the cemf of self-induction is equal and opposite to the impressed emf. Since the impressed emf is at its maximum value at 0°,

while the current attains its corresponding value at 90°, **the current lags behind the impressed emf by 90°, or the impressed emf leads the current by 90° when there is only inductance in the circuit.**

Energy and power. Whenever current flows in a circuit in the direction of the applied emf, energy is being supplied to the circuit. If, on the other hand, current flows in a part of a circuit **against** the emf across that part of the circuit, then energy is being stored up or dissipated in the part of the circuit under consideration. Thus, when a battery is being discharged, the flow of current is in the direction of the battery emf while, when being charged, the flow of current is against the battery emf.

If the current and the emf are variable, the rate at which energy is being supplied, or used, will usually vary; in fact, at one instant, the emf may be doing work while, at a succeeding instant, it may be that work is being done upon the source of emf. The rate at which energy is being supplied, or used, at any instant, is equal to the product of the instantaneous values of the emf and current. That is, **the instantaneous value of the power which is being delivered, or received, is**

$$p = ei$$

Referring to figure 104, the impressed emf and the current are both positive and, hence, are acting in the same direction during the first quarter of the cycle. This indicates that the impressed emf is supplying power to the circuit. On the other hand, during this interval, the cemf of self-induction is opposite to the current, indicating that energy is being dissipated, or stored in the inductance. The latter is really the case for, as was pointed out in Chapter VII, Part 2, when the current in an inductance is increasing, energy is being stored in the magnetic field.

During the second quarter of the cycle, however, the conditions are reversed. The cemf of self-induction and the current have the same direction; hence, energy is being supplied by the inductance to the circuit. At the same time, the impressed emf and the current are oppositely directed and, therefore, energy is being supplied to the source of the impressed emf. The former condition is in accordance with the fact that, when current is decreasing in an inductance, the energy stored in its magnetic field is returned to the circuit. The latter condition means that the source of the impressed emf has changed its rôle and, instead of being a generator, has become a motor, and is being run by the energy returning from the magnetic field of the inductance.

It is evident that, for any interval in the first quarter of the cycle in which power is being supplied by the impressed emf, there is a corresponding interval in the second quarter of the cycle in which an equal amount of power is supplied to the source of the impressed emf.

Therefore, the average power supplied by the impressed emf during the first half of the cycle, or succeeding half-cycles is zero.

Therefore, **the average power which is taken by a circuit with inductance alone is zero.**

Figure 105 shows the instantaneous values of the power during **one cycle** of the impressed emf, using the values given in the last example and the formula $p=ei$. It will be seen from the figure that the sine

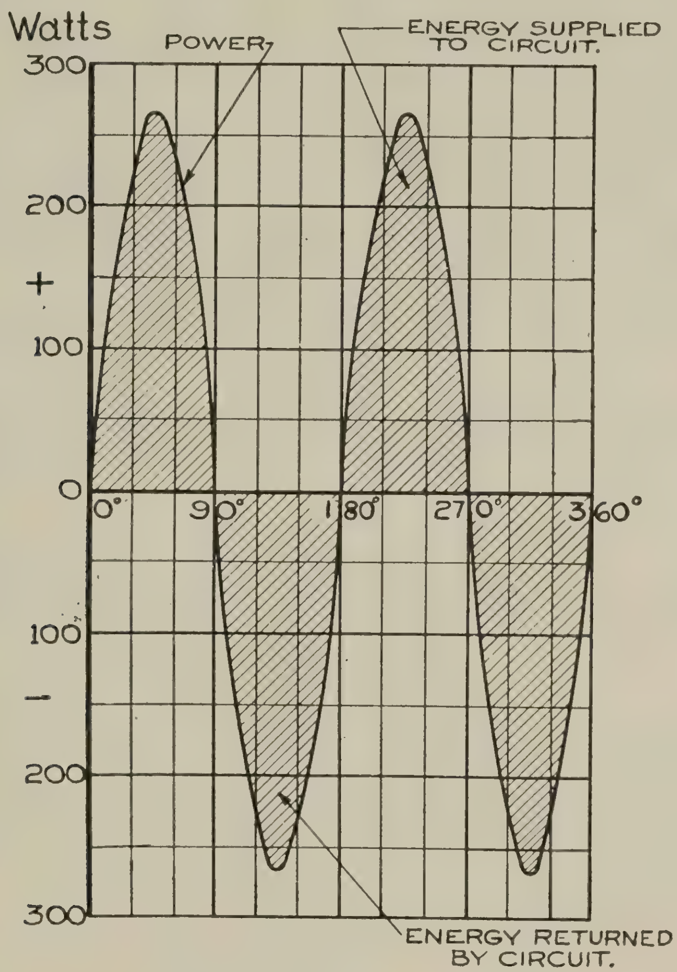


FIG. 105.—Energy and Power in a Circuit Containing only Inductance.

curve of the instantaneous power represents a variation in the power which has **twice the frequency** of the impressed emf. This curve also shows that the average power taken by such a circuit is zero.

The energy supplied to the circuit by the source, and also the energy returned to the source by the circuit, are represented by the **areas** enclosed by the curves. It is evident that the two amounts of energy mentioned are equal, so that the total energy taken by the inductive circuit during one cycle is zero. For these reasons, inductance without resistance limits the flow of alternating current without a loss of power and, in this respect, differs in its action from resistance.

This subject will again be treated.

CHAPTER V. AN ALTERNATING EMF IN A CIRCUIT CONTAINING CAPACITY ONLY.

Suppose that a sine wave of emf is impressed upon a perfect condenser. By a perfect condenser is meant a condenser which has no dielectric absorption, no series resistance in the leads and plates, and perfect insulation between the two sets of plates. It was pointed out in Chapter VIII of Part 2, that current flows into or out of a condenser only when the applied emf is varying. Under the action of the applied emf, the condenser C receives a charge q and, consequently, a difference of potential v is developed between its plates. This voltage of the condenser is opposite in direction to that of the applied emf e and, in the case when there is no resistance, is also exactly equal to it, that is,

$$\frac{q}{C}=v=e$$

or

$$q=Ce$$

Now, the charge q, which is a **quantity** of electricity, is conveyed at a definite rate which is the **current** i; that is, the time rate of change of the charge is the current, or

$$i=\frac{dq}{dt}$$

substituting for q in the above equation its value Ce

$$i=\frac{d(Ce)}{dt}$$

Now, the capacity C is invariable; hence, the time rate of change of a product containing a quantity that does not vary is the same as the product of the constant quantity and the time rate of change of the variable quantity, or

$$i=C\frac{de}{dt}$$

Also

$$e=E_0 \sin \omega t$$

Substituting this value of e in the previous equation and solving

$$i=\omega C E_0 \cos \omega t$$

From which

$$I_0=\omega C E_0$$

The effective value of the current is

$$I=E\omega C$$

or

$$I=\frac{E}{\frac{1}{\omega C}}$$

where I = effective current in amperes,
E = effective emf in volts,
C = capacity in farads,
$\omega=2\pi f$

On the assumption made that there was no resistance in the circuit, the voltage v of the condenser will be equal to the applied emf e. Hence, the above equation gives the value of the resulting current when an alternating emf is applied to a circuit containing only capacity.

Capacitive reactance. The similarity between the equation

$$I=\frac{E}{\frac{1}{\omega C}}$$

and Ohm's law is apparent. In this case, the term

$$\frac{1}{\omega C}$$

takes the place of R and **limits** the flow of alternating current. The term $\frac{1}{\omega C}$ is called the **reactance of capacity, or capacitive reactance, and is defined as the limitation placed upon the flow of alternating current by capacity.** The symbol of capacitive reactance is X_C, and is expressed in **ohms.** Thus,

$$I=\frac{E}{X_C}$$

Example:

A condenser has a capacity of 2.0μf. What will be the effective value of the current if the effective value of the applied emf is 1,000 volts and the frequency is 60 cycles?

Solution:

First find the capacitive reactance.

Formula $\quad X_C=\frac{1}{\omega C}$

substituting $\quad =\frac{1}{2\times 3.1416\times 60\times 2\cdot 10^{-6}}=\frac{1}{7.54\cdot 10^{-4}}$

$\quad =1.325\cdot 10^{3}$

whence $\quad X_C=1{,}325$ ohms.

Formula $\quad I=\frac{E}{\frac{1}{\omega C}}=\frac{E}{X_C}$

susbtituting $\quad =\frac{1.10^{3}}{1.325\cdot 10^{3}}=0.754$

whence $\quad I=0.754$ ampere.

Phase relation of the applied emf, current and cemf of capacity in a circuit containing only capacity. Assume that a sinusoidal emf has been applied to a circuit containing only capacity long enough for conditions to have become steady. Due to the variations in the applied emf, the current will also vary. It was pointed out in Chapter VIII of

Part 2 that, as the condenser becomes charged, the voltage of the condenser (cemf of capacity) becomes greater and greater until, when the condenser is fully charged, this voltage equals, but is oppositely directed to, the applied emf. At the same instant, the charging current becomes zero. Hence, the applied emf E and the cemf of capacity v are at a maximum, but displaced 180° from each other, at the instant that the current I is zero. The current, therefore, leads one emf by 90° and lags behind the other emf by 90°, or is a maximum when these two emfs are passing through zero and are, therefore, changing most rapidly.

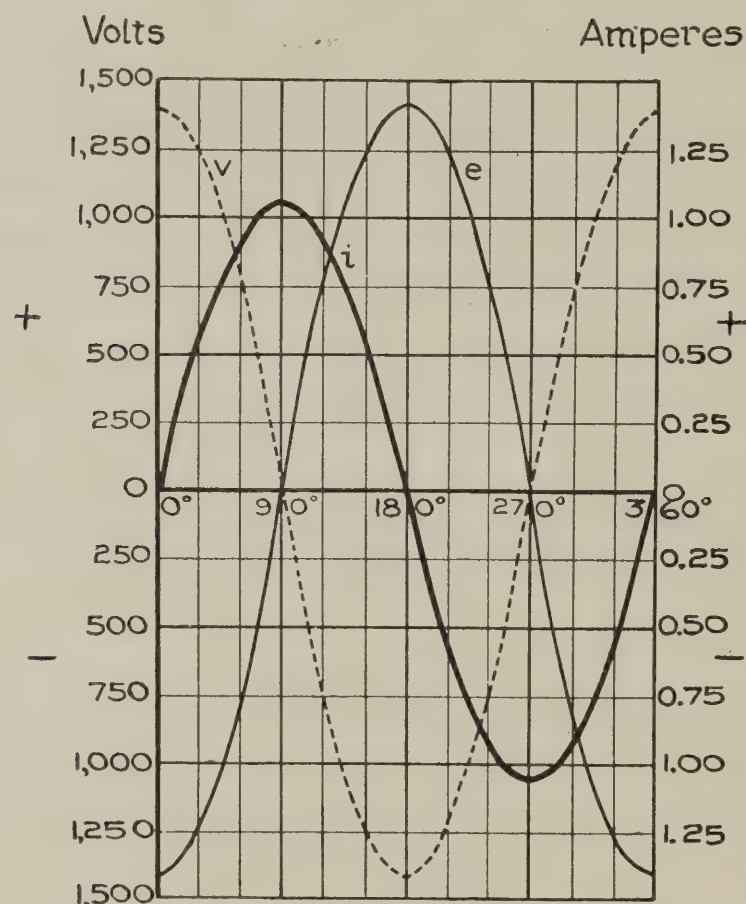


Fig. 106.—Current, Applied Emf and Condenser Voltage in a Circuit Containing only Capacity.

However, since it is the applied emf that causes the current to flow, and the current flows only when this emf is changing and, further, since the charging current flows in the same direction as that in which the applied emf is acting then, when the applied emf E is passing through zero in a positive direction, the current I will have a maximum positive value and be on the point of decreasing to zero. This means that the current leads the applied emf by 90°.

This relation in phase of the current and applied emf during this interval is shown in figure 106 between 90° and 180°. The instantaneous values of the current i are represented by the heavy-line sine curve, while those for the applied emf e are given to the light-line minus cosine

curve. Since the cemf of capacity v is oppositely directed to the applied emf, it may be represented by the cosine curve v in the figure. The values used in plotting figure 106 are those given in the previous example.

Therefore, **in a circuit containing only capacity, the current is 90° in advance of the applied emf, but lags 90° behind the condenser voltage.**

Difference between effect of inductance and capacity on alternating current. It has been shown that inductance limits the flow of current. Capacity also limits the flow of current by its reactance. In the case of inductance, the effective value of the current is

$$I = \frac{E}{\omega L}$$

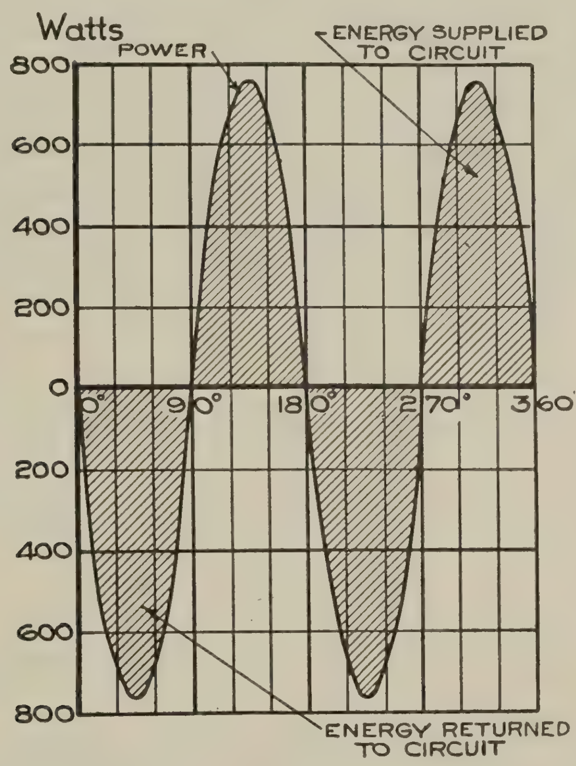


FIG. 107.—Energy and Power in a Circuit Containing only Capacity.

Now, the higher the frequency is the greater will be the numerical value of ω. Since ω is in the denominator of the fraction, the numerical value of the fraction will be decreased, provided E and L remain constant, and, hence, I will decrease in value. Thus, **with a given value of applied emf and a given inductance, the current will decrease as the frequency is increased.**

With capacity, the effective value of the current is

$$I = E\omega C$$

Here, any increase in frequency will increase the numerical value of the second member of the equation and, hence, I. Therefore, **with a given**

value of applied emf and a given capacity, the current will increase as the frequency is increased.

It is seen from the foregoing that the effect of inductance in limiting the flow of alternating current is least at those frequencies where the limiting effect of capacity is greatest.

Energy and power. The energy considerations are similar to those discussed in the case of inductance. During the interval between 0° and 90°, figure 106, the condenser is discharging and returning its store electrostatic energy to the circuit. During the interval between 90° and 180°, the condenser is being charged with an opposite polarity. The applied emf is in the same direction (positive) as the current, and energy is being supplied by the source and stored in the electrostatic field. This energy is returned again to the circuit in the next quarter cycle. The action occurring between 0° and 180° is duplicated in the interval between 180° and 360°.

The **average power** taken by the circuit from the source of emf is **zero.** Figure 107 shows the energy and power for such a circuit. The curve itself represents the instantaneous value of the power, while the hasded areas represent the energy. The curve is plotted from the instantaneous values given in figure 106.

CHAPTER VI. GRAPHICAL REPRESENTATION OF ALTERNATING QUANTITIES BY VECTOR DIAGRAMS.

Alternating emfs and currents have been represented up to this point by sine and cosine curves. Such curves show how an alternating emf or current varies with respect to time, or angular position; therefore, instantaneous values can be read from such curves. Phase relations have also been shown by these curves. However, sine curves are not entirely satisfactory, being rather difficult to draw and to interpret. Further, the radio engineer is usually concerned with the effective and maximum values rather than with the instantaneous values. A much simpler method of representing alternating quantities is by the use of **vectors,** which are discussed under **Graphs** in Section III. The method of applying vectors specifically to either the representation or the solution of ac problems is given in the following.

Any sine or cosine curve can be represented by a vector r rotating in a **counter-clockwise** direction for **positive** angles and in a **clockwise** direction for negative angles, the angle under consideration being the

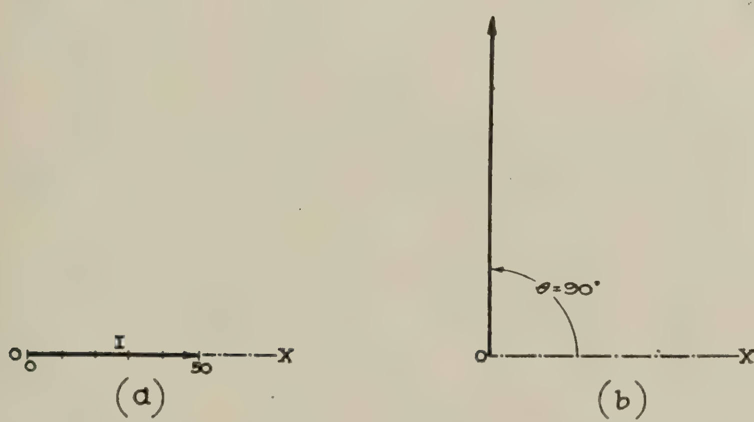


FIG. 108.—Vector Diagrams of (a) Current and (b) Rate of Change of Current

angle θ that the vector makes with the **initial line.** The length of the vector is generally used to represent the **effective** value of the emf or current, while the angle θ shows the **phase relation** of two quantities provided that one quantity is represented by a portion of the initial line.

In figure 108, (a) represents a sinusoidally varying current having an effective value $I=50$ amperes. The rate of change of this sinusoidal current is a maximum when the current is passing through zero and, hence, would be represented by the cosine. Since the cosine **leads** the sine by 90°, this fact would be shown by a vector displaced in the positive direction from OX by an angle $\theta=90°$ as shown in (b).

Figure 109 is a vector diagram of the magnitude and phase relations of the applied emf, current and cemf in a circuit containing only resistance. The effective values used are the same as those appearing in figure 103. In this case, both E and I are **in phase**; hence, the two vectors representing E and I are laid off on the initial line,

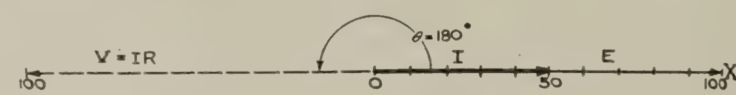


Fig. 109.—Vector Diagram Showing Magnitude and Phase Relations of Applied Emf, Current and Cemf in a Circuit Containing only Resistance.

thus showing that they are in phase. The IR drop is, of course, 100 volts and, since it is also opposite in direction to the applied emf, it will be represented by a vector equal in length to vector E displaced by an angle $\theta = 180°$, as shown in the figure. Thus, the vector IR represents a minus sine curve.

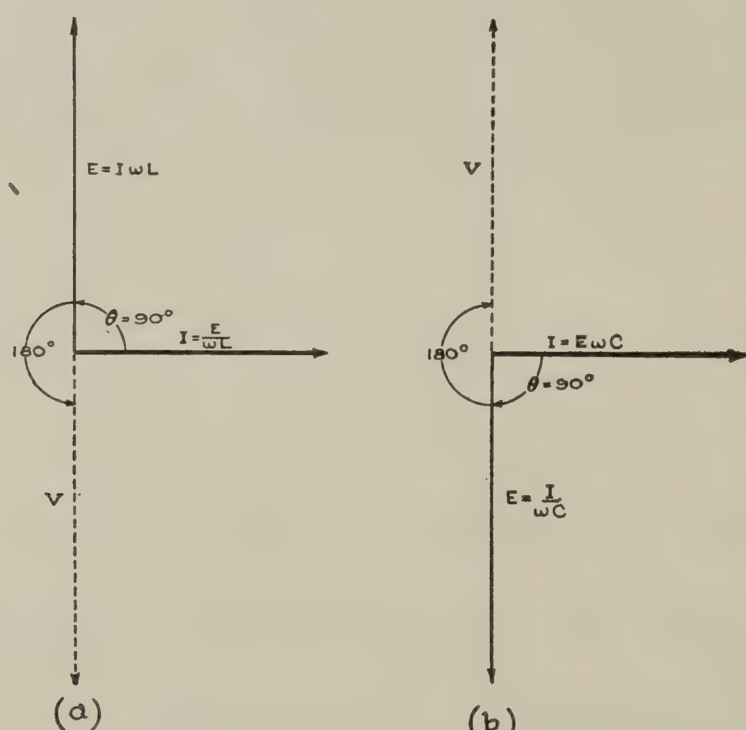


Fig. 110.—Phase Relations of Applied Emf, Current and Cemf in a Circuit Containing (a) Inductance only (b) Capacity only.

The phase relations and magnitudes of the applied emf, current and emf of self-induction in a circuit containing only inductance are shown in figure 110 (a). The current **lags behind** the applied emf by 90°; therefore, if the vector representing I is in the initial line, the vector for E will be as shown in the figure.

Also, since the emf of self-induction V is equal to and oppositely directed to E, it will be represented by the vector V.

Figure 110 (*b*) is a vector diagram of the phase relations of the same three alternating quantities for a circuit containing only capacity. Here, the current **leads** the applied emf by 90°, that is, the applied emf **lags behind** the current by 90°. Therefore, if the vector I lies in the initial line, the vector E will be located as shown. The vector representing the voltage of the condenser V will be equal and oppositely directed to E.

The vector diagrams in figure 110, although not drawn to any scale, represent the phase relations shown by the sine and cosine curves in figures 104 and 106, respectively.

Resolution of emfs and currents. If an emf E_1 is in series in a circuit with an emf E_2 having the **same frequency,** the resultant of these emfs can be obtained in the same manner as the resultant of two mechanical forces is obtained. Thus, in figure 111 (*a*), the vector E_1 represents one emf, while E_2 represents the other. The angle θ between them shows their phase relationship. Thus, in the figure E_2 leads E_1 by an angle θ. The combined effect of the two emfs, or their **resultant,** is found by completing the parallelogram $OABX$ by drawing AB and

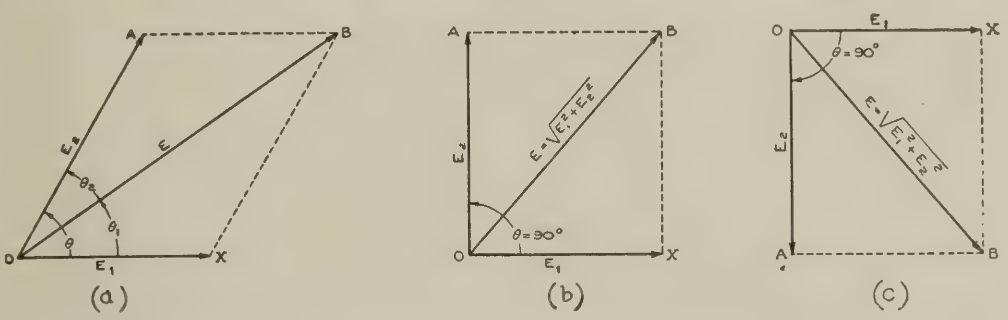


FIG. 111.—Vector Diagrams Showing Resolution of Emfs in a Series Circuit.

XB equal and parallel to OX and OA, respectively. The diagonal OB of the parallelogram then gives the resultant emf E, both with respect to its magnitude and phase. It will be seen that the resultant emf lags behind E_2 by the angle θ_2 and is in advance of E_1 by the angle θ_1.

If the quantities E_1, E_2 and θ are known, the vector diagram can be drawn to scale and the problem can be solved by the graphical method. The solution would consist in finding the values of E, θ_1 and Θ_2. Or, the problem can be solved by trigonometrical methods.

The interest lies, however, in the vector diagrams shown in figure 111 (*b*) and (*c*). It will be noted that the two emfs E_1 and E_2 are 90° out of phase, and that the parallelogram is, therefore, a **rectangle.** The diagonal OB of such a figure divides the rectangle into two equal **right triangles** OXB and OAB. Therefore, knowing the sides

$$OX = E_1 \text{ and } XB = E_2$$

the diagonal $\quad OB = E$

is given by $\quad E^2 = E_1^2 + E_2^2$

or $\quad E = \sqrt{E_1^2 + E_2^2}$

The resultant emf in the last two cases can, therefore, be found by the arithmetical method, or by the graphical method.

Similarly, if two currents I_1 and I_2 differing in phase, but of the same frequency, combine in a circuit, the resultant current I can be found.

The principles underlying the use of vector diagrams in the graphical representation of alternating quantities, and which have been given in the foregoing, will be utilized in the succeeding Chapters in connection with the problems appearing therein.

CHAPTER VII. SERIES COMBINATIONS.

The more usual alternating-current circuit will consist of resistance and inductance in series, of resistance and capacity or of resistance, inductance and capacity. The solution of such circuits requires only an extension of the results already obtained. When there are several counter emfs in a circuit, the impressed emf is balanced by the resultant of all the counter emfs, or the impressed emf can be considered to be split up into components, each of which balances one of the counter emfs. Extending the earlier results so as to render them applicable to portions of a circuit, the following facts are obtained:

(a) The counter emf across a resistance is 180° out of phase with the current. The component of the impressed emf which balances the counter emf of resistance is in phase with the current and has a value

$$E = IR$$

(b) The counter emf across an inductance lags 90° behind the current. The component of the impressed emf which balances the counter emf of inductance is 90° in advance of the current and has a value

$$E = I\omega L$$

(c) The counter emf across a capacity is in advance of the current by 90°. The component of the impressed emf which balances the counter emf of capacity lags behind the current by 90° and has a value

$$E = \frac{I}{\omega C}$$

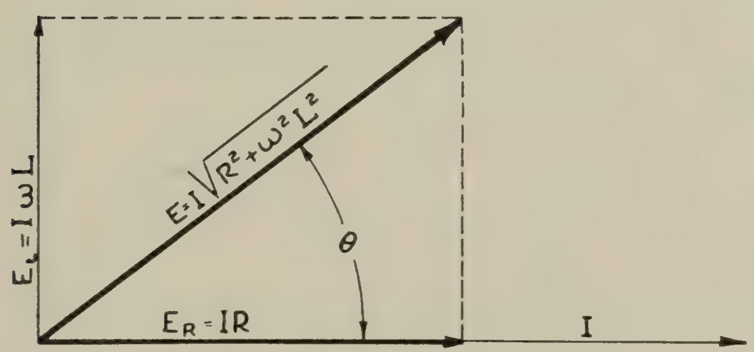


FIG. 112.

Series circuit with resistance and inductance. Let the circuit consist of a resistance R and inductance L. The resistance of the inductance itself may be the total resistance or a part of it. Assume the current I to be the vector I shown in figure 112. The component E_R of the impressed emf overcoming the resistance of the circuit is in phase

with the current, the component E_L overcoming inductance is 90° in advance of the current, the resultant or total impressed voltage E is the diagonal of the rectangle and is, therefore, equal to the square root of the sums of the squares of the two sides.

$$E=\sqrt{I^2R^2+I^2\omega^2L^2}$$

or

$$E=I\sqrt{R^2+\omega^2L^2}$$

or

$$I=\frac{E}{\sqrt{R^2+\omega^2L^2}}$$

This gives the relation between the current and voltage which again is similar to Ohm's law, the term $\sqrt{R^2+\omega^2L^2}$ which is called the **impedance** of the circuit takes the place of the resistance. Impedance is denoted by Z and is measured in ohms. If the frequency is very low so that ω is small and if the value of L is low, the term ω^2L^2 is negligible and may be considered to be zero. Then Ohm's law is obtained. On the other hand, if R is negligible the expression becomes the same as that for a circuit with inductance only.

The current I lags behind the voltage by the angle θ. From the figure, this phase angle is given by

$$\tan\theta=\frac{I\omega L}{IR}=\frac{\omega L}{R}$$

Example:

A series circuit contains an inductance of 0.5 henry and a resistance of 50 ohms. What current will flow in the circuit when a voltage of 100 volts is impressed at 60 cycles frequency? What will be the phase angle?

Solution:

Formula $Z=\sqrt{R^2+\omega^2L^2}$

substituting $=\sqrt{(50)^2+(2\times3.1416\times60\times0.5)^2}=\sqrt{38030}$

whence $Z=195$ ohms.

Formula $I=\frac{E}{Z}$

substituting $=\frac{100}{195}$

whence $I=0.513$ ampere.

Formula $\tan\theta=\frac{\omega L}{R}$

substituting $=\frac{2\times3.1416\times60\times0.5}{50}=3.77$

from Table 17 $\theta=75°\ 9'$

Referring to figure 112, it is noted that each of the vectors representing ems are multiplied by the current I. If each side is divided by I, the vector diagram of figure 113 is obtained. This shows that the impedance can be considered to be the vector sum of the resistance R and reactance ωL, the reactance being a vector at an angle of 90° from the resistance in a positive direction. Another equivalent way of representing the addition of the vectors is shown in figure 114. This is called an impedance triangle and is also used in the vector addition of forces in mechanics.

Impedances in series. Suppose two coils are in series in a circuit, one having an inductance L_1 and resistance R_1 while the other has an inductance L_2 and resistance R_2. The impedance diagram would be

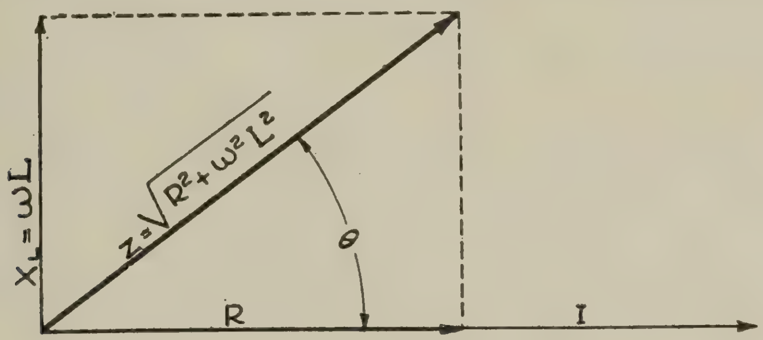


Fig. 113.

that of figure 115. The impedance Z_1 represents the impedance of one coil, while Z_2 represents the other. The resultant impedance will be Z, the **vector sum** of Z_1 and Z_2 and **not the arithmetic sum unless the two vectors happen to have the same phase angle.** Thus, if a problem were given to find the current for a given emf in a circuit with two coils in series, one having an impedance of 50 ohms and the other an impedance of 100 ohms, the problem could not be solved and it would be incorrect to assume that the total impedance of the circuit is 150 ohms unless

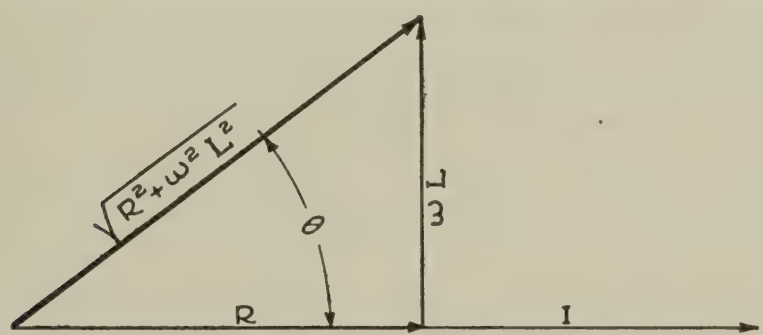


Fig. 114.

the two impedances happened to have the same phase angle. It would be necessary to give the resistance and inductance (or reactance) of each coil, or the impedance of each coil and its phase angle. Figure 115 can be also drawn in the simpler form of figure 116 with the same resultant impedance Z and phase angle θ. The resistances R_1 and R_2 are

added, and the reactances ωL_1 and ωL_2 also added but drawn at right angles to the resistances. The resultant Z is then given by

$$Z = \sqrt{(R_1+R_2)^2+\omega^2(L_1+L_2)^2}$$

and the phase angle θ is given by

$$\tan \theta = \frac{\omega(L_1+L_2)}{R_1+R_2}$$

This shows that in a series circuit all of the resistances can be added arithmetically to give the total resistance and the inductances can be

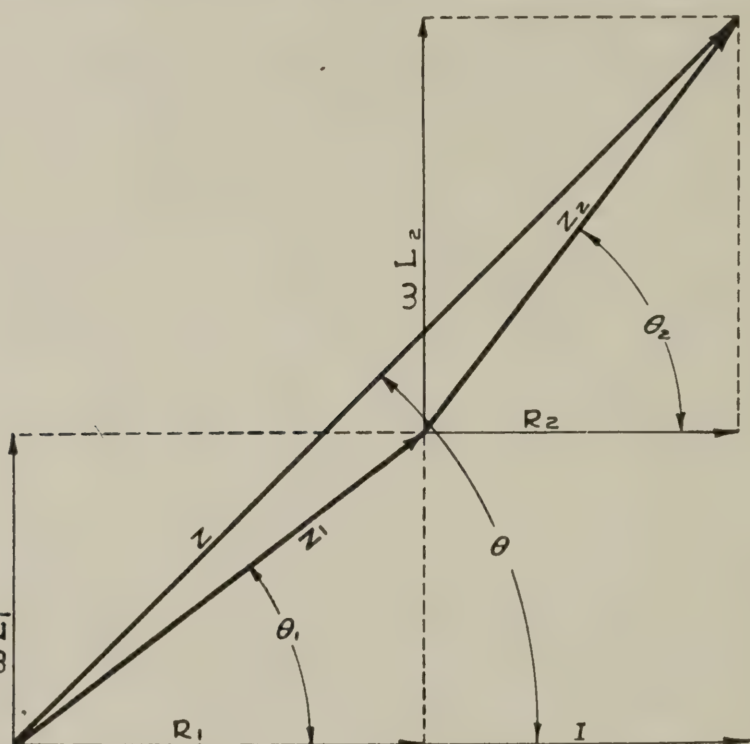


FIG. 115.

added to give the total inductance. These total values can then be combined to find the circuit impedance and phase angle in the same manner that they would be combined in the case of a single coil. The current is then given by the impressed emf E divided by the circuit impedance

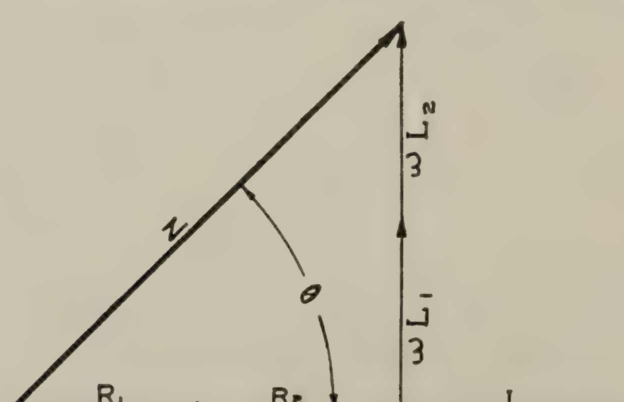


FIG. 116.

Z, and the phase angle θ gives the angle by which the current lags behind the impressed emf.

Power relations. In the case of the circuit with resistance only, the current and emf were in phase and the average power supplied to the circuit and dissipated therein was given by the product EI. In the case of other circuits, where the emf and current are not in phase, the current can be resolved into two components, one which is in phase with the emf and one at right angles to the emf. When the current leads or lags by 90°, the component in phase with the emf is zero. The power under these conditions is zero, as was pointed out in the considerations of circuits with inductance only or capacity only. Thus, the component of the current which is in phase with the emf determines the power. If the phase angle is θ, the component of the current which is in phase with the emf is $I \cos \theta$. **The average power is given by the product of the in-phase component of the current and the emf;** thus,

$$P = EI \cos \theta$$

The quantity $\cos \theta$ is called the power factor and is denoted by a. When $\theta = 0°$), $\cos \theta = 1$, the power factor is then said to be unity or 100 per cent. This represents the conditions for the circuit with resistance only. When $\theta = 90°$, $\cos \theta = 0$, the power factor is zero and the average power supplied or utilized is likewise zero. This is the condition corresponding to the circuits assumed to have inductance alone or capacity alone, though in actual fact such circuits cannot be realized in practice.

Example:

Assuming the data and results of the preceding problem, find the power factor of the circuit and power delivered to the circuit by the alternator. Solution:

Formula	Power factor $= a = \cos \theta$
substituting	$a = \cos 7509'$
whence	$a = 0.256$
or	$a = 25.6$ per cent.
Formula	$P = EI \cos \theta = EI\, a$
substituting	$= 100 \times 0.513 \times 0.256$
whence	$P = 13.1$ watts.

The product EI is frequently designated **volt-amperes.** One thousand volt-amperes is called a kilovolt-ampere and is sometimes used to rate the capacity of electrical machinery. It does not indicate the power unless the power factor is unity.

Series circuit with capacity and resistance. The voltage diagram and corresponding impedance triangle for a circuit with a capacity C and resistance R in series is given in figure 117. The current is determined by the equation

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

where again the quantity in the denominator is called the impedance. The phase angle is given by

$$\tan \theta = \frac{1}{\omega CR}$$

where θ is the angle by which the current leads the impressed emf. It will be noted in the impedance triangle that the capacity reactance $\frac{1}{\omega C}$ can be represented as a vector which is $-90°$ out from the resistance and, hence, oppositely directed from the inductive reactance, ωL, previously treated. The power supplied and dissipated is

$$P = EI \cos \theta = EI\, \alpha\, \theta$$

as before.

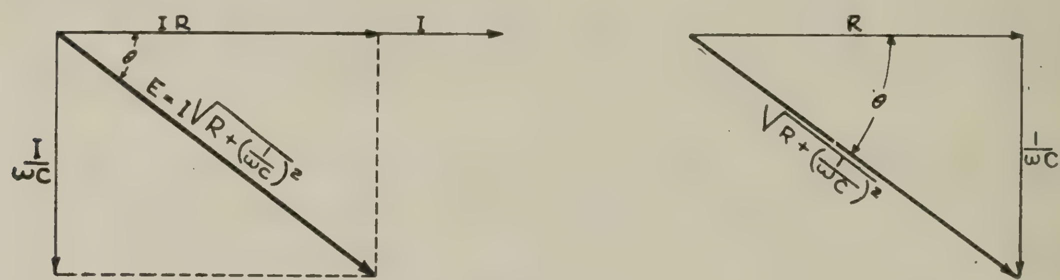


FIG. 117.

Dielectric loss. In the treatment of a **perfect condenser** without series resistance it was shown that the current was in advance of the emf by 90°. Such a condenser cannot be realized in practice though a condenser which has air as a dielectric and has very low resistance leads and plates and a very high insulation resistance between the plates approximates a **perfect condenser** very closely. Normally, a condenser will show a phase angle somewhat less than 90°, usually because of dielectric loss. Such a condenser will absorb power and is called an **absorbing condenser.** If the dielectric is poor, the departure from the 90° phase angle will be considerable and the power taken by the condenser can cause it to heat up. Frequently, the term **phase difference** is used to denote the difference between the actual phase angle of a condenser and the ideal 90° angle. Thus, the **phase difference** ψ is

$$\psi = 90° - \theta$$

By trigonometry

$$\sin \psi = \sin (90° - \theta) = \cos \theta$$

Hence $\sin \psi = \alpha =$ power factor.

Also $\tan \psi = \cot \theta = \omega CR.$

For small angles $\psi = \omega CR$

At a given frequency, therefore, a condenser with dielectric loss will dissipate the same amount of power and can be replaced by a perfect condenser of the same capacity but having a certain series resistance R which gives it the same phase difference as the absorbing condenser. The phase difference of the latter combination will, however, increase

in proportion to the frequency if R is constant, since ψ is proportional to ω. Now it is an experimental fact that the phase difference of a condenser with dielectric loss does not vary considerably with the frequency, in fact, at high frequencies, the phase difference is generally very constant. In order, therefore, to represent the absorbing condenser by a perfect condenser of the same capacity with a series resistance R, the value of R would have to vary with the frequency. Since $R=\frac{\tan\psi}{\omega C}$, it is clear that if ψ is constant, the value of R will have to vary inversely with ω or with the frequency and, therefore, directly with the wave length. On this basis, therefore, the resistance of a mica or glass condenser will increase with increasing wave length and in proportion to the wave length. If, therefore, the resistance is R_1 at a wave length λ_1, the resistance R_2 at a wave length λ_2 will be

$$R_2=R_1\frac{\lambda_2}{\lambda_1}$$

Example:

A leyden jar has a capacity of 0.002 microfarad and a phase difference of 2°. How much will the insertion of this jar in a series in a circuit increase the resistance of the circuit at wave lengths of 600 meters and 1000 meters?

Solution:

Formula $$\tan\psi=\omega CR$$

or $$R=\frac{\tan\psi}{\omega C}=\frac{\psi}{\omega C}\text{ approx.}$$

From the tables tan 2° = 0.03492. Also 2° = 0.03491 radians. Hence, for an angle this small, $\tan\psi=\psi$ very closely. For 600 meters wave length, $\omega=3.14\cdot10^6$.

Substituting $$R_1=\frac{3.49\cdot10^{-2}}{3.14\cdot10^6\times2\cdot10^{-9}}=5.56$$

hence $R_1=5.56$ ohms (600 meters)

Formula $$R_2=R_1\frac{\lambda_2}{\lambda_1}$$

substituting $$=5.56\frac{1000}{600}$$

whence $R_2=9.27$ ohms (1000 meters)

An antenna is a condenser having, primarily, air as a dielectric and should therefore be a perfect condenser excepting for the ohmic resistance of the conductors and ground and the so-called radiation resistance. In general, however, dielectric losses do occur in an antenna because of poor dielectrics in its field, which may be the antenna insulators, trees, buildings etc. This is shown in a curve of antenna resistance against wave length by the long wave portion of the curve which,

because of dielectric losses exhibits a linear increase in resistance with increasing wave length.

Series condensers. When several condensers are put in series in a circuit, the resistances are additive in determining the total resistance and likewise the sum of the reactances of the separate condensers gives the total reactance of the circuit. The impedance is then calculated as if the circuit consisted of a single resistance and capacity. Suppose several condensers, C_1, C_2, C_3,—are in series and that the combined effect is equivalent to a single condenser C. Then the reactance of C must equal the sum of the reactances of C_1, C_2, C_3—. Thus

$$\frac{1}{\omega C}=\frac{1}{\omega C_1}+\frac{1}{\omega C_2}+\frac{1}{\omega C_3}$$

or

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

Thus the capacities themselves are not additive to give the equivalent single capacity but the sum of the reciprocals of the separate capacities gives the reciprocal of the resultant capacity.

Example:

What is the equivalent capacity of three condensers, of 0.001, 0.002 and 0.004 microfarad when connected in series?

Solution:

Formula $$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

substituting $$=\frac{1}{1\cdot 10^{-9}}+\frac{1}{2\cdot 10^{-9}}+\frac{1}{4\cdot 10^{-9}}$$

$$=\frac{7}{4\cdot 10^{-9}}=\frac{1}{\frac{4}{7}\cdot 10^{-9}}$$

whence $$C=0.00057 \text{ microfarad.}$$

This is smaller than the smallest of the separate condensers, as is always the case.

Series circuit with resistance, inductance and capacity. A series circuit of resistance, inductance and capacity is the usual radio frequency circuit and is therefore of prime importance. In figure 118 is shown the emf diagram and impedance triangle. In the emf diagram, the three components of the impressed emf E are the emf IR in phase with the current, the emf $I\omega L$ which is 90° in advance of the current and the emf $\frac{I}{\omega C}$ which lags 90° behind the current. The resultant of the emfs $I\omega L$ and $\frac{I}{\omega C}$ is the vector $I\left(\omega L-\frac{1}{\omega C}\right)$. This combines with IR to give

E. It results, therefore, that

$$E^2 = (IR)^2 + I^2 \left(\omega L - \frac{1}{\omega C}\right)^2$$

or

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

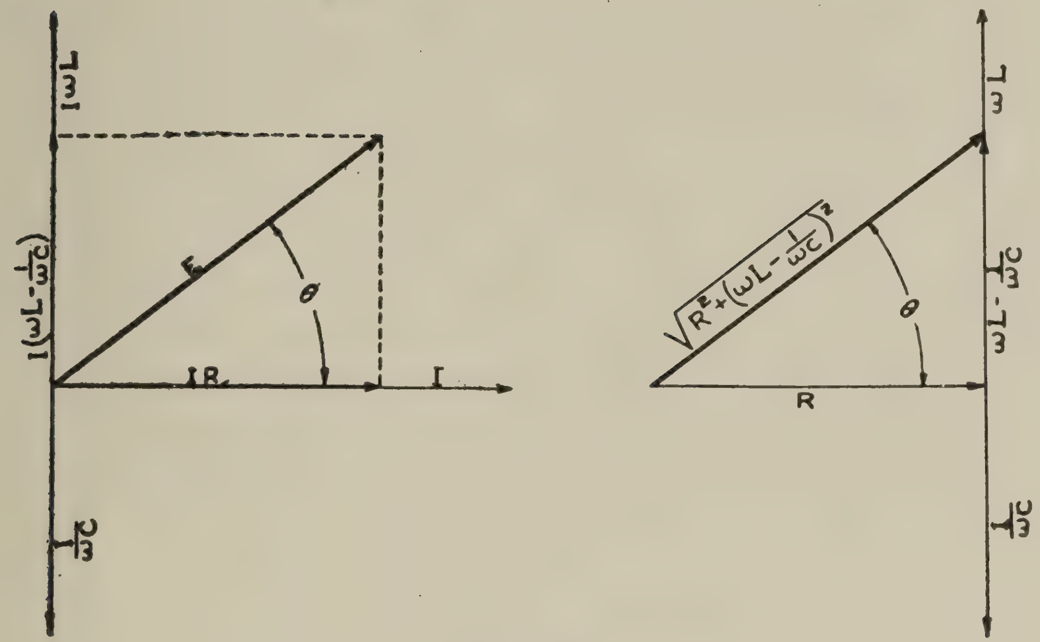


FIG. 118.

and

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Again the quantity $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2}$ is called the impedance.

In the impedance triangle, the resultant reactance $\left(\omega L - \frac{1}{\omega C}\right)$ is combined vectorially with the resistance R to give the circuit impedance. In case $\omega L = \frac{1}{\omega C}$ then the resultant reactance $\left(\omega L - \frac{1}{\omega C}\right)$ becomes equal to zero. In this case, no component of the impressed emf is required to overcome the reactance of the circuit, but the impressed emf is balanced entirely by the IR drop. This is called the **condition of resonance.** The expression $\omega L = \frac{1}{\omega C}$ which determines resonance can also be written

$$\omega^2 LC = 1$$

or

$$\omega^2 = \frac{1}{LC}$$

The previous equations for the current I and phase difference θ become, when $\left(\omega L - \frac{1}{\omega C}\right)$ is made equal to zero,

$$I = \frac{E}{R}$$

$$\theta = 0$$

Thus, when the circuit is in resonance with the impressed emf the current is determined by Ohm's law just as in the case of a circuit with resistance only. The current is also in phase with the impressed voltage and the **power factor is unity.**

The counter emfs of inductance and capacity exactly neutralize each other. **These emfs still exist,** however, **and can be many times as great as the impressed emf.** In the customary receiving circuit, the voltage across the condenser in the resonant circuit is applied to the input of a radio-frequency amplifier or the detector. This voltage can be very large as compared with the emf impressed in the circuit. Also the voltage between an antenna and ground is this high resonant voltage and the insulation of the antenna must be designed for this voltage. The voltage across the capacity is

$$V_C = \frac{I}{\omega C}$$

but the current is

$$I = \frac{E}{R}$$

whence

$$V_C = \frac{E}{\omega CR}$$

Hence, the ratio of the voltage across the condenser to the emf impressed upon the circuit is

$$\frac{V_C}{E} = \frac{\frac{1}{\omega C}}{R} = \frac{X_C}{R}$$

or since

$$\omega L = \frac{1}{\omega C}$$

$$\frac{V_C}{E} = \frac{\omega L}{R} = \frac{X_L}{R}$$

The rise in resonance voltage is therefore greater, the lower the resistance of the circuit.

Example:

What will be the ratio of the resonant voltage to the impressed emf at 800 meters wave length for a coil having an inductance of 3.4 mh and a resistance of 3.2 ohms, assuming the condenser to be perfect?

Solution:

$$\omega = 2.36 \cdot 10^6 \text{ for 800 meters wave length.}$$

Formula $$\frac{V_C}{E} = \frac{\omega L}{R}$$

substituting $$= \frac{2.36 \cdot 10^6 \times 3.4 \cdot 10^{-3}}{3.2}$$

whence $$\frac{V_C}{E} = 2510$$

The voltage across the condenser in this resonant circuit will be 2510 times the emf impressed upon the circuit.

CHAPTER VIII. PARALLEL COMBINATIONS

In the case of series combinations just treated, the current through each of the elements of the circuit is the same but the voltages across the elements may be different. When there are two or more parallel paths for the current to flow between two points in a circuit and the impressed emf is external to the portion of the circuit between the two points, the emf between the two points will be a definite quantity and is therefore the same for all of the parallel paths, but the current in the various paths may be different. These currents will however, add up vectorially to the value of the current which is flowing in the external circuit.

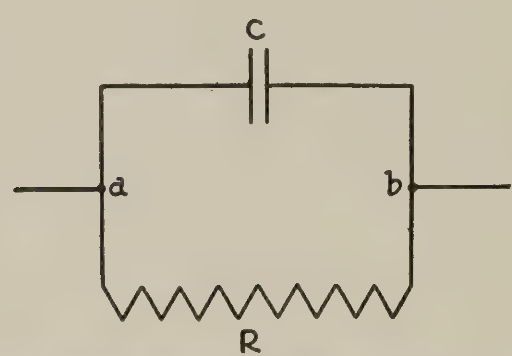


FIG. 119.—Circuit with Resistance and Capacity in Parallel Between Points a and b.

Circuit with resistance and capacity in parallel. Suppose that a condenser having a capacity C and a resistance R are in parallel between the points a and b as in figure 119. Assume the emf across the parallel combination to be the vector E, as in figure 120. The current I_C in

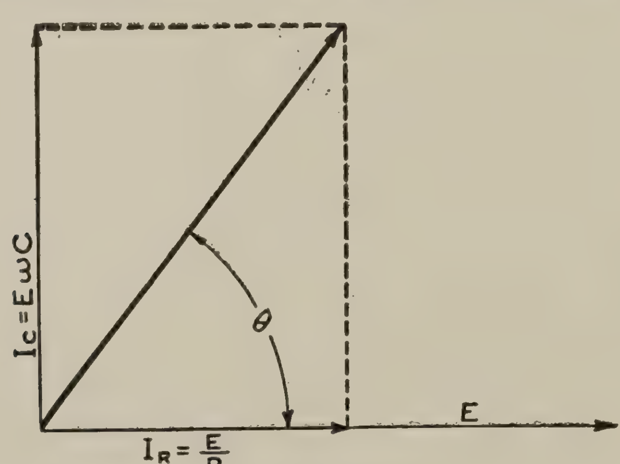


FIG. 120.

the condenser will be 90° in advance of the voltage E and will have a value $I_C = E\omega C$. The current I_R in the resistance R will be in phase with E. The current I, which is the current in the external circuit,

will be the resultant of I_C and I_R. It will be in advance of the emf by the angle θ.

From the figure, the current I is

$$I = E\sqrt{\frac{1}{R^2} + \omega^2 C^2}$$

and the phase angle θ is given by

$$\tan\theta = \frac{\omega C}{\dfrac{1}{R}} = \omega CR$$

The term $\sqrt{\dfrac{1}{R^2} + \omega^2 C^2}$ is called the **admittance** of the parallel circuit. corresponding to the conductance g in Ohm's law when this law is written $I = Eg$. The reciprocal of the admittance gives the impedance of the circuit.

The above parallel combination can be taken to represent a leaky condenser, or one which has appreciable leakage through the insulation between the two sets of plates, or it can represent an antenna with defective insulation. The resistance R in shunt with the condenser causes a power loss which is given by

$$P = EI\cos\theta$$

or since

$$I\cos\theta = I_R = \frac{E}{R}$$

$$P = EI_R = I_R^2 R = \frac{E^2}{R}$$

Such a leaky condenser can be considered as the equivalent of a perfect condenser with a series resistance. The capacity C_e of the perfect condenser and the series resistance R_e would have to be such as to give the same impedance and phase angle as the actual condenser. It can be readily shown that these values are

$$R_e = \frac{R}{1 + \omega^2 C^2 R^2}$$

and

$$C_e = C\left(1 + \frac{1}{\omega^2 C^2 R^2}\right)$$

These are the values which would be obtained for the capacity and increase in circuit resistance due to leakage in an antenna resistance measurement. Normally, R would be a high resistance, high as compared with the reactance $\dfrac{1}{\omega C}$. In this case, the apparent capacity of the antenna would be unchanged, for the term $\dfrac{1}{\omega^2 C^2 R^2}$ would be very small. The expression for R_e would become

$$R_e = \frac{1}{\omega^2 C^2 R^2}$$

since $\omega^2C^2R^2$ in the denominator of the expression for R_e would be large as compared to unity. Hence the increase in resistance of the circuit is inversely proportional to the leak resistance. For a constant leak resistance, the increase in resistance of the circuit due to leakage would be inversely proportional to ω^2 or directly proportional to the wave length squared. In a curve of antenna resistance against wave length, the resistance at the longer waves would increase rapidly with the wave length were the leakage appreciable, very much more rapidly than the linear rate of increase which is characteristic of dielectric loss. Normally, rather low values of leakage resistance are required to produce appreciable effect. Thus, with an antenna of 0.001 microfarad capacity and at a wave length of 2,000 meters the leakage resistance would have to be as low as one megohm in order to increase the antenna resistance by about one ohm.

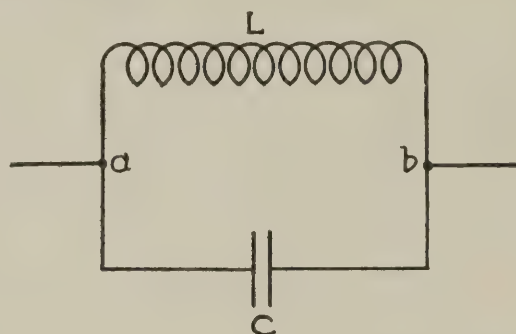


FIG. 121.—Circuit with Inductance and Capacity in Parallel Between Points a and b.

Circuit with inductance and capacity in parallel. Assume the inductance L and capacity C of figure 121 to be of neglible resistance and in parallel between the points a and b. In figure 122, E is the voltage between a and b, $I_C = E\omega C$ is the current in the condenser branch and $I_L = \dfrac{E}{\omega L}$ the current in the inductance branch. If, as shown in the figure, I_C is greater than I_L, the resultant current I which flows in the external circuit will be $I_C - I_L$ and will be 90° in advance of the voltage. On the other hand, if the frequency were reduced so as to make I_L greater than I_C, the current I would lag 90° behind the voltage E. There will be a frequency at which $I_C = I_L$, in which case under the assumed condition of zero resistance in the inductance and capacity, no current would flow in the external circuit, the currents I_L and I_C being 180° out of phase with each other and adding vectorially to zero. When the current I_L is a maximum and directed from a to b, the current I_C will be a maximum and directed from b to a so that the current will flow around the circuit in the same direction or circulate around it. The condition for $I_L = I_C$ is

$$\omega C = \frac{1}{\omega L}$$

or

$$\omega^2 LC = 1$$

This is the condition for **parallel resonance** and it will be noted that it is the same as that for series resonance.

If the frequency is considerably higher than that for resonance, the current I_L will be very small compared with I_C; in fact, the circuit will be nearly the equivalent of the condenser branch alone. On the other hand, if the frequency is considerably lower than the resonant frequency, the current I_L will preponderate and the condenser could

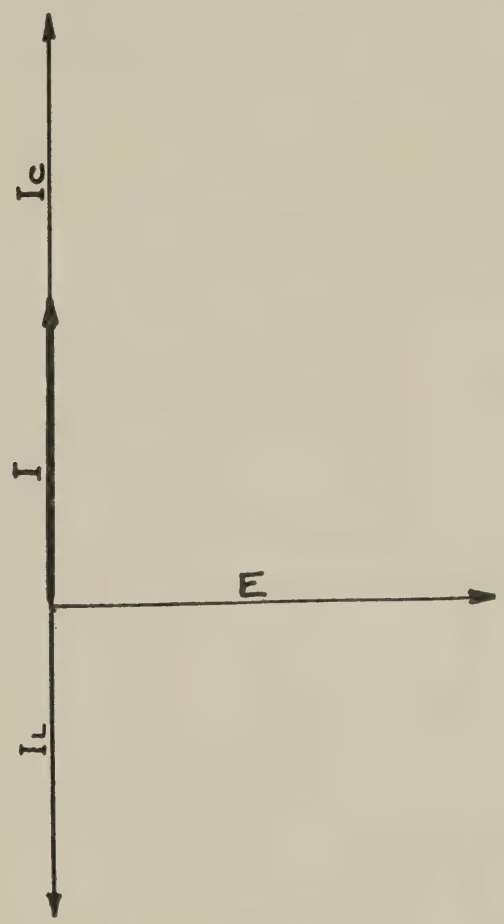


FIG. 122.

be removed without appreciably affecting the current in the external circuit. Increasing the capacity C and decreasing the inductance L tends to make the currents I_C and I_L heavier. At resonance, the two currents still neutralize each other so that the current in the external circuit is zero; but on either side of resonance a very large current will flow in the external circuit due to the preponderance of either I_C or I_L. Thus, such a combination of large condenser and small inductance in parallel serves effectively to stop the flow of a current in the external circuit having the resonant frequency, but will readily permit the flow of current of other frequencies even though only slightly different in frequency. This is the basic principle underlying the operation of the so-called rejector circuit which is used to reduce interference.

In actual practice it is of course impossible to realize circuits of zero resistance. In general, the resistance plays an important part only

near resonance. The resistance of the condenser branch can be made practically negligible, but the resistance of the inductance branch will be appreciable. Such a circuit is represented in figure 123, where R

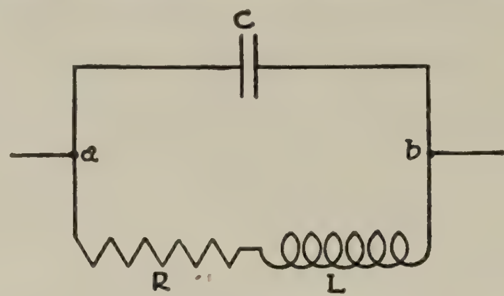


FIG. 123.

represents the resistance of the inductance L. The current I_L through this branch will, as shown before, be

$$I_L = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}$$

This current will lag behind the voltage by the angle given by

$$\tan\theta = \frac{\omega L}{R}$$

The vector diagram for resonance will be that of figure 124. The current I_L lags by an angle θ behind the emf E. The current I_C is 90° in

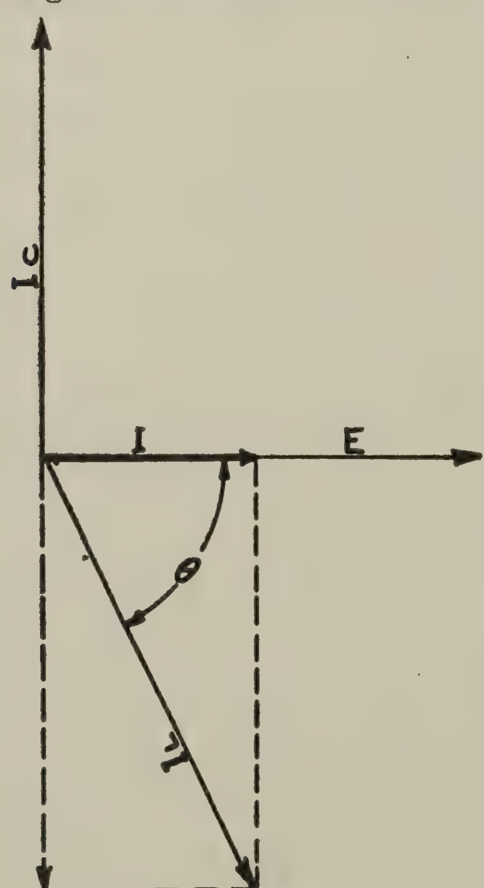


FIG. 124.

advance of E. If I_L is split into two components, one perpendicular to E and one in phase with E, the component perpendicular to E will, in the case of resonance, be equal to I_C and will neutralize I_C. The resultant current I will be the component of I_L which is in phase with E. Hence, the current in the external circuit will be small and in phase with the emf and thus the parallel circuit behaves as a very high resistance.

The component of I_L which is in phase with E will be $I_L \cos\theta$, that which is in quadrature will be $I_L \sin\theta$. Referring to figure 112, it is evident that

$$\cos\theta = \frac{R}{\sqrt{R^2+\omega^2L^2}}$$

and

$$\sin\theta = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}$$

thus

$$I_L \cos\theta = \frac{ER}{R^2+\omega^2L^2}$$

and

$$I_L \sin\theta = \frac{E\omega L}{R^2+\omega^2L^2}$$

For resonance $I_L \sin\theta$ must equal I_C, hence,

$$\frac{E\omega L}{R^2+\omega^2L^2} = E\omega C$$

or

$$\frac{1}{R^2+\omega^2L^2} = \frac{C}{L}$$

In this case the resultant current

$$I = I_L \cos\theta = \frac{ER}{R^2+\omega^2L^2}$$

Substituting $\frac{C}{L}$ for $\frac{1}{R^2+\omega^2L^2}$ the equation

$$I = \frac{ERC}{L} = \frac{E}{\frac{L}{RC}}$$

is obtained. The parallel circuit therefore behaves as if it had an effective resistance R_e given by

$$R_e = \frac{L}{RC}$$

Thus the parallel combination will offer a higher resistance to resonant currents flowing in the external circuit, the lower the resistance of the inductance. As pointed out above, the use of a large capacity and small inductance in a rejector, permits currents of frequencies other than the resonant frequency to flow readily through the rejector circuit. The

rejector must, however, oppose the flow of currents of the resonant frequency. This opposition is determined by the value of R_e. It will be noted that a large C and small L leads to a low value of R_e. It is necessary, therefore, that R be very small in order to make R_e sufficiently large. This is the explanation of the requirement of a heavy conductor of low resistance for the inductance L as used in the rejector.

Circuits with condensers in parallel. Suppose first that two condensers C_1 and C_2, which are each perfect condensers, are connected in parallel. The current I_1 through the condenser C_1 will be $I_1 = E\omega C_1$ if E is the applied emf. The current I_2 will be $I_2 = E\omega C_2$. Both currents will lead the voltage by 90°. The resultant current will be merely the sum of I_1 and I_2 thus,

$$I = I_1 + I_2 = E\omega(C_1 + C_2)$$

This is the same current that would flow into a single condenser of capacity $C = C_1 + C_2$. Hence, the capacity of condensers connected in parallel is additive.

Suppose next that a small resistance R_1 is in series with C_1 while another small resistance R_2 is in series with C_2. Or the problem will be the same if C_1 is considered to be a condenser with dielectric loss and having a phase difference

$$\psi_1 = \omega C_1 R_1$$

and, correspondingly, C_2 has a phase difference

$$\psi_2 = \omega C_2 R_2,$$

the phase difference in each case being a small angle. The currents I_1 and I_2 will be

$$I_1 = \frac{E}{\sqrt{R_1^2 + \dfrac{1}{\omega^2 C_1^2}}}$$

and

$$I_2 = \frac{E}{\sqrt{R_2^2 + \dfrac{1}{\omega^2 C_2^2}}}$$

but, since R_1 and R_2 are small

$$I_1 = E\omega C_1$$
$$I_2 = E\omega C_2$$

The components of I_1 and I_2 in phase with E will be $I_1 \sin \psi_1$ and $I_2 \sin \psi_2$ but, since ψ_1 and ψ_2 are small angles these components will be $I_1 \psi_1$ and $I_2\psi_2$. The total current in phase with E will be $I_1\psi_1 + I_2\psi_2$. The components at right angles will be $I_1 \cos \psi_1$ and $I_2 \cos \psi_2$ which are approximately I_1 and I_2; hence, the total quadrature current will be: $I_1 + I_2$. The phase difference of the combination will be ψ where

$$\tan \psi = \frac{I_1\psi_1 + I_2\psi_2}{I_1 + I_2}$$

or substituting for I_1 and I_2,

$$\psi = \frac{C_1\psi_1 + C_2\psi_2}{C_1 + C_2}$$

Hence, the phase difference of two condensers in parallel is what may be called the **weighted mean** of the phase differences of the separate condensers, that is, the capacity of each condenser is multiplied into its phase difference, these products are added and the result divided by the sum of the capacities. This law holds for any number of parallel condensers, provided the phase difference of each is small. The capacity of the parallel combination will be very nearly the sum of the capacities of the separate condensers. In terms of resistances, assume R to be the resistance of the parallel combination, from which $\psi=\omega CR$, where $C=C_1+C_2$. Substituting in the above formula for ψ, then

$$\omega CR=\frac{\omega C_1^2R_1+\omega C_2^2R_2}{C}$$

and

$$R=\frac{C_1^2R_1+C_2^2R_2}{C^2}$$

It is of interest to apply the above results to a variable air condenser. In such a condenser, the capacity through the insulating bushings etc., can usually be considered to be a fixed capacity having a certain dielectric loss. The capacity through air is the variable portion and is a perfect capacity. The variable condenser can therefore be regarded as a fixed condenser C_1 with a constant phase difference ψ_1 in parallel with a variable condenser C_2 for which $\psi_2=0$. Substituting in the above equation for ψ, the phase difference of the variable air condenser is

$$\psi=\frac{C_1\psi_1}{C}$$

and its resistance is

$$R=\frac{C_1^2R_1}{C^2}$$

When the setting of the variable is changed, C is varied but C_1 and ψ_1 remain unchanged. Thus, **the phase difference ψ of a variable air condenser is inversely proportional to the capacity.** If the wave length is kept constant, R_1 will also be constant. Thus, **at a given wave length, the resistance of a variable air condenser will be inversely proportional to the square of the capacity.**

At a given setting of the variable air condenser, **its resistance will increase in proportion to the wave length,** as pointed out before, the resistance R being given by

$$R=\frac{\psi}{\omega C}$$

These laws permit the resistance of a variable air condenser to be estimated for any wave length or capacity setting when its resistance is known at any one wave length and setting. Thus, if a variable air condenser at 1,000 meters and 1,000 micro-microfarads capacity has a resistance of 0.1 ohm, it will have 10 ohms resistance at 100 micro-

microfarads setting, 1,000 meters wave length, and 20 ohms resistance at 100 micro-microfarads setting and 2,000 meters wave length. It is clear that at low capacity settings and long waves, the dielectric loss in a variable air condenser becomes very important.

When a coil is connected across a condenser in a radio circuit, the coil capacity is put in parallel with the condenser. Usually the capacity of a coil shows considerable dielectric loss, which is occasioned by the coil form and the insulation on the wire. This capacity can be considered to be a fixed absorbing capacity and, hence, produces effects similar to the imperfect part of the capacity of a variable air condenser. With low condenser settings, even when the variable air condenser has neglible losses, the resistance of the circuit will be increased on account of the loss in the coil capacity. Thus, a coil which is to be used with a small capacity across it, should be designed to have both low capacity and small dielectric losses. Coils such as the spider-web or other types in which the coil form is a frame work and the turns are separated from each other with air between give the best results when used with very low condenser capacities.

CHAPTER IX. TRANSFORMERS.

General types and purpose. In radio practice, transformers are widely used for various purposes. They are of the step-up type when used with the spark type of transmitter since it is desirable to charge a condenser, such as that used in an oscillatory circuit at a high potential, generally about 15,000 volts. Transformers used in vacuum-tube circuits are of both the step-up and step-down types. The step-up type serves to increase the applied voltage to a value suitable for a vacuum tube plate supply. The step-down type is used to supply current suitable for lighting the filaments of the tubes. Transformers, in receiving circuits or amplifiers are used to transfer energy from one circuit to another where the impedances of both circuits are different, and an output voltage of the transformer is desired of different value from the input voltage. In commercial light and power practice the problem of generating power at one central plant and then distributing the power to a local community or, in some cases, generating power at a hydroelectric station and distributing power to cities many miles distant, has been solved and made practical by the use of high voltages. High voltage is necessary for the transmission of power, since power is equal to voltage multiplied by current; and power loss is equal to the resistance of the circuit multiplied by the current squared. It is impossible to increase the size of a conductor beyond limits set by strength of suspension and reasonable cost, and since the loss depends on the current squared, it is advisable to keep the current value low. For a given power any reduction in current value must be accompanied by a voltage increase; hence it is evident that high voltage is desirable for power transmission. In the case of direct current, rotative machinery must be used, and the voltage is very limited due to difficulties experienced with commutation.

When alternating currents are dealt with, the problem becomes comparatively simple since it is possible, by means of mutual induction, to transfer electric energy from one circuit to another, the ratio of the number of turns of magnetically coupled windings determining the amount by which the applied emf is stepped up or stepped down. A mutual induction device for changing a voltage or current value is called a **transformer.** Such a device is shown in the elementary diagram, figure 125.

This diagram shows a two-circuit transformer in its simplest form, consisting of a circular iron core, a primary winding and a secondary winding.

The **primary winding,** for a step-up transformer, is usually made up of a small number of turns of very low resistance. If the iron core were

not present, (in a low-frequency type), the primary winding would have no reactance except that due to its resistance, and since this resistance value is very low, the transformer would form a short circuit if placed directly across the terminals of a generator. The presence of the iron core, when the secondary winding is open, causes the primary to become

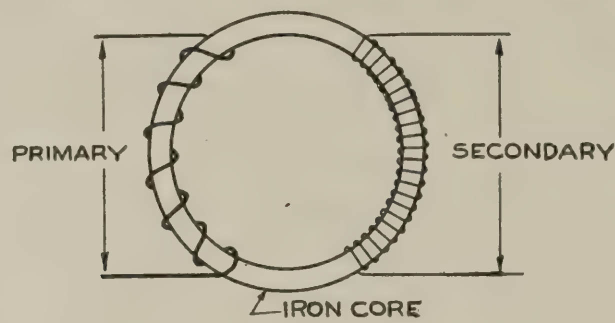


Fig. 125.

a self inductance of such a value that for the frequency used, the counter emf of self-induction will equal the impressed emf. Under this condition, if the primary is connected directly across the terminals of a generator, with the secondary circuit open, only a small lagging current will flow. This current represents a small power loss. It is made up of two components. The larger of the two, which serves to set up the

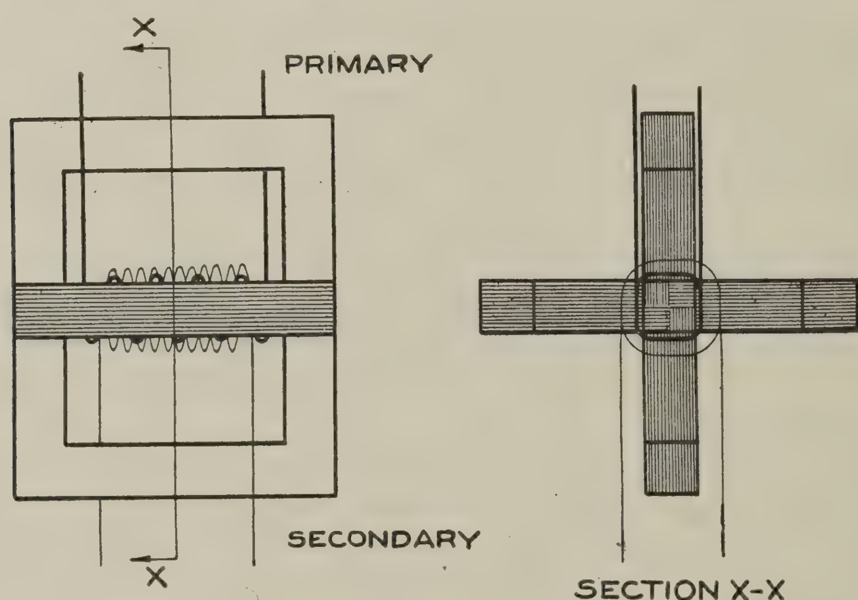


Fig. 126.—Shell-Type Transofrmer.

magnetic flux, is wattless and lags the applied emf by 90°; the other, which is in phase, represents power loss since it supplies the energy used up as I^2R, hysteresis and eddy current losses.

The core is usually made up of a laminated steel section and is of such size and shape that the proper charging current will flow when the secondary circuit is open and the desired amount of energy will be transferred to the secondary circuit when it is closed. Cores are,

in general, of either the shell or the core type. The shell type of core is one in which the windings are practically surrounded by a magnetic circuit of iron or steel as shown in Figure 126.

The core type of transformer is one in which the principal branches of a magnetic circuit are embraced by both primary and secondary windings, as shown in figure 127.

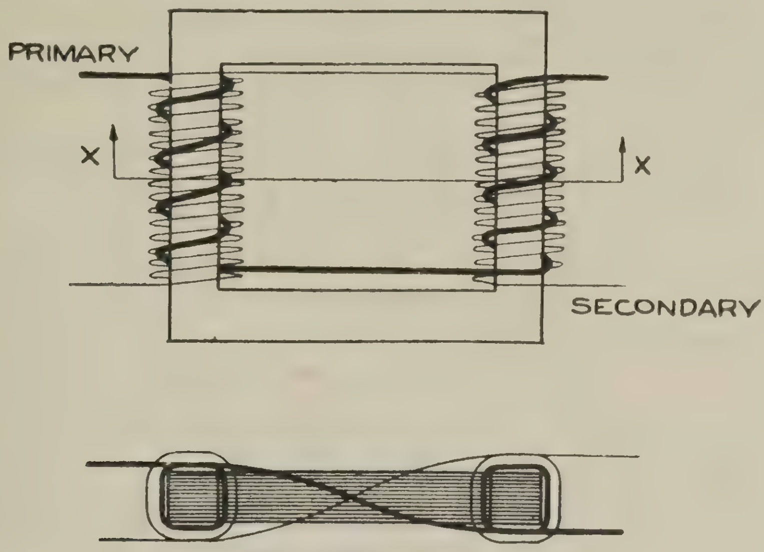


FIG. 127.—Core Type Transformer.

The voltage produced across the secondary terminals when they are open depends upon the ratio of the number of turns of wire in the primary to the number of turns of wire in the secondary and upon whether or not all the lines of force produced by the primary, link all the turns of the secondary. This is true since the voltage induced in each turn of wire, regardless of whether it is in the primary or in the secondary circuit, will depend upon the rate at which the **flux** cuts each turn, the flux being the same for each turn, regardless of the winding of which it is a part. Because of the fact that an alternating current creates this flux, it will appear and disappear once each quarter cycle of the current, and in effect it will be the same as four times the number of lines if the change were but once per cycle. The total number of lines of force which will link each turn per second is therefore

$$\Phi = BS4f$$

where

Φ = total flux in maxwells $= BS$,
B = flux density in maxwells per cm^2,
S = area of core in cm^2.,
f = frequency in cycles of applied emf.

It has previously been stated that when 10^8 lines of force per second link or cut a conductor, one volt will be produced at the terminals of the conductor; therefore, the voltage induced in **one turn** is given by

$$E_{ps} = BS4f \cdot 10^{-8}$$

the total voltage (counter emf) of the primary is

$$E_p = E_{ps} N_p$$

and of the secondary is

$$E_s = E_{ps} N_s$$

where

$$E_p = = \text{voltage of primary}$$
$$E_s = \text{voltage of secondary}$$
$$E_{ps} = \text{volts per turn}$$
$$N_p = \text{turns of primary}$$
$$N_s = \text{turns of secondary}$$

since

$$E_{ps} = \text{is the same for both windings}$$

$$\frac{E_s}{N_s} = E_{sp} = \frac{E_p}{N_p}$$

or

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

This formula shows that when it is desired to lower the current and increase the voltage, the number of turns on the primary must be less than the number of turns on the secondary. If all the lines of force produced by the primary do not cut the secondary, the voltage produced will be of a value less than that given by the above ratio.

Example:

A transformer is so designed that all the lines of force produced by the primary link the secondary; the applied line voltage is 100 volts 60 cycle, the core area is 521 sq. cm. and is to be worked at a density of 8,000 lines cm². Find the output voltage when 100 turns are on the primary and 2300 turns are on the secondary.

Solution:

Formula	$E_{ps} = BS4f \cdot 10^{-8}$ (volts)
substituting	$= 8 \cdot 10^3 \times 5.21 \cdot 10^2 \times 4 \times 6 \cdot 10^1 \cdot 10^{-8}$
whence	$E_{ps} = 1.00$ volt
Formula	$E_p = E_{ps}\ N_p$
substituting	$= 1 \times 1 \cdot 10^2 = 100$
whence	$E_p = 100$ volts
Formula	$E_s = E_{ps}\ N_s$
substituting	$= 1 \times 2.3 \cdot 10^3 = 2{,}300$
whence	$E_s = 2{,}300$ volts.

Performance of secondary. The action which takes place when the secondary is closed may be explained briefly as follows: When the secondary circuit is completed a current will flow, the value of which will be determined by the constants of the secondary circuit, the primary applied emf remaining constant. This flow of current in the secondary circuit will produce a flux which is opposite in direction to that produced by the primary and, consequently, a diminution of the total flux linking

the two circuits. Since the counter emf of the primary depended on the value of the flux, the inductance and impedance of the primary circuit will be diminished with a consequent increase in primary current. This action will continue until the power output of the transformer equals the power input minus a small amount of power which is used in overcoming primary and secondary I^2R losses and hysteresis and eddy current losses. These losses usually are from two to eight per cent of the total power handled, depending on the size of transformer used. Large commercial transformers are about 98% efficient.

Regardless of the use to which a transformer is put or of its design, the winding on the input side is called the primary winding and that of the output side the secondary winding. In any transformer, either winding will act as primary or secondary. Ordinarily, in radio work, the primary winding is the low-voltage side. In power distribution work nearly the same number of step-up transformers are in use as those designed for step-down work.

If the **secondary winding** is open and an emf is applied to the primary winding, the small current which flows, multiplied by the number of primary turns, gives the exciting ampere-turns and represents the number of ampere-turns which are necessary to set up the magnetic flux of the primary and supply the energy which is consumed in eddy current and hysteresis loss. In commercial, low-frequency transformers, the percentage of cross-sectional area occupied by insulating material amounts to from 9 to 15 per cent. For radio-frequency transformers this ratio is much greater.

The **flux density of a transformer** magnetic circuit varies with the core design used but for ordinary calculations the following flux density values are ordinarily used

25 cycles, 12,000 lines per cm²
60 cycles, 8,000 lines per cm²
500 cycles, 2,500 lines per cm²

Although these figures are used in design work, it is permissable to operate a transformer designed for one frequency, at another. In some cases a decrease in operating efficiency will be noted, but if the operating frequency is increased the operating efficiency is liable to increase and the efficiency of installation and material to decrease.

The flux density in a particular transformer varies inversely as the frequency of the impressed emf, and the eddy current loss is proportional to the square of the flux density. Therefore, variation of frequency leaves the eddy current loss substantially unaffected provided the paths through which the eddy currents flow do not change.

The **regulation of a transformer** is defined as **the ratio of the rise of secondary terminal voltage from rated noninductive load to the secondary terminal voltage at rated load.** Regulation is dependent on

primary voltage drop due to leakage reactance and to the resistance in the primary and secondary coils of the transformer.

Leakage inductance is a term applied to that portion of self-inductance of either primary or secondary circuit which appears due to the leakage of lines of force passing through one circuit and not through the other. It may be represented, in effect by inserting a pure self-inductance in either circuit. The reactance caused by this inductive effect is called **leakage reactance.**

Leakage also causes the output voltage of a transformer to vary from that value which would be anticipated from the turn ratio (ratio of transformation).

Auto-transformers. In addition to the type of transformers having two separate windings, there is another type called the **auto-transformer.** This type of transformer has a single winding, a part of which is common to both the primary and secondary circuits, the remainder of the winding serving either as the continuation of the primary or secondary winding according to whether the transformer is used for step-up or step-down purposes. An auto-transformer will be cheaper than an ordinary two winding transformer of the same rated output and efficiency. The saving however, is only apparent when the ratio of transformation is near unity. Auto-transformers are only used when the voltages of both primary and secondary are low or when the ratio is near unity and both are near ground potential. The difficulty arises from the fact that the two circuits are electrically connected and consequently the danger from short circuit is very great.

The **induction coil,** or make and break **spark coil** is, in reality, a special transformer of the open core type. It is always of the step-up

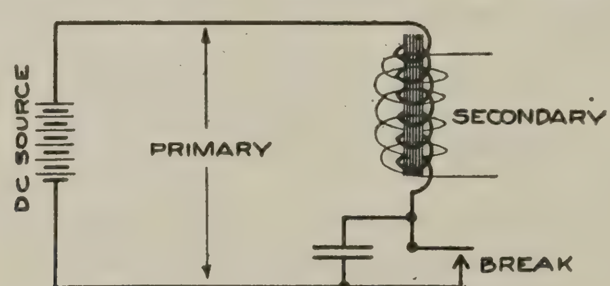


FIG. 128.—The Induction Coil.

design, usually to very high voltages with practically no secondary current or power. From the following diagram, figure 128, it may be seen that the source of supply is direct current. The control of flux variation and, consequently, output frequency and voltage is by means of the interrupter or break.

The interrupter is generally a vibrating armature controlled by the flux of the coil core. The condenser is placed across the interrupter break in order that the voltage induced in the primary winding by the falling magnetic field, when the break is opened, will serve to charge

the condenser and not cause sparking at the contacts. It also serves to absorb the energy which would be used up in sparking. When the contacts are open, the energy is fed back to the circuit in a direction opposite to that of the original current flow. This causes a rapid demagnetization of the iron core.

Frequency changers. Frequency changers are sometimes used for doubling or tripling the available frequency. For radio-frequency purposes, a frequency of some 10,000 cycles is generated and then by means of a transformer arrangement, the generated frequency is either doubled or tripled, corresponding to wave lengths of about 15,000 and 10,000 meters. The system may also be used at lower frequencies but this is seldom done due to the ease with which a generator of low frequencies may be designed.

Frequency doublers and triplers make use of special transformers. For single phase operation, two transformers are used. If it is desired to double the frequency, two transformers containing three windings are used and connected as shown in Figure 129.

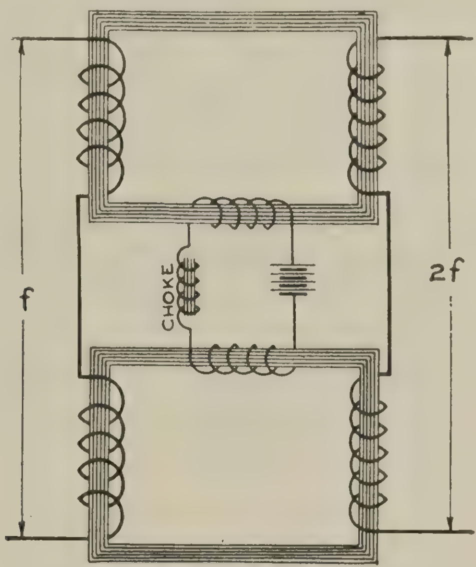


Fig. 129.—Frequency doubler.

The third winding on each transformer is supplied with direct current for the purpose of bringing the core of each transformer to a point near saturation. When the core is under this condition, it is evident that there will only be a flux change in the transformer when the impressed emf is in such a direction that the current forced through the primary winding will produce a flux in opposition to that produced by the dc winding. This change in flux will produce a current in the third or secondary circuit, the frequency of which will be double that of the primary circuit. Exactly the same action will take place in the second transformer during the next half cycle of the primary emf. The windings of the second transformer are so arranged that the current induced in the secondary is a continuation of that produced in the first.

The choke coil is placed in the dc circuit to prevent any reaction between the two circuits. The current of the secondary circuit is generally increased by inserting a condenser in that circuit of such size that the circuit impedance is materially decreased.

Frequency triplers make use of two transformers so designed that the core of one will become saturated when the primary current reaches approximately one-half its maximum value, the other being so designed that for maximum current, the core is far below the saturation point. When the primaries of these transformers are joined in series and in opposition and the secondaries are joined in series and opposition and adjusted for nearly equal emf output, the two transformers will produce a current in the secondary circuit of three times the applied frequency. This effect is strengthened by tuning the secondary circuit and is due to nearly flat wave produced by the saturated core and the peaked wave of the non-saturated core, the two effects combining to form the triple frequency of the secondary circuit.

Energy losses in the transformer copper or iron depend upon the design of transformer used; and the power loss ratio of these losses, or the type and design of core used, are based on experience. The cores of transformers are made up of **transformer steel** which usually has a silicon content of 2 1/2 to 4 per cent. The cores are made up of laminations stamped from sheets of this steel about 0.014 inch in thickness for low-frequency work. For **high frequency** work the **laminations** are stamped from cleaned sheets of about 0.002-inch thickness. It is necessary to provide electrical insulation between these laminations and in order to accomplish this, for low-frequency work, the scale formed during the sheet rolling process is left on the laminations and a coat of insulating varnish added. For radio-frequency work, the scale is removed and the insulating varnish applied.

CHAPTER X. RECTIFIERS.

It is sometimes desirable to provide direct current when the only available source of energy or convenient means of transporting energy is alternating current. The direct current is generally desired for charging batteries, providing plate current for vacuum tubes, etc. If any considerable amount of power is desired, a rotary converter or motor-generator is usually used.

The mercury arc is, in effect, a true rectifier allowing the entire one-half cycle of current to pass through the circuit just as though an automatic switch had been inserted to open and close when the current is at zero value, the switch remaining open while the applied emf is negative. The above action takes place when the mercury arc is supplied with a single anode; when two electrodes are provided, each will act independently and pass both sides of alternating current as shown by the following:

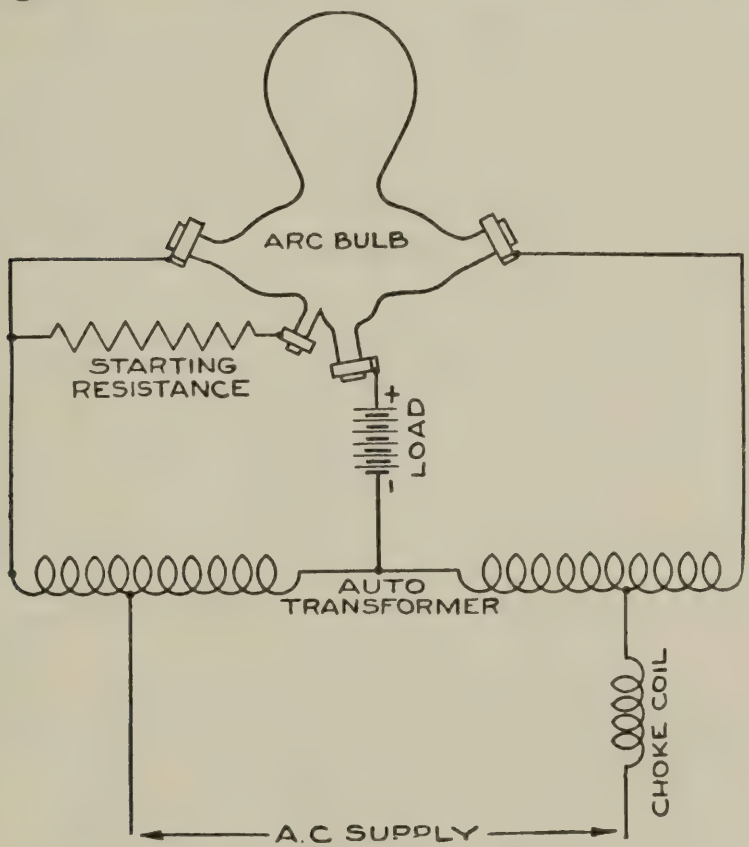


Fig. 130.—Mercury Arc Rectifier.

The mercury rectifier shown in the above diagram consists of a hermetically sealed glass bulb filled with air and provided with four electrodes. The two lower ones are of mercury while the upper two are of graphite or other suitable material. The two upper ones are the anodes; the lower one connected to the load is the cathode, and the one

connected to the starting resistance is the starting electrode. The lower electrodes are not connected when the tube is in a vertical operating position, the bulb being so mounted that a slight tilt will bring these two electrodes in contact for starting. Before starting, there is a high resistance film over all electrodes; that over the anodes is only apparent when the cathode is positive with respect to one of the anodes. The resistance of the mercury pool cathode, when once broken down, will disappear so long as a current flows. Should the current stop for an instant, the arc will extinguish, and must be reestablished. This arc, as has been stated, is established by tilting the tube so that the two pools of mercury unite and form a circuit for current flow; when the tube is again righted the pools will separate, causing a spark which breaks down the resistance and allows a current to flow alternately from the two anodes since, at each instant, the electrodes are at opposite potentials because of the split transformer connection. The reactance coil shown in the supply line serves to regulate the arc for battery charging work. If it were not provided, the voltage applied to the batteries would remain constant, causing a wide variation in the charging current. With the reactance in the circuit, the dc voltage changes as the battery becomes charged and the current flow remains practically constant.

A double-electrode rectifier of this type gives a direct current with a slight ripple but no breaks, as would occur in a single electrode tube. Some tubes are provided with electrically controlled cradles which will automatically establish the arc on first start, or re-establish it should it become extinguished.

The efficiency of the mercury arc is quite high, since there is only a small loss in the ac circuit and only a small resistance loss in the tube. Vacuum-tube and gas filled rectifiers are classed as two-electrode tubes, the theory and detailed operation of which have been described in detail in Chap. IV, Part 10. They are made in various sizes suitable for battery charging and plate current supply for vacuum tubes. The former are generally capable of taking care of loads of from 25 to 1,000 watts and are of the gas filled type and are not very efficient. The latter type is an ordinary two-electrode vacuum tube exhausted to a high vacuum. These tubes are used in radio installations and are made in sizes from 250 watts to 100 kilowatts and for handling voltages from 1,000 to 25,000. The ordinary three element tube is sometimes used as a rectifier, but for the larger sizes considerable expense is involved due to insulation and mechanical difficulties encountered in supporting the third element (grid). If a low or commercial frequency is rectified, the resultant direct current will have to be smoothed out by means of filters and chokes until there is less than one per cent ripple; otherwise the current cannot be used for radio purposes because the carrier wave will be modulated, and, therefore, cause interference due to the side

frequencies of modulation. This precaution is not necessary for battery charging.

The following diagram shows the fundamental circuit of a two-electrode tube used as a rectifier:

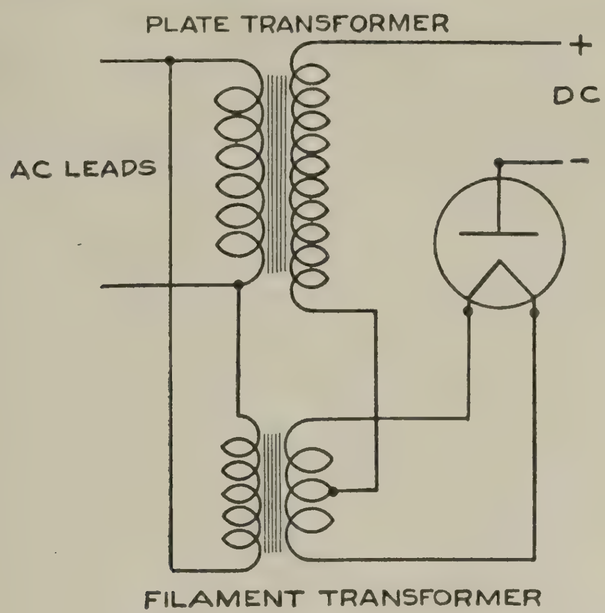


FIG. 131.—The Two-Electrode Tube Rectifier Circuit.

This connection gives an intermittent direct current, since the tube will only pass current when the plate is positive with regard to the filament. This action has been explained in detail in Chapter I, Part 10.

If a continuous current having only a slight ripple is desired, two tubes must be used and may be connected as shown in figure 132:

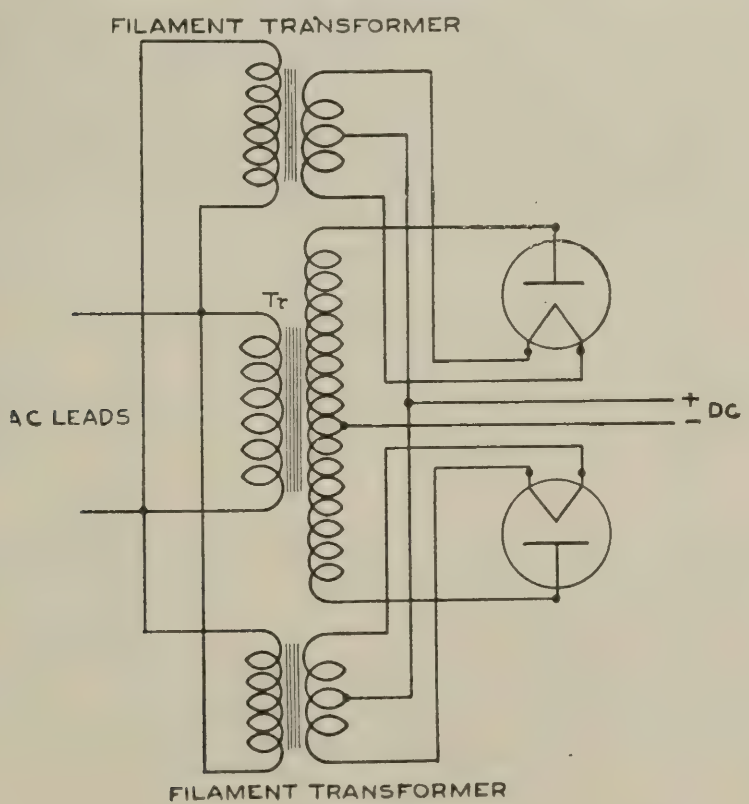


FIG. 132.

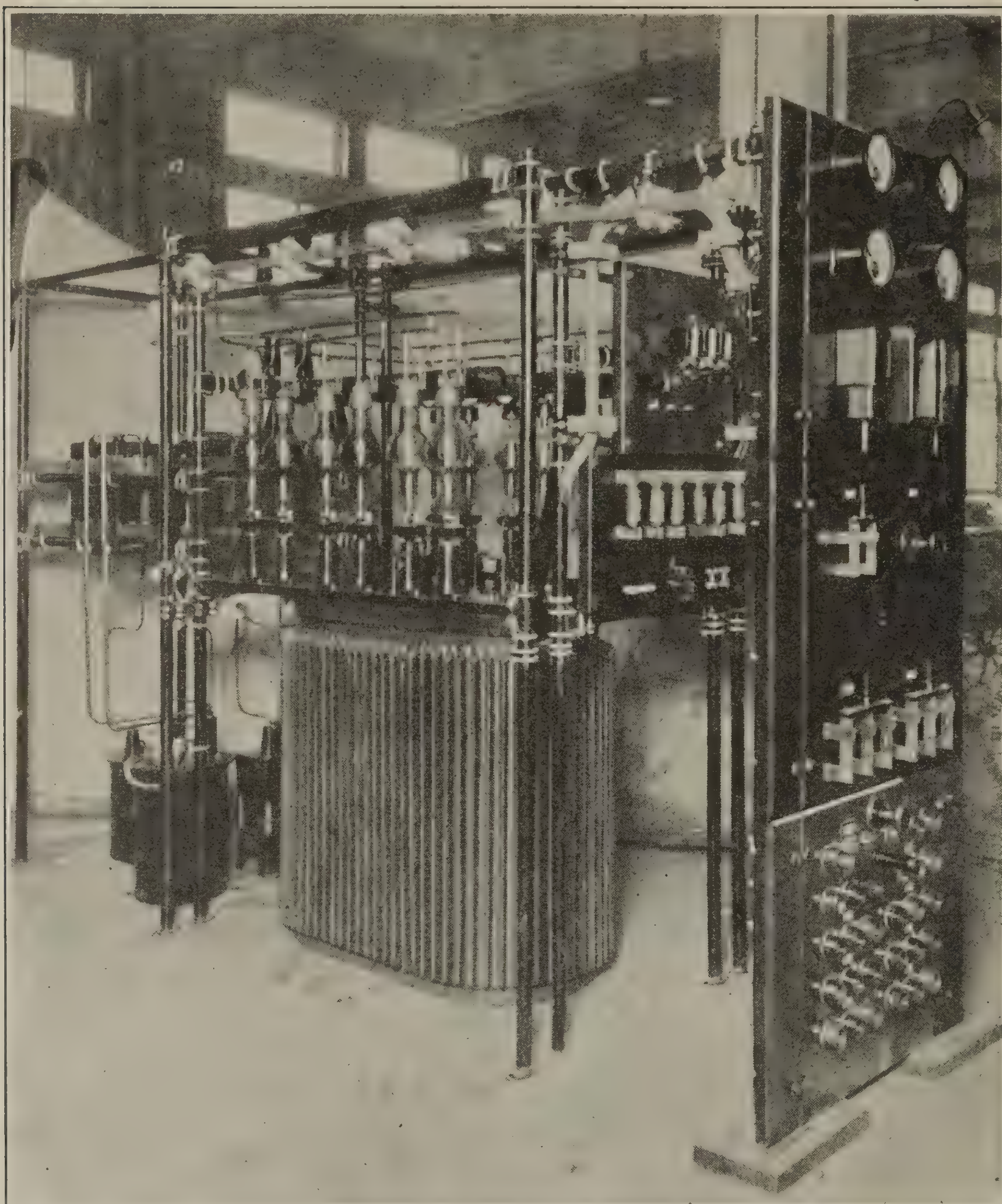


Fig. 133.

High-Power Vacuum-Tube Rectifier.

Figure 133 shows a 30-Kw. 15,000-volt rectifier. This rectifier is designed for converting 6-phase current into direct current at any voltage from 7,500 to 15,000 volts with a ripple of one-tenth of one per cent. It makes use of twelve 2½-Kw. Kenotrons.

A rectifier of this type is automatic and of very good efficiency. The life of the tube is approximately the same as that for a three-electrode tube of equal power rating.

Mechanical rectifiers have been developed for charging small batteries and are very satisfactory for currents small enough to permit a make and break contact in the circuit. The scheme is shown in figure 134.

The device consists of a vibrating reed which alternatingly makes contact with the open ends of the transformer. This reed is a specially designed electrically controlled mechanical device which makes contact at the proper instant. Equipments of this type are made quite rugged and inexpensive as well as efficient.

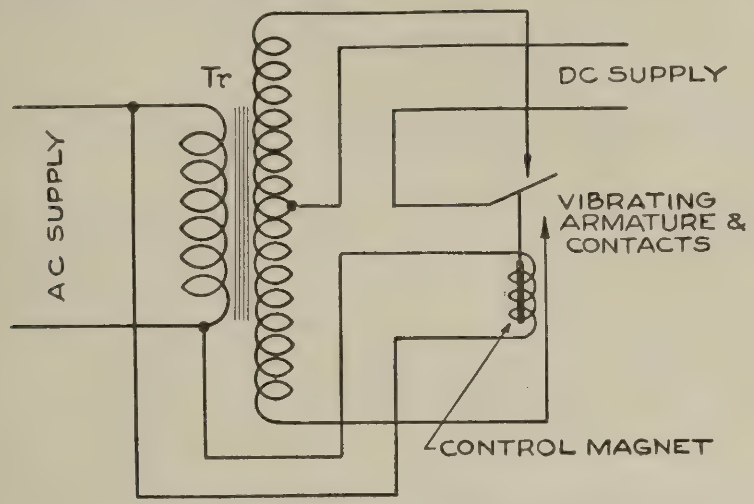


Fig. 134.—Mechanical Rectifier.

Chemical rectifiers have become an important part of low-power radio transmitters where it is not desired to purchase rather expensive high-voltage dc generators, batteries, or use vacuum tubes for the purpose. These rectifiers are generally made up of a number of units, depending on the voltage desired and upon the amount of current to be handled. Since this current value is generally less than one ampere, the area of the electrodes need not necessarily be very large. Each cell usually contains two plates, one of lead and one of aluminum. The solution should be of boric acid, sodium bicarbonate, sodium phosphate or some other such chemical. The solution should be saturated and a slight amount of ammonia added. Only distilled water should be used in making up the solution or as replacement for evaporation. One method of connecting the cells is given in figure 135. Not over 40 volts per cell should be allowed and under any conditions the adjustments should be such that there will only be a slight glow on the plates during operation.

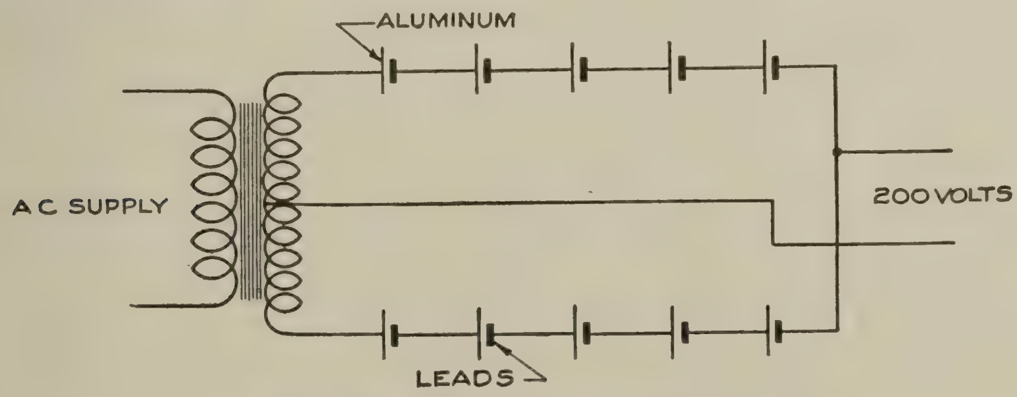


Fig. 135.—Lead-aluminum Rectifier.

When the cells are first assembled, it is necessary to **form** the cell; this is done by passing approximately one-half ampere direct current through each cell for a short time. It is also possible to form the cells by passing a small alternating current through each cell.

PART 4.

APPLICATION OF ALTERNATING-CURRENT THEORY TO RADIO CIRCUITS

CHAPTER I. SIMPLE SERIES CIRCUITS.

The principles of alternating-current theory given in Part 3 are directly applicable to those frequencies which are above the audio or commercial limit of 10,000 cycles.

These radio frequencies are those which fall within a band from 10,000 cycles per second to a present practical upper limit of about 3,000,000. Laboratory methods have produced frequencies as high as $3 \cdot 10^{10}$ cycles per second.

The importance of inductive and capacitive reactances at radio frequencies is clearly demonstrated by the fact that a few turns of conductor, which at low frequencies might act in a circuit only as a conductor, will act at radio frequencies as a high reactance, and a small capacity which would act as an open circuit in a low-frequency circuit, will act as a path of low resistance at radio frequencies.

In the simple series circuit containing inductance and resistance, it is probable that the reactance produced by the inductance at radio frequency will be greater than the resistance of the circuit. It should be remembered, however, that the resistance value of the circuit is probably greater at radio frequencies than it is for direct current, due to **skin effect.**

The inductance takes on a large reactive value at radio frequencies because of the extremely rapid rate of change of the lines of magnetic force threading the inductance. This rate of cutting lines of force and the self-induced emf (reactance) are proportional to the frequency. Example:

The following table gives an example of the preponderance of the reactance of a coil over its resistance as the frequency is increased. The resistance R of an inductance coil of $1.88 \cdot 10^{-4}$ henries was measured at the various frequencies given in the first column. Its reactance

f	λ	X_L	R	X_L/R
60	$5 \cdot 10^6$	0.068	0.2	0.34
10,000	3.10^4	11.3	0.3	37.6
100,000	$3 \cdot 10^3$	112.8	1.0	112.8
300,000	$1 \cdot 10^3$	339	2.5	135.4

was calculated by the formula $X_L = 2\pi fL$. The last column gives the ratios of X_L to R.

It is seen that this ratio increases from .34 at 60 cycles to 135.4 at a frequency of 300,000 cycles.

Similarly the reduction in the reactance of a capacity with increasing frequency is shown in the table below. The condenser was assumed to have a capacity C of .005 μf and its reactance calculated by the formula $X_L = \frac{1}{2\pi fC}$. The reactance falls from a value of $5.32 \cdot 10^5$ ohms at 60 cycles to a value of 94 ohms at 300,000 cycles.

f	λ	X_C
60	$5 \cdot 10^6$	$5.32 \cdot 10^5$
10,000	$3 \cdot 10^4$	$2.8 \cdot 10^3$
100,000	$3 \cdot 10^3$	$2.82 \cdot 10^2$
300,000	$1 \cdot 10^3$	$9.4 \cdot 10^1$

The simple series circuit usually consists of an inductance and capacity in series with an applied emf. Such a circuit is commonly employed for radio purposes, since it is very important to reduce or eliminate reactance caused by inductance or capacity at radio frequencies. Under conditions of no reactance, a radio-frequency current flowing in the circuit is impeded only by the radio-frequency resistance of the circuit.

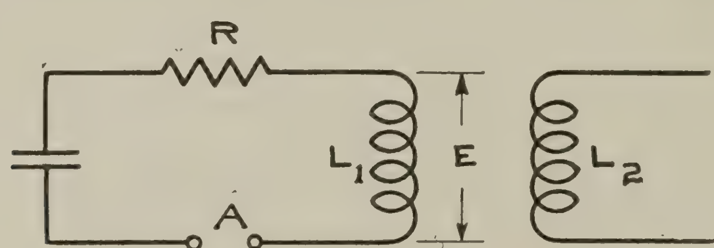


Fig. 136.

The above diagram, figure 136, shows a resistance R, an inductance L_1, and an applied voltage E, all in series. The resistance R represents the total resistance of the circuit and is generally the radio-frequency resistance of L_1. It may include such losses as those due to eddy currents and hysteresis.

The action within the circuit is exactly the same whether the emf E is applied at A, or if A is closed and E is applied through L_1 from some other inductance L_2. This is true only when the natural period of the coil L_1 is far below the wave length corresponding to the frequency of E.

The frequency at which the inductive reactance of a circuit is exactly equal to the capacitive reactance of the same circuit is called **the natural period** or **resonant frequency** of the circuit.

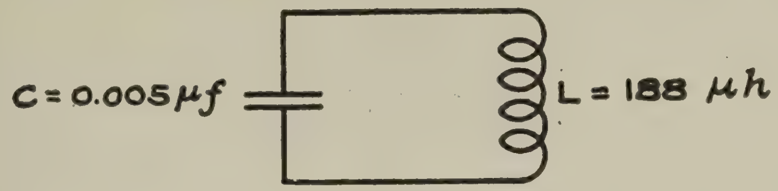


FIG. 137.

Example:

The following problem (figure 137) illustrates how the natural period of a circuit is found. The values of the electrical constants are the same as those used in the previous example.

The total impedence of the circuit is given by

$$Z=\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}$$

where $2\pi fL=$ Inductive reactance,

$$\frac{1}{2\pi fC}=\text{Capacitive reactance,}$$

L and C being in henries and farads, respectively. As has been stated, for resonance, the inductive reactance is equal to the capacitive reactance, or

$$2\pi fL=\frac{1}{2\pi fC}$$

then $$f^2=\frac{1}{4\pi^2LC}$$

and $$f=\frac{1}{2\pi\sqrt{LC}}$$

substituting $$=\frac{1}{6.28\sqrt{1.88\cdot 10^{-4}\times 5\cdot 10^{-9}}}$$

whence $f=164{,}200$ cycles (resonant frequency).

The resonant frequency of a circuit may also be determined graphically as shown in figure 138. This graph has been developed by plotting reactance against frequency. The inductive reactance is taken as positive and the capacitive reactance negative. The respective values are obtained from the reactance formulas given in the previous problem and are given in Table I.

The resultant reactance curve is obtained by plotting the algebraic sum of the inductive and capacitive reactances. It will be noted that

this curve cuts the zero axis at the resonant frequency, since at this frequency the sum of the reactances is zero.

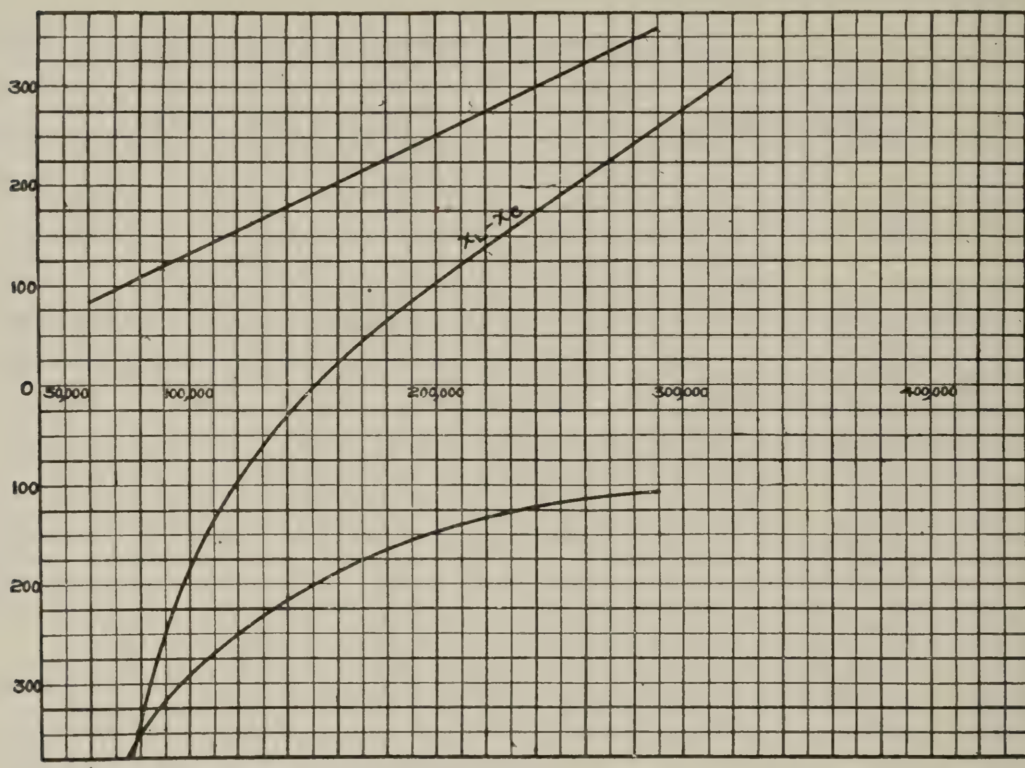


FIG. 138.

TABLE I.

f	λ	X_L	X_C	$(X_L - X_C)$
100,000	3,000	118.0	318.0	−200.0
135,100	2,220	159.4	235.2	− 75.8
150,000	2,000	177.1	212.0	− 34.9
164,200	1,827	193.6	193.6	0
180,700	1,660	213.3	175.8	+ 37.5
230,800	1,300	272.4	137.8	+134.6
300,000	1,000	354.0	107.0	+247.0

From these figures and the curves it will be noted that for frequencies below resonance the predominating reactance is capacitive while the inductive reactance predominates at frequencies above that of resonance.

This means that a certain amount of reactance or impedance will be added to any series circuit containing inductance or capacity or both, when the current flowing in the circuit is of any other frequency than that for which the circuit is resonant. The value of the impedance at any frequency, for this particular problem, is shown by the curve $X_L - X_C$.

The wave length corresponding to the frequency of resonance is easily obtained when it is remembered that the velocity of an electromagnetic wave equals the frequency of the wave multiplied by its length, or

$$V = f\lambda$$

or
$$\lambda = \frac{V}{f}$$

substituting
$$= \frac{3 \cdot 10^8}{1.642 \cdot 10^5}$$

whence
$$\lambda = 1{,}827 \text{ meters.}$$

When the circuit is used at this wave length, the only resistance to current flow, as has been explained, will be the radio-frequency resistance of the circuit. Referring to the previous problems on reactance, the resistance of the circuit at resonance will be between 1 and 2.5 ohms. If only the inductance were in series with the emf it would offer between 112.8 and 339 ohms and if capacity alone were present its reactance would be between 94 and 282 ohms. This same frequency or wave length may be obtained by reference to the LC Table 13.

At resonant frequency, the power supplied to a series circuit is given by I^2R, where I is the radio-frequency current measured between the inductance and the capacity and R is the radio-frequency resistance of the circuit. If the current is measured at some point within the

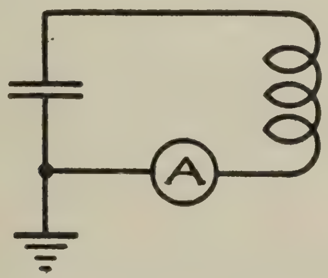


FIG. 139.

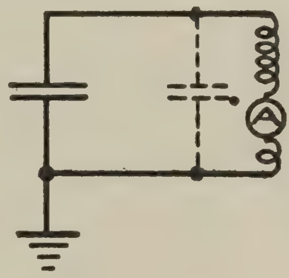


FIG. 140.

inductance there is a possibility that a portion of the current will pass through the distributed capacity of the coil and not through the meter. To avoid this condition, the current should be measured as shown in figure 139, and not as in figure 140. The circuit should be grounded as shown.

Example:

If a resonant current of 10 amperes flows through the series circuit of the previous problem. Find the watts dissipated.

Formula $P = I^2R$

substituting $= 10^2 \times 1.7$

whence $P = 170$ watts.

Resonance curves are important as a means for observing the response of a tuned circuit to a given exciting frequency. They indicate

the value of the current which will flow in the circuit at various frequencies. A resonance curve (figure 141) may be plotted for the previous example, as follows.

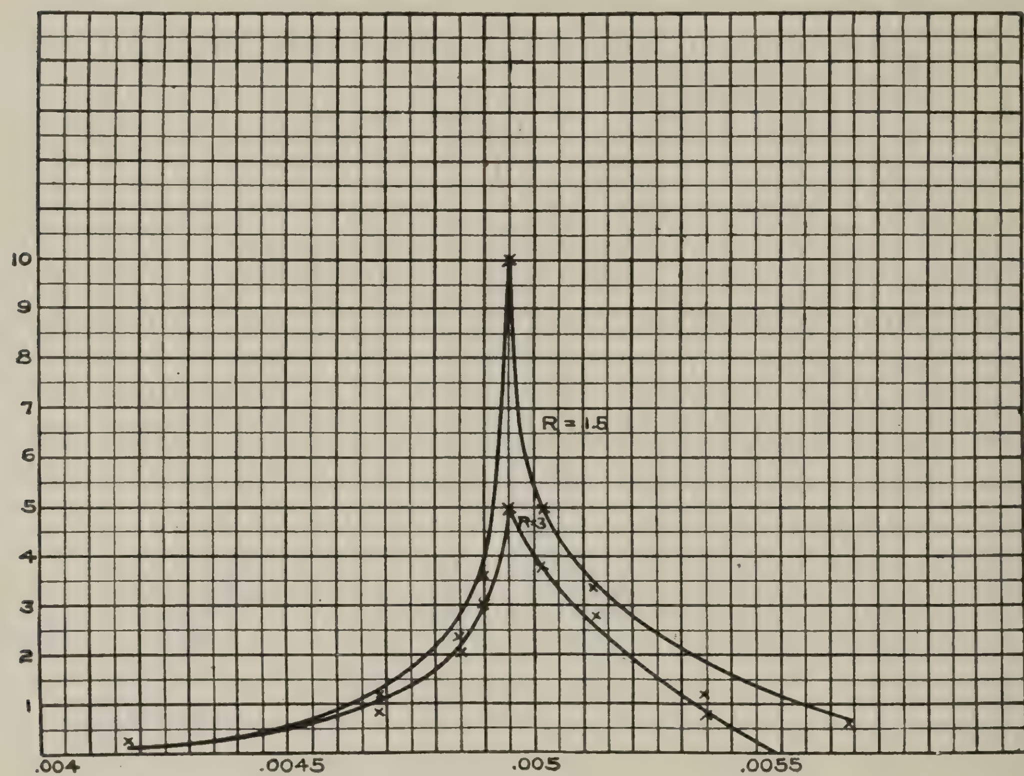


FIG. 141.—Resonance Curves.

Considering the inductance and exciting resonant frequency as remaining fixed, the impedance of the circuit will vary with a change in capacity value, as follows:

$$Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

For resonance

$$Z=\sqrt{1.5^2+(193.6-193.6)^2}$$

whence $\quad Z=1.5\Omega$

For all other values of L the reactance of the circuit will vary as shown in table 2 below.

In the calculation of the table, it is assumed that 15 volts at resonant frequency are applied to the circuit, say by the inductance; then, the current which will flow in the circuit for various values of capacity will be given by

$$I=\frac{E}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

Z_1 is calculated for a resistance of 1.5 ohms and Z_2 for a resistance of 3 ohms. The curves of figure 141 have been plotted for the various values of capacities and current for an applied voltage of 15 volts resonant frequency. From these curves it will be clearly seen that the ratio of current at resonance to that at any other capacity value, is much greater for 1.5 ohms than for 3 ohms.

TABLE II.

C	Z_1	I_1	Z_2	I_2
.00569	22.6	.65	24.00	.63
.0054	14.50	1.34	16.65	.90
.00512	4.36	3.44	5.07	2.96
.00507	3.0	5	3.87	3.88
.00500	1.5	10	3.00	5
.00490	4.27	3.51	5.00	3
.00485	6.18	2.43	6.7	2.24
.00469	12.9	1.16	15.45	.97
.00412	41.6	.36	41.7	.36

This property of a circuit is very important when it is desired to eliminate interference by sharp tuning. When the condition of resonance is obtained, it is important that the voltages of the circuit be understood. In the problem, the driving emf was assumed to be 15 volts. This emf produced a current of 10 amperes at resonance with the circuit resistance of 1.5 ohms. This resonance current of 10 amperes will produce a voltage across the condenser which is exactly balanced by the voltage produced by the inductance. These voltages are equal and opposite in phase since they are produced by the same current flowing through equal and opposite reactances. The voltage value across the condenser is given by

$$V = IX_C$$

Formula $$V = I\frac{1}{\omega C}$$

substituting $$= \frac{10}{6.28 \times 3 \cdot 10^4 \times 5 \cdot 10^{-9}}$$

whence $$V = 10{,}615 \text{ volts.}$$

In the problem of insulation, these resonant voltage values must be further considered because of the fact that the voltage may not be sinusoidal. The above problems are based on a sinusoidal current and voltage and the values given are effective and not maximum. The maximum value of the condenser voltage in the last two problems is given by

$$V_0 = 1.414\ V$$

For the frequency of 164,200 cycles.

$$V=\frac{1}{\omega C}$$

substituting $$=\frac{10}{6.28\times 1.642\cdot 10^5\times 5\cdot 10^{-9}}=1.938\cdot 10^4$$

whence $V=1938$ volts.

and then $V_0=1.414\ V=1.414\times 1.938\cdot 10^4$

whence $V_0=2740$ volts.

The voltage across the inductance is given by

Formula $V=I\omega L$

substituting $=10\times 6.28\cdot 1.642\cdot 10^5\cdot 1.88\cdot 10^4=1.938\cdot 10^3$

whence $V=1938$ volts and $V_0=2{,}740$ volts.

These voltages are not effective for producing current through the circuit since they neutralize each other. However, the potential difference, at any instant, across the condenser is 1,938 volts and this voltage would be effective for any circuit connected to the condenser terminals. Because of this resonant voltage effect it is always important to guard against voltage troubles when dealing with radio-frequency circuits carrying relatively large currents. This is especially true if the frequency of the current is low, since the condenser voltage varies inversely as the frequency of the current through the condenser. In the above problem, the frequency was taken as 164,200 cycles which corresponds to a wave length of 1,827 meters. Suppose the inductance is increased so that the circuit is resonant for 10,000 meters, this corresponds to a frequency of 30,000 cycles. If the current of 10 amperes at 30,000 cycles now passes through the .005 μf condenser, the voltage across its terminals will be

$$V=I\frac{I}{\omega C}$$

For the frequency of 30,000 cycles

$$V_0=1.414\times 10{,}615=15{,}000 \text{ volts.}$$

In the case of a transmitting antenna having a capacity of 0.005ωf and a resonant current of 80 amperes at 30,000 cycles, the voltage on the antenna, or voltage strain on the insulators would be

Formula $$V=\frac{I}{\omega C}$$

substituting $$=\frac{80}{6.28\times 3\cdot 10^4\times 5\cdot 10^{-9}}$$

whence $V=84{,}900$ volts

and $V_0=1.414\times 84{,}900=119{,}000$ volts.

CHAPTER II. PARALLEL CIRCUITS.

The treatment of a parallel circuit is quite similar to the treatment of a series circuit, the chief difference being in the case of the parallel circuit where the source of emf must be considered as applied simultaneously to both inductance and capacity.

It will be remembered that the series circuit was considered as having an emf supplied as either E_a or E_b, figure 142.

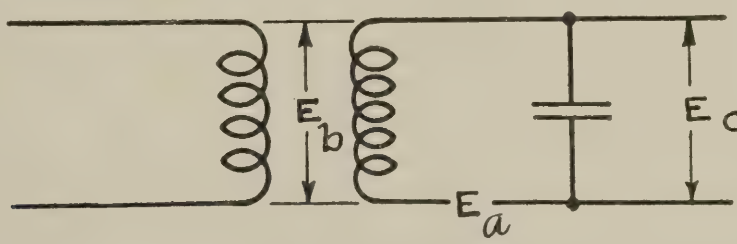


FIG. 142.

E_b, being considered as made up of the sum of the individual voltages induced by the supply circuit in each element of the circuit under consideration, has the effect of a voltage such as E_a which is in series with the inductance and capacity of the circuit.

A parallel circuit (or parallel resonance) is usually considered as having the exciting voltage, such as E_c, applied directly to the condenser. This, in effect, is the voltage applied to the inductance and capacity and, therefore, there is no tendency for the inductive reactance to counteract the capacitive reactance, so far as the external or supply circuit is concerned. The applied voltage, in the case of a parallel circuit without resistance, is balanced by the combined reactive voltages of the inductive and capacitive branches. These reactive voltages are equal and opposite in case the applied voltage is of resonant frequency. Therefore, the total reactive voltage is equal to the numerical voltage value of either the inductance or the capacity. This is shown to be true by the following example.

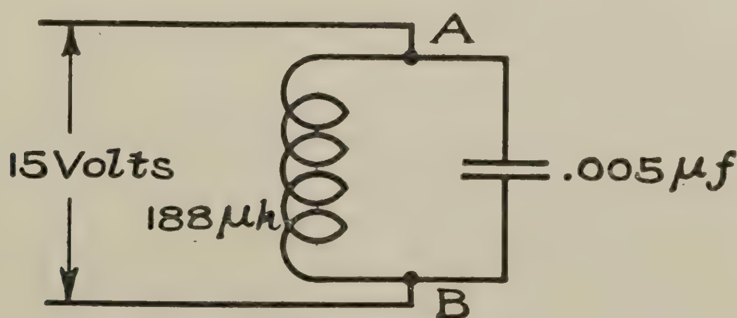


FIG. 143.

If 15 volts resonant frequency are applied at A and B in figure 143 the reactance of the inductive branch will be

$$X_L = \omega L = 193.6 \text{ ohms}$$

and the reactance of the capacitive branch will be

$$X_C = \frac{1}{\omega C} = 193.6 \text{ ohms}$$

The current through each branch will be

$$I_L = I_C = \frac{E}{X_L} = \frac{E}{X_C}$$

$$= \frac{15}{193.6}$$

whence $I_L = I_C = .0775$ ampere.

This current is known as the circulating current, and is the amount of current necessary to produce the counter emf of the reactances, since

$$I_L X_L = I_C X_C = .0775 \times 193.6 = 15 \text{ volts.}$$

So far as the parallel circuit is concerned, the reactive voltages are neutralized, but at any instant the voltage across the condenser is equal and opposite to the applied emf. If a greater emf is applied, more current will flow within the reactances and the required counter emf will be built up.

The formula, which has been developed for parallel circuits, shows that the larger the capacity of a parallel combination, the greater will be the value of the circulating current I_{cir}.

If either of the reactive branches contains resistance, the supply circuit will have a current flow as shown by the following:

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X_L^2}}$$

where R represents the ohmic resistance of the inductive branch of the circuit, IR the in-phase voltage component and IX_L the inductive counter-emf which is exactly equal to the emf of the capacitive branch. From this formula it will be seen that as the circuit resistance increases, the circulating current will decrease, due to the applied emf being reduced by the IR voltage drop of the circuit. This current component in phase with the applied emf is the current which flows in the supply circuit. The out-of-phase current due to the reactance X_L will be equal to the capacity current in value, but 180° out of phase with it. If the capacity branch contains resistance, it is to be treated in the same manner as the inductive branch. The in-phase components of the currents through the capacitive and inductive branches add together and form the current which flows through the supply circuit.

The impedance, or effective resistance, which the parallel circuit offers to the supply circuit is given by

$$R = \frac{L}{r_C}$$

where R = the effective resistance of the combination.
r = the circulating resistance of the combination $(r_L + r_C)$
L = inductance of combination,
C = capacity of combination.

This formula is only useful in finding the effective resistance to the resonant frequency of the combination, the sharpness of the circuit or the resistance which it offers to other frequencies. This will vary inversely as the circulating resistance and the ratio of L/C.

When the applied emf is of greater frequency than the resonant frequency of the parallel circuit, that circuit will offer a capacitive reactance to the supply circuit or, in effect, will be a simple capacity. If the frequency of the supply emf is lower than the resonant frequency of the parallel circuit, the reactance will be inductive and will have the effect of an inductance in the supply circuit.

CHAPTER III. COUPLED CIRCUITS.

General. When there is a transfer of power from one circuit to another circuit, it is said that the two circuits are coupled. This transfer of power can be accomplished by electromagnetic or electric induction, or by resistance coupling. The coupling between circuits whether it is resistance, electromagnetic (inductive), or electric (capacitive), is divided into two classes, i.e., the direct and the indirect coupling.

Direct coupling refers to circuits where the coupling element is common to both circuits, whether it is an inductance, a capacity or a resistance. See figures 144, 145 and 146.

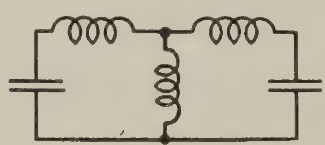


FIG. 144.—Direct Coupling by Inductance.

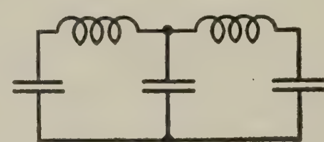


FIG. 145.—Direct Coupling by Capacity.

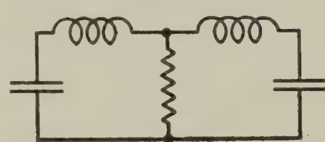


FIG. 146.—Direct Coupling by Resistance.

Indirect coupling is accomplished between circuits by magnetically coupling the inductances of each circuit or by capacitive coupling using condensers between the circuits, as shown in figures 147 and 148.

FIG. 147.

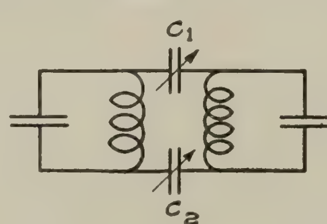


FIG. 148.

In figure 147, the transfer of power is accomplished magnetically and this power is proportioned to the distance between the two inductances, the closer the inductances the greater the transfer, while the farther apart the inductances are, the smaller the transfer. In other words, the closer the inductances the greater the electromagnetic induction while the farther apart the inductances are the smaller the electromagnetic induction.

Figure 148 shows a method of indirectly coupling circuits by capacities. An increase in the value of the coupling capacities (C_1 and C_2)

will **increase** the amount of power transferred from one circuit to another, while a **decrease** will **reduce** the power transfer.

In both methods of indirect coupling referred to, it is customary to refer to the condition of maximum power transfer as **tight** coupling and the condition of minimum power transfer as **loose** coupling. Coupling and coefficient of coupling are explained in detail in Chapter 7, Part 2.

If the coupling between two circuits is small, the two circuits will be resonant to one frequency but if the coupling is increased, both circuits, due principally to the effect of mutual inductance, will no longer be resonant to the original frequency, but will be resonant to two frequencies, one frequency being lower than the original frequency and the other being higher than the original frequency. With loose coupling it is possible to have each circuit resonant to one frequency, while if the coupling is tightened or increased in value, each circuit will be resonant to two frequencies.

The difference in frequencies referred to is dependent on the degree of coupling.

Figure 149 shows the resonance curve for a typical circuit when loosely coupled to another circuit, while figure 150 shows the same circuit under influence of a tight coupling.

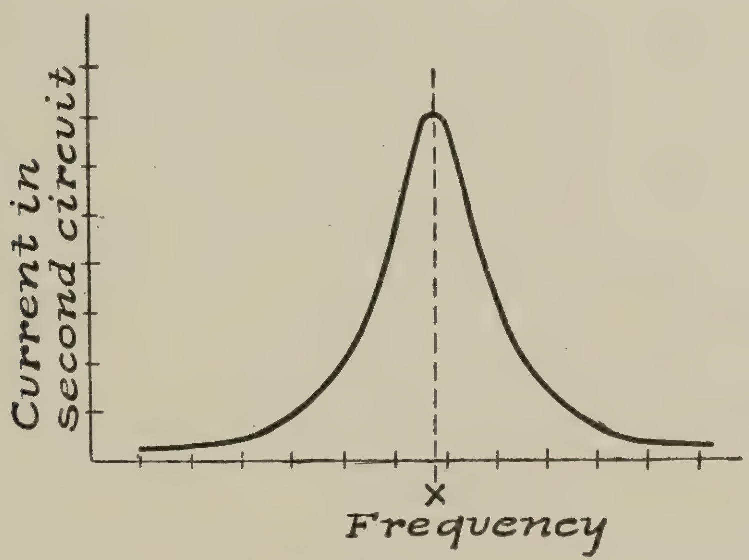


FIG. 149.

In figure 149, X represents the resonant frequency of the circuit, while X_1 and X_2 in figure 150 are the two frequencies to which the circuit is resonant when affected by tight coupling.

The question naturally arises why are the circuits resonant to two frequencies? The two circuits are resonant to two frequencies because of the fact that the mutual inductance of the circuits momentarily adds or subtracts from the inductance of each circuit.

The value of the inductance in each circuit varies momentarily and the circuits are resonant to two frequencies. The difference in

the two frequencies depends on the value of mutual inductance or, in other words, on the coupling between the circuits. This may be represented

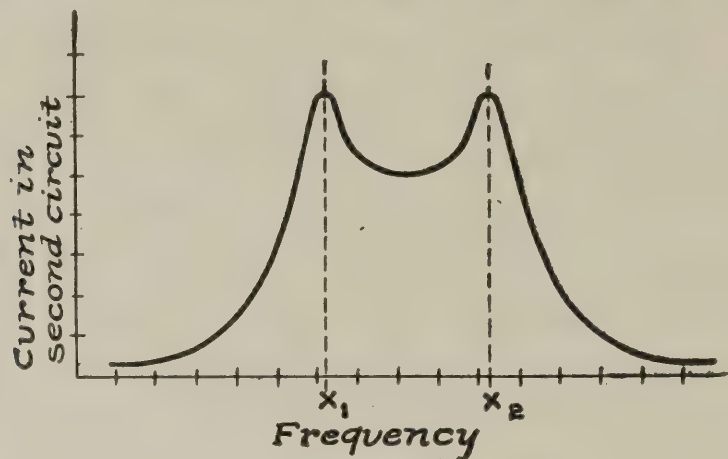


FIG. 150.

by the following *LC* equations which, in turn, have a definite relation to the frequency and wave length to which the circuits are resonant.

For loose coupling $$f=\frac{1}{2\pi\sqrt{LC}}$$

For tight coupling $$f_1=\frac{1}{2\pi\sqrt{(L+M)C}}$$

and

$$f_2=\frac{1}{2\pi\sqrt{(L-M)C}}$$

where

$$M=k\sqrt{L_1\,L_2}$$

L_1 being inductance of one circuit and L_2 the inductance of the other circuit.

When the coefficient of coupling is known, the two periods of resonance of the coupled circuits are found by the following:

$$\lambda_1=\lambda\sqrt{1+k}$$

and

$$\lambda_2=\lambda\sqrt{1-k}$$

where λ is the wave length to which the circuit is resonant to when acting singly, or very loosely coupled.

This phenomena of coupling and resonance of the circuits produces an effect in a spark transmitter where signals on **two** wave lengths are transmitted when tight or medium coupling is resorted to. In vacuum-tube or other undamped transmitters, although the two resonant frequency conditions exist with tight or medium coupling, the vacuum tube tends to oscillate on **one frequency only.** This frequency is the frequency at which the circuit has minimum resistance and, therefore, can be higher or lower than the natural frequency of the circuit.

Direct Inductive Coupling. The action of a direct inductively coupled circuit is identical with that of the indirect inductively coupled circuit.

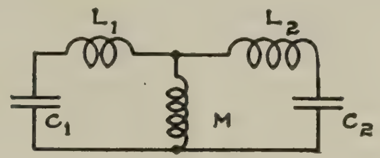


FIG. 151.

This may be better understood by referring to figure 151 where the inductance N is common to both circuits and is considered as the mutual inductance of coupling.

The coupling between circuits L_1C_1 and L_2C_2 can be represented as follows:

$$k = \frac{M}{\sqrt{L_1 \ L_2}}$$

Calculation of the coefficient of coupling k in the case of direct coupled circuits is comparatively easy in view of the fact that it is easy to obtain values of the mutual inductance.

Direct coupled circuits are not often used in transmitting or receiving systems, because of the fact that sharp tuning or loose coupling is extremely hard to obtain with efficient power transfer from one circuit to the other. It is used only in certain types of spark transmitters which depend upon a high damping or quick cutting off of power in the primary circuit, after which the primary circuit is practically open and therefore has no effect on the secondary.

Capacitive Coupling. Circuits coupled by capacity may be divided into two classes, one as shown in figure 152 and the other as shown in figure 153.

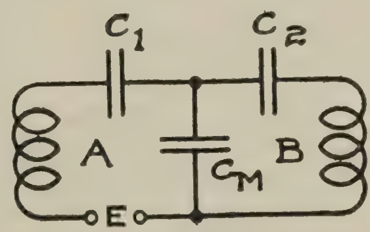


FIG. 152.

In figure 152, the transfer of power from circuit A to circuit B is controlled by the capacity C_M. By increasing the value of C_M, the power transfer is reduced and by reducing the value of C_M, the power transfer is increased.

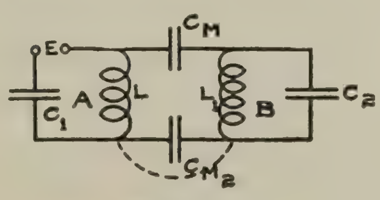


FIG. 153.

In figure 153, the opposite effect is to produce an increase in the value of C_M or C_{M_2} which will produce a corresponding increase in the power transferred. The usual way of connecting a capacity coupled circuit of the type shown in figure 153 is to eliminate capacity C_{M_2} and substitute a conductor similar to that shown by the dotted lines.

The coupling between the circuits shown in figure 152 may be stated as follows:

$$k=\sqrt{\frac{C_1\,C_2}{(C_1+C_M)\,(C_2+C_M)}}$$

In the case of figure 153, the coupling may also be represented as follows:

$$k=\frac{C_3}{\sqrt{(C_1+C_3)(C_2+C_3)}}$$

where $$C_3=\frac{C_M\,C_{M_2}}{C_1+C_2}$$

Referring to figure 153, an increase in coupling capacities from one circuit to another produces a change in the constants of the circuits; that is, the circuits are resonant to a frequency different from that obtained at extremely loose coupling.

This may be explained by the fact that any incresae in the values of C_M and C_{M_2} means an increase in the value of the resultant or total capacity of each circuit, which results in an increased LC value. This increased capacity, which is mutual to each circuit, is equal to

$$C \text{ or } \frac{1}{\frac{1}{C_M}+\frac{1}{C_{M_2}}}=\frac{C_M\,C_{M_2}}{C_M+C_{M_2}}$$

The circuit B of figure 151 is resonant to a frequency which may be represented as follows:

$$f=\frac{1}{2\pi\sqrt{L_1\left(C_2+\frac{C_M\,C_{M_2}}{C_M+C_{M_2}}\right)}}$$

Capacity coupled circuits similar to figure 153 are used principally in receiving and amplifier circuits, where one condenser functions in transferring the power from one circuit to another.

Inductive and Capacitive Coupling. The preceding Chapters treat the true inductive coupling or the true capacitive types of coupling. There are, however, in radio engineering practice, conditions where both inductive and capacitive coupling are met with in the same coupled circuits. Such a circuit is shown in figure 154.
Here L and L_1 are coupled inductively and the circuits A and B are coupled by the capacity C_3.

The combination of capacitive and inductive coupling produces two results, i.e., when the induced currents due to the two kinds of coupling are in phase the coupling is increased; but when the induced currents are out of phase, the coupling is reduced in proportion to the difference in the currents. The change in the position of L_1 and L, and also the method of winding the inductances, will produce currents which will differ in phase with the current due to capacitive coupling.

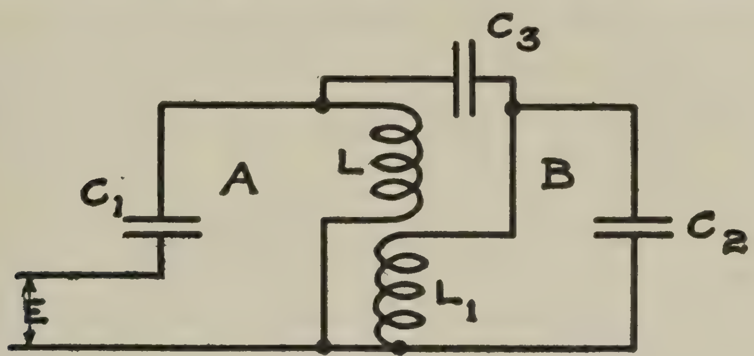


FIG. 154.—Circuits Coupled Inductively as well as by Capacity.

It is good practice to use one type of coupling and, therefore, eliminate the trouble met in circuits which have the two types of coupling. The advent of screening of radio circuits has helped to a great extent in obtaining radio circuits which contain only one type of coupling, preferably the inductive coupling.

Resistance coupling between two circuits is very convenient when it is desired to measure the amount of emf applied to a given circuit. Care must be taken that the resistance is the only means by which the two circuits are coupled.

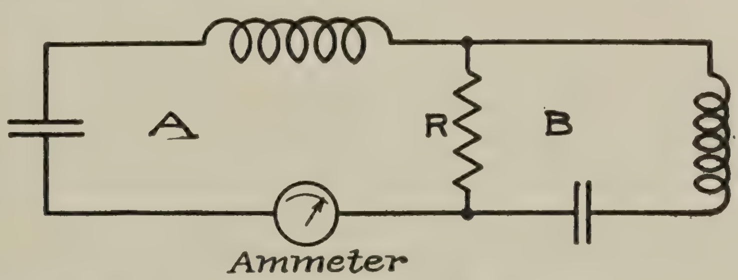


FIG. 155.—Resistance Employed to Insert a Known Emf in Another Circuit.

In the figure 155, the circuit A is coupled to the circuit B by means of the noninductive resistance R. If the current in the circuit A is measured by means of the ammeter and the value of R is known, then the emf applied to the circuit B is IR. Care must be taken to keep R as small as possible since it is always important to keep the resistance value of a radio-frequency circuit at a minimum.

CHAPTER IV. ANTENNA REACTANCE

A modern radio antenna usually consists of two portions: the **lead-in** and the **aerial.** See figure 156. Such an antenna has distributed inductance, capacity and resistance. The distributed inductance is due to the lines of magnetic force which are set up by the current in the antenna. The distributed capacity consists of the sum of the capacities of the various elements of the antenna to earth, and the resistance of the antenna is that due to all the losses taking place, including radiation.

Because of the fact that all the current which flows in the lead-in does not flow to the open end of the aerial portion, but returns through the distributed capacity to ground, it may be assumed that the antenna is analogous to commercial circuits such as cables, telephone and transmission lines, and may be treated by the same theory.

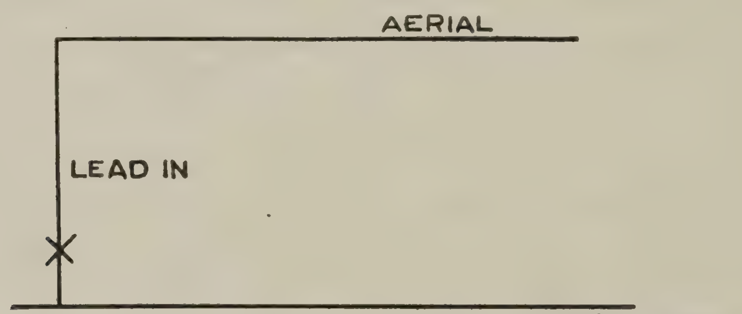


Fig. 156.

The unloaded antenna may be considered as being made up of two parallel conductors having uniformly distributed resistance, inductance and capacity. In the following diagram, the distributed inductance and capacity are shown without resistance since the resistance of an antenna is usually small and has no appreciable effect on the change of antenna inductance and capacity with frequency.

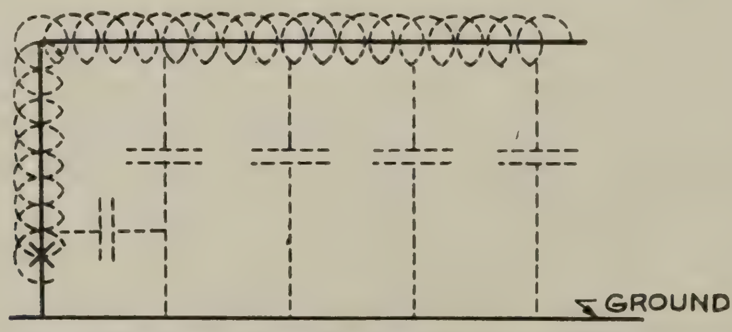


Fig. 157.—Antenna Represented as a Line with Uniformly Distributed Inductance and Capacity.

In the theoretical treatment of an antenna, the lead-in is neglected and the horizontal wires are treated as a transmission line with respect to earth. For practical purposes, the lead-in is considered as being a part of the flattop, and the total inductance which would be measured at low frequency from end to end of the antenna, as L_0. This inductance L_0 is assumed to be the inductance per unit of length, multiplied by the length of the antenna and lead-in. The capacity of the antenna is taken as C_0 and represents the sum of the capacities of each unit to ground. Under these conditions, the values of L_0 and C_0 are approximately related to the reactance of the antenna at various frequencies by the following formula:

$$X=-\sqrt{\frac{L_0}{C_0}}\cot \omega \sqrt{L_0 C_0}$$

This formula shows that the reactance of an antenna is **negative** or **capacitive,** at frequencies below the fundamental, and becomes zero at $f=\frac{1}{4\sqrt{L_0 C_0}}$. Above this frequency the reactance is **positive,** or **inductive,** up to a frequency of $f=\frac{1}{2\sqrt{L_0 C_0}}$, where the reactance becomes infinite, and again negative, and approaches zero at the first harmonic frequency of the fundamental frequency.

At the frequency $f=\frac{1}{4\sqrt{L_0 C_0}}$, the reactance becomes zero since $\cot \omega\sqrt{L_0 C_0}=\cot\frac{2\pi l}{\lambda}$, and the values of the cotangent will be zero for $\frac{\pi}{2}, \frac{3\pi}{2}$ or $(2n+1)\frac{\pi}{2}$ (The cotangent for an angle of $\frac{\pi}{2}=90°$, and $\frac{3\pi}{2}=270°$ is zero). The value of $\frac{\pi}{2}, \frac{3\pi}{2}$ or $(2n+1)\frac{\pi}{2}$ is obtained when

$$\frac{2\pi l}{\lambda}=\frac{\pi}{2}, \frac{3\pi}{2} \text{ or } (2n+1)\frac{\pi}{2} \text{ or } l=\frac{\lambda}{4},\ l=\frac{3\lambda}{4} \text{ or } l=\left(\frac{2n+1}{4}\right)\lambda.$$

These values show that the antenna will be free to oscillate wherever the reactance is zero, or at the fundamental of the unloaded antenna, and at the 3rd and 5th harmonics of the fundamental frequency. For the even harmonics of the fundamental frequency, the reactance is infinite and, therefore, such harmonics cannot exist.

The use of l is based on the assumption that the antenna is a single vertical conductor and, therefore, calculations by its use are only approximate. The formula is here given for a brief explanation of antenna reactance.

Cotangent curves illustrating these conditions are shown in figure 158.

The loaded antenna is one in which either an inductance or capacity has been inserted in the lead-in. If an inductance is inserted, its reactance X_L is added to the circuit.

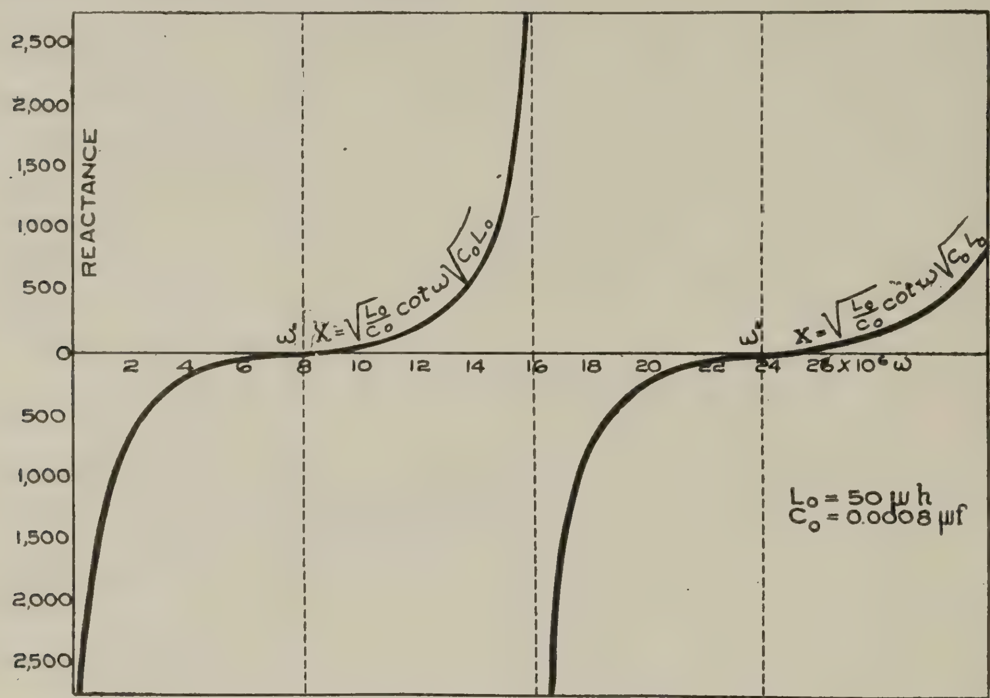


FIG. 158.

The resultant reactance, since it is inductive, is positive and increases linearly with the frequency, and is represented in figure 159 by a solid line.

The frequencies at which the system will oscillate with the inserted coil is given graphically in figure 159 by the intersection of the $-X_L$, or

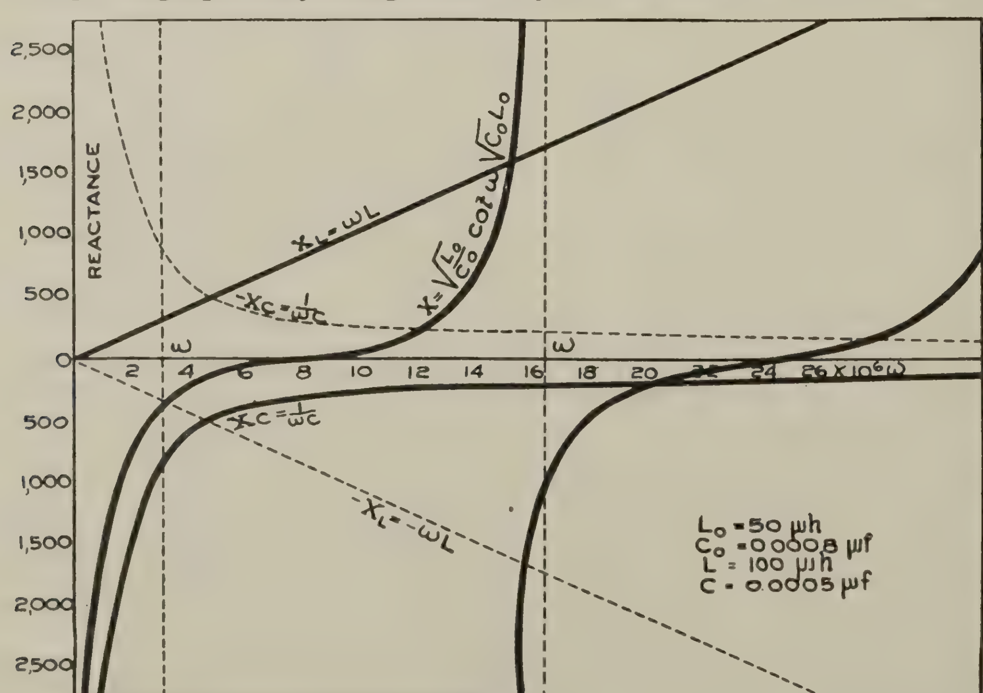


FIG. 159.

dashed line, with the cotangent curves. When a condenser is inserted in a lead-in, the fundamental frequency of the antenna system will be decreased.

If a condenser C is inserted, its reactance, which is added to the circuit, is given by $X_C = -\frac{1}{\omega C}$. The reactance values of this capacity for the various frequencies are shown by the dotted hyperbolic curve shown in figure 159. This curve is the negative of the actual reactance curve shown by the full-line hyperbolic curve. The intersection of the dashed line with the cotangent curve gives the new fundamental of

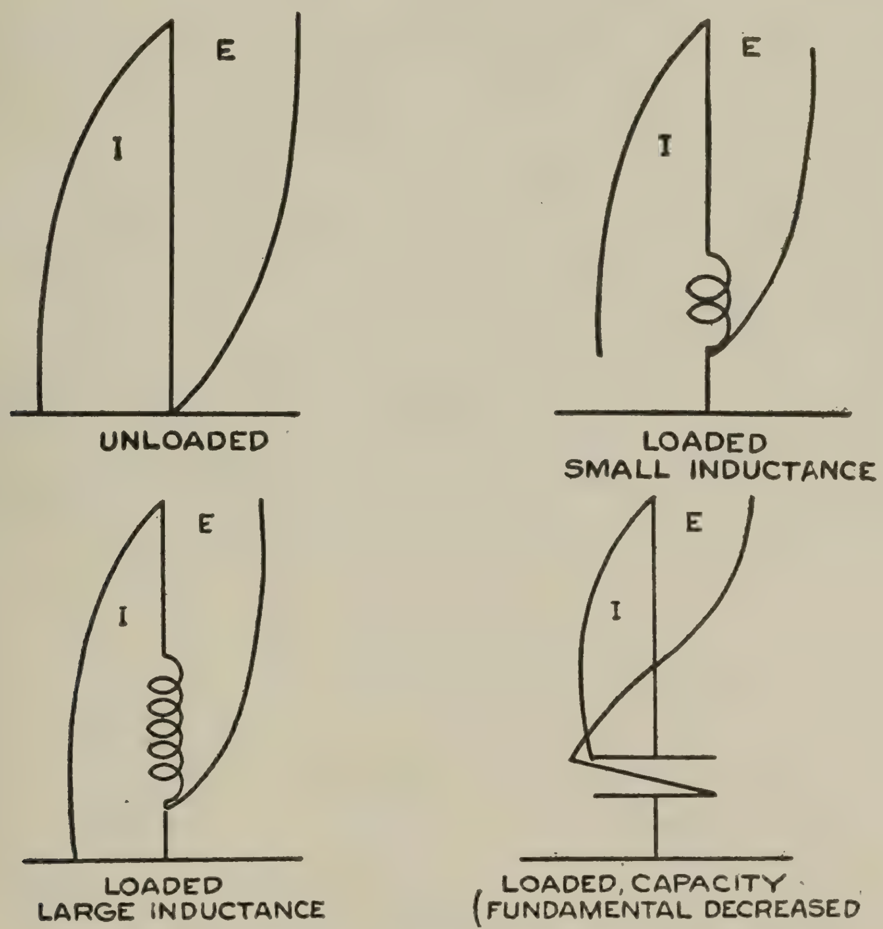


FIG. 160.—Current and Voltage Distribution in Antennas.

the system. It will be noted that, in either case of loading the antenna, the resultant harmonic frequencies are not integral multiples of the fundamental, as was the case of the unloaded antenna made up of distributed inductance and capacity.

The equivalent circuit of an antenna is sometimes desired for experimental work, and since it is true that the constants of an antenna change with frequency, only an approximate equivalent circuit may be calculated. The formula $\lambda = 2\pi c\sqrt{LC}$ is quite accurate for calibration of antenna inductance and capacity values, provided

the ratio of $\frac{L}{L_0}$ is large. The percentage of error in using this formula varies from about ten per cent, for a ratio of one to one for $\frac{L}{L_0}$, to 0 per cent for a four to one ratio. Beyond this ratio, the formula is accurate for all practical purposes.

If it is desired to calculate the natural period of an antenna loaded to several times its fundamental, the ordinary computation, using an *LC* table may be made, where *L* is the inserted inductance and *C* the capacity of the antenna.

The voltage of an unloaded antenna, made up of distributed inductance and capacity, is considered a maximum at the open end of the antenna, and zero at the ground end of the lead-in. In actual practice, this distribution of voltage does not hold true, since the antenna is usually loaded to increase its natural frequency.

It is not an uncommon belief that the maximum voltage on a transmitting antenna is at the open end. This is not true when the antenna inductance is small compared to the loading inductance; and, if an antenna is loaded to several times its fundamental, it is safe to assume that the voltage is the same at all parts of the antenna flattop.

The current distribution and voltage along an antenna are conveniently shown by the above diagrams, where the vertical wire is assumed to represent a combination of lead-in and areial having uniformly distributed inductance and capacity throughout its length.

PART 5.

THEORY OF DAMPED OSCILLATIONS

CHAPTER I. OSCILLATORY DISCHARGE OF A CONDENSER

General. Continuous, or undamped oscillations were defined in Part 3 as being simply alternating currents of high frequency having a sine wave form and a constant amplitude. Also, the oscillations referred to in Part 4 were assumed to be undamped. The necessity of understanding alternating-current theory and its application to radio circuits is more than ever apparent in view of the fact that continuous waves are being employed in radio transmission to a greater extent than heretofore.

Forced oscillations. When an alternating emf is applied in a constant manner to a circuit, the circuit **is forced** to oscillate in the period of the applied emf. Such oscillations are called **forced oscillations.** Also, whatever losses may occur in the circuit will be constantly supplied, and the alternating current in the circuit will have a constant amplitude. The effect produced when such a circuit has the same period as that of the applied emf has already been pointed out.

Free oscillations. However, oscillations can be produced in a circuit without the use of such an alternating emf. Figure 161 shows

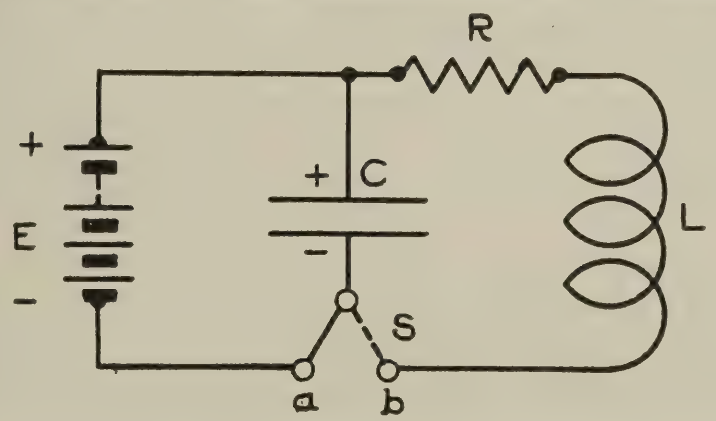


FIG. 161.—Circuit for Producing Oscillations using Dc Supply.

one method of producing oscillations. Referring to this figure, it will be seen that there are two distinct circuits, depending upon the position of switch *S*. The circuit *ECSa* is the **charging circuit,** while circuit *CRLbS* is the **discharge circuit** in which the oscillations are to be produced. Condenser *C* is common to both circuits.

The manner in which the condenser receives its charge was explained in Chapter VIII, of Part 2. It will be remembered that the potential difference *V* of the charged condenser equals the impressed

emf E, and that a definite amount of energy W is then stored in the form of a strain in the dielectric. This energy is

$$W = \frac{1}{2} CV^2 \qquad \text{(joules)}$$

and represents the contribution of electrical energy made by the battery E.

If the switch S is now opened, the charge will remain in the condenser and the battery E need no longer be considered.

Now assume that the switch S is thrown to position b. The charged condenser will then be connected into the discharge circuit. Let this instant be $t=0$. Suppose, also, that R represents the total

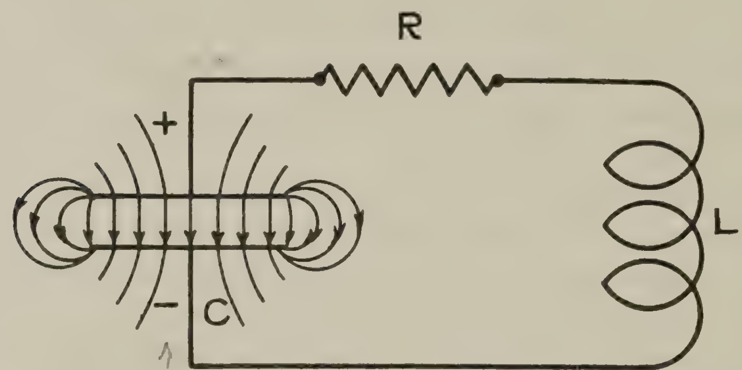


Fig. 162.—All of the Energy is Stored in the Electric Field. Time $t=0$.

resistance of the discharge circuit. The condition at this instant is shown in figure 162. As stated previously, all of the energy is stored in the electrostatic field of the condenser and there is no motion of the charge. Immediately upon closing the switch, a conducting path for the charge is provided, and the charge begins to move through the discharge circuit in the form of a current. The direction of the current around the circuit is from the positive to the negative plate of the condenser.

Now, a magnetic field is always produced by current, and energy is stored in a magnetic field when the current is increasing. Thus, as the current rises in value under the action of the emf of the condenser, energy is continually leaving the condenser and is being stored in the magnetic field about the circuit, especially in the immediate vicinity of the inductance L. At the instant that the condenser becomes discharged, practically all of the energy that was in the electric field is in the magnetic field, which has a maximum intensity. The condition of the circuit at this instant is shown in figure 163. The discharge current is now at its maximum. This completes the first quarter of the cycle. The energy stored in the magnetic field has a value

$$W = \frac{1}{2} LI^2$$

The magnetic field now operates to prevent any change in the current, and the energy in the magnetic field is now restored to the circuit, with the result that a current flow in the **same direction** is maintained. This current recharges the condenser to a polarity opposite to that of the initial charge. During this recharging period practically

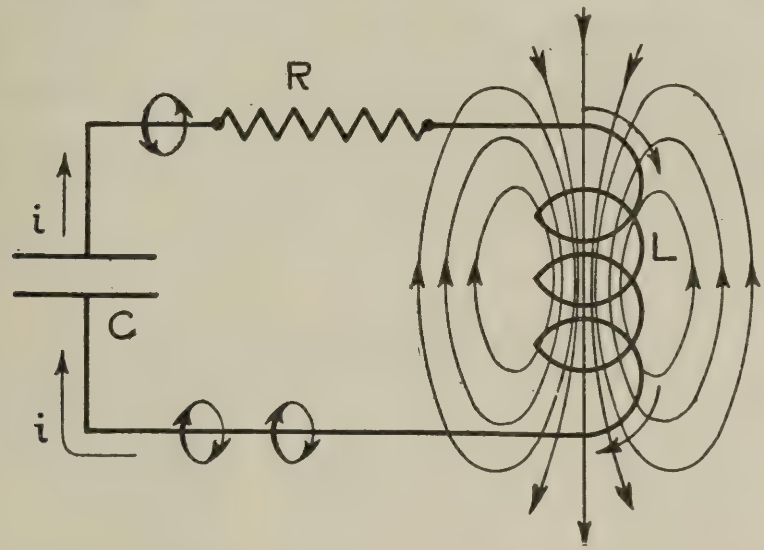


FIG. 163.—All of the Energy Stored in the Magnetic Field. Time $t=\frac{T}{4}$

all of the energy stored in the magnetic field is transferred to the condenser by means of the current, so that at the end of one-half cycle, the energy is again stored in the electric field of the condenser, and the current has ceased to flow. This condition is shown in figure 164.

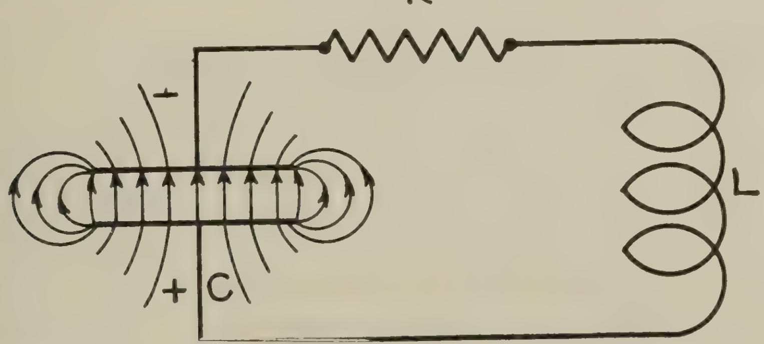


FIG. 164.—All of the Energy is Stored in the Electric Field. Time $t=\frac{T}{2}$

The recharged condenser immediately begins to discharge in the manner previously described, but the discharge current flows in a direction opposite to that of the first discharge, due to the change in polarity. This discharge continues until time $t=\frac{3\,T}{4}$, when the current and the magnetic field again attain maximum values. This condition is shown in figure 165. From this instant to the end of the cycle, time $t=T$, the energy in the magnetic field is restored to the circuit in the form of a current, which again charges the condenser to the same

polarity as existed at the beginning of the cycle, the condition being as shown in figure 162.

This cycle of events is repeated over and over again, until the energy initially stored in the condenser by the battery has been transformed into heat by the I^2R losses, or has otherwise been lost to the circuit.

It will be seen that **there is an oscillating exchange of energy between the electric and magnetic fields** during this interval. **The circuit is said to be oscillating**; that is, an **oscillating current** having a definite period is flowing in the circuit, due to this periodic exchange of energy between the two fields.

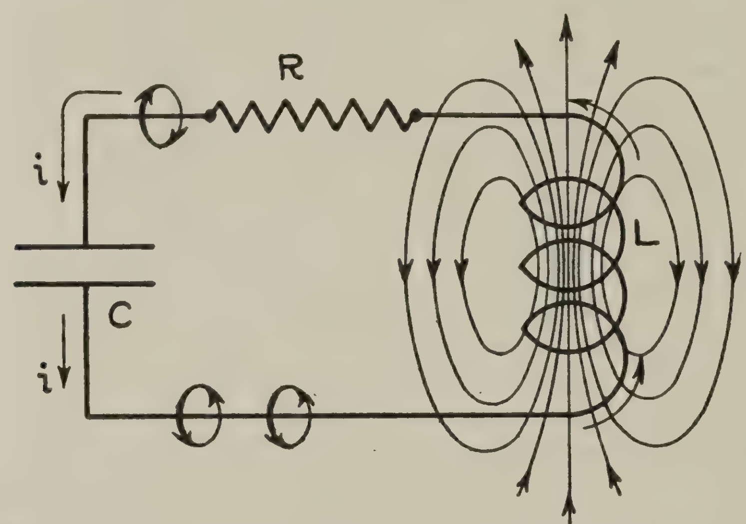


FIG. 165.—All of the Energy Stored in the Magnetic Field. Time $t=\frac{3T}{4}$

Period of the oscillating discharge. If the discharge circuit had no resistance, and did not radiate electromagnetic waves, the exchange of energy between the electric and magnetic fields would continue indefinitely, because there would be no transformation of electrical energy into other forms and, hence, the same amount of energy would exist in the circuit at all times. However, since every circuit has resistance, and does radiate to some extent, some of the energy is lost in heat and radiation during each exchange. Consequently, the energy initially stored in the condenser by the battery is lost to the circuit more or less rapidly. In the following discussion, loss of energy by radiation will be considered as included in the loss due to the resistance of the circuit.

If the resistance is very high relatively to the inductance of the circuit, all of the energy initially stored in the condenser is transformed into heat energy in the resistance during the first discharge, and no oscillations can occur. The discharge of the condenser is then similar to that which would occur if no inductance were in the circuit; that is, the discharge current would rise abruptly from zero to a maximum value, and then decrease more or less rapidly to zero, the flow of current being only in one direction, or **unidirectional.** Such a discharge is said to be

aperiodic, and is obtained when

$$R > 2\sqrt{\frac{L}{C}}$$

The smallest value of the resistance which still results in an aperiodic discharge is when

$$R = 2\sqrt{\frac{L}{C}}$$

For this condition, the circuit is said to be **critically damped.**

However, the interest lies in the condition necessary to obtain an oscillatory discharge. It exists when

$$R < 2\sqrt{\frac{L}{C}}$$

This inequality can also be written in the form

$$\frac{R}{2L} < \frac{1}{LC}$$

The value of the oscillating current at any instant can be shown to be

$$i = \frac{V_0}{\omega L} \epsilon^{-\frac{R}{2L}t} \sin \omega t$$

where

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

and i = instantaneous value of oscillating current in amperes,
V_0 = initial voltage of the condenser in volts,
C = capacity of circuit in farads,
L = inductance or circuit in henries,
R = rf resistance of circuit in ohms.

In the above formula the term $\sin \omega t$ represents an oscillation just as in the ordinary ac case. The first part represents the amplitude of the oscillation which, on account of the factor $\epsilon^{-\frac{R}{2L}t}$ is continually decreasing as time goes on. For $\epsilon^{-\frac{R}{2L}t}$ can be written $\frac{1}{\epsilon^{\frac{R}{2L}t}}$ and as t becomes greater and greater, the denominator increases; thus, the value of the fraction becomes less and less. Therefore, the whole equation represents an oscillation which is decaying with time.

In the formula, $\frac{V_0}{\omega L}$ is the theoretical maximum value of the oscilat-

ing current, $\epsilon^{-\frac{R}{2L}t}$ is called the **damping factor,** and the numerical value of $\frac{R}{2L}$ is the **damping coefficient.**

Since $$\omega = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

This is the **natural frequency** of the free oscillations occurring in a circuit containing inductance, capacity and resistance. Expressing C and L in microfarads and microhenries, respectively, the formula becomes

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} = \frac{R^2}{4L^2}} \cdot 10^6$$

In the case of the usual radio circuit, the term $\frac{R^2}{4L^2}$ is small compared to the term $\frac{1}{LC}$ and, hence, has a negligible effect on the frequency. Therefore, the equation for f can be written

$$f = \frac{1}{2\pi\sqrt{LC}} \cdot 10^6$$

This expression for the natural frequency is the same as that for the resonant frequency. Thus, the frequency of the free oscillations in a given circuit will be the same as the frequency of the impressed emf which, in the case of forced oscillations, leads to the maximum current in the circuit.

The **quantity LC** is called the **oscillation constant** of the circuit. It should be noted that the frequency is expressed in terms of this product; hence, L may be made large compared with C, or vice versa, but so long as their product remains the same the natural, or resonant, frequency of a radio circuit will not be changed.

It was the usual practice to speak of the **natural wave length** of a radio circuit rather than of its **natural period** or **frequency.** The relation between frequency and wave length is fully explained in Part 7, Chapter I. It is

$$\lambda = \frac{v}{f}$$

where v = velocity of progagation of electromagnetic waves $= 3 \cdot 10^8$ meters per second,

f = frequency in cycles per second.

Substituting in this equation the value of f given above, and the numerical value of v

$$\lambda = 2\pi \times 3 \cdot 10^8 \sqrt{LC} \cdot 10^{-6}$$

whence

$$\lambda = 1.885 \sqrt{LC} \cdot 10^3$$

where
λ = wave length in meters.
L = inductance in microhenries,
C = capacity in microfarads.

Values of 1, f and ω for any value of the oscillation constant LC can be found directly, or by interpolation, using Table 13. Examples showing how this formula is used have previously been given.

CHAPTER II. DAMPING, DECREMENT.

Damping. It has just been shown that resistance, in general, has a negligible effect on the natural frequency of an oscillatory circuit and that the free oscillations will have a period, or frequency, determined primarily by the inductance and capacity of the circuit. However, the resistance of a circuit does determine the rapidity with which the amplitude of the oscillatory discharge decreases toward zero.

Assume that the rf resistance of such a circuit is constant, and that the initial discharge of the condenser is just commencing. Call this instant $t=0$. During the first discharge, the current is forced through the constant resistance, and a definite amount of energy is lost to the circuit. Consequently, the energy stored in the magnetic field at time $t=\frac{T}{4}$, although a maximum, is less than the amount initially stored in the condenser by this definite amount. Likewise, the energy stored in

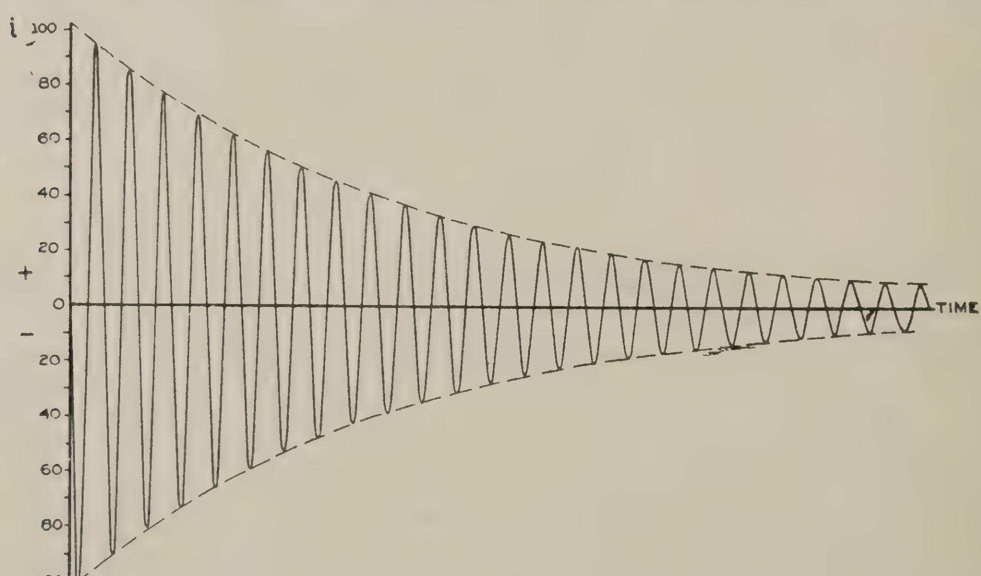


FIG. 166.—A Wave Train of Moderately Damped Oscillations. Ratio of Successive Amplitudes in Same Direction $=\frac{10}{9}=1.11$ Decrement $=0.1$.

the condenser during the second charge will be less than that in the magnetic field, and so on. Thus, the total amount of energy in the circuit steadily decreases. As a result, the voltage of the condenser on each charge is less than on the previous charge. For this reason, the successive amplitudes of the oscillating current decrease in value, and the oscillating current is said to be **damped.** A series of damped oscillations is shown in figure 166. The complete series of current oscillations that occur during the oscillatory discharge of a condenser is called a **wave train.**

The damping of oscillations is also dependent upon the inductance of the circuit, as is shown by the damping coefficient

$$\frac{R}{2L}$$

Thus, the greater the rf resistance and the smaller the inductance, the greater is the damping effect and, therefore, the higher the rate of decrease in the value of the oscillating current. Figure 167 shows a highly damped oscillatory discharge.

Logarithmic decrement. The curve, such as the dash-line curves in figures 166 and 167, drawn through the successive amplitudes in the same direction of a damped oscillating current, is a logarithmic curve and

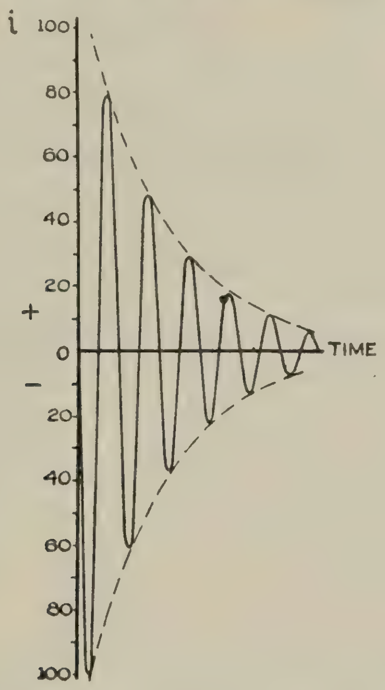


Fig. 167.—Highly Damped Wave Train. Decrement = 0.5.

will approach the axis of abscissas more rapidly as the damping coefficient is increased. The figures also show that the ratio of any maximum to the next succeeding maximum in the same direction is a constant. Thus, if the amplitude of the first maximum is assumed to be 100 units and the second is 90 units, the ratio between the two will be 0.9. The third maximum will also be 0.9 of the second, or 0.81 of the first. The fourth maximum will be 0.9 of the third, or 0.729 of the first, etc. This is the ratio used in figure 166.

Instead of using this constant ratio as a measure of the rate of decrease in current amplitude, it is simpler to employ the Napierian logarithm of this ratio. This is a number, and is called the **logarithmic decrement,** or **decrement per cycle,** or **per complete oscillation.** The manner in which the logarithmic decrement depends upon the characteristics of the circuit will be derived in the following. Use will be made

of the relation that, if T is the natural period of oscillation in seconds,

$$T=\frac{1}{f}$$

where f = natural frequency of the oscillations,

and, since

$$\omega=2\pi f$$

$$T=\frac{2\pi}{\omega}$$

Assume that the rf resistance, inductance and capacity of a circuit are constant. Also, assume that the condenser is charged, and at time $t=0$ begins to discharge in the circuit. The formula for the instantaneous value of the oscillating current is

$$i=-\frac{V_0}{\omega L}\epsilon^{-\frac{R}{2L}t}\sin \omega t$$

Hence, at time $t=0$, the value of the oscillating current is zero, and is so shown in figure 167, in which the values of the oscillating current are plotted for successive instants.

At time $t=\frac{T}{4}=\frac{\pi}{2\omega}$, the quantity $\sin \omega t$ will be unity and the current will be at its first maximum in the negative direction. The next maximum of current in the **same direction** will occur one period later than time $t=\frac{T}{4}$, or at time $t=\left(\frac{T}{4}+T\right)=\frac{5T}{4}=\frac{5\pi}{2\omega}$. Also, the $\sin \omega t$ will again be equal to unity. Substitute these values for t, in the equation for the instantaneous current.

At time $t=\frac{\pi}{2\omega}$

$$i_1=-\frac{V_0}{\omega L}\epsilon^{-\frac{R}{2L}\cdot\frac{\pi}{2\omega}}\sin\frac{\pi}{2}=-\frac{V_0}{\omega L}^{-\frac{\pi R}{4\omega L}}$$

and at time $t=\frac{5\pi}{2\omega}$

$$i_2=-\frac{V_0}{\omega L}\epsilon^{-\frac{R}{2L}\cdot\frac{5\pi}{2\omega}}\sin\frac{5\pi}{2}=-\frac{V_0}{\omega L}\epsilon^{-\frac{5\pi R}{4\omega L}}$$

The ratio

$$\frac{i_1}{i_2}=\frac{\epsilon^{-\frac{\pi R}{4\omega L}}}{\epsilon^{-\frac{5\pi R}{4\omega L}}}=\epsilon^{\frac{\pi R}{\omega L}}$$

If the ratios of the amplitudes $\frac{i_2}{i_3}$, $\frac{i_3}{i_4}$, etc., are obtained in the manner shown above for the ratio $\frac{i_1}{i_2}$, it will be found that they all have the same value $\epsilon^{\frac{\pi R}{\omega L}}$; hence, the ratio of one amplitude to the next succeeding amplitude in the same direction is constant.

Instead of using this ratio as the measure of the decay of oscillations, it is customary and convenient to employ the **Napierian logarithm** of this ratio, which is called the **logarithmic decrement** δ. Hence, remembering that

$$\log_\epsilon \epsilon^x = x$$

then
$$\delta = \log_\epsilon \frac{i_1}{i_2} = \log_\epsilon \epsilon^{\frac{\pi R}{\omega L}} = \frac{\pi R}{\omega L}$$

Using the relation

$$\omega = \frac{1}{\sqrt{LC}}$$

the logarithmic decrement can be written in the following equivalent forms:

$$\delta = \pi \frac{R}{\omega L} = \pi R\sqrt{\frac{C}{L}}\pi R\omega C = \frac{R}{2fL}$$

where R = rf resistance of circuit in ohms,
C = capacity of circuit in farads,
L = inductance of circuit in henries.

For convenience in calculations, the following forms are used:

$$\delta = 3.1416 \cdot 10^6 \frac{R}{\omega L} \qquad \delta = 5 \cdot 10^5 \frac{R}{fL}$$

$$\delta = 3.1416\, R\sqrt{\frac{C}{L}} \qquad \delta = 1.67 \cdot 10^{3-} \frac{R\lambda}{L}$$

$$\delta = 3.1416 \cdot 10^{-6}\, R\omega C \qquad \delta = 5.918 \cdot 10^3 \frac{RC}{\lambda}$$

where C = capacity of circuit in microfarads,
L = inductance of circuit in microhenries,
R = rf resistance of circuit in ohms,
λ = natural wave length of circuit in meters.

The formulas given above for calculating the logarithmic decrement apply to a **single circuit,** the rf resistance of which has a constant value. In many cases, it is much more convenient and accurate to measure the logarithmic decrement of a circuit by one of the methods given in Measurement No. 26 rather than to calculate it.

Plotting a wave train. The logarithmic decrement can also be defined as **the constant decrease in the Napierian logarithms of successive amplitudes in the same direction.** Thus, in plotting a wave train, it is customary to assume that the first amplitude is negative and is also the maximum amplitude of the wave train, and to assign to it an arbitrary value of 100 units. The natural logarithm of 100 is 4.6052. The logarithm of the next negative amplitude is obtained by subtracting the numerical value of δ from 4.6052. The number corresponding to

this new logarithm is the numerical value of the second negative amplitude.

Example:

Let $\delta = 0.05$. Determine the value of the second negative amplitude, assuming that the first negative amplitude is 100.

Solution:

		$\log_\epsilon 100 = 4.6052$
		$\delta = 0.05$
subtracting,	$\log_\epsilon$ of 2nd neg. amplitude	$= 4.5552$
whence	2nd negative amplitude	$= 95.1$

To find the numerical value of the first positive amplitude simply subtract $\frac{\delta}{2} = 0.025$ from 4.6052 for the logarithm of the number, and obtain the corresponding number.

This procedure is continued until the amplitudes are negligible in value.

Number of oscillations in a wave train. Since the decrease in the current amplitudes of a damped oscillatory discharge is logarithmic, the number of oscillations in a wave train is infinitely great. Practically, however, the wave train can be considered as ended when the oscillations are reduced to one per cent, or even ten per cent, of the maximum amplitude. In spark transmission, it is usual to neglect amplitudes of less than ten per cent of the maximum. The number of oscillations in a wave train, before the amplitude is reduced to ten per cent of the maximum, is

$$n = \frac{2.3026 + \delta}{\delta}$$

Table 7 gives the number of oscillations in a wave train for various decrements.

Duration of a wave train. The practical limit placed on the number of oscillations in a wave train permits the duration of any wave train to be determined. It is necessary to know both the frequency and the decrement. From the first is obtained the periodic time in seconds of each complete oscillation, while the latter gives the number of oscillations in the wave train.

Example:

Given $\delta = 0.05$ and $\lambda = 1{,}000$m. Find the duration of a wave train.

Solution:

From Table 13, $f = 3 \cdot 10^5$ cycles per second

and $\quad T = \frac{1}{f}$

substituting $\quad T = \frac{1}{3 \cdot 10^5} = 3.33 \cdot 10^{-6} = 3.33$ microseconds.

From Table 7 $\quad n = 47$

The duration of a wave train equals the periodic time of one oscillation multiplied by the number of oscillations in the wave train; hence,

duration $= nT$

substituting, $= 47 \times 3.33 \cdot 10^{-6} = 156.5 \cdot 10^{-6}$

whence duration $= 156.5 \cdot 10^{-6}$ second, or 156.5 microseconds.

Overlapping wave trains. If there are 1,000 wave trains per second, such as are emitted by a properly adjusted 500-cycle quenched-spark transmitter, the time interval between the beginning of one wave train and the beginning of the next is $\frac{1}{1,000}$ second. The duration of the wave train given in the above example is very short compared with the time interval between successive wave trains. In this instance, oscillations are present during only 15.7 per cent of the time; that is, the circuit is idle 84.3 per cent of the time, and there can be no overlapping of the wave trains.

However, if the decrement is reduced to 0.02, and the wave length is increased to 3,000 meters, there will be 116 oscillations in each wave train, the duration of the train being $1.16 \cdot 10^{-3}$ second. In this case there will be an overlap between trains of 16 oscillations. This will produce a **mushy** note in the receiver.

Other methods of supplying energy to an oscillatory circuit. In the previous discussion, it was assumed that the circuit, in which the oscillatory discharge took place, derived its energy from a condenser, which was charged but once by a battery. In practice, the condenser is charged from the charging circuit many times per second. Thus, if an interrupter, such as a buzzer or a commutator, were substituted for the switch S, figure 161, the condenser could be charged several hundred times per second. However, during the intervals that the commutator closes the discharging circuit, the condenser would discharge in the manner just described. Thus, if the commutator permitted the condenser to be charged by the battery 500 times per second, and the condenser discharged into the oscillatory circuit each time, there would be 500 wave trains per second.

The condenser can also be charged from an alternating current source, which is the usual method employed in spark transmitters. If the frequency of the alternating current is 500 cycles, and the condenser is permitted to discharge once per alternation, and at the instant that its voltage is a maximum, there will be 1,000 wave trains per second in the oscillatory circuit.

The oscillatory circuit can also have its initial amount of energy stored in the magnetic field by an interrupted direct current, as in the case where a buzzer is used to excite a wavemeter circuit, figure 89; or the circuit can be acted upon inductively for a brief interval of time, and then permitted to oscillate in its own period. This latter method is

practically achieved in the 500-cycle, quenched-spark transmitter where the primary oscillatory circuit is the inducing circuit, and the antenna circuit is the one which is allowed to oscillate in its natural period, after the primary circuit has been opened by the action of the gap.

Effective value of the current in damped wave trains. The energy in each wave train is, for all practical purposes, dissipated in heat and radiation before the beginning of the next wave train. The effective value of the current I, can, therefore, be obtained from the energy of the initial charge of the condenser in the following manner.

The condenser is charged by the source n times per second, and its energy on each charge is

$$W_1 = \frac{1}{2} C V_0^2$$

Hence, the total energy put into the condenser in one second, or **the power,** is

$$P = \frac{1}{2} C V_0^2 n$$

However, since all of this energy is dissipated in heat and radiation, and letting R = rf resistance of the circuit,

then $$\frac{1}{2} C V_0^2 n = I^2 R = P$$

from which $$I = \sqrt{\frac{P}{R}} = V_0 \sqrt{\frac{nC}{2R}}$$

where I = effective (rms) value of damped current, in amperes, and is given by the steady deflection of a hot-wire ammeter,
V_0 = maximum voltage of the condenser in volts,
C = capacity of condenser in farads,
R = rf resistance of circuit in ohms.

It is apparent that the first maximum of a damped oscillating current may be a great many times larger than the value indicated by an ammeter in the discharge circuit. It is likewise apparent that the rate at which work is done in the discharge circuit can be very high for an exceedingly short period. Thus, in the example previously given, the condenser is charged during 0.0005 second, and then discharges the whole amount of energy in 156.5 microseconds.

Example:

An oscillatory discharge occurs 1,000 times per second in a circuit containing a capacity of 0.032 μf in series with an inductance of 8.78 μh when the condenser is charged to a potential of 17,000 volts. The decrement of the circuit is 0.05. Calculate (a) the maximum current, (b) the rate of doing work, and (c) the effective current.

Solution:

The LC product is 0.281

From Table 13 $\lambda = 1{,}000\text{m}$,

$f = 3 \cdot 10^5$ cycles per second,

$\omega = 1.885 \cdot 10^6$

The rf resistance of the circuit is found by using the formula

$$\delta = 5 \cdot 10^5 \frac{R}{fL}$$

from which $R = 2 \cdot 10^{-6} \delta f L$

substituting, $= 2 \cdot 10^{-6} \times 5 \cdot 10^{-2} \times 3 \cdot 10^5 \times 8.78 = 0.263$

whence $R = 0.26\,\Omega$

(a) The maximum current amplitude occurs at time $t = \frac{T}{4} = \frac{\pi}{2\omega}$

At this instant

$$i_1 = -\frac{V_0}{\omega L} \epsilon^{-\frac{\pi R}{4\omega L}} \sin\frac{\pi}{2}$$

Solving the exponential term

$$\epsilon^{-\frac{\pi R}{4\omega L}} = \epsilon^{-\frac{3.1416 \times 0.26}{4 \times 1.885 \cdot 10^{-6} \times 8.78 \cdot 10^{-6}}} = \epsilon^{-0.012}$$

An inspection of Table 12 shows that the numerical value of $\epsilon^{-0.012}$ is 0.99. Since it is practically equal to unity, it need not be considered. Also, $\sin\frac{\pi}{2} = 1$. The formula for the instantaneous current then becomes

$$i_1 = \frac{V_0}{\omega L}$$

substituting, $= \frac{1.7 \cdot 10^4}{1.885 \cdot 10^6 \times 8.78 \cdot 10^{-3}} = 1.026 \cdot 10^3$

whence $i_1 = 1{,}026$ amperes.

This is the **maximum current** that will flow in the circuit during the oscillatory discharge. The necessity for providing conductors having ample current carrying capacity for this rf current is apparent.

(b) The energy stored in the condenser just before the discharge begins is

$$W = \frac{1}{2} C V_0^2 \qquad \text{(joules)}$$

substituting, $= \frac{3.2 \cdot 10^{-8} \times (1.7 \cdot 10^4)^2}{2} = 4.62$

whence $W = 4.62$ joules.

The decrement and wave length are the same as in the previous example. Therefore, the duration of the oscillatory discharge is

$156.5 \cdot 10^{-6}$ second. In this brief period, 4.62 joules are dissipated in heat and radiation. If this continued for one second, the energy in joules lost to the circuit would be

$$4.62 \times \frac{1}{1.565 \cdot 10^{-4}} = 2.95 \cdot 10^4$$

or 29,500 joules.

Thus, work is done in the oscillatory circuit during the discharge at the rate of 29,500 joules per second, which is the same as 29.5 kw.

(c) This rate of doing work, or power of 29.5 kw, is dissipated in the rf resistance of the circuit. Since

$$P = I^2R$$

The effective current is $I = \sqrt{\frac{P}{R}}$

substituting, $= \sqrt{\frac{2.95 \cdot 10^4}{0.26}} = \sqrt{11.35 \cdot 10^4} = 3.37 \cdot 10^2$

whence $I = 337$ amperes.

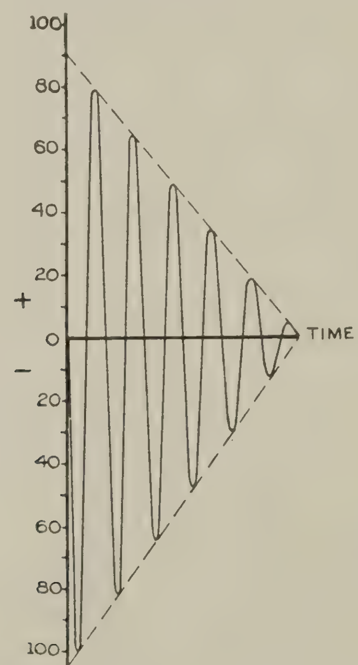


Fig. 168.—Linear Decrease of Current Oscillations in a Discharge Circuit Which Includes a Spark Gap.

This is the effective value of the oscillating current **during the wave train.**

If the condenser is charged and discharged 1,000 times per second, the power put into the condenser would be 4.62 kw, and this would be dissipated mostly in heat in the rf resistance of the circuit. The effective value of the current for this condition would be equal to

$$I = \sqrt{\frac{P}{R}}$$

substituting, $$=\sqrt{\frac{4.62\cdot 10^3}{0.26}}=1.33\cdot 10^2$$

whence $$I=133 \text{ amperes.}$$

This would be the reading of the ammeter in the circuit.

Linear decrement. If the discharge circuit includes a spark gap, the resistance of this circuit is no longer a constant quantity. This is due to the action of the spark gap, which offers a lower resistance to large currents than to small currents; in fact, this varying resistance of the spark gap usually preponderates. The relation between spark-gap resistance and current is linear. Hence, the decrease in successive current oscillations is **linear,** as shown in figure 168. The number of oscillations that will occur in such a circuit is largely influenced by the type of gap employed. The quenched-spark gap, which deionizes very rapidly, permits only a few highly damped oscillations to occur before it opens the discharge circuit.

CHAPTER III. DAMPED OSCILLATIONS IN COUPLED CIRCUITS

Effect of a nearby circuit on the frequency and decrement of the inducing circuit. The first two Chapters of this Part dealt only with a single isolated circuit in which an oscillating discharge occurred. The presence of a closed circuit, which is coupled sufficiently to the one in which the discharge is taking place so that current is induced in it, will react on the inducing circuit to change its inductance and rf resistance. The **inductance** of the inducing circuit is **decreased,** while the **rf resistance** is **increased.** This operates to **increase** the **decrement.** In addition, the natural frequency of the circuit is decreased due to the decrease in the inductance.

The above conditions are undesirable, unless the power transferred to the nearby circuit is to be utilized in some manner. Therefore, care should be exercised to reduce this effect to a minimum in whatever manner that may be possible. In this category may be included: tapped inductance coils, only a part of which is in the active circuit, idle inductance coils, which are lying in the immediate vicinity of the circuit, closed circuits, parts of which may be resonant to the natural frequency of the active circuit, neighboring antennas in case the active circuit is an antenna circuit, etc.

Free oscillations in coupled circuits. The usual and desired condition encountered in radio engineering is that of two circuits intentionally coupled and tuned to have the same natural frequency, with the oscillatory discharge occurring periodically in the primary circuit These requirements must be fulfilled in order that power may be transferred from the inducing (primary) circuit to the coupled (secondary) circuit.

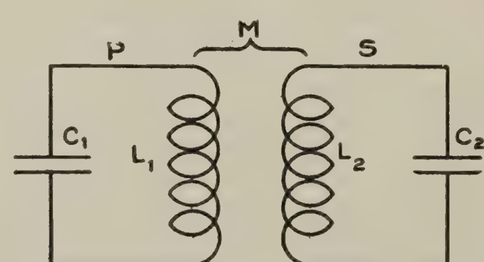


FIG. 169.—Inductively Coupled Circuits.

Figure 169 shows two circuits coupled inductively, which is the method of coupling usually employed. P is the primary circuit in which the oscillatory discharge occurs, while S is the secondary circuit. It is assumed that the oscillation constant of P equalled that of S when the circuits were not coupled together; that is, $L_1C_1 = L_2C_2 = LC$, where LC is the oscillation constant corresponding to the natural frequency of each.

Assume that an oscillatory discharge is taking place in P. The varying magnetic flux due to oscillating current flowing in P, threads S and induces an emf in S. The resulting oscillating current in S induces an emf in P and another oscillating current is produced in P. In this manner, energy is transferred from P to S and then re-transferred by S to P. This action continues until all the energy initially stored in condenser C_1 has been dissipated in heat and radiation.

Effect of tight coupling. At first glance, it would seem that the transfer of energy from the primary to the secondary circuit would be accomplished most efficiently with the tight coupling. However, this is not exactly the case.

Assume that the primary circuit does not include a spark gap and, therefore, remains closed all the time. For this condition, the magnetic effect of the inducing current in P is sometimes aided and sometimes opposed by that of the current induced in it by circuit S. The result of this is that the effective inductance of circuit P is alternately increased and decreased, and the oscillation constant of P is changed to **a new value** differing from L_1C_1. It can be shown that **two currents of different frequencies,** corresponding to the two new values of L_1C_1 **flow in circuit** P. These frequencies are connected with the natural frequency f of the circuit by the following relations:

$$f_1=\frac{f}{\sqrt{1+k}} \text{ and } f_2=\frac{f}{\sqrt{1-k}}$$

where k = coefficient of coupling between circuits P and S.

Expressed in terms of the natural wave length of circuit P, the wave lengths λ_1 and λ_2 corresponding, respectively, to f_1 and f_2 are:

$$\lambda_1=\lambda\sqrt{1+k} \text{ and } \lambda_2=\lambda\sqrt{1-k}$$

Similarly, the reaction on circuit S produces **two currents of different frequencies in S.** The natural frequency f of this circuit also disappears, and in its place appear the two frequencies f_1 and f_2, and their corresponding wave lengths λ_1 and λ_2, which are equal to and identical with those of circuit P; that is, instead of both P and S oscillating in the same natural frequency f, and the corresponding wave length λ, each one now oscillates in two frequencies f_1 and f_2 which are the same for each circuit.

If the coupling is not extremely loose, but is sufficiently loose to make

$$k^2<\left(\frac{\delta_1-\delta_2}{2\pi}\right)^2$$

a slight reaction is noticeable. This reaction results in a change in the damping of the oscillations of the two circuits. The decrement of the more weakly damped oscillation is increased and that of the more highly damped oscillation is reduced, the two approaching more closely to the same value.

If the decrements of the two oscillations are widely different, the amplitude curve of the resultant oscillation tends to become identical with the oscillation having the lower damping.

As previously stated, all the energy to be transformed into oscillatory power is originally stored in the primary condenser. When the primary condenser circuit is closed, this energy starts surging back and forth from the primary capacity to the primary inductance, and also starts to flow in the secondary circuit, due to mutual induction. If the two circuits are properly tuned, with 20% coupling, all of the energy will have been transferred to the secondary circuit in $\frac{1}{2k}$ cycles. Unless prevented from doing so, the energy will then start to flow back to the primary circuit. If, however, the primary circuit is opened by some device at an instant when all of its energy has been transferred to the secondary circuit, the re-transfer of energy to the primary circuit is made impossible because no current can flow in the primary circuit if that circuit is open.

Such action of opening the primary circuit is accomplished by a quenched-spark gap when properly adjusted. The forms of current in the primary and secondary circuits are as indicated in the following curves, Fig. 170 which represent circuits with a 20 per cent coupling.

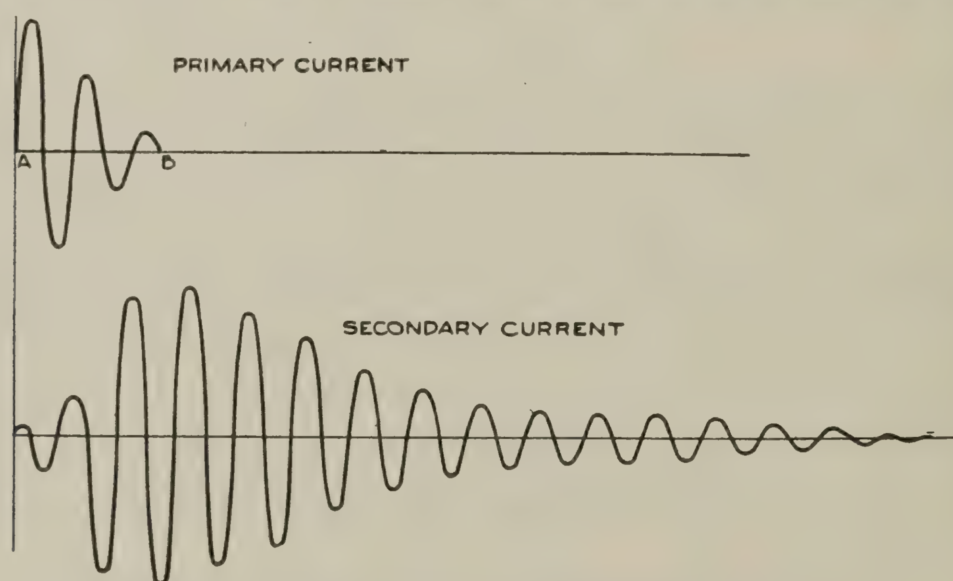


FIG. 170.—Relation of Current in Primary and Secondary.

The number of cycles per beat for such a coupling is 5; therefore the time during which the energy is being transferred to the secondary will be equal to two and one-half cycles. At the time the primary circuit is opened, and from this time on, the secondary circuit oscillates just as if the primary circuit were not present.

The disturbance band of a modulated wave. It is of importance to note that an alternator, or any other source that might be used at a

transmitting station to produce an undamped current of constant amplitude and single frequency, except for any harmonics that might be present, will produce, when modulation takes place, a current flowing into the transmitting antenna which may be shown to be equivalent to a number of component harmonic currents of different amplitudes and frequencies.

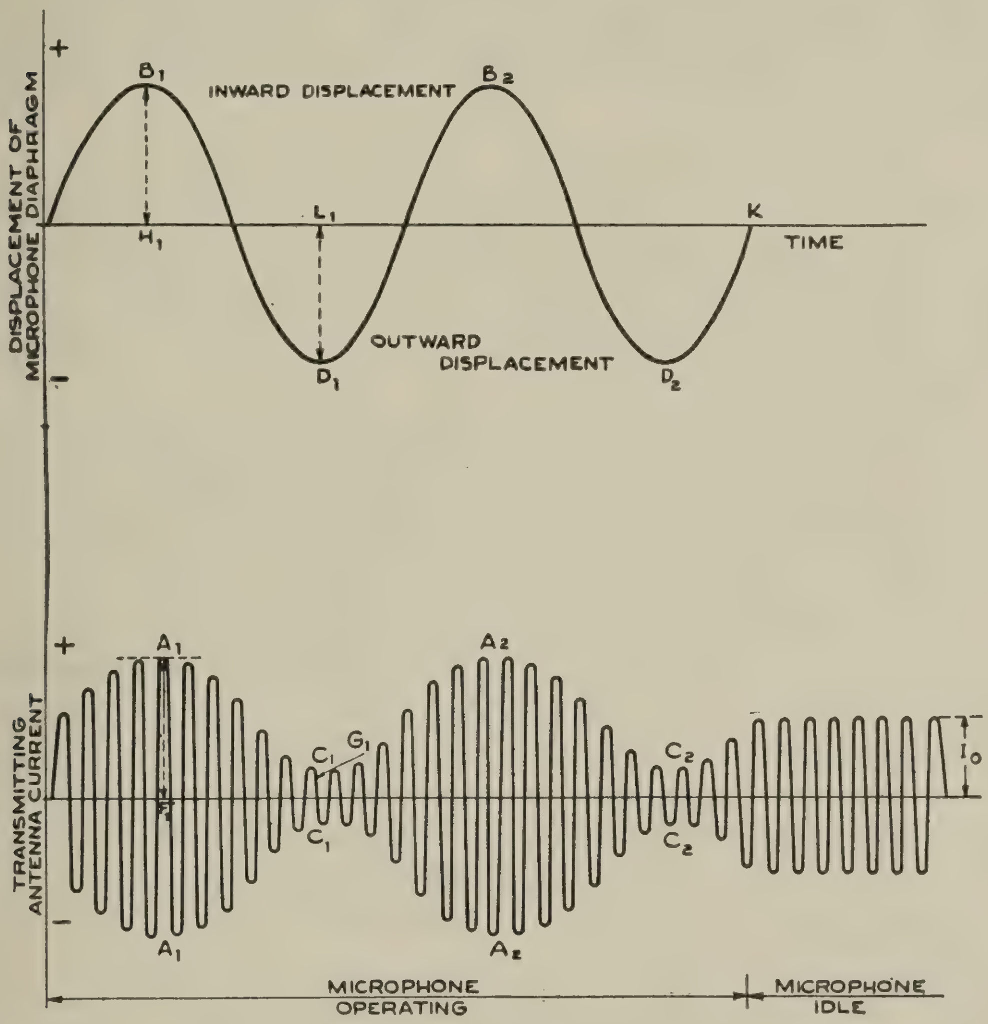


FIG. 171.—Modulated Current in Antenna.

The current represented by the graph of figure 171 is made up of three harmonic currents of the following frequencies:

	Amplitude	Frequency
Component No. 1	I_0	f
Component No. 2	$\frac{I_0{}^1}{2}$	$(f+f_1)$
Component No. 3	$\frac{I_0{}^1}{2}$	$(f-f_1)$

If $f=300{,}000$ and $f_1=1{,}000$

then the three frequencies will be 300,000, 301,000 and 299,000, which means a difference between the lowest and highest frequency of 2,000

cycles, or about two-thirds of one per cent of the frequency of the unmodulated carrier wave. On the other hand, if

$$f = 20{,}000 \qquad \text{and} \qquad f = 1{,}000$$

the three frequencies will be 20,000, 21,000 and 19,000, which means a difference between the lowest and the highest frequency of about ten per cent of the frequency of the modulated wave.

Speech modulated current is made up of currents of a very large number of frequencies, one of which is the frequency of the carrier wave and the others are

$$(f+f_1),\ (f-f_1),\ (f+f_2),\ (f-f_2), \ldots\ldots (f+f_n),\ (f-f_n)$$

where f_1, f_2, f_3, etc., are all the frequencies included in the human voice. The higher the frequencies of f_1, f_2, etc., and the lower the frequency of f, the larger becomes the difference between the highest and the lowest frequency, expressed as a percentage of the frequency of the carrier wave.

For the transmission of voice, or other musical sounds, it is necessary that the receiving antenna reproduce the characteristics of the transmitting antenna. It therefore follows that the receiving antenna current must have the same frequencies as the transmitting antenna current. The receiving antenna circuit must not be sharply tuned to any one frequency, to the partial or entire exclusion of all others, but must be so designed as to be able to pick up all the modulating frequencies equally well. This means that if the difference between the maximum and minimum frequencies, expressed as a percentage of the carrier frequency, is very large, the tuning of the receiving circuit must be broad in order for it to respond equally well to a wide range of frequencies.

In the tuning of the receiving circuit for voice reception, it will be noted that if the receiver has a certain degree of sharpness of tuning, a high-pitched voice will be less distinct than a low-pitched one. The effect is well illustrated when listening to radiotelephone transmission on long wave lengths in the neighborhood of 15,000 meters. It is quite possible to adjust the receiving circuit to such sharp resonance that the speech is unintelligible, although very loud. It is generally best to decrease the strength of the signal and at the same time broaden the tuning of the receiver, thus permitting an improvement in the quality of the received speech.

These effects of voice producing a broadening of the carrier wave are also true for interrupted continuous wave signaling and keying of continuous waves. The frequency at which the interrupter of an *ICW* transmitter breaks up the continuous waves will produce a broadening of the transmitted wave, the width of the disturbance band being dependent upon the frequency of the breaks. In the case of keying of continuous waves, the broadness of the carrier wave depends upon the speed of transmission, that is, the speed at which dots and dashes are

made. For very high speeds the wave becomes very broad, and in some cases so broad that a great deal of interference is produced. This is especially true if the carrier wave is of the order of 15,000 meters.

There are other conditions which must be considered when the transmission or reception of modulated continuous waves is concerned. Since the continuous waves are of varying amplitude, and may even be completely stopped, it is necessary to consider the decrements of the circuits through which the currents flow just as though the same circuits were going to be used for damped-wave telegraphy.

PART 6.

THEORY OF VACUUM TUBES

CHAPTER I. GENERAL.

Current flow in conductors. When a battery is connected into a circuit and a current flows through the circuit, it is ordinarily said that the current flows from the positive terminal of the battery through the circuit to the negative terminal. The early experimenters in electricity assumed that the current was a flow of positive electricity from the positive terminal of the battery through the circuit to the negative terminal. It is now believed that the current is a flow of negative charges of electricity in the opposite direction around the circuit.

More exactly, as explained in a previous Chapter, the flow of current consists in the motion of extremely small charges of negative electricity, called electrons, and the action of the battery voltage is to drive these electrons around the circuit. Electrons are present in very great numbers in all metals and ordinarily move around inside the metal in a haphazard manner, bumping into the atoms of the metal. When the battery voltage is applied, the electrons are forced to take up a general motion around the circuit in addition to their haphazard motion. This drift of the electrons around the circuit constitutes the current flow.

Emission of electrons. The haphazard motion of the electrons depends upon the temperature of the metal. When the metal is heated, the electrons jump around more rapidly, some of them are slowed up by collision with the atoms, and some of them are hit in such a way as to be speeded up. At the surface of the metal, the atoms cling together very strongly and form a sort of tough skin which ordinarily prevents the electrons from leaving the metal. When the metal becomes quite hot, however, some of the more rapidly moving electrons attain a high enough speed to shoot through the restraining skin and leave the metal completely. This is what happens when the filament of a vacuum tube is heated, and the filament is then said to be emitting electrons. Since, in general, the motion of the electrons within the metal becomes more rapid the higher the temperature of the metal, it is evident that a greater number will be able to escape in each second, the hotter the filament becomes. The filament of an ordinary incandescent lamp is emitting enormous numbers of electrons every second. It is natural to ask what becomes of them. In this case, where a filament alone is present, the electrons form a sort of cloud around the filament. Since they are all charged negatively and the filament is charged positively

because of having lost these negative charges, the tendency is for the electrons to be driven back into the filament again, both because of the repulsion of the other electrons in the cloud and the attractive force of the filament. In time there will be just as many going back into the filament per second as are being emitted. The number of electrons emitted depends upon the temperature and the material of the filament. Tungsten emits electrons quite freely, can be heated to a high temperature, and is, therefore, used in many types of vacuum tubes. Platinum does not emit electrons freely, but can be coated with certain oxides, such as calcium oxide, barium oxide, and strontium oxide, which emit a very great number of electrons even at a low heat.

Thorium is also a metal which emits electrons freely. It is possible to increase the emission from a tungsten filament by adding a small amount of thorium to the tungsten before drawing the filament. When operated at a proper temperature, the thorium will coat the surface of the filament producing, in effect, a filament of thorium. Such filaments are called **thoriated** filaments.

Since each electron emitted from the filament has a definite electrical charge, the emission of electrons from the filament constitutes an electric current which flows through the surface of the filament. The current per square centimeter of filament surface is given by the product of the number of electrons emitted per second from a square centimeter of surface and the charge on each electron. The direction of the current is **opposite** to the motion of the electrons, that is, it flows from the outside to the inside of the filament. When the number of electrons returning to the filament per second is the same as the number emitted, then the total current will be zero. This is the condition of equilibrium of a filament by itself as discussed above. It is customary to speak of the emission from a filament in terms of current rather than the number of electrons emitted in a second.

Law of Emission. The emission from a filament increases very rapidly as the temperature of the filament is increased. The law connecting the emission with the temperature is:

$$i = aT^{1/2}\epsilon^{-\frac{b}{T}}$$

where i is the emission per unit area or the current per unit area, a and b are constants which depend upon the material and surface conditions,

T = the absolute temperature of the filament,

ϵ — the base of the natural system of logarithms.

The rapidity of increase of emission with temperature is shown in figure 309. For tungsten, the emission is 0.3 of a milliampere per cm^2 at 1800° Abs, rising to 4 ma at 2000°, to 48 ma at 2,200° and to 365 ma at 2,400° which is roughly the operating temperature in vacuum tubes.

Initial velocity. Inside the metal, the velocity of the thermal motion of the electrons varies greatly from one electron to another. The average velocity, however, is definitely related to the temperature and increases with the temperature. As a result of the variation in velocity of the individual electrons inside the metal, some of them will shoot through the restraining surface skin with a very high velocity while others will have barely enough velocity to get them through. Thus the velocity of the emitted electrons, or what is called their

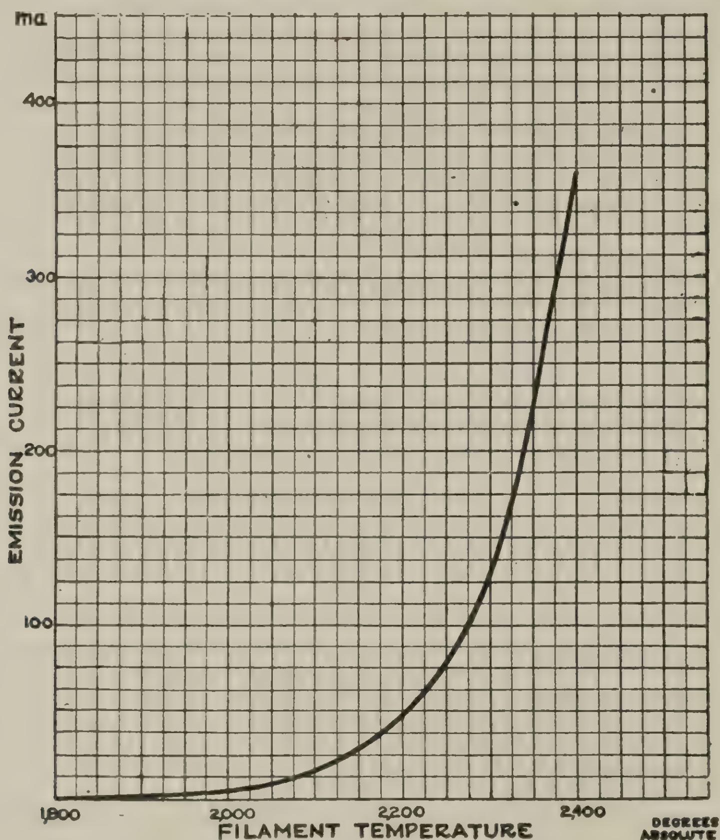


FIG. 172.—Curve Showing Increase of Emission from a Tungsten Filament with Increase in Temperature.

initial velocity, varies widely from one electron to another; some may be just barely moving when they leave the metal while others may leave the metal surface with many times the velocity of a rifle bullet. The **average initial velocity** is dependent upon the temperature and increases with the temperature just as in the case of the electrons within the metal. The average velocity is such that the average kinetic energy of the emitted electrons is the same as the kinetic energy which a molecule of gas would have, if the gas were at the same temperature as the heated metal. Now, the velocity of a molecule of gas at a temperature corresponding to that of the filament of a vacuum tube is very high, and the mass of a molecule of any gas is several thousand times the mass of an electron. Hence, to have the same kinetic energy, the initial velocity of emitted electrons must be extremely high, many miles per

second. On the other hand, when electrons are acted upon by electrical forces, it is not difficult to give them velocities which are very much higher than this initial velocity, so that usually the effects of the initial velocity are minor in determining the behavior of vacuum tubes. It is quite customary to speak of the velocity of electrons in terms of the velocity which would be imparted to them by the action of an electrical force. If an electron moves from one point in an electrical field to another point at a higher potential, the work done upon the electron will be the product of its charge and the difference in potential between the two points. The electron will have its kinetic energy increased by the amount of work done upon it and, hence, its velocity will be increased by a definite amount which depends upon the difference in potential between the two points. Difference in potential is measured in volts; hence, the velocity of an electron can be spoken of as a velocity of a certain number of volts, meaning the velocity imparted to the electron by moving between two points in the field between which there is an increase in potential of so many volts.

In terms of volts, the **average velocity** of the electrons emitted from a vacuum tube filament under normal operating conditions is of the order of tenths of a volt, which is usually small compared to the voltage applied to the tube.

Example:

Suppose an electron to be moved from rest by electrical force from one point to another which is higher in potential by 10 volts. What will be its velocity?

Solution:

The work done, W, upon the electron will be the product of its charge Q and the potential difference V. Hence,

$$W = QV$$

If Q and V are expressed in practical units, viz. coulombs and volts, then W is obtained in joules or 10^7 ergs. Therefore,

$$W = QV \cdot 10^7 \text{ ergs.}$$

Now, the kinetic energy given to the electron will be equal to W. The kinetic energy is equal to $\frac{1}{2}mv^2$ and is also expressed in ergs if m is the mass in grams and v the velocity in centimeters per second.

Therefore,

$$\tfrac{1}{2}mv^2 = QV \cdot 10^7$$

and

$$v = \sqrt{\frac{2QV}{m} \cdot 10^7}$$

It is only necessary to substitute in this formula, the value of V in volts as given in the problem and the values of Q and m for an electron in order to find the required velocity v.

Given $V = 10$ volts and for an electron $Q = 1.6 \cdot 10^{-19}$ coulomb, $m = 8.8 \cdot 10^{-28}$ gram.

Hence $$v=\sqrt{\frac{2\times1.6\cdot10^{-19}\times10\cdot10^{7}}{8.8\cdot10^{-28}}}=1.9\cdot10^{8}$$

$$v=1.9\cdot10^{8}\text{ cms per second}$$

or $$v=1{,}900\text{ km per second.}$$

Thus, an electron moving through a difference in potential of ten volts attains a velocity of 1,900 kilometers per second. This is more than two thousand times the muzzle velocity of a projectile from a fourteen inch gun.

In the case of high-power vacuum tubes where the plate voltage may be 10,000 volts, the electrons hit the plate with a velocity of 60,000 kilometers per second or 37,000 miles per second, or one-fifth the velocity of light. Assuming the average initial velocity of emitted electrons to correspond to 0.2 volt, this initial velocity will be 270 kilometers per second or 170 miles per second.

The two-electrode vacuum tube. Suppose that inside a highly exhausted vacuum tube, as in figure 173, there is a filament F which is heated by the current from a battery A so as to make the filament emit electrons. Suppose that inside the tube there is also a metal plate P and that outside the tube this plate is connected through a battery B and ammeter A to the filament. If, as shown in the figure, the

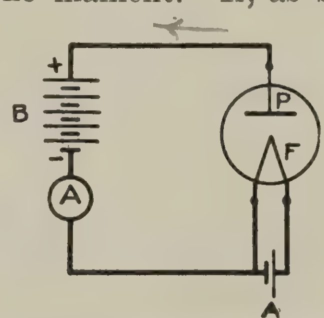


Fig. 173.—The Two-Electrode Vacuum Tube.

positive terminal of the battery B is connected to the plate, the ammeter will indicate a current flowing. Current flows because the positive plate attracts the electrons emitted by the filament so that they move to the plate and enter it. They then flow around the external circuit from the plate back to the filament, constituting a current around the circuit.

According to convention, the direction of the current flow is opposite to the motion of the electrons, and the ammeter which accords with convention would indicate a current flowing from filament to plate outside the tube.

Characteristic curves. It is of interest to investigate how this current flow depends upon the polarity and magnitude of the B battery voltage. Assume that the filament current, and hence the filament temperature, is kept constant. If the B battery is reversed so that the plate is made negative, no current will flow; because then the plate will

repel rather than attract the electrons emitted by the filament. As the plate is made more and more positive, the current starting at zero at approximately zero voltage will increase as the *B* battery voltage increases. For lower voltages, the increase in current with the voltage will be more and more rapid; but, finally, the increase in crruent will be less rapid until at last increases in the battery voltage will produce no further appreciable increase in the current. This is quite different from the way ordinary conductors, such as a coil of wire or rheostat, behave. These conductors obey Ohm's law, and the current increases as fast as the voltage increases. A curve showing how the current through any device increases as the voltage increases is called a **characteristic curve.** The characteristic curve of an ordinary resistance is a straight line. Such a curve is shown in figure 174, and represents the characteristic curve of a 2-ohm resistance.

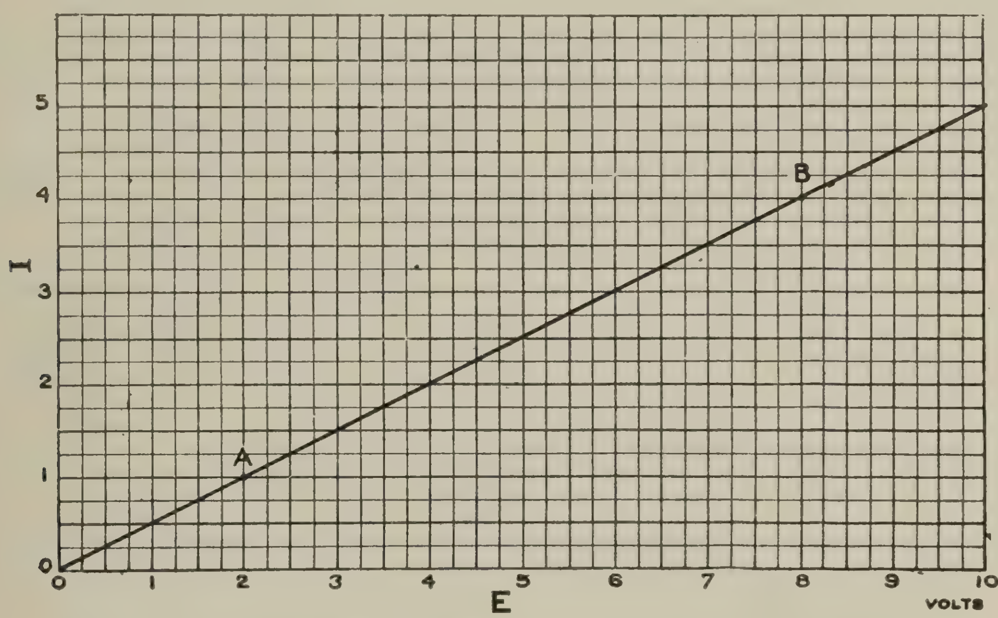


FIG. 174.—Characteristic Curve of a Noninductive Resistance.

By Ohm's law, for 2 volts a current of 1 ampere flows, corresponding to point A. For 8 volts, 4 amperes will flow, as given by point B.

In figure 175 is shown in solid line the characteristic curve of a two-electrode tube, showing the variation of current with *B* battery voltage as described above. The flat part of the curve, *CD*, represents the maximum current that can flow through the tube. The current can not go higher, because all of the electrons emitted by the filament flow to the plate as fast as they are emitted. For lower plate voltages the current is less; and, since the number of electrons emitted by the filament is unchanged, it follows that only a portion of the emitted electrons are flowing across to the plate. It is of interest to inquire why this is so; why all of the emitted electrons do not flow over to the plate excepting when the plate voltage becomes high? It was said that the electron flow to the plate starts because the plate is positive and attracts the

electrons away from the immediate vicinity of the filament. When the current is flowing, however, there will be electrons which are in motion over to the plate and which will, after a fashion, fill up the space between filament and plate. These electrons are negatively charged and, therefore, tend to repel those electrons which are about to start over to the plate. Thus the force exerted by the electrons in the space between the filament and plate tends to neutralize the force exerted by the plate. In fact, there will be an imaginary surface surrounding

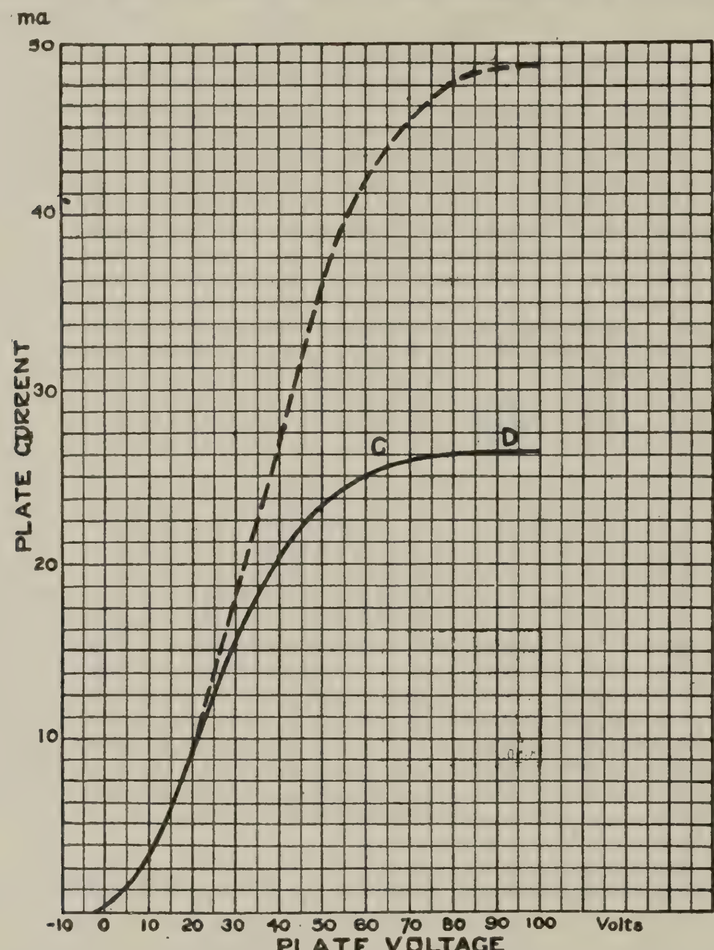


FIG. 175.—Characteristic Curve of a Two-Electrode Vacuum Tube.

the filament at a certain distance away and at this surface the force of the plate will be completely neutralized. The force in the region between this imaginary surface and the surface of the filament will be such as to draw the emitted electrons back to filament, while on the other side of the surface the force will be such as to pull the electrons over toward the plate. Any electrons which are emitted with sufficient velocity to overcome the opposing force exerted in the region near the filament, and hence to pass the imaginary surface, will flow over to the plate under the influence of the aiding force on the other side. Those electrons which have not sufficient velocity to reach the surface of zero force will be pulled back into the filament again. Thus the location of this imaginary surface of zero force determines the percentage of the

electrons which flow over to the plate. The nearer the surface lies to the surface of the filament, the greater the number of electrons which can get by and hence the greater will be the plate current. The location of this surface depends upon the **plate voltage** and the number of electrons which are in motion to the plate or the **space charge.** An increase in the plate voltage tends to move the surface of zero force nearer the filament, while an increase in the plate current has the effect of moving the surface away from the filament. When the voltage of the plate is negative with respect to the filament, the surface coincides with the glass wall of the tube; as the voltage becomes increasingly positive, the surface shrinks toward the filament and an increasing number of electrons flow to the plate. When the plate voltage reaches a high positive value, the surface coincides with the surface of the filament and all of the emitted electrons flow to the plate. In general, the plate current cannot exceed this value, which is called the **saturation current,** excepting for the possibility at very high voltages of actually pulling electrons out of the filament and hence increasing the effective emission from the filament.

Effect of filament temperature. If the temperature of the filament is increased, the rising portion of the curve of figure 175 is very nearly unchanged. The curve will, however, rise to a higher value before flattening out, as shown by the dotted line. This higher value corresponds to the increased emission of electrons from the filament.

Rectification. Because of the fact that current can only flow to the plate of a two-electrode tube when the plate is positive, the tube can be used to rectify alternating current into direct current. Thus, if instead of using a plate battery as shown in figure 173, an alternating voltage from a generator or radio receiver is inserted in the plate circuit, current will flow only during the half cycle when the plate is positive; and this current will consist of pulses which are all in the same direction. A galvanometer in the plate circuit would then show a deflection corresponding to the average current flowing through it. In radio reception, these pulses can act upon the telephones, giving a signal. A similar rectifying effect can be obtained in virtue of the fact that the curve has bends in it and is not straight as for an ordinary resistance. This will be dealt with more fully later under the considerations of the use of a three-electrode vacuum tube as a detector.

Laws for two-electrode vacuum tubes. I. Langmuir has developed theoretical expressions for the law connecting the variation in plate current with the plate voltage in two-electrode vacuum tubes which apply to the rising portion of the curve or for current below the saturation current. In order to simplify the theory, certain assumptions are made, primarily that the potential of the filament is the same at all points, which of course is not the case when the filament is heated by a current, and that the initial velocity of the electrons is so low as to be negligible. Langmuir treated two particular cases of two-electrode vac-

uum tubes. In the first of these the filament is supposed to be plane of infinite extent emitting electrons, and the plate is also an infinite plane parallel to the other at a distance x and maintained at a voltage V_b with respect to the filament. The current I_b in amperes per square centimeter of surface is given by:

$$I_b = 2.33 \cdot 10^{-6} \frac{V_b^{\frac{3}{2}}}{x^2}$$

In the second case, the filament is assumed to be a fine cylindrical wire of infinite length and the plate is a cylinder of infinite length surrounding the wire and concentric with it; the flow of current between the electrodes per centimeter of length is given by:

$$I_b = 1.465 \cdot 10^{-5} \frac{V_b^{\frac{3}{2}}}{r}$$

where r is the radius of the cylindrical plate.

In both of the above examples, it is to be noted that the flow of current, when limited by space charge, is proportional to the three-halves power of the plate voltage. In fact, Langmuir demonstrates that, in general, whenever the flow of electrons takes place between two equipotential surfaces this law of variation holds.

The three-electrode vacuum tube. In the three-electrode vacuum tube there is an additional electrode, called the **grid,** which is interposed between the plate and the filament. As its name indicates, the grid is ordinarily in the form of a mesh, or network of wires, through which the stream of electrons flows to the plate. The insertion of the grid electrode, which is due to Dr. Lee DeForest, gave the three-electrode tube the properties of amplifying and oscillating, which are not possessed by the two-electrode tube and which are primarily responsible for the important position of the vacuum tube in modern radio.

Action of the grid. It is possible to shield a piece of apparatus, such as a wavemeter or receiver, from the electric field of a transmitter by enclosing it in a box lined with sheet copper which is connected to ground. The electric field, due to external electric charges, can not penetrate into the interior of an enclosure which is surrounded by grounded conducting walls. The effect of an electric field is to induce charges on the wall of the enclosure, and the field of these induced charges neutralizes that of the external charges. If the shield is made of wire screen, the shielding effect is somewhat less complete and the electric forces will penetrate to the interior of the enclosure to a greater or less degree, depending upon the coarseness of the mesh.

In the three-electrode vacuum tube, the grid acts as a sort of screen around the filament and prevents the full force of the positive plate voltage from being exerted upon the electrons which are being emitted from the filament. If the grid is connected to the filament, so as to

bring it to the same potential, it will itself exert no force and, hence, the total force at the filament will be less under these circumstances than if the grid were removed. It was shown before, in the case of the two-electrode vacuum tube, that when the electron flow to the plate is less than that corresponding to the total emission from the filament, the current to the plate depends upon the electric force which is acting near the filament; that is, the current is limited by space charge. Under these conditions, the presence of the grid will, when it is at filament potential, result in a lower value of the plate current than would flow if the grid were absent. Suppose now that a batter, usually called the **C battery,** is inserted in the lead connecting the filament and grid, by means of which the grid can be made positive or negative with respect to the filament by any desired voltage. If the grid is made somewhat positive, it will itself exert an attractive force upon the electrons near the filament and a greater number will leave the filament and start across the tube. In fact, the current flow will increase until the space charge effect neutralizes not only the force of the plate but also that of the grid. Very nearly all of this electron current will flow through the spaces between the grid wires and go to the plate so that, as a result of a positive voltage on the grid, the plate current will increase. A negative voltage on the grid produces the opposite effect, for then the force exerted by the grid will oppose that of the plate and the plate current will be reduced. In fact, the grid voltage can readily be made negative enough to stop completely the current flow through the tube.

It is evident, therefore, that the grid acts as a control over the current flow to the plate in that, by variation of the voltage of the grid, the current to the plate can be increased or decreased at will. The electrons pass through the grid to the plate and, unless the grid voltage is made quite positive, very few hit the grid wires. The current to the grid is, therefore, practically zero and only a feeble current flows from the *C* battery, so that very little power is required to regulate the voltage of the grid.

Grid voltage--plate current characteristic curve. The control which the grid exerts on the plate current is shown by the characteristic curve representing the relation between plate current and grid voltage. This curve is shown in figure 176 for a type of tungsten filament receiving vacuum tube. The plate voltage is 60 volts and the filament current is 0.65 ampere. The current which heats the filament also produces a drop in potential along the filament so that when one speaks of the grid or plate being so many volts positive with respect to the filament, a particular point on the filament must be specified. Usually it is customary to take the terminal of the filament which is connected to the negative terminal of the filament battery as the point of zero potential, and the grid and plate voltages are then measured relative to this point. Thus, in figure 176 the grid and plate voltages are measured relative to the

negative filament terminal. The grid is at zero voltage when it is connected to the negative filament terminal without any C battery.

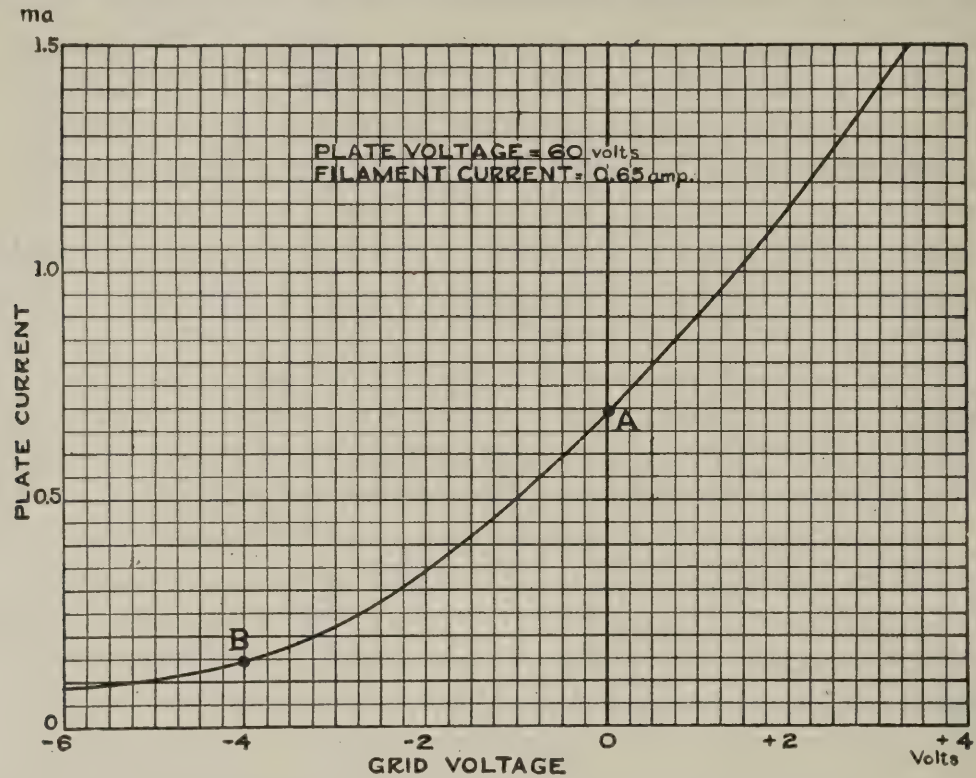


FIG. 176.—Curve Showing How the Grid Voltage Controls the Plate Current.

The curve of figure 176, which shows how the grid voltage controls the flow of current in the plate circuit, is of the main importance in

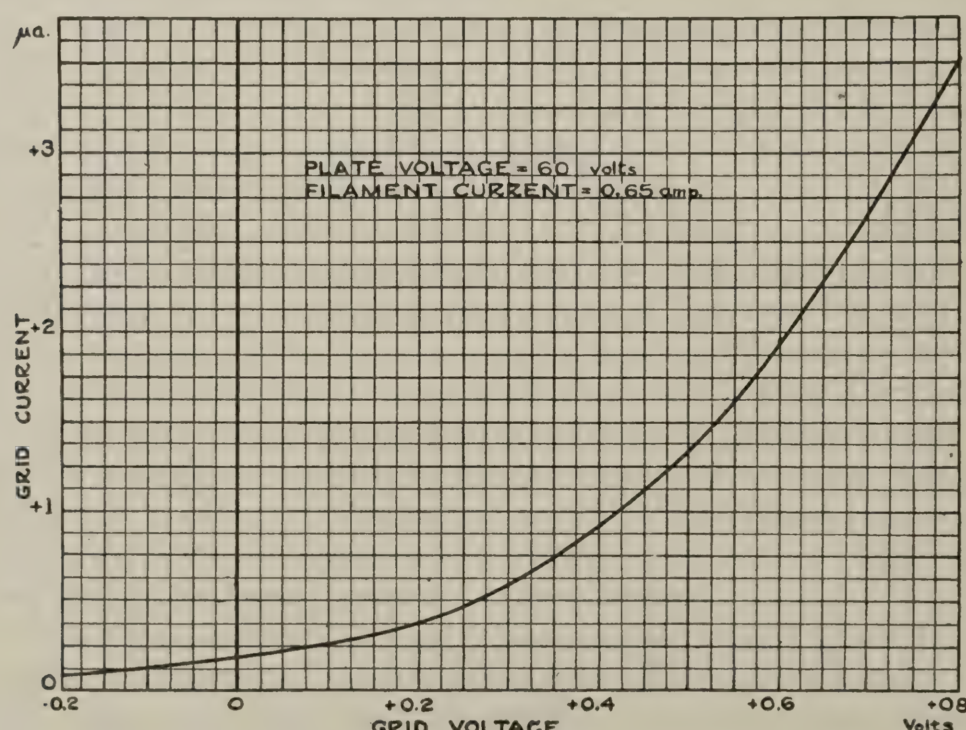


FIG. 177.—Grid Voltage—Grid Current Characteristic Curve.

explaining how the vacuum tube operates and will be referred to a number of times later.

Grid voltage—grid current characteristic curve. Another curve, which is important, shows the current which flows to the grid for different values of grid voltage. Such a curve is given in figure 177 for the same vacuum tube at the same plate voltage and filament current. As stated before, practically no current flows to the grid when it is negative, but it commences to flow more and more as the grid becomes positive, because some of the electrons, which are on their way to the plate, are attracted to the grid. It is to be noted that the currents to the grid, as shown in figure 177, are very small compared to the currents in the plate circuit; the grid currents are measured in micro-amperes, or millionths of an ampere, while the plate currents are measured in milliamperes, or thousandths of an ampere. The grid current is of importance in the action of the vacuum tube as a detector when a grid condenser and grid leak are used, and also is of some importance in connection with amplifiers.

Effect of an alternating voltage. The main interest in the use of vacuum tubes is in their operation with alternating currents and voltages, for the received signals are radio-frequency alternating currents and the telephone currents are audio-frequency alternating currents. The vacuum tubes are required to amplify and detect these alternating currents. In doing this, alternating voltages are applied to the grid of the vacuum tube and act in addition to the steady battery voltages.

Since the current which flows through the vacuum tube is carried by electrons and these electrons are very light and easily influenced, changes in the grid voltage can take place very rapidly, and the plate current will respond instantly. Even if the frequency of the alternating voltage applied to the grid is 50,000,000 cycles per second, the plate current can follow them readily, so that vacuum tubes can be used to amplify and detect at any frequency.

At present, these actions will be treated in an elementary manner in order to give a general idea of the functioning of the vacuum tube. The matter will be handled with greater detail in the Chapters which follow.

Assume that the vacuum tube, for which the grid voltage—plate current characteristic curve is given in figure 176, is operated under the conditions of plate voltage and filament current there stated. Suppose, in the first place, that no grid battery is used so that the grid voltage is zero. The operating conditions of the vacuum tube will then correspond to the point marked A on this figure, and the plate current will be about 0.7 ma. If, now, an alternating voltage is impressed upon the grid so that its voltage varies above and below the steady value then, as

the grid voltage varies up and down, the plate current will similarly rise and fall.

In figure 178, the curve (*a*) shows the assumed sinusoidal variation in the grid voltage which is taken to have an amplitude which carries the grid up to +2 volts and down to −2 volts. From figure 313 it is seen that for +2 volts the plate current will be a little over 1.0 ma, and for −2 volts will be about 0.3 ma. Hence, the plate current will vary about as shown in curve (*b*) of figure 178. The plate current curve is of the same form as the voltage curve, because the region of the variation was taken over a part of the characteristic that was nearly straight.

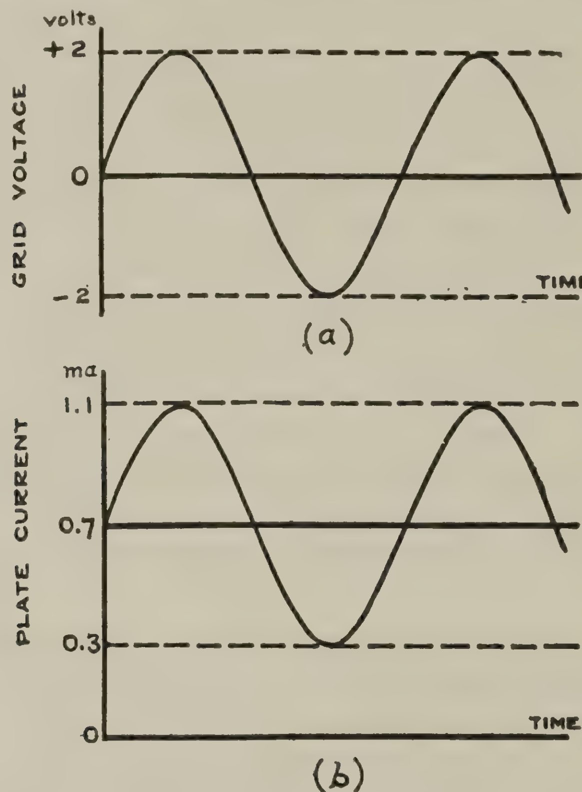


FIG. 178.—Curve Showing Amplification Without Distortion.

Now let the grid battery be such as to make the grid 4 volts negative, corresponding to the point *B* of figure 176. Then, with the same alternating voltage as before impressed upon the grid, the grid voltage will vary between −2 and −6 volts. The plate current for −4 volts grid voltage is about 0.1 ma; the increase in grid voltage to −2 volts will carry the plate current to 0.3 ma, but the decrease to −6 volts will produce only a slight reduction in the plate current below the value for −4 volts because of the bend in the curve. The variation in the plate current in this case will be a curve such as that shown in figure 179, which is quite distorted in form from that of the grid voltage. The distortion results for the reason that the variation takes place over a bending portion of the curve.

If, therefore, the steady voltages acting on the vacuum tube are so chosen that the operating point is on a straight portion of the grid

voltage-plate current curve, then variations in the grid voltage will produce similar variations in the plate current. This will be true no matter whether the variations of grid voltage are simple sinusoidal variations produced by an alternator, as shown, or of a complex form, such as the speech currents produced by a telephone transmitter. This illustrates the use of a vacuum tube as an amplifier. The varying current of figure 178 is the same as would result if an alternating and a direct current were both flowing in the plate circuit. The plate current can therefore be considered to have two components,—one an alternating component and the other a direct component. The latter is the average value of the varying current. In figure 178, the average current is 0.7 ma, the same as the steady current flowing when no alternating voltage is impressed upon the grid. The alternating component has an amplitude of 0.4 ma corresponding to an effective current of 0.3 ma. Because of the distorted form of the wave in figure 179, however, it is evident that the average value of the current is greater when the alternating voltage is applied than it would be if no such voltage were acting on the vacuum tube. Since direct current instruments respond to the average value of the current flowing through them, a direct current ammeter in the plate circuit of the vacuum tube, under the

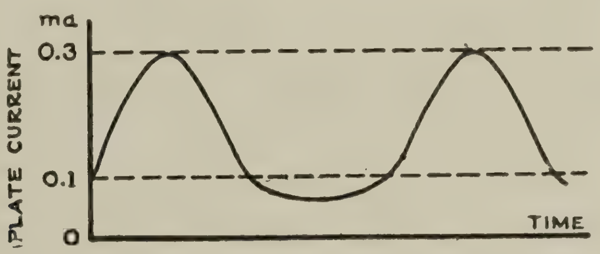


FIG. 179.—Curve Showing Amplification with Distortion.

conditions assumed for figure 178, would show no change in reading when the alternating voltage is applied to the vacuum tube; it would, however, show an increased reading under the conditions of figure 179 upon the application of the alternating voltage. This change in average value of the current is of vital importance in the operation of the vacuum tube as a detector, and in this use the distortion produced by operating the vacuum tube on a bending portion of the characteristic serves a useful purpose.

Input and output power. For purposes of illustration, the amplitude of the alternator voltage used above was assumed much larger than would correspond to the actual normal use of the vacuum tube in radio reception. It is clear that in the case of operation on the straight portion of the characteristic, or the amplifier case, the amplitude of the variations of plate current will be proportional to the amplitude of the variations of grid voltage. Assuming, therefore, the grid voltage to vary by only one-twentieth of the value assumed before, or with an amplitude of 0.1 volt instead of 2 volts, the amplitude of the

alternating component of the plate current will be 0.02 ma instead of 0.4 ma. The effective value of the current will be about 0.014 ma or 14 μa. Sensitive telephones, such as are used in radio work, will give a signal of about a thousand audibility for one micro-ampere at a frequency of 800 to 1,000 cycles, so that it is evident that even this latter current is fairly large as compared with the normal currents met with in radio reception.

Assuming then that an alternating voltage with an amplitude of 0.1 volt is impressed upon the grid, consider what takes place in the grid circuit of the vacuum tube using the grid current—grid voltage curve of figure 177. The grid current will vary up and down in accordance with the variations in grid voltage and, if the normal voltage of the grid is zero as assumed in the amplifier case discussed above, the grid voltage will vary between $+0.1$ and -0.1 volt. The grid current will vary between 0.3 and 0.1 μa roughly, hence the alternating component will have an amplitude of 0.1 μa. The ac power in watts which is taken from the apparatus that supplies the alternating voltage to the grid, is given by the product of the effective values of the alternating voltage in volts and alternating current in amperes. In this case the effective voltage is $7 \cdot 10^{-2}$ volt and the effective current is $7 \cdot 10^{-8}$ ampere, hence the power taken by the vacuum tube or the input power is $5 \cdot 10^{-9}$ watt. This power supplied to the vacuum tube causes a current having an effective value of $1.4 \cdot 10^{-5}$ ampere to flow in the plate circuit. In order to find out how much output power is represented by this latter current, either the voltage acting in the plate circuit or the resistance of the circuit must be known. It will later be shown how either of these quantities can be determined; at present, let it suffice to state that the voltage acting in the plate circuit for the particular vacuum tube used in this discussion is roughly ten times the voltage acting in the grid circuit or, in effective value, 0.7 volt. The power represented by the ac component of the plate current, or the output power, is $0.7 \times 1.4 \cdot 10^{-5}$ watt or roughly $1 \cdot 10^{-5}$ watt. The output power is, therefore 2,000 times as great as the input power. There is no violation of the law of conservation of energy in this result, for the output power is supplied by the plate battery, the grid of the vacuum tube acting merely as a control over the current being drawn from this battery. It is now justifiable to say that the vacuum tube is acting as an amplifier because it has been shown that:

(a) The output current is of the same frequency and wave form as the input voltage;

(b) The output power exceeds the input power.

It is evident from the shape of the grid current—grid voltage characteristic, that the current in the grid circuit can have a distorted wave form relatively to the wave form of the applied voltage in a similar manner to the case discussed above in connection with the plate current.

Similarly, also, when the operation takes place over a curved portion of the characteristic the average value of the grid current will be different when the alternating voltage is impressed from that flowing when the voltage is not impressed. This action is also utilized for the purpose of detection in circuits in which a grid condenser and grid leak are employed.

Ac conductance and resistance. In an alternating-current circuit in which the current is in phase with the electromotive force, the same law holds for the relation between the current and the electromotive force as for direct current. In other words, Ohm's law applies to such circuits. The current and voltage are in phase in circuits which have resistance only, or in circuits in which the inductive reactance and capacitive reactance neutralize each other for the frequency of the emf, that is, tuned circuits. In such circuits, then,

$$I=\frac{E}{R}$$

$$I_0=\frac{E_0}{R}$$

$$I=Eg$$

$$I_0=E_0g$$

where I and E = the effective values of current and emf, respectively,

I_0 and E_0 = the maximum values, which are $\sqrt{2}$ times the effective values,

R = the resistance of the circuit for the frequency of the emf,

$g=\frac{1}{R}$ = the conductance.

These relations hold for a portion of a circuit provided that the emf and current are in phase for the portion of the circuit considered.

In the foregoing pages it was shown that the application of an alternating emf on the grid circuit of a vacuum tube produces variations in the grid and plate currents of the vacuum tube, and that such varying currents can be looked upon as made up of a steady component and an alternating component. If the variations in current do not lag behind the variations in voltage applied to the vacuum tube, then the alternating component of the current will be in phase with the applied alternating voltage. This is always the case in vacuum tubes with a high vacuum where ionization is absent. Also if the characteristic curve is straight, the alternating current will be undistorted and proportional to the applied voltage. In any case, the characteristic curve will be practically straight if only a limited portion of it is utilized; so that, for very small alternating voltages, it can be considered to be straight. Regarding only the alternating component of the current and the alternating

voltage applied to the vacuum tube, it is clear that the vacuum tube behaves as a pure conductance or resistance for small voltages.

It will now be shown how the numerical values for the ac conductance or resistance of such a device are obtained from the characteristic curves. In figure 180 is shown a portion of the grid voltage—grid current curve of figure 177 but on an enlarged scale. Suppose the steady grid voltage to be +0.4 volt corresponding to the point *a* on the curve. At *a* draw a tangent to the curve. It is clear that the tangent nearly coincides with the curve in a limited region around the point of tangency. If now an alternating voltage having an amplitude of 0.025 volt is superimposed upon the steady voltage acting in the grid circuit, the total voltage will vary from *a* to *b* in one direction and an equal

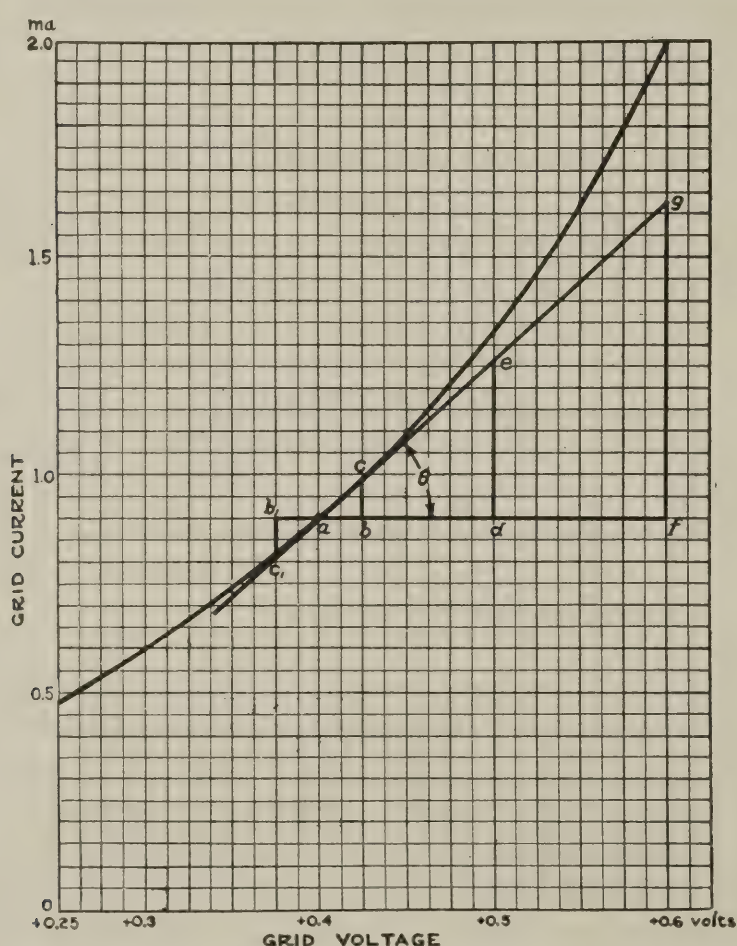


Fig. 180.—Curve Showing how the Input Conductance or Resistance is Derived from the Grid Voltage—Grid Current Characteristic.

amount on the other side, that is, from *a* to b_1. Correspondingly, the current will increase by an amount equal to *bc* and then decrease by an equal amount b_1c_1. The amplitude of the voltage is given by *ab* and the amplitude of the current by *bc*. The conductance *g* is given by $\frac{I_0}{E_0}$, that is, the ratio of maximum current to maximum emf. Thus

$g=\frac{bc}{ab}=\tan\theta=$ slope of the tangent to the curve. Hence the conductance for small oscillations at any point of the characteristic curve is given by the slope of the tangent to the curve at that point, or, the resistance is given by the reciprocal of the slope. In order to get a fairly accurate value of the conductance, it is more convenient to use the ratio $\frac{de}{da}$ or $\frac{fg}{fa}$ which is the same as $\frac{bc}{ab}$. Using the ratio $\frac{fg}{fa}$, the length of fg corresponds to 0.72 μa, while fa corresponds to 0.2 volt. Hence,

$$g=\frac{0.72 \cdot 10^{-1}}{0.2}=3.6 \cdot 10^{-6} \text{ reciprocal ohm (mho)}$$

and

$$R=\frac{1}{g}=2.8 \cdot 10^{5} \text{ ohms.}$$

The smaller the angle θ which the tangent to the curve makes with the horizontal, the less will be the conductance and the higher will be the resistance. Thus, if the steady value of the grid voltage is made less positive, the resistance will increase or, for a given alternating voltage applied to the grid, a smaller alternating current will flow in the grid circuit. Since the input power to the vacuum tube is dependent upon the product of the input voltage and input current, it is desirable, in order to make this input power very small, to make the grid resistance high by adjusting the steady voltage of the grid to be near zero or even somewhat negative.

The previous discussion has made use of the grid voltage—grid current characteristic of the vacuum tube for the purpose of illustrating ac conductance and resistance. The resulting conductance or resistance in this case measures the alternating current which flows in the grid circuit for a given impressed alternating voltage. These particular values are sometimes called the **grid conductance** or **grid resistance**, and sometimes the **input conductance** or **input resistance** of the tube. There are also two other values of conductance or resistance which are of very great importance in connection with the operation of the three-electrode vacuum tube. One of these is obtained from the grid voltage—plate current characteristic. The slope of this curve at any point gives what is called the **mutual conductance** of the tube and is a measure of the alternating current which flows in the plate circuit of the tube for a given applied voltage on the grid provided there is no appreciable resistance or reactance in the plate circuit of the tube external to it. Another conductance is derived from a type of characteristic which has not been considered earlier but which is very important in the theory of operation of the tube. This characteristic is the plate voltage—plate current characteristic, and shows how the plate current varies with the plate voltage on the tube when the grid voltage is kept constant.

The slope of this curve at any point gives the **plate conductance,** and its reciprocal gives the **plate resistance.** These quantities measure the alternating current which flows in the plate circuit for a given impressed emf in that circuit in case there is no appreciable resistance or reactance in the circuit other than that of the tube itself.

CHAPTER II.

ACTION OF THE VACUUM TUBE AS AN AMPLIFIER

Input and output circuits. In dealing with the vacuum tube as an electrical device, two circuits external to the vacuum tube must be considered: (*a*) the **input circuit** connecting the grid and filament, and (*b*) the **output circuit** connecting the plate and filament. In the former is impressed the varying voltage or current which is to be amplified. This circuit may also include a steady voltage to fix the value of the steady grid potential. The output circuit contains the apparatus by which the reproduced and amplified variations are utilized, and also the steady voltage which supplies the power. The electrical behavior of the device will be determined if the relations between the instantaneous values of the current in these two circuits and the instantaneous voltages impressed between the electrodes are known. The current in each circuit will depend upon the voltages in both. Thus, the grid current at any instant depends upon both the grid voltage and plate voltage acting upon the vacuum tube at that instant. Similarly, the plate current depends upon both the grid and plate voltages.

Input power. As has been pointed out before, the input power to the vacuum tube, or the power which the tube takes from the circuit connected to the input vacuum tube, depends upon the alternating grid current. In general, it is desirable to reduce this power as much as possible in order to increase the efficiency of the vacuum tube as an amplifier. Thus, it is customary to operate the grid at a steady voltage which is slightly negative, so that both the direct and alternating grid currents are nearly zero for small oscillations.

Output power. The alternating plate current, however, represents the output of the vacuum tube and is, therefore, of very great importance in considerations of the action of the vacuum tube as an amplifier. It has been shown that the **mutual conductance** of the tube determines the alternating current flowing in the plate circuit for a given impressed alternating voltage **on the grid.** Since, however, the grid voltage—plate current characteristic curve is obtained by varying the grid voltage and reading the corresponding values of plate current **with the plate voltage kept at a constant value,** the mutual conductance, which is derived from this characteristic curve, determines the alternating plate current only for the condition that **the plate voltage is kept constant.**

Effect of a load on plate current. Now, in the actual use of the vacuum tube as an amplifier, a resistance, or the primary of a transformer, or telephones, or other apparatus is inserted in the plate circuit

of the vacuum tube. When such apparatus is traversed by a varying plate current, the voltage drop across the apparatus also varies, and the **actual voltage applied to the plate of the vacuum tube is not constant.** Therefore, in the actual use of the vacuum tube the alternating plate current cannot be determined from the value of the mutual conductance alone.

The variations in plate voltage due to the voltage drop in the apparatus in the plate circuit of the vacuum tube can, however, be considered to be the equivalent of an alternating voltage impressed in that circuit. As pointed out before, the alternating plate current produced by an alternating voltage **acting in the plate circuit** is determined by the **plate conductance,** or the slope of the plate voltage—plate current curve. By using **both the mutual and plate conductances,** it is clear that the combined effect of **both an impressed alternating grid voltage and a varying plate voltage** can be determined. The equation for the alternating plate current I_p produced by an alternating grid voltage E_g is, therefore,

$$I_p = E_g g_m + E_p g_p$$

where g_m = mutual conductance of the vacuum tube,
g_p = the plate conductance,
E_p = the alternating plate voltage.

Resistance load. This equation will now be applied to the case of a resistance coupled amplifier in which it is assumed that a pure resistance R is present in the plate circuit of the vacuum tube. The alternating voltage E_p across the resistance R, when traversed by an alternating current I_p, is I_pR. It is evident that this voltage opposes the action of the voltage on the grid, for, as the grid is going positive and hence increasing the plate current, the drop across R will be increasing and will thus reduce the plate voltage acting on the vacuum tube, thereby tending to offset the action of the grid emf. Hence, the phase of E_p is such as to oppose E_g, and this is expressed by writing

$$E_p = -I_pR$$

The above formula for I_p then becomes

$$I_p = E_g g_m - g_p I_p R$$

and

$$I_p = \frac{E_g g_m}{1 + g_p R}$$

Dividing numerator and denominator of the above equation by g_p

$$I_p = \frac{\dfrac{g_m}{g_p} E_g}{\dfrac{1}{g_p} + R}$$

Now, $\dfrac{1}{g_p}$ is the **internal plate circuit resistance** of the vacuum tube

which will be designated by R_p, while $\frac{g_m}{g_p}$ is called the **amplification coefficient** or **amplification constant** of the vacuum tube which will be designated by μ. Hence, the formula finally becomes

$$I_p = \frac{\mu E_g}{R_p + R}$$

Thus, it is seen that **the alternating current in the plate circuit for a given impressed voltage on the grid is the same as would flow in a simple circuit with an emf μ times as great and having a resistance R_p equal to that of the vacuum tube plus the additional inserted resistance** R. Thus, the effect of impressing the voltage E_g upon the grid of the vacuum tube is to cause a voltage of μE_g to act in the plate circuit of the tube. This latter emf is inside the tube and acts through the tube resistance and external resistance in series.

Because of the mobility of the electrons, the voltage μE_g does not lag behind the impressed voltage E_g in a high vacuum tube, but reaches a maximum value in a direction so as to cause current to flow from the plate to the filament inside the vacuum tube at the same instant that the grid voltage reaches a maximum in a direction tending to make current flow from grid to filament inside the vacuum tube. Adopting this convention as to directions, **the impressed and resultant emfs are exactly in phase with each other.** Also, in the case of a pure resistance in the external circuit, **the alternating current in the plate circuit is exactly in phase with both emfs.**

General theorem for plate current. The above simplifying theorem can be extended to the case where, in place of a pure resistance, **any impedance whatsoever** is inserted in the plate circuit of the vacuum tube. The magnitude and phase of the plate current can be calculated just as simply as in the case of ordinary simple ac circuits. It is necessary only to assume that the emf μE_g is acting in a circuit which has in series the resistance R_p and the inserted impedance.

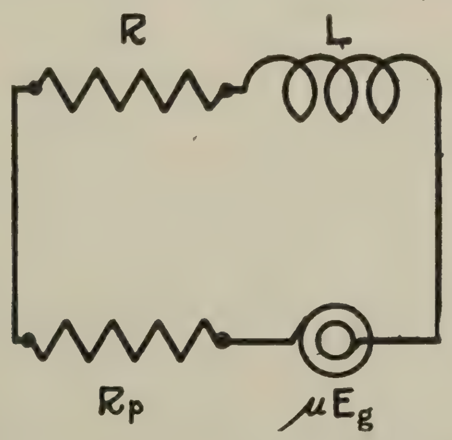


FIG. 181.—Simple Ac Circuit, the Equivalent of the Plate Circuit.

Suppose, for example, that the inserted impedance is a telephone receiver having a resistance R and inductance L. The equivalent

simple circuit will be that of figure 181. The total impedance of the circuit will be

$$Z=\sqrt{(R_p+R)^2+(\omega L)^2}$$

The current will be equal to the voltage divided by the impedance; hence

$$I_p=\frac{\mu E_g}{\sqrt{(R_p+R)^2+(\omega L)^2}}$$

and this current will lag behind the emf by the angle

$$\phi=\tan^{-1}\frac{\omega L}{R_p+R}$$

Recapitulating, there exists the following important theorem which is applicable to a vacuum tube **used as an amplifier,** or in any case where the amplitude of the oscillations is small and the characteristic curves can be considered to be straight lines. **The alternating current in the plate circuit of a three-electrode vacuum tube for a given impressed voltage on the grid of the tube is the same in magnitude and phase as that which would flow in a simple series circuit with an impressed voltage μ times as great as the impressed voltage on the grid, acting, through a resistance R_p and an impedance Z, which latter is the same as the external impedance in the plate circuit of the vacuum tube.**

Regenerative amplification. It is possible to increase very decidedly the amplification obtained by a single vacuum tube by what is known as

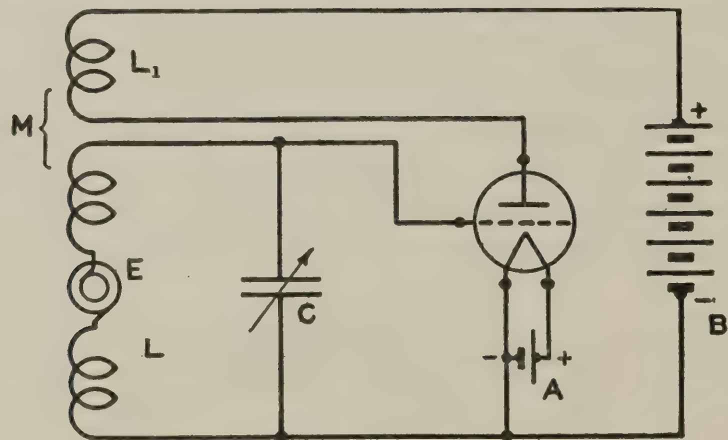


FIG. 182.—Type of Regenerative Circuit.

regeneration. One type of circuit which illustrates the principle involved is that shown in figure 182. The circuit containing the inductance L and condenser C can be considered to be the tuned secondary of a receiver. The emf E represents the voltage induced in the coil L from the primary circuit as the result of a received signal. In the plate circuit of the vacuum tube is a coil L_1 which is coupled to coil L, the mutual inductance between the two coils being M. Coil L_1 is called the **tickler,** or **feed-back** coil. In words, the action is as follows:

The emf E causes an alternating current to flow in the tuned circuit and gives rise to a voltage E_g across the condenser C which is applied between the grid and filament of the vacuum tube. The voltage on the grid of the vacuum tube causes an alternating current to flow in the plate circuit. This current flows through the coil L_1, and by reason of the coupling with L, induces an emf in coil L which, with proper coupling, will be very nearly in phase with and will add to the original emf E_g. The addition of this voltage leads to an increase in the current in the LC circuit, and also to an increase in the voltage applied to the vacuum tube. The alternating plate current is likewise increased and a still larger emf is fed back to the LC circuit. This cycle is repeated over and over, but usually in a short time a stable condition is reached in which **much larger currents and voltages are present in all parts of the circuit than would be present if the feed-back feature were absent.** The stable condition is reached if the emf fed back by the coil L_1 in the first cycle is less than the original emf. Suppose this to be

$$\tfrac{1}{2}E_g$$

Then the total emf will be

$$E_g+\tfrac{1}{2}E_g$$

In the second cycle, $\frac{1}{2}$ of this increase will likewise be fed back, raising the total emf to

$$E_g+\tfrac{1}{2}E_g+\tfrac{1}{4}E_g$$

and, finally, the emf acting in the LC circuit will be

$$E_g+\tfrac{1}{2}E_g+\tfrac{1}{4}E_g+\tfrac{1}{8}E_g \ldots\ldots\ldots\ldots \text{etc.}$$

The summation of this series is $2E_g$. Hence, the final impressed emf, and with it the currents and voltages in all portions of the circuit, will be doubled.

Generation of oscillations. If the coupling is increased so as to feed back a greater emf, the multiplication of currents and voltages will be increased up to the point where the emf fed back in the first cycle is equal to the original emf, in which case the series becomes

$$E_g+E_g+E_g+E_g \ldots\ldots\ldots\ldots \text{etc.}$$

The summation of this series is infinity, which means that a stable condition is not possible. What actually happens is this: Any initial oscillation in the LC circuit is built up to a greater and greater amplitude until limited by a change in the characteristics of the vacuum tube. The oscillation is then maintained at this limiting value or, in other words, **the vacuum tube self-generates oscillations.** This will be treated more fully later. Theoretically, the amplification becomes higher and higher without limit as the oscillating point is approached but, practically, after a certain degree of amplification is obtained the adjustments become so critical that any slight disturbance, such as static or variation in battery voltage, will start the vacuum tube oscillating.

CHAPTER III.

MULTIPLE-STAGE AMPLIFIERS

It has been pointed out before that the power in the output circuit of an amplifying vacuum tube can be a great many times the power required by the input to the vacuum tube. Instead of feeding back some of this power to the input of the vacuum tube in order to enhance the amplification, which is the regenerative process described above, it is possible to feed into the input circuit of another vacuum tube and apply to this vacuum tube a voltage many times greater than the voltage originally supplied to the first vacuum tube. Similarly, the output of this second vacuum tube can be made to apply a multiplied voltage to the input of another vacuum tube, and so on. This process leads to what is called **multiple-stage amplification.**

When feeding from the plate circuit of one vacuum tube to the grid circuit of a succeeding vacuum tube, it is desirable to apply the maximum possible voltage to the latter.

The various types of multiple-stage amplifiers are named in accordance with the kind of device used to couple one vacuum tube with the succeeding vacuum tube. The amplifiers are thus designated as **resistance type, reactance type** and **transformer type.** Further, since it is possible to amplify either radio frequencies, such as are used in radio transmission, or audio frequencies, such as the ordinary speech currents in a telephone line or the radio signals which have been acted upon by a detector, it is also customary to distinguish between radio and audio-frequency amplifiers or stages of amplification.

Resistance type amplifier. In the **resistance type amplifier,** the coupling units are high resistances such as R and R_1 of figure 183. The figure shows two vacuum tubes of such an amplifier. The filament and plate batteries (A and B in figure) are common to all of the vacuum tubes. The input voltage E_g, which is applied between the filament and grid of tube 1, causes an alternating current to flow in the plate circuit of that tube and, hence, through the resistance R which may be of the order of fifty or a hundred thousand ohms. The voltage drop across this resistance is applied to the input of the next vacuum tube through the coupling condenser C to the grid and through the B and A batteries to the filament. The condenser C is required in order to prevent the steady positive voltage on the plate of the first vacuum tube from being applied to the grid of the second vacuum tube. The resistance r, called the **grid leak,** serves to fix the steady potential of the grid of the second vacuum tube at the same value as that of the negative filament

terminal and takes care of any slight direct-current leakage through the condenser C. Since it is desired that the full alternating voltage drop in the resistance R shall be applied between the grid and filament of the next vacuum tube, it is evident that no appreciable drop should occur in the condenser C. Therefore, condenser C must be large enough so that its reactance, at the lowest frequency for which the amplifier is to be used, is small compared to the impedance between grid and filament of the vacuum tube with the resistance r in parallel. Further, in order that the amplification shall not be reduced by reason of the fact that this latter impedance is in parallel with R, the value of this impedance should be high relatively to R. This condition cannot

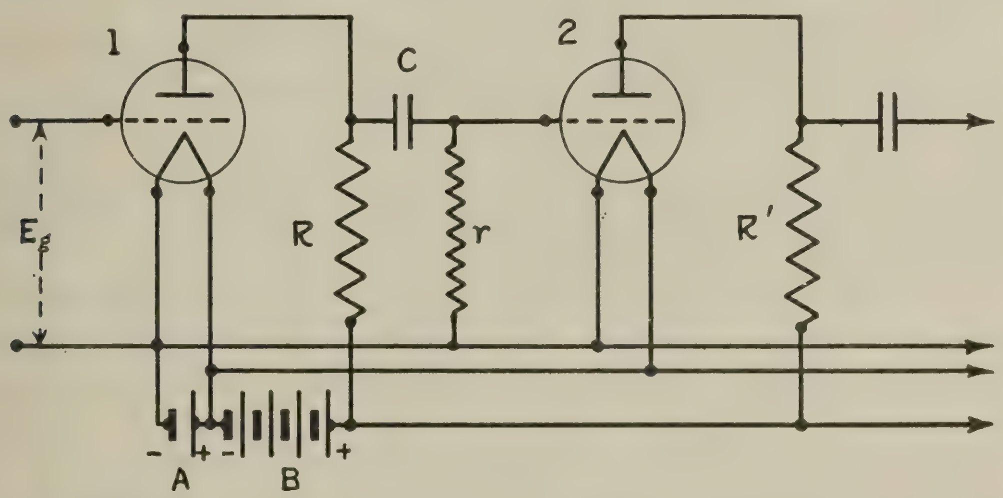


FIG. 183.—Diagram Showing Method of Connecting Amplifying Stages by High Resistances. Resistance Type Amplifier.

always be fulfilled, particularly at high radio frequencies, when the reactance of the capacity between the grid and filament of the vacuum tube becomes a shunt of such a low value on the resistance R as to reduce seriously the amplification which is obtained.

The amplification of an amplifier can be expressed as a voltage amplification of so much per stage. This is given by the ratio of the voltage between the grid and filament of one vacuum tube to the corresponding voltage on the vacuum tube ahead. This amplification can be very readily computed in the case of the resistance type amplifier. It will be assumed that the reactance shunting R is very high. As has been shown above, the alternating plate current I_p is given by

$$I_p = \frac{\mu E_g}{R_p + R}$$

and the voltage E'_g, which is handed on to the next tube is

$$E'_g = I_p R = \frac{\mu E_g R}{R_p + R}$$

The voltage amplification is

$$A = \frac{E'_g}{E_g} = \frac{\mu R}{R_p + R}$$

Example:

Suppose that the vacuum tubes used in a resistance type amplifier have an amplification constant $\mu=10$ and an internal plate resistance $R_p=40{,}000\Omega$. What amplification will be obtained with a coupling resistance R of $80{,}000\Omega$?

Solution:

Formula $$A=\frac{\mu R}{R_p+R}$$

substituting $$=\frac{10\times 8\cdot 10^4}{4\cdot 10^4+8\cdot 10^4}=\frac{8\cdot 10^5}{1.2\cdot 10^5}=6.67$$

whence $$A=6.7$$

The above formula shows that, if the resistance R is very large compared with R_p, R_p can be neglected and the formula becomes

$$A=\mu$$

Thus, the amplification constant of the vacuum tube determines **the maximum voltage amplification per stage which can be obtained with a resistance coupled amplifier.**

It is well to note, however, that while the value of the amplification constant for a given vacuum tube is fairly independent of the steady plate voltage applied to the vacuum tube, the value of the internal plate resistance depends upon the plate voltage and increases as the plate voltage drops. Now, the steady plate current has to flow through the resistance R and, as a result of the voltage drop in this resistance, the actual voltage applied to the plate of the vacuum tube is less than that of the plate battery. Hence, if R is increased without increasing the voltage of the plate battery, R_p will also increase somewhat and, hence, the increase in amplification is not as rapid as might be expected. This is shown in figure 184 which gives experimental curves for the variation of μ and R_p with varying values of the inserted resistance R and also a computed curve of the voltage amplification A. The vacuum tube is a Western Electric Co., Type J operated at a constant plate battery voltage of 40 volts and a negative grid voltage of 1.5 volts. The amplification constant varies only from about 6.3 to 6. The internal plate circuit resistance increases, however, from about $20{,}000\Omega$ to $48{,}000\ \Omega$, the latter value corresponding to a value of $R=140{,}000\ \Omega$. Even with this high inserted resistance, the amplification is only 4.4, or about 75 per cent of μ. An increase in plate battery voltage to, say, 80 or 100 volts would decidedly improve the amplification. The normal operating plate current for the usual amplifying vacuum tubes is roughly 0.5 ma. Hence, each $10{,}000\ \Omega$ in R would require the addition of 5 volts to the plate battery voltage in order to maintain the normal operation of the vacuum tube.

It might be expected that a vacuum tube constructed so as to have a very high amplification constant would be very much superior for a

resistance type amplifier. It is a fact, however, that any variation in tube construction which leads to a higher value of μ also leads to a higher value of R_p. Therefore, a vacuum tube with a high amplification constant requires a high plate battery voltage in order to keep the plate resistance at a reasonable value. For example, the Type J tube of the Western Electric Company has a value of μ of about 6.4 and a resistance of about 20,000 Ω with a plate voltage of 20 volts, while the Type V tube of the same Company has a μ of 30 and a resistance of 40,000 Ω, even with 100 volts on the plate. While very high voltage

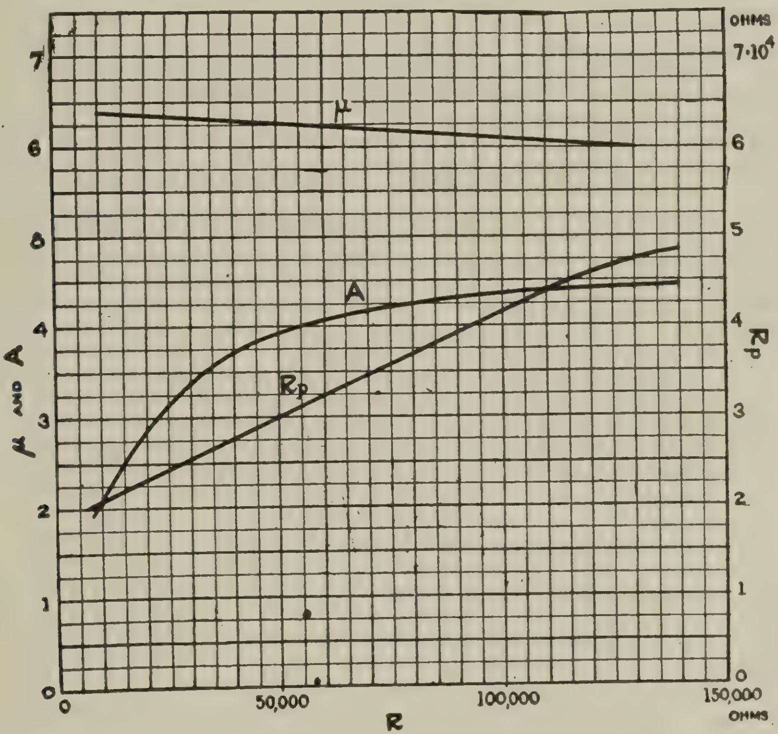


FIG. 184.—Amplification Constant, Voltage Amplification and Internal Plate Circuit Resistance with Varying Resistance Load. W. E. Co. Type J Tube.

amplification per stage could be secured with this latter tube using about 150 bolts in the plate battery, the use of such a high voltage would usually be inconvenient.

Resistance type amplifiers can be used for the amplification of radio frequencies, audio frequencies or even lower frequencies, such as are used in ordinary power circuits. At radio frequencies, the resistance type amplifier will give a uniform amplification over the whole range of wave lengths excepting at the shorter waves where, as has been mentioned above, the capacity of the vacuum tube acts as a low reactance shunt to the coupling resistance and thereby lowers the amplification. In fact, with vacuum tubes of customary construction, satisfactory amplification is not secured for wave lengths much shorter than 1,000 m. Since a resistance type amplifier will amplify audio frequencies as well as radio frequencies, the amplifiers are usually noisy.

As audio-frequency amplifiers, the resistance type is, in general, far inferior to the transformer type with respect to the amplification

secured. Because of the uniformity of the amplification for various frequencies, the resistance type gives less distortion when used for the reception of music in radio telephony.

At very low frequencies, for which very large transformers would by required, the resistance type is advantageous.

Reactance type amplifier. In place of the resistance R and R' of figure 183, the reactance type amplifier makes use of coils or a combination of coils and condensers in parallel, for the coupling units between stages.

In computing the amplification secured by such an amplifier, it will be assumed that the coupling unit has a reactance X at the frequency of use and that its resistance is negligible.

For an emf E_g applied to the input of the vacuum tube, an emf μE_g will act in the plate circuit. This latter emf acts through the plate circuit resistance R_p of the vacuum tube and the reactance X in series, the impedance of which is $\sqrt{R_p^2+X^2}$. The alternating plate current will be

$$I_p=\frac{\mu E_g}{\sqrt{R_p^2+X^2}}$$

and the voltage across the reactance X, which is handed on the next vacuum tube will be

$$E'_g=I_p X=\frac{\mu E_g X}{\sqrt{R_p^2+X^2}}$$

whence the voltage amplification is

$$A=\frac{E'_g}{E}=\frac{\mu X}{\sqrt{R_p^2+X^2}}=\frac{\mu}{\sqrt{1+\left(\frac{R_p}{X}\right)^2}}$$

If X is large compared to R_p, a voltage amplification equal to μ will be secured.

It is easier to secure a high amplification with reactance coupling than with resistance coupling, for two reasons. First, the reactance X can be very high and still the coupling unit can have a low dc resistance. In this case, the full plate battery voltage will be applied to the vacuum tube with a resultant low value for R_p. Second, when reactance is inserted in the plate circuit of a vacuum tube, the total impedance of the circuit is less than the total resistance of the circuit when an equal number of ohms of resistance is inserted. Hence, the alternating component of the plate current is greater in the former case. It is this current multiplied by the reactance, or resistance, in ohms which gives the voltage handed on to the next vacuum tube.

Example:

In a previous example on the resistance type amplifier, the vacuum tube was supposed to have an amplification constant $\mu=10$ and a

plate resistance $R_p = 40{,}000\ \Omega$. What amplification will be obtained with a coupling reactance of 80,000 Ω as compared with that for a coupling resistance of 80,000 Ω in the previous example?

Solution:

Formula $$A = \frac{\mu X}{\sqrt{R_p^2 + X^2}}$$

substituting $$= \frac{10 \times 8 \cdot 10^4}{\sqrt{16 \cdot 10^8 + 64 \cdot 10^8}} = \frac{8 \cdot 10^5}{\sqrt{80 \cdot 10^8}} = \frac{8 \cdot 10^5}{8.94 \cdot 10^4} = 8.9$$

whence $A = 8.9$

Thus, a voltage amplification of 8.9 is obtained as against 6.7 in the previous example. This superior result is also obtained with a lower plate battery voltage because of the low voltage drop in the reactance as compared with that in the resistance.

The type of coupling unit used in a reactance coupled amplifier is usually an inductance coil and condenser in parallel. For frequencies considerably lower than the natural period of the circuit, the coupling unit behaves as an **inductive reactance** which is equal to the reactance of the coil alone. As the frequency is increased, the inductive reactance increases at first in proportion to the frequency, as would be the case for a coil alone; but, as the natural period is approached, the inductive reactance rises to a very high value and then suddenly decreases to zero at the natural period. At the same time, the effective resistance of the circuit also increases and, at resonance, usually has attained a very high value. Thus, at resonance, the circuit behaves as a very high pure resistance for alternating current. As the frequency is still further increased, the reactance becomes a **capacitive reactance** and goes rapidly to a very high maximum value. For still higher frequencies the reactance becomes smaller again, and finally corresponds to that of the condenser alone. As resonance is passed, the effective resistance reaches a maximum value and then falls off rapidly toward zero as the frequency is still further increased. The maximum value of the **effective resistance** R_{eff} of the parallel circuit, which occurs very close to the resonant frequency, is given by

$$R_{\text{eff}} = \frac{L}{CR}$$

where L = the coil inductance in henries,
C = the condenser capacity in farads,
R = the resistance in ohms of coil and condenser in series, or what is normally spoken of as the rf resistance of the circuit or circulating resistance of the circuit.

The lower this resistance is, the higher will be the effective resistance of the parallel combination.

When the capacity C is not very small and the inductance L not very high, the reactance or resistance of the combination will be high only over a very narrow band of frequencies.

Example:

Assume the inductance L to be 1 mh and the capacity C to be 0.002 μf. Let the coil resistance be 1 Ω and the condenser of negligible resistance. The natural period of the circuit will correspond to a frequency of $f=112{,}000$ and a wave length $\lambda=2{,}670$m. What will be the value of the effective resistance R_{eff} of this parallel combination at the resonant frequency?

Solution:

Formula $$R_{\text{eff}}=\frac{L}{CR}$$

substituting $$=\frac{1\cdot 10^{-3}}{2\cdot 10^{-9}\times 1}=5\cdot 10^{5}=500{,}000$$

whence $$R_{\text{eff}}=500{,}000\ \Omega$$

This value of effective resistance is so high compared to the internal plate circuit resistance R_p of the usual vacuum tube that, for the frequency of resonance, a voltage amplification very nearly equal to the amplification constant of the tube will be secured.

Example:

What will be the reactance of the above combination at $\lambda=1{,}000$m. and $\lambda=9{,}000$m?

Solution:

At $\lambda=1{,}000$m., the reactance will be a capacity reactance X_c equal very nearly to that of the condenser alone. This will be given by

$$X=X_C=\frac{1}{\omega C}$$

where X_c = capacitive reactance in ohms,
C = capacity in farads.

The value of ω for $\lambda=1{,}000$ m is $1.885\cdot 10^{6}$ (Table 13)

Substituting $$X_C=\frac{1}{1.885\cdot 10^{6}\times 2\cdot 10^{-9}}=\frac{1}{3.77\cdot 10^{-3}}=2.65\cdot 10^{2}=265.$$

whence $X_C=265\Omega$

This reactance is so very low compared with R_p of the vacuum tube that no amplification will be secured.

At $\lambda=9{,}000$m., the reactance will be practically that of the coil that is, it will be inductive. Hence,

$$X=X_L=\omega L$$

where X_L = inductive reactance in ohms,
L = inductance in henries.

At this wave length

$$\omega = 2.094 \cdot 10^5 \qquad \text{(Table 13)}$$

and $$X_L = 2.094 \cdot 10^5 \times 1.10^{-3} = 2.09 \cdot 10^2 = 209$$

whence $$X_L = 209\ \Omega$$

Similarly, therefore, no amplification will be secured at this wave length.

In fact, with the values of inductance and capacity assumed above, amplification would only be obtained in the immediate vicinity of the resonant wave.

If a coil of 200 mh and a capacity of 10 $\mu\mu$f are used as the coupling unit, in which case the capacity can be that of the coil itself plus the capacity contributed by the vacuum tube, the wave length of resonance will be the same, the effective resistance at resonance will be even higher than before and the reactances at the wave lengths of 1,000 m. and 9,000 m. will be 200 times those computed above, or 52,000 Ω and 42,000 Ω, respectively. With these values of reactance, a fairly high amplification would be secured and, hence, the whole wave length range from 1,000 to 9,000 meters could be covered.

The first type of amplifier has the advantage of high selectivity, which is particularly marked when several stages of amplification are utilized. Suppose, for example, that the voltage amplification at resonance is 6 per stage, while on an adjacent wave length the amplification is 3 per stage. With three stages, the voltage amplification of a signal on the resonant wave would be 6×6×6 or 216 times, while an interfering signal on the adjacent wave length would be amplified only 27 times.

The disadvantages of this type of amplifier are: the multiplicity of tuning requirements when various wave lengths are to be received, and the great tendency for the amplifier to be unstable and self-generate oscillations, in which case the amplifier is rendered useless.

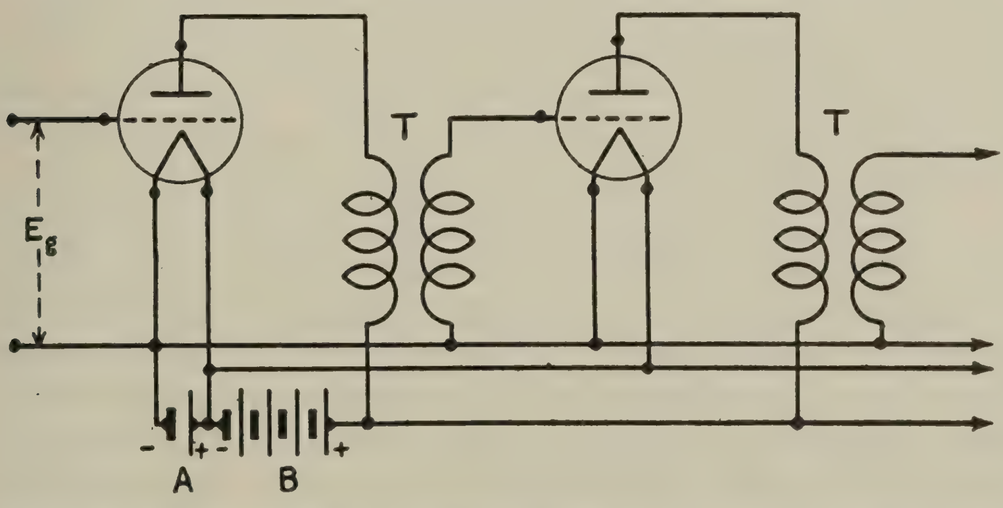


Fig. 185.—Typical Circuit Used in Transformer Type Amplifiers, Showing Method of Coupling Stages.

Transformer type amplifier. The typical circuit used in transformer type amplifiers is, in general, that shown in figure 185. The primary winding of the transformer T is included in the plate circuit of one vacuum tube, while the secondary is connected between the grid and filament of the succeeding tube. The transformer type amplifier is used for the amplification of both radio and audio frequencies. For radio-frequency amplification it is customary to use a 1:1 transformer ratio excepting for very long wave lengths, where a step-up ratio is found to be advantageous. For audio-frequency amplification it is customary to use a step-up ratio which varies in different designs from about 1:3 up to 1:10. Both air and iron-core transformers are used in both radio- and audio-frequency amplifiers. Usually, however, the air core is used for radio and the iron core for audio frequencies.

The problem of transformer design with vacuum-tube amplifiers is quite different from ordinary transformer design. In the ordinary use

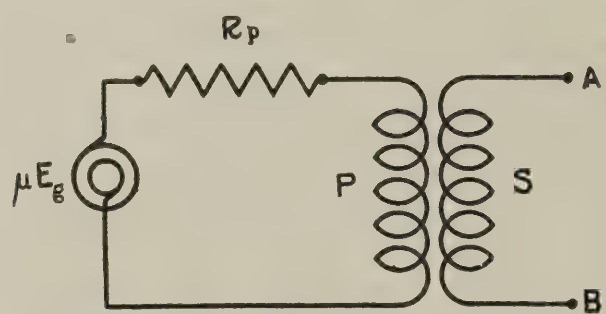


FIG. 186.—Equivalent Plate Circuit Including Transformer Primary.

of transformers, the primary is connected across the line, and the resistance and reactance in series with the primary is small. In vacuum-tube amplifiers the source of emf is inside the vacuum tube and, in series with this emf, is the plate circuit resistance R_p of the vacuum tube. Thus, in figure 186 the emf μE_g acting in the plate circuit of the tube is in series with R_p and the primary P of the transformer. With the customary receiving vacuum tubes, the value of R_p varies between 20,000 and 60,000 Ω. The voltage across the primary will be low unless its impedance is high compared with R_p. This means that a high impedance primary will be required. The secondary winding S is connected to the input of the next vacuum tube. It is desirable to obtain a high voltage across the terminals A and B of this winding. If the grid of the following vacuum tube is at a negative potential sufficient to render the grid current practically zero, the secondary is closed through the capacity between the grid and filament of the vacuum tube. If appreciable grid current flows, the secondary is closed through a resistance which decreases as the grid of the following vacuum tube is made more and more positive. Usually, this resistance must be very high or the amplification will be seriously reduced.

The capacities of the primary and secondary windings and between these windings are usually of very great importance in determining the mode of operation of the transformer as well as the leakage of flux between the windings. If it were possible to build transformers without these capacities and the vacuum tubes were free from capacity effects, it would be possible to obtain any desired amount of amplification. It would then be necessary to use sufficient primary turns to obtain impedance which would be high compared to the plate resistance of the vacuum tube over the range of frequencies to be covered, and any desired amplification could be secured by using a high step-up ratio.

Practically, however, it will be found that, with any chosen primary winding and frequency, the voltage applied to the next vacuum tube will at first increase with increase in secondary turns up to a maximum, and then decrease again. This is a resonance effect caused by the capacities of the transformer and vacuum tube which is similar in action to the resonance transformer used with quenched-spark transmitters. The primary of the transformer will have a high impedance only in the range of frequencies about resonance and, hence, it is only within this range that an appreciable voltage is applied to the primary terminals by the vacuum tube. The higher the step-up ratio of the transformer, the greater will be the amplification at resonance, but the range over which high amplification is obtained will be reduced. At radio frequencies this would result in an amplifier which would be useful only over a narrow band of wave lengths. In an audio-frequency transformer for the reception of a note of a particular frequency, such as a 1,000 cycle spark note, there would be no disadvantage. For telephony, however, particular component frequencies of the speech or music would be amplified more than others, resulting in distortion. Hence, a sacrifice in amplification would be required to preserve the quality of the transmitted speech or music.

Advantages of radio-frequency amplifiers. The function of a detector is to transform radio-frequency oscillations which can not produce a note in the telephones over into audio-frequency oscillations which can be heard. The way the vacuum tube does this will be taken up later. At present it is desirable to state that the efficiency of a detector for spark signals or telephony is greater for strong signals than it is for weak signals. There is, therefore, an evident advantage in increasing the strength of a weak radio-frequency signal before applying it to the detector tube. In fact, if the amplitude of a weak signal is amplified twenty times before it is applied to the detector tube, the resulting audibility will be as great as would be obtained by applying the signal to the detector tube directly without amplification, and then using an audio-frequency amplifier to amplify the audio frequency four hundred times. Another reason for using radio-frequency amplification is because only two or three stages of audio-frequency amplification are

feasible. It is difficult to prevent an audio-frequency amplifier having a greater number of stages from howling, and, furthermore, the amplifier is likely to be very noisy because it will amplify noises due to bad contacts, variable batteries, and noisy tubes.

Radio-audio-frequency amplifiers. It is customary to combine radio- and audio-frequency amplification in the same instrument, in which case it is also necessary to include a detector in order to convert the radio-frequency oscillations over into audio-frequency. Thus, frequently three tubes and transformers are used to amplify the radio-frequency oscillations, another tube acts as a detector, and two more tubes and transformers amplify the audio frequency.

CHAPTER IV.

ACTION OF THE VACUUM TUBE AS A DETECTOR.

The function of a detector. Because of the high frequency of the electrical oscillations used in radiotelegraphy and telephony, it is not possible for the received currents to flow in appreciable amount through the inductive winding of the telephones; the diaphragms of the telephones could not vibrate with a frequency so high even if the currents did flow through the windings, and the ear could not hear a note of such high frequency even if the diaphragms could vibrate with a frequency so high. It is necessary, therefore, to convert the received radio-frequency oscillations over into audio-frequency oscillations in order that the signals will be heard by the operator. The detector performs this operation, but it is necessary that the received radio-frequency oscillations vary in amplitude at an audio-frequency rate in order to permit the detector to convert the radio-frequency oscillations over into audio. In ordinary spark transmission the oscillations do vary at an audio-frequency rate. Each spark at the transmitter generates a single train of radio-frequency oscillations and, since the spark frequency is ordinarily about a thousand per second, the wave trains are sent out at this rate.

The current in the receiving antenna is shown roughly in figure 187, where 1, 2 and 3 represent the trains of oscillations produced by three sparks at the transmitter, the time between the wave trains being

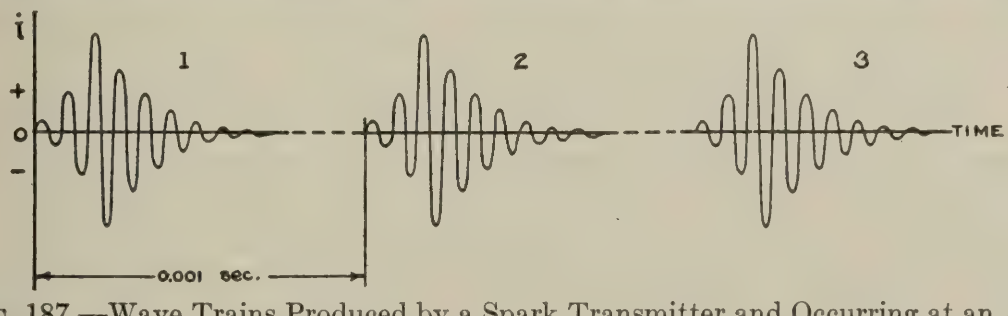


Fig. 187.—Wave Trains Produced by a Spark Transmitter and Occurring at an Audio-Frequency Rate. Each Wave Train Consists of Radio-Frequency Oscillations which Decrease Logarithmically in Amplitude.

about a thousandth of a second. The figure, of course, shows a fewer number of wave trains than occur in a dot unless the speed of transmission is very rapid. The detector will convert the wave trains of oscillations over into oscillations having a frequency of a thousand per second, so that the note in the telephones will correspond to the frequency of the spark of the transmitter. In arc or vacuum-tube transmission the waves are continuous, so that the received oscillations are about as shown in figure 188. These oscillations do not vary at

an audio-frequency rate, and it is necessary to supply this feature in order to obtain a note in the telephones. This can be done by interrupting the oscillations by means of a chopper either at the transmitter or receiver.

Beat reception. The more usual and better method of converting radio-frequency oscillations into audio-frequency oscillations is the **beat method of reception.** An oscillating vacuum-tube circuit is employed at the receiving end to generate continuous-wave oscillations which differ slightly in frequency from that of the incoming, or signal oscillations, and the two sets of oscillations are then superimposed in the receiver. Being slightly different in frequency, the signal oscillations will at one instant be in step with, and will add themselves to the local oscillations. A little later, the former will be out of step with the latter, and will subtract from them. Thus, the amplitude of the combined oscillations will rise and fall. The number of times per second that the local and signal oscillations get into step, or phase, and produce

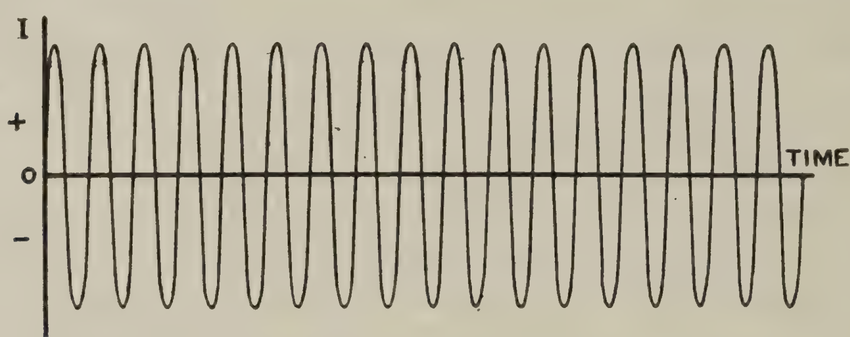


FIG. 188.—Unmodulated Continuous or Undamped Waves.

what is called a **beat** depends upon the difference in frequency of the two sets of oscillations.

Figure 189 shows how beats are produced by superimposing two sine waves of different frequencies. Thus, (*a*) shows an oscillation having a frequency of 18 cycles per second, and (*b*) one having a frequency of 16.5 cycles, the amplitudes of the latter being twice those of the former. Both wave trains are shown starting at zero and then increasing in the positive direction; that is, both start **in phase.** The resultant at any instant can be found by **adding algebraically** their values at the same instant as is shown in the figure. (*c*) is the resultant obtained by thus superimposing the two sine waves in (*a*) and (*b*). The resultant, or beat, frequency should be equal to the difference of the two frequencies: 18—16.5, or 1.5. Curve (*c*) shows this to be the case, the beats being indicated by the dash-line **envelope** which is drawn through the amplitudes of the resultant oscillations. The length of one beat is determined by the time elapsing between either the instants at which the interfering waves come into phase and produce a maximum resultant oscillation, or are 180° out of phase and produce a minimum resultant oscillation.

It will be seen that in the figure there are only a few oscillations in the duration of a beat. This is because a high percentage difference in the two frequencies was chosen for the purpose of illustration in the example. Normally in radio reception the percentage difference in the two frequencies would be small, and there would be a large number of oscillations in the duration of a beat. For example, if *CW* transmission is being received on a wave length $\lambda = 6{,}000$ m. for which $f_1 = 50{,}000$, and the local oscillations are adjusted to a frequency $f_2 = 49{,}000$ or 51,000, the resultant beat note would have a frequency $f_b = f_1 - f_2$ or

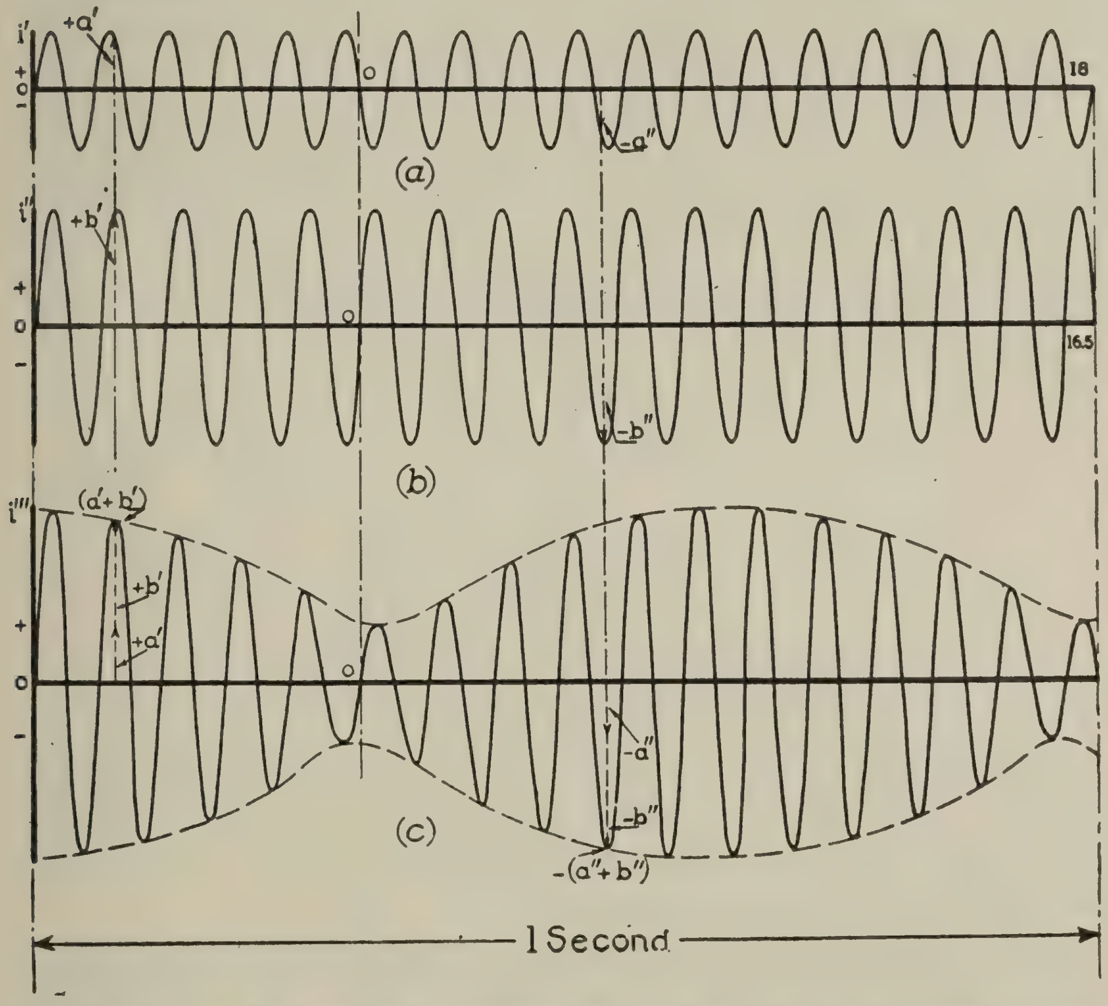


FIG. 189.—Beats Produced by Superimposing Two Sine Waves of Different Frequencies. Curve (a) Represents Incoming or Signal Oscillations; Curve (b) Shows Local Oscillations; Curve (c) Gives the Algebraic Sum of (a) and (b), and Shows the Resultant Beats.

1,000. In one second, the envelope of the resultant oscillations would pass through a minimum value (not necessarily zero) 1,000 times, and each beat would consist of oscillations occurring at a radio-frequency rate, but they will have the requisite variation in amplitude at an audio-frequency rate. The detector will then convert the oscillations into audio-frequency oscillations as will be explained later, and the note given out by the telephones will correspond to the beat frequency. In

practice, the amplitude of the local oscillations is much greater than that of the signal oscillations.

When receiving *CW* transmission by the beat method, the note heard in the telephones will be clear and musical, and have a frequency corresponding to the beat frequency. This is because the two superimposed frequencies are sinusoidal and do not vary in amplitude. When modulated radio-frequency oscillations, such as those emitted by a spark transmitter or a radiotelephone, are received by the beat method, the beat note will be non-musical and rough in the case of the former, while with the latter the speech or music will be rendered unintelligible, for the reason that the local audio-frequency modulation is being superimposed on that occurring at the transmitter.

The local oscillations may be self-generated in the secondary circuit of the receiver by an oscillating vacuum-tube detector, or they may be induced in the receiver by a separate oscillating vacuum-tube circuit. The method of producing the local oscillations by the receiver secondary circuit is called the **autodyne** method, while that using a separate circuit is called the **heterodyne** method.

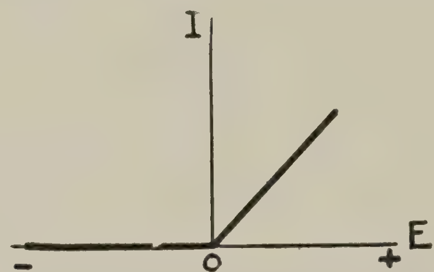


Fig. 190.—Characteristic Curve of an Ideal Detector.

In radiotelephony, the amplitude of the radio-frequency oscillations sent out by the transmitter is made to vary in accordance with the voice oscillations, which are very complex. The detector converts these oscillations over into complex audio-frequency currents which reproduce the voice in the telephones.

The operation of an ideal detector. An ideal detector would have a characteristic curve somewhat as shown in figure 190. For positive voltages, the current increases with the voltage with a straight line relationship and, hence, the detector behaves for positive voltages as an ordinary ohmic resistance. For negative voltages, the current is always zero, that is, the resistance is infinite. If an alternating voltage were applied to such a detector so as to cause the voltage to alternate around zero, current would flow when the voltage is on the positive swing, but no current would flow on the reverse. Such a detector would act as a complete rectifier. Suppose that, in the reception fo spark signals, the voltage across the secondary condenser were applied to such a detector. The form of the voltage would be similar to that shown in (*a*) of figure 191. Current would flow when the voltage is in one direction, so that

the current would consist of half-waves in one direction as shown in (*b*) figure 191. In case the customary telephones and bypass condenser are in series with the detector, the pulses of current of (*b*) will accumulate on the condenser, charging it and then discharging through the telephones so that the current in the telephones will resemble that shown in (*c*), figure 191. Every wave train will cause current to flow through the telephones, which will move the diaphragm, and between wave trains, the diaphragm will move back again, so that the frequency of vibration of the diaphragm and, hence, the note emitted will correspond to the spark frequency of the transmitter. In the case of a 500-cycle quenched-spark transmitter, properly adjusted, one spark occurs

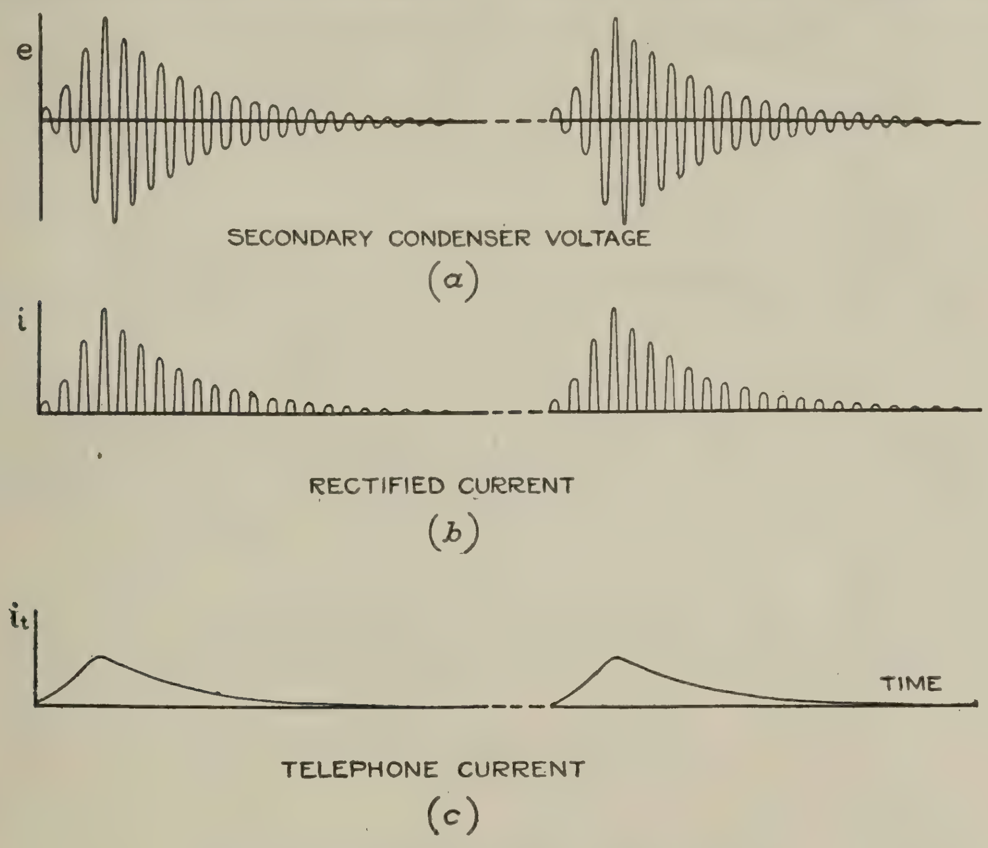


FIG. 191.—Operation of an Ideal Detector.

per alternation, or two sparks per cycle, so that the note has a frequency of 1,000.

It is apparent that, if the amplitude of the signal oscillations of (*a*) were increased, the half-wave pulses of (*b*) would be increased by a proportional amount, as would also the amplitude of the oscillations in (*c*). This leads to the conclusion that, in the case of an ideal detector, the audio-frequency current in the telephones would be directly proportional to the intensity of the radio signal. Such a detector would be very efficient compared to the detectors actually used for spark reception, especially for weak signals.

Three-electrode vacuum tube as detector, without grid condenser and grid leak resistance. The circuit which is used with the three-electrode vacuum tube for detection, when the grid condenser is not

employed, is shown in figure 192. The grid of the vacuum tube is connected through the inductance coil of the secondary circit of the receiver to the terminal of a resistance r in the negative filament lead,

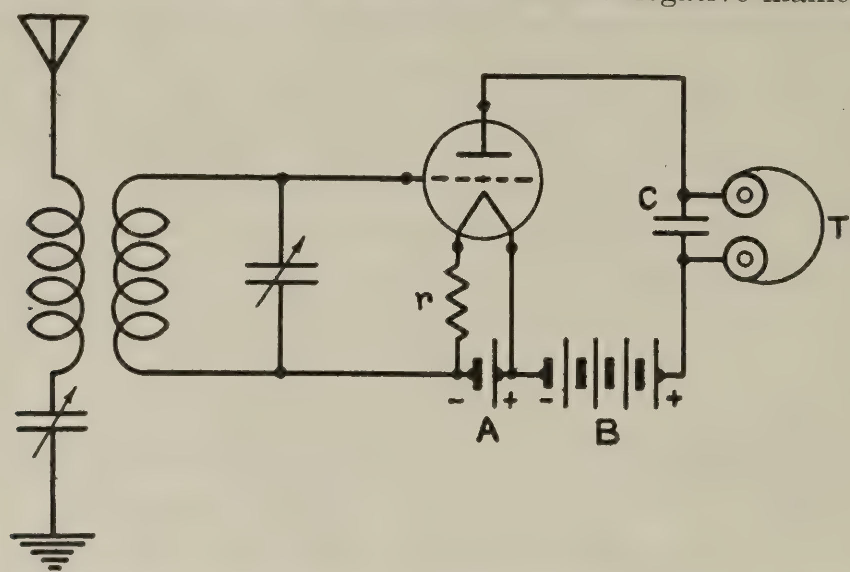


FIG. 192.—Circuit Diagram Showing Method of Connecting a Nonoscillating Vacuum-Tube Detector without Grid Condenser and Grid Leak Resistance, but Using a Grid Bias.

which renders the grid negative with respect to the filament. This negative voltage is frequently called the **grid bias,** or **biasing voltage,** and

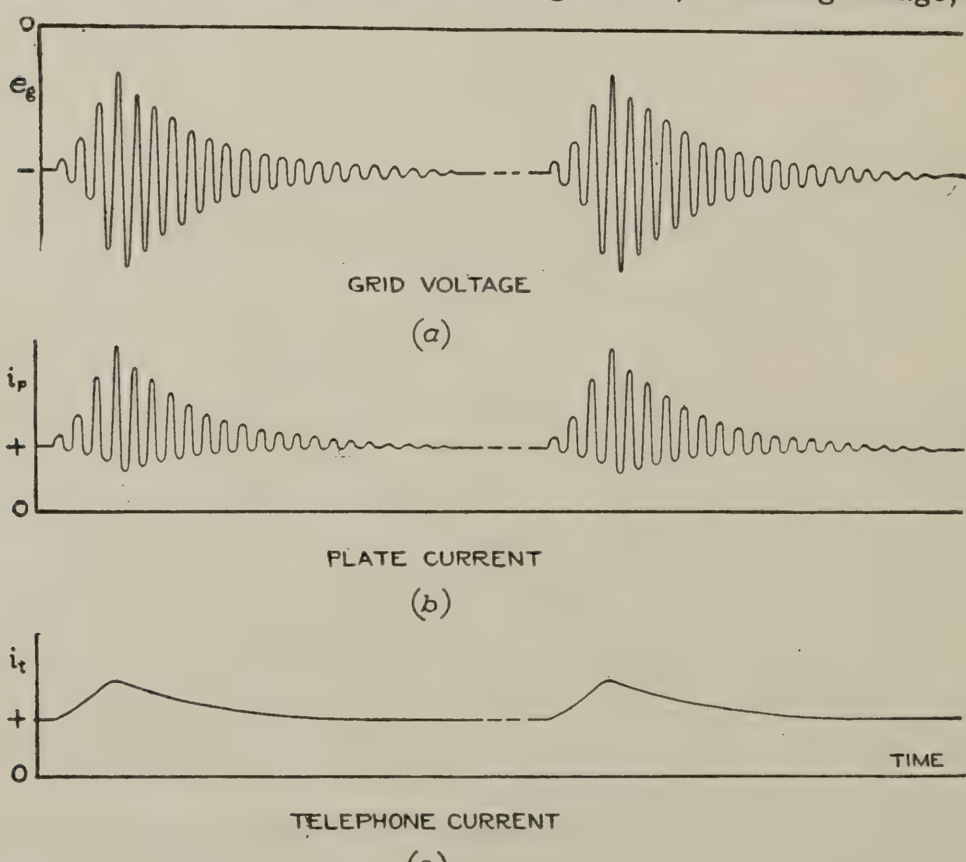


FIG. 193.—Operation of a Detector without Grid Condenser and Grid Leak.

has the result of fixing the operating point on the curved portion of the plate current—grid voltage characteristic. It has already been pointed out in Chapter I that, under this condition, when an alternating voltage is applied to the grid of the vacuum tube, the wave form of the current in the plate circuit is distorted from that of the grid voltage, and that the plate current increases more above the normal value when the grid voltage is on the positive swing than it falls below on the negative swing. In the reception of spark signals, the voltage across the secondary condenser and, hence, the voltage on the grid of the vacuum tube will be like (*a*) of figure 193. The resulting variations in plate current will be similar to (*b*), differing from those of the ideal detector discussed above, in that the oscillations run somewhat below the line of normal current, so that the rectifying action is not complete. The radio-frequency variations in plate current flow through the bypass condenser C of figure 192; the audio-frequency pulses flow through the telephones as shown in (*c*) of figure 193. In the case just discussed, the detecting action of the vacuum tube is a result of the distortion resulting from the bending or curvature of the plate current—grid voltage characteristic. When the grid condenser and the grid leak resistance are used, the detecting action depends upon the curvature of the grid current—grid voltage characteristic.

Three-electrode vacuum tube as detector, with grid condenser and grid leak resistance. The circuit ordinarily used is shown in figure 194. In the lead to the grid is inserted a small condenser C_g having a capacity of 100 to 200$\mu\mu$f which is shunted by a high resistance r of 1 to 4 megohms. The other connection from the secondary condenser runs to the positive terminal of the filament. This renders the grid a few tenths of a volt positive; less positive than the positive terminal of the filament because of the high resistance of the grid leak. The voltage which the grid will assume can be determined from the grid current—grid voltage characteristic curve. Figure 195 is such a curve, and **shows the current flowing from grid to filament for different voltages of the grid** relatively to the negative terminal of the filament. Assume the voltage drop on the filament to be four volts; then, if the grid were connected directly to the positive end of the filament, its voltage would be +4 volts, and a current of about 31 μa would flow from the grid to the filament inside the vacuum tube and from filament to grid on the outside. If a grid leak resistance r is inserted in the connection between the grid and the positive end of the filament then, when the grid current starts to flow, there will be a voltage drop in this resistance. This voltage drop will oppose the current, making the grid end of the resistance negative with respect to the filament end and will, therefore, reduce the amount by which the grid is positive. The straight line in the figure shows the **current through the grid leak resistance for different voltages on the grid** assuming the resistance to have a value of 2 meg-

ohms. If the grid were at +4 volts, then both ends of the resistance would be at +4 volts and no current would be flowing through the resistance. This, of course, is an impossible condition because there would be 31 μa flowing from grid to filament inside the vacuum tube and no return current through the resistance on the outside. If the grid were at zero, then the voltage across the resistance would be 4 volts, one end being at zero potential and the other at +4 volts. The current through the resistance would be

$$\frac{4}{2 \cdot 10^6} \text{ ampere or } 2\mu\text{a.}$$

This is also an impossible condition, since the grid current inside the vacuum tube would be only about 0.2 μa.

The only possible condition is that which makes the grid current inside the vacuum tube the same as the return current on the outside

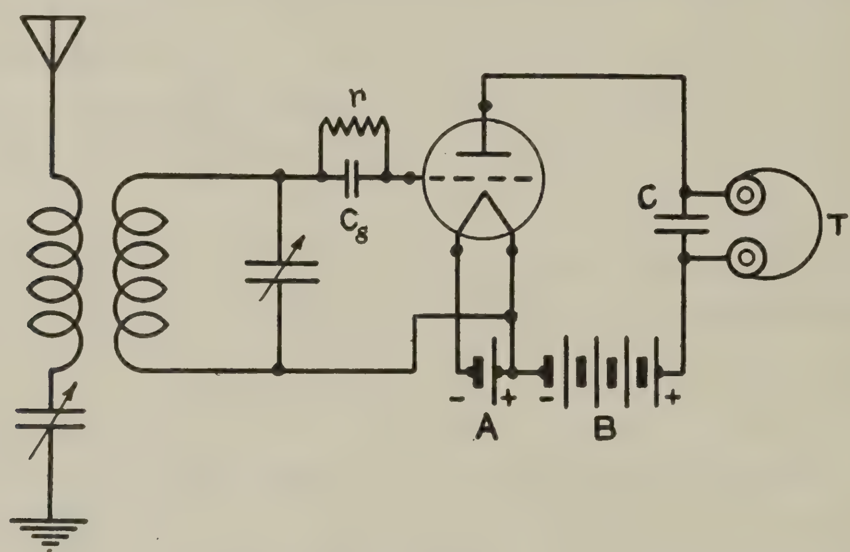


FIG. 194.—Circuit Diagram Showing Method of Connecting a Nonoscillating Vacuum-Tube Detector with Grid Condenser and Grid Leak Resistance.

through the leak resistance. This condition is given by the intersection of the straight line with the grid current characteristic. This point is marked *A* on the figure, and corresponds to a grid voltage of about +0.65 volt. Thus, with a vacuum tube having the characteristic shown in the figure, if the grid is connected through a 2 megohm resistance *r* to the positive terminal of the filament it will assume a voltage of only 0.65 volt more positive than the negative terminal of the filament. This grid voltage is such as to make the operation of the vacuum tube take place on a curved portion of the grid current—grid voltage characteristic. A small current will be flowing. Now, when a train of oscillations comes in, the grid voltage will swing positive and negative; on the positive swing the grid current will increase considerably, while on the negative swing the reduction in grid current will be slight. In effect, the average grid current will increase during the

oscillations. This increased grid current will disturb the condition of equilibrium discussed above. The average grid voltage will have to become less positive, causing a reduction in the steady grid current to offset to some extent the increase in average grid current produced by the oscillations. The average grid voltage will decrease to, say, the point *B* where the steady grid current given by the ordinate *Bc* plus the increase *Ba*, caused by the oscillations, will equal the current *ac* through the leak resistance. When the oscillations stop, as occurs between the wave trains, the grid voltage will return to the equilibrium value given by the point *A*.

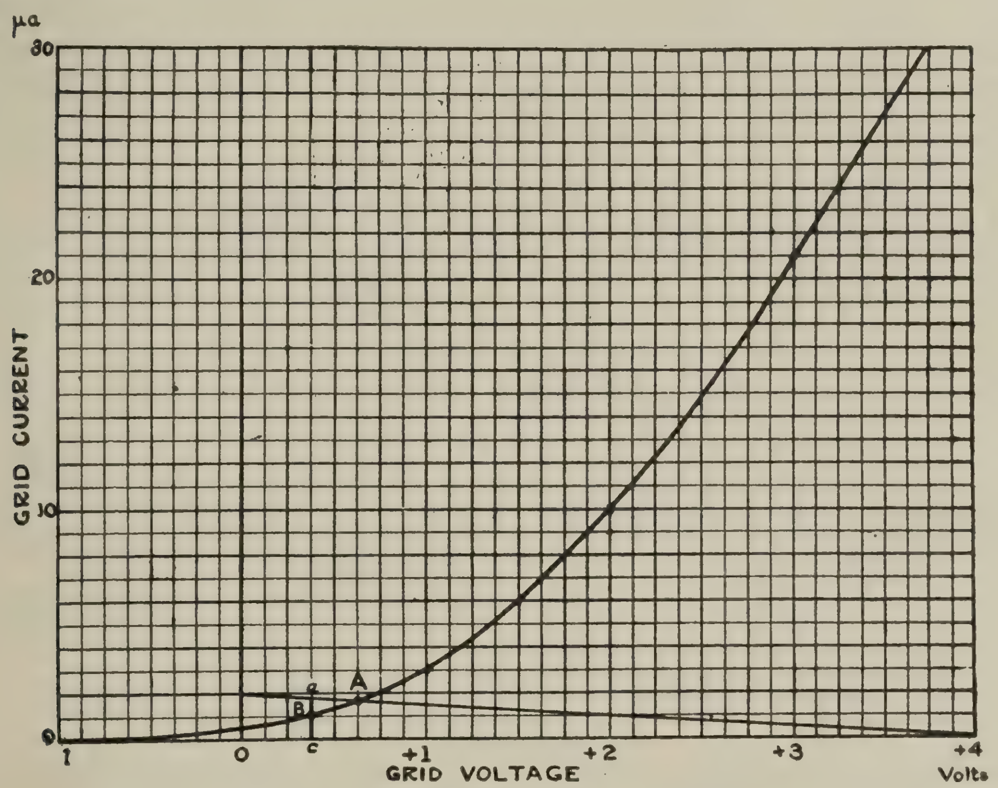


FIG. 195.—Grid Current—Grid Voltage Characteristic Curve for a Receiving Vacuum Tube.

The function of the grid condenser is to permit the radio-frequency oscillating voltage to be applied to the grid in full value without having to act through the high leak resistance, which would reduce the voltage. To bring this about, the condenser must have a reactance less than the impedance between grid and filament of the vacuum tube. On the other hand, the capacity of the grid condenser must not be too large, for, whenever the average grid voltage is altered, the voltage and charge on this condenser must be varied accordingly. With a large condenser, a considerable charge would be required and the changes in voltage of the grid would be retarded and might not be fully completed during a wave train. This would be undesirable, because it is the fluctuation in average grid voltage which produces the detecting action.

The action, as explained up to this point, is illustrated in figure 196. Curve (*a*) represents the voltage variations across the secondary

condenser produced by the signal wave trains. Curve (*b*) shows the voltage variations of the grid of the vacuum tube, the heavy line denoting the average grid voltage. Between wave trains, the average value corresponds to point *A* of figure 195. During the wave train it drops to correspond to point *B*.

Now, it must be remembered that the plate current will follow all changes in the grid voltage, so that the wave form of the plate current will resemble the grid-voltage curve of (*b*). The radio-frequency portion of the plate crrrent oscillations will flow through the telephone condenser, but, corresponding to the decrease in the average value of the grid voltage, the current through the telephones will be reduced during each wave train as shown in (*c*), producing a note in the telephones of the same pitch as the wave train frequency. It is to be noted that **there is,**

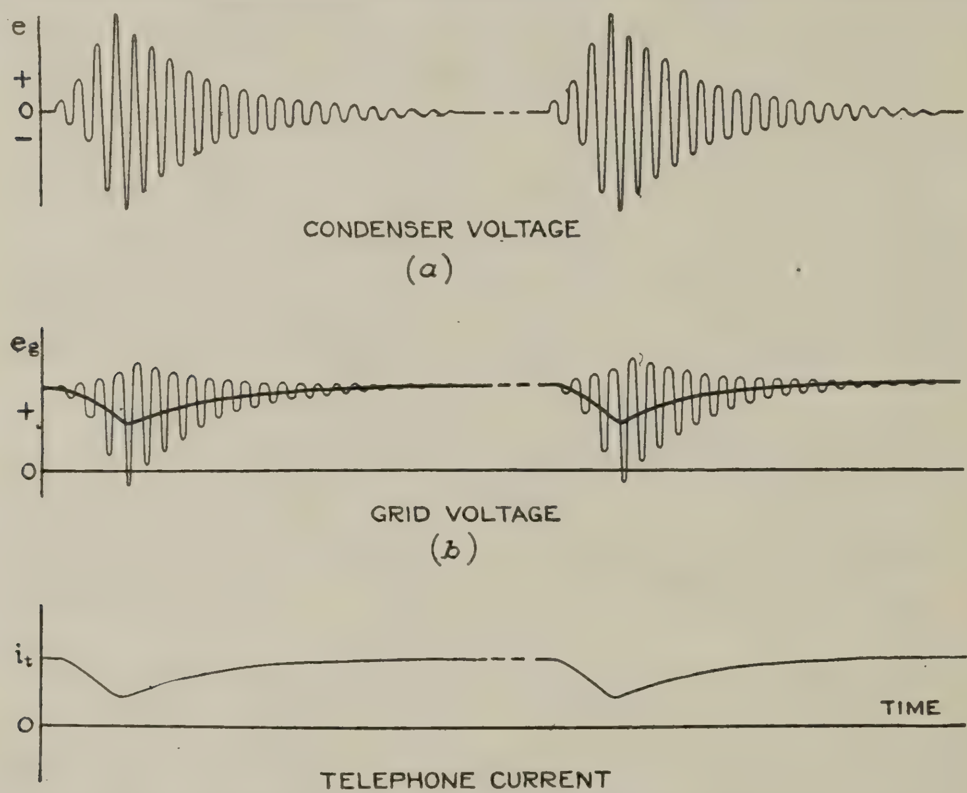


FIG. 196.—Operation of a Detector with Grid Condenser and Grid Leak.

in the case of detection with grid condenser and grid leak, a reduction in telephone current during the wave train while, in the previous case of detection without grid condenser, the telephone current increased during the wave train. Also, the detection with grid condenser results from the curvature of the grid current characteristic, while detection with a biasing voltage results from the curvature of the plate current characteristic.

When the grid leak and condenser are not used, the action of the vacuum tube is the equivalent of **a stage of radio-frequency amplification followed by a detector,** because the radio-frequency voltage applied to the input is amplified into the plate circuit and there converted into

audio frequency. On the other hand, when the grid leak and condenser are used, the action is similar to **a detector followed by one stage of audio-frequency amplification,** for now the conversion to audio frequency takes place in the grid circuit and the audio-frequency variations are amplified into the plate circuit.

The detecting property itself of a hard vacuum tube must be poor as compared with a good crystal detector, because the combined result of detection and amplification with the vacuum tube leads to results perhaps inferior to those obtained with a crystal in which detection alone occurs. This, of course, is true only when the amplification of the vacuum tube is not enhanced by regeneration.

Regenerative vacuum-tube detector. Either type of detection described above can be used in combination with the regenerative amplifying circuit, such as that discussed in Chapter II and illustrated in figure 182. For example, in the circuits of figures 192 and 194, it is only necessary to insert a feed back coil in the plate circuit of the vacuum tube and to couple this coil to the secondary inductance coil of the receiver. As explained before, regeneration leads to larger radio-frequency currents and voltages in all parts of the circuit for a given strength of received signal, so that the audio-frequency response in the telephones will likewise be louder. Up to a certain point, as the feed back coupling is increased, the response will increase and the note from a spark station will be clear. Beyond this point, however, the vacuum tube will start oscillating and the clear spark note will be lost, being replaced by a mushy note which will, however, be much louder than the clear note when receiving feeble signals.

Oscillating vacuum-tube detector or the autodyne. The vacuum tube in an oscillating state, as just described, is extremely valuable for the reception of the continuous oscillations of an arc or vacuum-tube transmitter. The frequency of the oscillations generated by the vacuum tube will depend primarily upon the inductance and capacity of the secondary circuit of the receiver, so that, if the secondary is slightly out of tune with the incoming oscillations, there will be present in this circuit oscillations of two frequencies and, hence, beats as described above and as shown in figure 189. The beat frequency and hence the note in the telephones can be varied at will by tuning the secondary.

The detection of these beats takes place in a manner similar to that described above for spark reception. This will be explained later in greater detail.

Heterodyne reception. In heterodyne reception, instead of using the same vacuum tube to generate the local oscillations to produce beats, and also to act as a detector, the local oscillations are furnished by another vacuum tube or other source which is coupled to the secondary of the receiver or the antenna. The ordinary detector tube or crystal detector performs the detecting function. When the coupling to the

source is such as to supply local oscillations of the right amplitude, this method of reception becomes as sensitive and, perhaps, a little more sensitive than the autodyne. The law of reception is the same as that for the autodyne. The disadvantage is the requirement of tuning the source of oscillations in addition to the other tuning adjustments.

Laws of detection. Suppose that the characteristic curve of a detector, a crystal or vacuum tube, is similar to that of figure 197 (a), and that the steady operating voltage is such as to fix the operating point at A. Now, let an alternating emf of maximum value E_0 be impressed upon the device; then the voltage will vary about the point A by an amount E_0 and the current will increase to point B and then decrease to point C. The variations in i will be as shown in figure 197 (b); the solid line shows on the left the steady current flowing when no

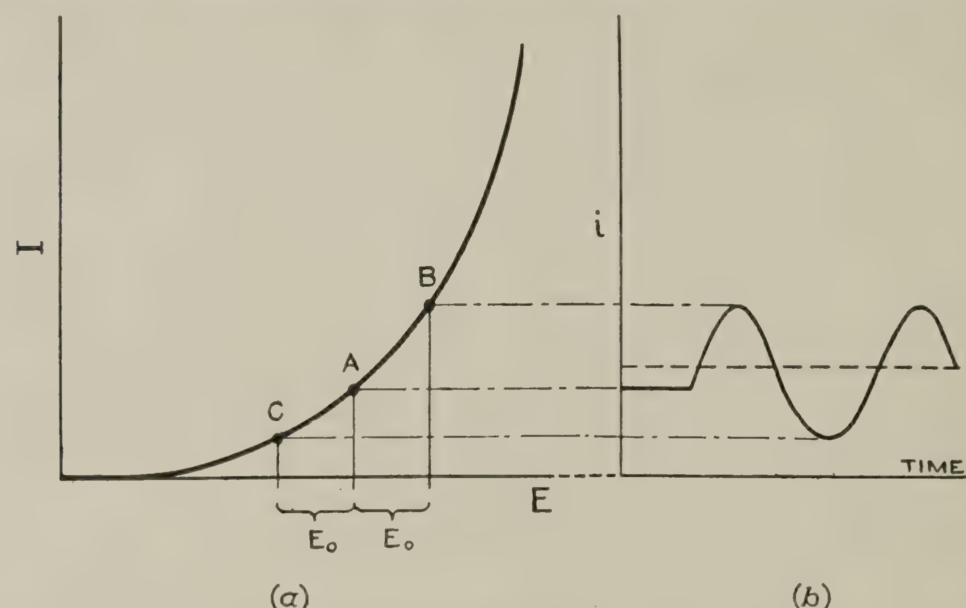


Fig. 197.—Curve Showing Rectifying Action of a Detector.

alternating emf is applied, and the dash line shows the increased average value of the current due to the oscillations. It is this change ΔI in average value of the current which acts upon the telephones. The telephone current and audibility of the received signal are proportional to ΔI. The more rapidly the curve is bending at the point A, or the more rapidly the slope of the curve is changing at A, the greater will be ΔI. It can be shown that, for feeble oscillations or weak signals,

$$\Delta I = \frac{E_0^2}{4}\left(\frac{d^2 I}{dE^2}\right)$$

where the quantity is parentheses is the rate of change of the slope.

This law shows that with any detector, in which the detecting action depends upon the bending of the characteristic curve the **audibility of the signal for weak signals is proportional to the square of the radio-frequency voltage applied to the detector.** The voltage applied to the

detector is proportional to the current or voltage in the receiving antenna, so that the law can also be expressed in terms of these quantities. The law applies in general to all of the detectors used for the reception of spark or telephone signals. The inefficiency of such detectors for weak signals is evident, for if the received antenna current is reduced to one-half, or the received power to one-quarter, the telephone current falls to one-quarter, or the output power to one-sixteenth. Thus, the efficiency is reduced to one-quarter.

For strong signals the efficiency of these detectors increases. The law of response then changes over to a first power law, or the audibility becomes proportional to the received current or voltage in the antenna.

In the case of autodyne or heterodyne reception, there is acting upon the detector a fairly strong local oscillation prior to the reception of the signal emf. When the latter is superimposed, it will, because of the difference in frequency of the two oscillations, at one instant add to

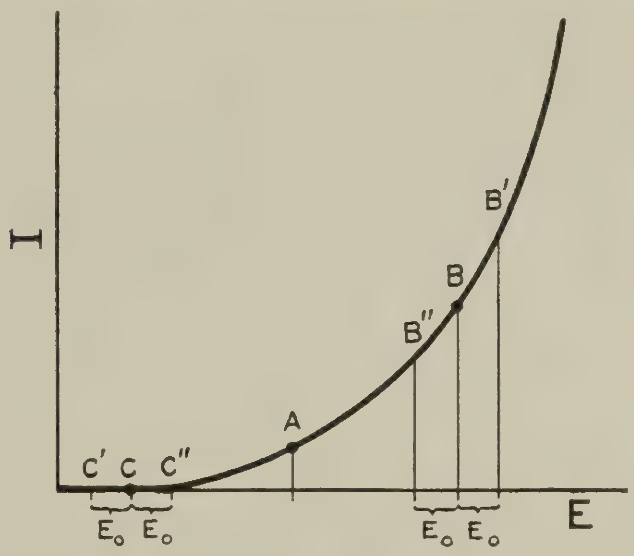


FIG. 198.

the local oscillations and then a little later be subtracted from the local oscillations. Thus, the amplitude of the combined oscillations will vary up and down by an amount equal to twice the amplitude of the received signal voltage. The average value of the current through the detector will also vary up and down, and it is this which actuates the telephones. Thus, in figure 198 the local oscillations will cause the current to vary from A to B on one side and from A to C on the other. When the signal oscillations of amplitude E_0 are superimposed, the combined oscillations will run from B' to C' when in phase, and from B'' to C'' when out of phase. Since the value of I for the points C', C and C'' are practically identical, the variations in the average current will be due primarily to the variations in amplitude between B'' and B' and will be practically proportional to the distance from B'' to B'. Therefore, for different values of E_0 the variations in the average current will be proportional to E_0 or the signal oscillation.

If a crystal detector with a heterodyne is being used, the changes in average current will directly constitute the telephone current. If a vacuum tube with grid condenser and leak is being used, either self-oscillating (autodyne) or with a separate source (heterodyne), the changes in average current will be changes in average grid current, which will bring about proportional changes in average grid current and, likewise proportional changes in the plate current through the telephones. Likewise, in the case of a vacuum tube with a biasing negative grid voltage, the changes in average plate current will occur as described above. In all these cases, therefore, the telephone current will be proportional to the amplitude of the received signal oscillation. **For these so-called oscillating detectors, the audibility of the signal is proportional to the radio-frequency voltage applied to the detector.** These detectors maintain their efficiency no matter how weak the signal.

Dr. L. W. Austin, who first pointed out this important difference in the laws of response of nonoscillating and oscillating detectors, has also determined that a spark or modulated continuous-wave signal which is just barely audible, or has unit audibility, when received with a plain detector vacuum tube, will have an audibility of about 800 when received on an oscillating detector. At this signal strength, the latter detector is the equal of the former plus two stages of audio-frequency amplification. For weaker signals, the nonoscillating detector is still more markedly inferior; a signal of 10 audibility on the oscillating vacuum tube would have an audibility of $\frac{1}{6,400}$ of unity with the nonoscillating detector and would require an audio-frequency amplification of 64,000 times in order to bring it up to equality.

In addition to the advantage of signal strength, the oscillating detector is also superior for reading through atmospheric disturbances, or static, which are more intense than the signal. If the intensity of the static is twice that of the signal, the response in the telephones will be four times as strong as the signal in the case of the plain detector, but only twice as strong in the case of autodyne or heterodyne reception.

CHAPTER V.

THE THREE-ELECTRODE VACUUM TUBE AS AN OSCILLATOR.

Analogy of oscillating vacuum tube and clock. The action of a vacuum tube when generating oscillations is similar to the action of a clock. The pendulum of a clock swings back and forth and works the escapement. During each oscillation, the escapement permits an impulse to be delivered to the pendulum by the main spring. This impulse is delivered to the pendulum in the direction in which the pendulum is swinging, thus tending to increase the amplitude of the swing. When the amplitude of oscillation of the pendulum increases to a certain value, the loss of energy due to friction during each oscillation becomes equal to the energy delivered to the pendulum by the impulse, so that the oscillation no longer increases in amplitude but is maintained at a constant value.

In the case of the three-electrode vacuum tube, the current in the oscillatory circuit is analogous to the swinging of the pendulum, the grid of the tube takes the place of the escapement of the clock, and the plate battery replaces the main spring. The current in the oscillatory circuit acts upon the grid of the tube, changing its potential. The changes in potential of the grid produce changes in the plate current supplied by the plate battery, and these changes in plate battery current act upon the oscillatory circuit in the proper direction to tend to increase the current in that circuit. When the vacuum tube is put in operation, any feeble oscillation will build up in amplitude until a final amplitude is reached where the power supplied by the vacuum tube is equal to the loss of power in heat and radiation. The oscillations will then continue at this amplitude.

A typical oscillatory tube circuit. A typical circuit for generating oscillations with vacuum tubes is the **Meissner circuit.** This is one of the earliest circuits, but is being employed to a considerable extent in modern transmitting tube apparatus. The circuit is shown in figure 199, the filament and plate batteries being omitted for simplicity. The oscillatory circuit consists of the coils L_3 and L_4 and the condenser C. In transmission, the condenser C would be replaced by the antenna. The coil L_1 is included in the plate circuit of the vacuum tube and is coupled to the coil L_3 while the coil L_2 is included in the grid circuit of the vacuum tube and is coupled to L_4.

Let is be assumed that feeble oscillations occur in the oscillatory circuit. These oscillations will induce an alternating voltage in coil L_2 which will act upon the grid, producing variations in the plate current flowing through L_1 and these will produce an alternating

voltage in the coil L_3 which, with the proper sign and sufficient strength of coupling, will reinforce the original oscillations, causing them to increase in amplitude. The increased oscillations will induce a still greater voltage in the coil L_2 and correspondingly greater variations in

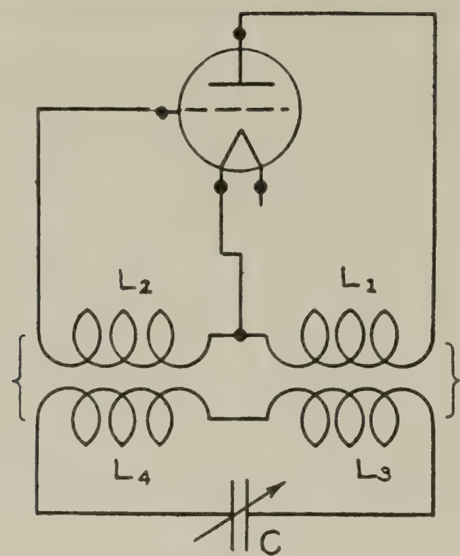


FIG. 199.—The Meissner Circuit.

the current through L_1, leading to a further increase in the oscillating current. This building-up process continues until the vacuum tube cannot supply enough power to the oscillatory circuit to increase further the amplitude of the oscillations and an alternating current of constant

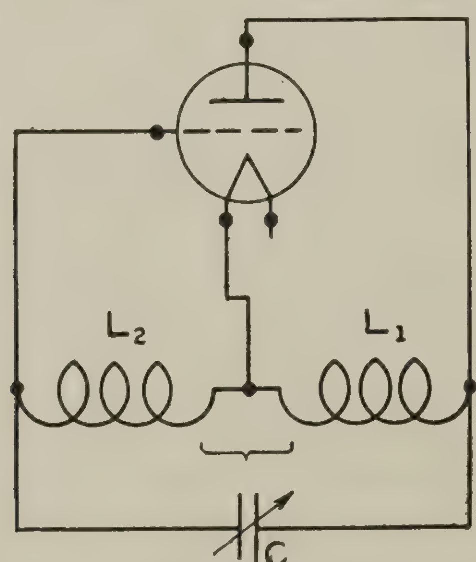


FIG. 200.—The Hartley Circuit.

amplitude will flow in the circuit having a frequency very nearly that of the natural period of the oscillatory circuit. Ordinarily, the final state is reached in a very small fraction of a second after the vacuum tube is put into operation.

Other well-known circuits for generating oscillations. It is possible to derive a number of well-known circuits from the Meissner circuit. For example, in the so-called **Hartley circuit** of figure 200, the plate cir-

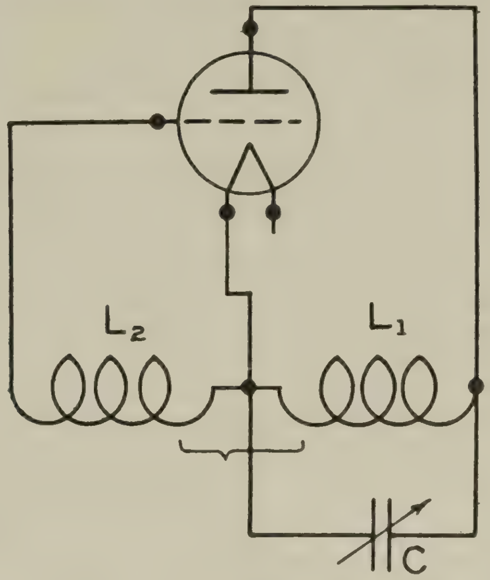


FIG. 201.—The Tuned-Plate Circuit.

cuit of the vacuum tube is directly coupled to the oscillatory circuit through the coil L_1, replacing the coupling between the coils L_1 and L_3 of the Meissner circuit. Similarly, L_2 couples the grid circuit directly to the oscillatory circuit and it is the voltage across this coil which actuates the grid. The varying plate current, which flows through L_1 and C, L_2 in parallel supplies the power to maintain the oscillations. Coils L_1 and L_2 can be coupled together, if desired.

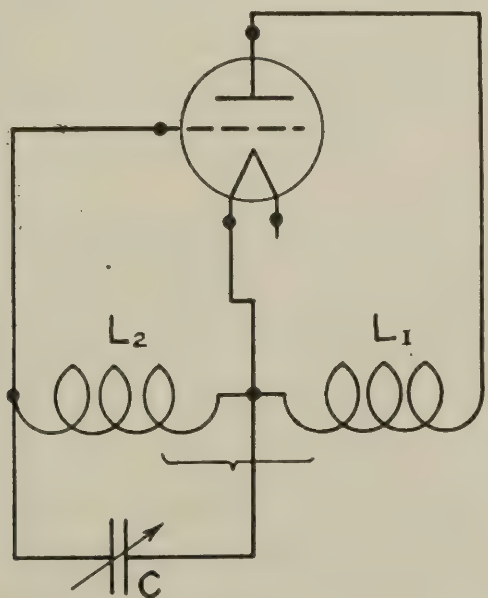


FIG. 202.—The Tuned-Grid Circuit.

The circuit frequently called the **tuned-plate circuit,** figure 201, is obtained by putting the condenser C across the coil L_1 alone. It is necessary in this circuit to couple the coil L_2 to L_1 in order to obtain the

controlling voltage on the grid, since the coil L_2 is no longer in the oscillatory circuit.

The **tuned-grid circuit,** which is used so extensively for receiving work, is obtained by connecting the condenser across the coil L_2 alone as in figure 202. In this circuit, coupling between L_1 and L_2 is ordinarily necessary, for it is by this means that the voltage is applied from the plate circuit of the vacuum tube to sustain the oscillations.

Another well-known type of circuit, usually designated the **Colpitts circuit,** shown in figure 203, is similar to the Hartley circuit in using direct coupling between the tube circuits and the oscillatory circuit. The coupling, however, is capacitive instead of inductive. When used for transmission, the capacity C_1 is replaced by the antenna while C_2 is in the set.

Phase requirements in oscillating vacuum-tube circuits. The alternating voltage on the grid of the vacuum tube produces the variations

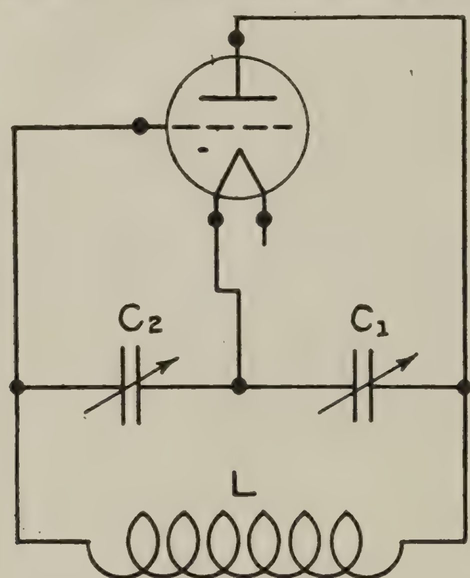


FIG. 203.—The Colpitts Circuit.

in the plate current when the tube is oscillating. As before, the varying plate current can be considered to consist of two components, one a continuous current and the other a superimposed alternating current. It is the alternating component of the plate current which supplies the voltage to the oscillatory circuits and sustains the oscillations. The voltage which works the grid is derived from the oscillatory circuit. Since the plate current increases when the grid is positive, the required coupling between the circuits is evidently the following. When the alternating component of the plate current reaches its maximum value in the positive direction, which makes the total plate current high, the current produced in the oscillatory circuit must be acting through the grid coupling so as to bring the grid voltage to its maximum positive value. Thus, the alternating plate current and the grid voltage must be in phase. If the connections are incorrect in the circuits of figures

199, 200, 201, 202, a reversal of the connections to any one coil will usually make them right.

Conditions for oscillation. In some cases, even when the circuit is such that the phase relations are correct, the vacuum tube will not generate oscillations. For example, a given vacuum tube will generate oscillations when the capacity in the oscillatory circuit is low, but will not oscillate when the capacity is higher than a certain value. Also, increasing the resistance of the oscillatory circuit tends to stop the vacuum tube from generating oscillations so that, when the capacity is low, the resistance can be higher and, when the resistance is low, a greater capacity can be used. When two vacuum tubes are connected in parallel, it is possible to get oscillations at a higher value of capacity or resistance than when only one vacuum tube is used.

It is possible to derive in an elementary fashion the conditions under which a vacuum tube will self-generate oscillations. As pointed out in Chapter II, when dealing with regeneration, if an emf E_g applied to the grid of a vacuum tube causes an equal or greater emf to be impressed upon the grid due to the feed back, then the vacuum tube will self-generate oscillations. Since the interest lies in the fact as to whether the oscillations will start, it is only necessary to consider very small oscillations. The characteristic curves can be considered to be straight lines and the conductances, resistances and amplification constant can be used.

Consider the tune-grid circuit of figure 202 and assume the mutual inductance between coils L_1 and L_2 to be M. Assume an emf E_g to be impressed upon the grid. As shown before, this will cause an emf μE_g to act in the plate circuit with which the tube resistance R_p and coil L_1 are in series. Assuming for simplicity the impedance of coil L_1 to be small compared to R_p, the alternating plate current is

$$I_p = \frac{\mu E_g}{R_p}$$

The emf impressed in the grid circuit through the mutual inductance M will be

$$E = I_p \omega M = \frac{\omega M \mu E_g}{R_p}$$

This emf produces a current

$$I = \frac{E}{R} = \frac{\omega M \mu E_g}{R_p R}$$

in the tuned-grid circuit where R is the resistance of that circuit. The emf applied to the grid by reason of this feed-back current will be

$$E'_g = I \omega L_2, \text{ or } E'_g = \frac{I}{\omega C}$$

whence

$$E_g' = \frac{M \mu E_g}{R_p R C}$$

Now, if as stated above, the emf E'_g due to the feed-back is equal to or greater than the original emf E_g then oscillations will take place. For E'_g to be equal to or greater than E_g, $\frac{M\mu}{R_qRC}$ must be equal to or greater than 1. The condition for oscillation therefore,

$$\frac{M\mu}{R_pRC} \geqq 1$$

Therefore, increasing M or μ, or reducing R_p, R or C favors the generation of oscillations.

If, in such a circuit, everything is kept constant but the capacity of the condenser C and this is increased in a continuous manner from a very low value, the oscillating current in the circuit will increase as shown in figure 204, up to a maximum and then decrease to zero at a

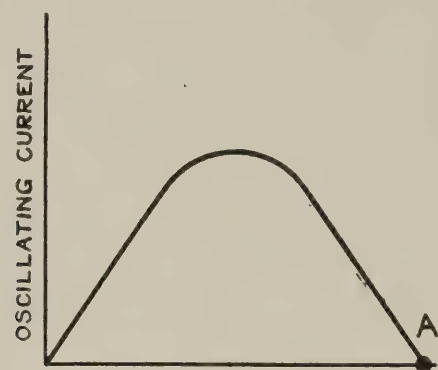


FIG. 204.—Variation in the Oscillating Current of a Vacuum Tube Due to Variation in the Oscillatory Circuit Capacity.
FIG. 205.

capacity C given by the point A. This is the limiting condition for oscillation, and is given by

$$\frac{M\mu}{R_pRC} = 1 \qquad \text{or} \qquad C = \frac{M\mu}{R_pR}$$

In general, it is not easy to obtain a complete curve such as that shown in the figure, for in the region near A the condition for oscillation is becoming unfavorable, and if there is a possibility for the vacuum tube to oscillate in some other more favorable way it will do so.

Almost any circuit will have several degrees of freedom, as it is called, or can oscillate in several different ways with several different frequencies which may be widely different. The tube will probably jump suddenly to one of these other modes of oscillation, usually of a very short wave length, and there may be no current of this frequency flowing through the ammeter whatsoever. This is what happens when the oscillation **breaks.** In the simple circuit of figure 205, because of the distributed capacity C_0 of the coil L and the inductances L_0 of the leads, two modes of oscillation are possible. The normal oscillation is

that corresponding primarily to the L and C of the circuit which would put current through the ammeter. However, there is also the possibility of an oscillation determined primarily by C_0 and the inductances L_0, which would be of very short wave length. For this oscillation, C would behave practically as a short circuit, and L as a choke or open circuit. No appreciable current would flow through the ammeter.

In some cases at short waves, the vacuum tube will refuse to oscillate at a low capacity value even when the connections are correct, or may even oscillate with a reversed connection from that which gives oscillations at higher capacity values. This is caused by incorrect phase relations between the grid voltage and plate current resulting from the capacities of the coils, leads and between the electrodes of the vacuum tube. In this respect, the circuit of figure 203 is valuable, for here, capacities between the tube electrodes and the leads are merely in parallel with the condensers C_1 and C_2 and hence do not cause trouble

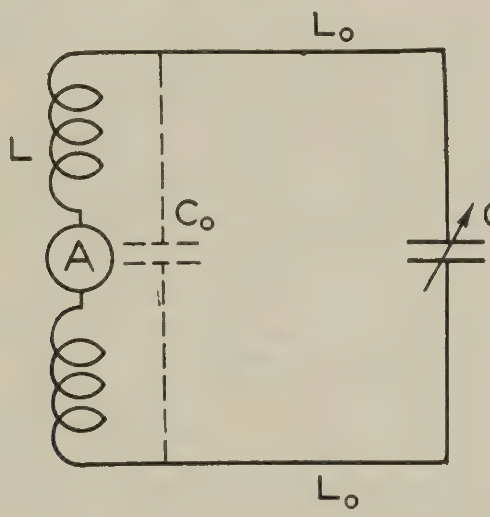


Fig. 205

Adjustment for maximum output. As pointed out before, the plate circuit of the vacuum tube supplies the power to the oscillatory circuit. This latter circuit acts as the load, and in fact, is very nearly equivalent to a resistance load in the plate circuit of the vacuum tube. In order to obtain the maximum current in the oscillatory circuit, it is necessary that the load be of the proper value to fit the vacuum tube. In the case of the tuned-plate circuit, for example, the load in the plate circuit depends upon the values of L_1, C and the rf resistance R of the oscillatory circuit, the load rf resistance being high when L_1 is great or C and R small. If C represents an antenna having a small capacity and if the rf resistance of the circuit is low, it is probable that, at a given wave length or value of L_1, the rf resistance of the load will be too high to suit the vacuum tube and, although oscillations will be obtained, the current will not be a maximum. The remedy for this is to change the point of connection between the plate and the oscillatory circuit to an intermediate tap on the coil L_1 so as to throw part of the

inductance into the antenna side of the circuit as shown in 206. A tap on the coil L_1, say tap 4, will be found for which the maximum current is obtained. This has the effect of reducing the load resistance to the proper value. It is also necessary to vary the coupling between the coil L_2 and L_1 so that the voltage applied to the grid of the vacuum tube is not excessive. When the antenna capacity is higher or the rf resistance of the circuit is greater, the maximum current will be obtained nearer the end of the coil, say on tap 2, or 1. When two vacuum tubes are used in parallel, or the filament current or plate voltage is changed, a different adjustment of the connection is required in order to fit the load to the new tube conditions.

Use of coupled circuits. Coupled tuned circuits are rarely used with vacuum tubes. The Meissner circuit does not come under this heading because only one circuit is tuned. The objection is loss of

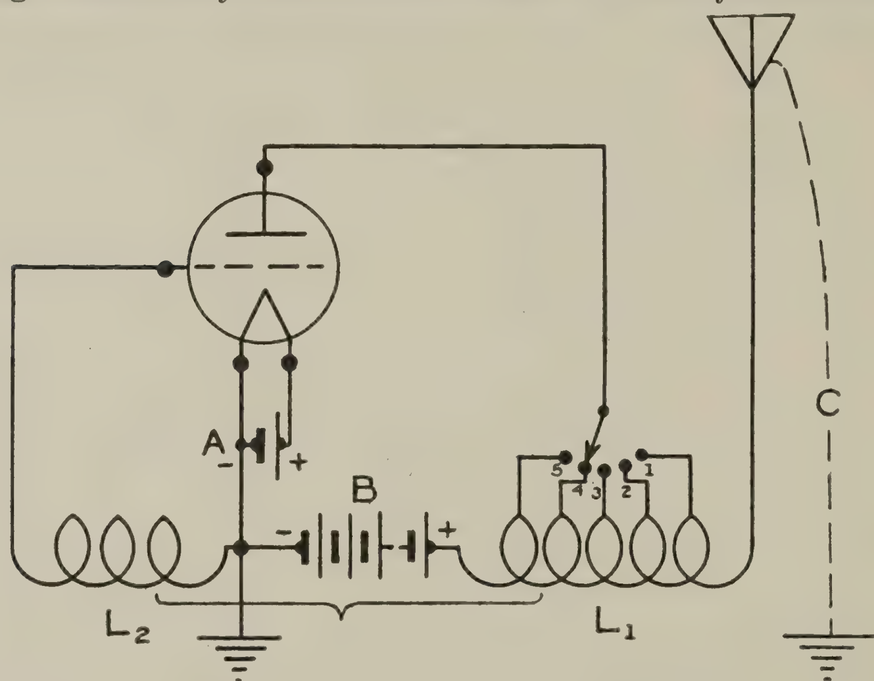


Fig. 206.—Tuned-Plate Transmitter Circuit Showing Means for Adjusting to Maximum Output.

efficiency and erratic operation. With close coupling between tuned circuits, it is possible for oscillations of two different frequencies to occur; the vacuum tube can jump from one of these frequencies to the other, in fact, it is possible to adjust the circuits so that the antenna current is high and definite wave length secured and then find that, on keying, the current in the antenna will jump to a low value and have a widely different frequency. However, with vacuum-tube generators as with arc generators, harmonics are present in the antenna current when the direct-coupled circuits are utilized. With high-power tube sets these harmonics can cause serious interferences, so that it is desirable to use coupled circuits with some sacrifice in efficiency in order to reduce the interference.

Position of batteries or generators. When an antenna is used with any of the direct-coupled circuits, the antenna ground is of necessity connected to some point on the vacuum-tube circuit. It is desirable to connect the batteries or generators, which supply the tube, to the same grounded point in the circuit. The reason for this is that the batteries and generators have large capacities to ground and, unless the connections are made as indicated above, these capacities to ground will be put in parallel with a part of the oscillatory circuit and will cause loss of power and possibly prevent oscillations. Ordinarily, therefore, the batteries or generators and the antenna ground are connected adjacent to the filament. For example, the tuned-plate circuit would ordinarily be connected as shown in figure 206. Here the A and B batteries and the ground connection all come adjacent to the filament.

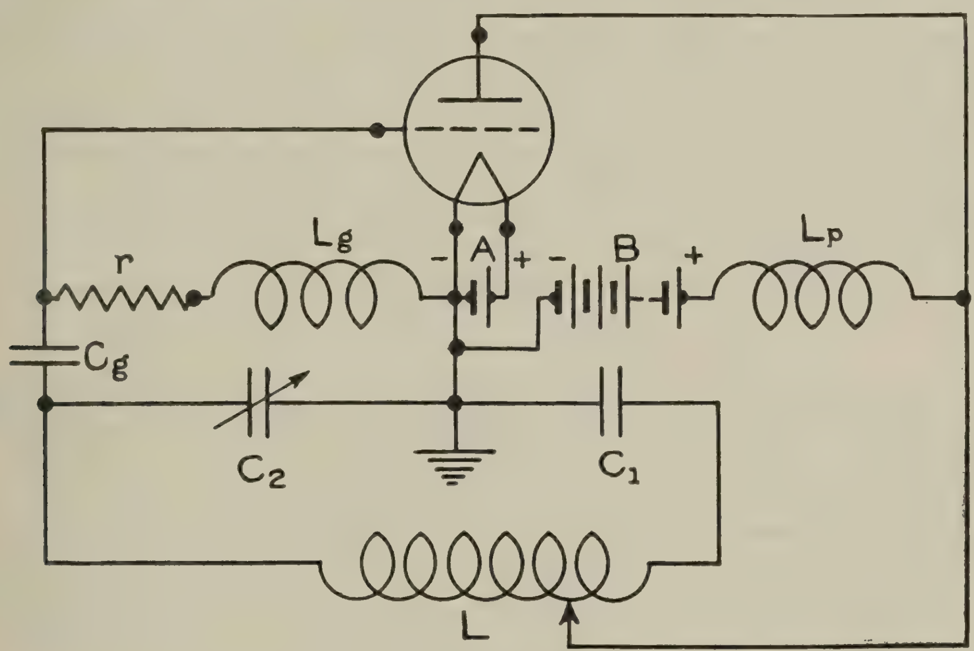


Fig. 207.—Colpitt's Transmitter Circuit.

In the case of the Colpitt's circuit, it will be noted, in figure 207, that the filament is insulated from the plate by the condensers C_1 and C_2 thus rendering a flow of direct current impossible from filament to plate through the oscillatory circuit. The plate battery or dynamotor is applied, as shown in figure 207, through a choke coil L_p, which acts as a very high impedance for radio-frequency currents but permits the steady plate current to flow. The alternating component of the plate current flows through the oscillatory circuit from the filament to the tap on the coil L and supplies the power for the oscillations. The tap permits the proper load adjustment. The condenser C_1 represents the antenna, the ground connection being adjacent to the filament. Condenser C_2 constitutes the grid coupling: the coupling to the grid decreasing as the capacity C_2 is increased. The condenser C_g in the grid lead is required because, without this condenser, the grid would be

directly connected to the plate and would be at a high positive potential. The condenser C_g prevents this. However, when the vacuum tube is oscillating, electrons flow to the grid when it is on its positive swing and, unless this current were permitted to flow away, the grid would become charged to a high negative voltage and the oscillations would stop. The resistance r and the choke coil L_g permit these grid charges to flow away without preventing the necessary radio-frequency voltage variations from being applied to the grid. During operation, the grid becomes somewhat negative because of the voltage drop across r. This is of advantage in most cases, as it leads to an increased efficiency of the vacuum tube as a generator.

Efficiency of a vacuum-tube generator. The efficiency of a vacuum-tube generator is usually defined in terms of the input power supplied by the plate battery and the output radio-frequency power in the oscillatory circuit. The power required to heat the filament is usually neglected, although in small vacuum tubes it may be several times the output power. The power supplied by the plate battery is the product of the dc volts and dc amperes. The output power is the product of the square of the radio-frequency current and the rf resistance of the oscillatory circuit. The efficiency is

$$\eta = \frac{\text{output power}}{\text{input power}}$$

For low-power vacuum tubes this may be 15 to 30 per cent; for medium-power vacuum tubes, 40 to 50 per cent. The maximum efficiency obtained in high-power vacuum tubes is about 80 per cent.

Power rating of vacuum tubes. Vacuum tubes are frequently rated in terms of the output power. For example, a certain vacuum tube will be rated as a 5-watt tube. It should, therefore, supply 1 ampere into an antenna having a rf resistance of 5 ohms. Thus, the output power

$$P_{\text{out}} = I^2R \qquad \text{(watts)}$$

substituting $\qquad = (1)^2 5 = 5$

whence $\qquad P_{\text{out}} = 5$ watts.

The normal plate supply would be about 50 ma at 350 volts. In this case, the input power

$$P_{\text{in}} = EI \qquad \text{(watts)}$$

substituting $\qquad = 0.05 \times 350 = 17.5$

whence $\qquad P_{\text{in}} = 17.5$ watts

and $\qquad \eta = \dfrac{P_{\text{out}}}{P_{\text{in}}}$

substituting $\qquad = \dfrac{5}{17.5} = 0.29$

whence $\qquad \eta = 29$ per cent.

Usually, efficiency is sacrificed to some extent in order to obtain the maximum output from the vacuum tubes; at reduced outputs somewhat greater efficiencies are obtainable. The power in the plate curcuit, which is not used in generating oscillations, is dissipated in heat in the plate of the vacuum tube. If the vacuum tube stops oscillating, then all of the plate circuit power is expended in heating the plate. In low-power vacuum tubes the plate of the tube can frequently be seen to heat up when oscillations stop. In high-power vacuum tubes, this heating can be sufficient to destroy the vacuum tube in a short time; hence, it is necessary to maintain oscillations or to shut off the vacuum tube immediately.

Variation of output with plate voltage. The output of a vacuum tube is limited primarily by the total emission of electrons from the filament and the plate voltage, provided that the design is such that the plates do not become overheated and the vacuum is high. With a given plate voltage, however, only a limited amount of emission from the filament can be used; the higher the plate voltage, the more emission is useful. On the other hand, with a given emission the output of the vacuum tube can be increased without limit by increasing the plate voltage, provided that the vacuum tube will stand it. Thus, high plate voltages are necessary for high-power vacuum tubes. In tungsten filament tubes, the filament emission is usually limited, for, at usual operating temperatures, it requires considerable filament power to maintain the temperature required to liberate the electrons in the coated filament tubes, the emission for the same filament power is very much greater, usually a number of times greater than is normally utilized.

When the filament emission is very great, the output current varies practically in proportion to the plate voltage while, with insufficient emission, the output current is approximately proportional to the square root of the plate voltage.

Modulation of the radio-frequency output in radiotelegraphy. It is this strong dependence of the radio-frequency output current upon plate voltage which is utilized in the usual circuits for radiotelephony. In radiotelephony it is required that the amplitude of the radio-frequency current in the antenna shall vary up and down in accordance with the variations of current in the microphone. Thus, in figure 208, if (*a*) represents the variations in the microphone current, the radio-frequency oscillations which are transmitted should vary in amplitude as shown in (*b*), where the envelope of the oscillations shown by the dotted lines is the same form of curve as the curve in (*a*). These oscillations are radiated by the antenna and, when received by a detector tube, the current through the telephones will duplicate the original microphone current and, hence, the speech or music will be duplicated. In the modulator-oscillator combination customarily used in radiotelephone transmitters, the microphone acts through a transformer upon

the grid circuit of the modulator tube as in figure 209 and varies its voltage in accordance with the audio-frequency vibrations, caused by speech or music. In the plate circuit of the modulator tube is a choke coil L_a of very high inductance for oscillations of speech frequencies. The variations in grid voltage tend to vary the plate current in the usual way, but these variations in plate current must flow through the choke coil which opposes the variations. On this account, the voltage across the choke coil varies. With proper design, these voltage variations can have an amplitude approximately equal to the plate voltage

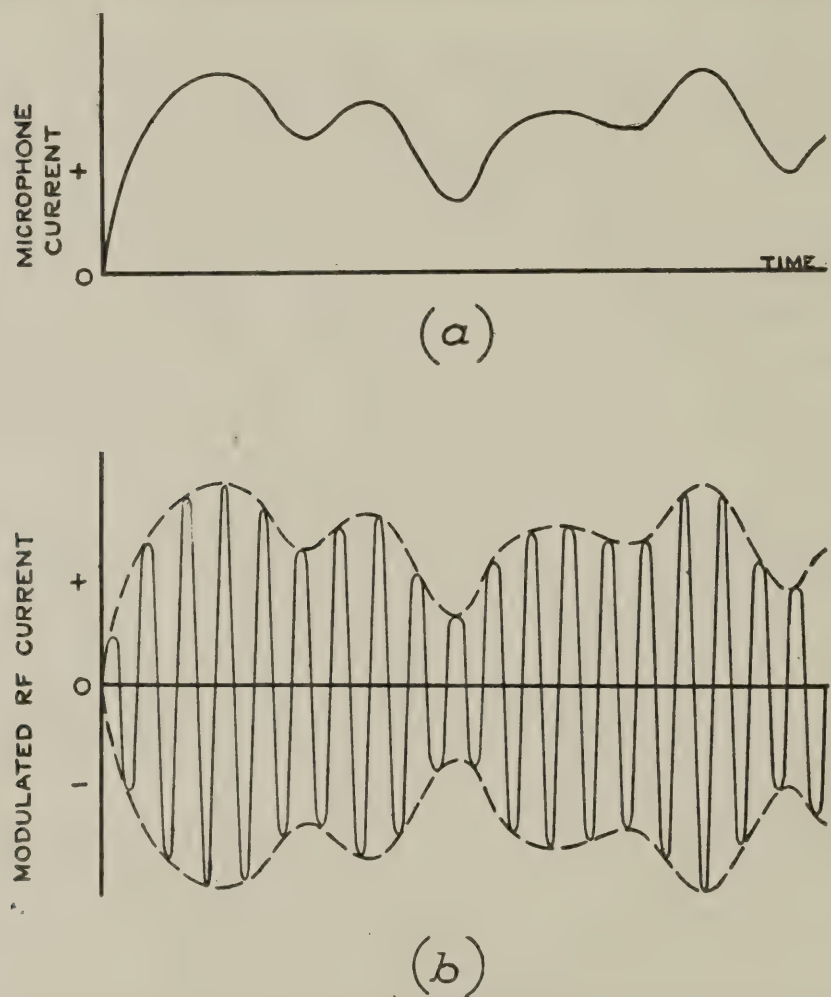


FIG. 208.—Radio-Frequency Oscillations as Modulated in Radiotelephony.

so that, on one swing of the grid voltage, the total voltage acting in the plate circuit may be reduced nearly to zero and on the other swing may be nearly double that of the plate-voltage supply.

The oscillator tube is fed from the same plate supply, and its plate circuit also includes the choke coil. When the microphone is not being spoken into, the voltage on the plate of this vacuum tube is very nearly equal to that of the plate supply since the dc resistance of the choke coil is low. It will, therefore, generate radio-frequency oscillations having an amplitude corresponding to this plate voltage. As soon as the

microphone is spoken into, however, the plate voltage varies up and down because of the action of the modulator tube, and the amplitude of the radio-frequency oscillations varies accordingly. As stated before, this is the requirement for radiotelephony.

In order that the best results may be obtained, it is desirable that, for speech or music of considerable intensity, the amplitude of the radio-frequency oscillations shall have a maximum variation from zero to

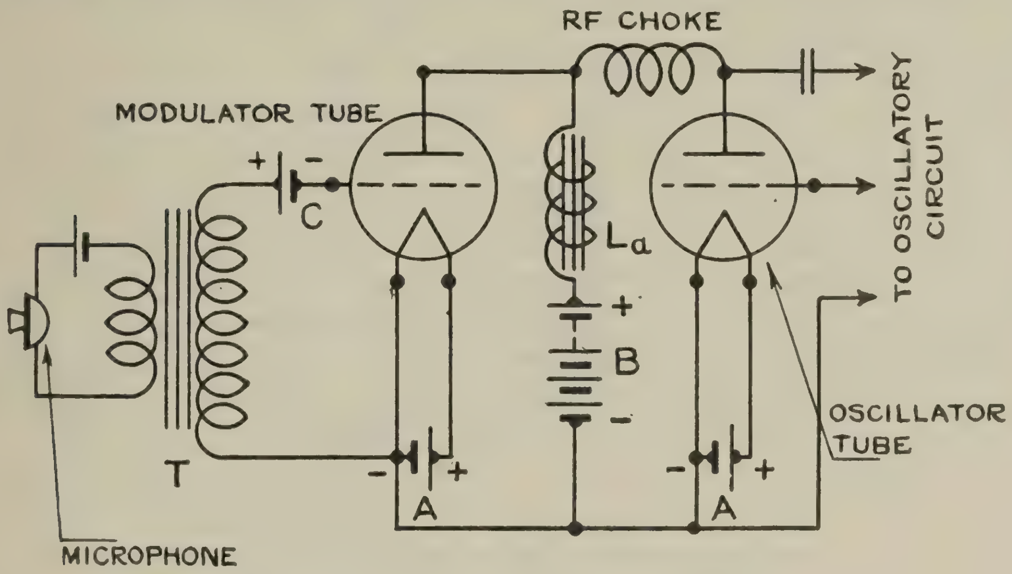


FIG. 209.—The Modulator-Oscillator Circuit.

double the amplitude corresponding to no speech. This is called complete modulation. The fluctuation of the antenna ammeter during speech is not a reliable indication as to whether good modulation is being attained or not.

It is a natural mistake to attempt to modulate the output of an oscillating vacuum tube by applying the microphone voltages to the gird of the oscillating vacuum tube itself. It will be found, however, that in general the amplitude of the oscillations generated by a vacuum tube does not depend strongly upon the grid voltage. It is obvious, therefore, that this is a poor method of modulation.

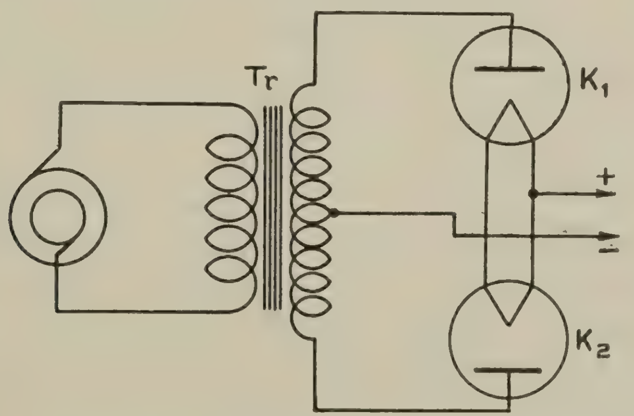


FIG. 210.—Diagram of Connections of Kenetron Rectifier System. Tr = Step-up transformer. K_1 and K_2 = Kenetrons.

Vacuum tube rectifiers. Vacuum tubes are used for converting high-voltage alternating current into direct current.

These rectifiers are divided into two classes, namely: the **Kenetron** or cold plate—hot filament vacuum tube, and the two cold electrode tube, commonly termed the **S-tube.** The former tube has been accepted commercially as a reliable rectifier for alternating current of voltages up to 20,000 volts, while the latter is commercially accepted for use on voltages under 1,000 volts. Figures 210 and 211 show circuit diagrams for the Kenetron and S-tube rectifiers:

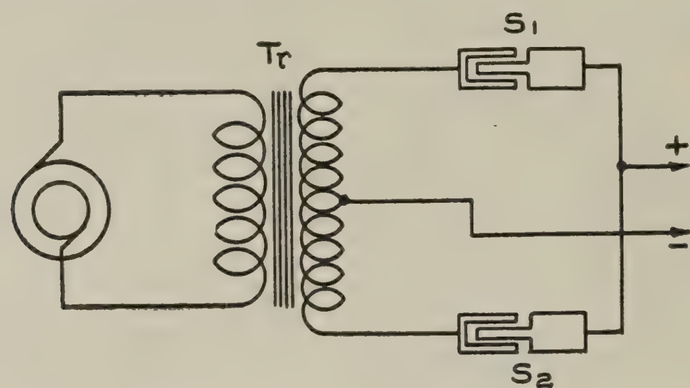


FIG. 211.—Diagram of S-tube Rectifier System.
Tr = Step-up transformer. S_1 and S_2 = S-tubes.

PART 7.

THEORY OF WAVE PROPAGATION.

CHAPTER I. WAVES AND WAVE MOTION.

Periodic and harmonic motion. When a particle goes through the same series of motions at regularly recurring intervals, the movement of the particle is said to be **periodic.** If, in addition, the motion of the particle is continually being reversed in direction, the motion is said to be **vibratory,** or **oscillatory.** The motion of a pendulum is vibratory.

If a particle that is revolving at a uniform rate in a circle is viewed from a very distant point in the same plane as the circle, the particle will appear to have a to-and-fro movement, that is, at right angles to the line of vision. This apparent motion will also be at a varying rate.

This is shown in figure 212. Suppose that the observer is at a very great distance in the same plane and to the right of the figure, and is

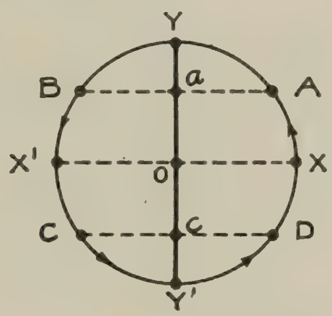


FIG. 212.—Particle vibrating with a Simple Harmonic Motion.

looking at the figure in such a way that the line YOY' is vertical. Let the particle be revolving in a counter-clockwise direction and be at X, when first seen. It will appear to be at O in the line YOY'. A little later it will be at A with an apparent position at a. Still later it will be at Y both actually and apparently. When it is at B it will appear to be at a, and so on throughout one revolution. The apparent motion of the particle will be most rapid as it passes through points X and X' and slowest at points Y and Y'. At Y and Y' the direction of the apparent motion is reversed.

Now, if another particle moves to and fro on line YOY' and at every instant occupies the apparent position of the revolving particle, such as point a when the revolving particle is at point A or B, the particle is vibrating on line YOY' with a **simple harmonic motion.**

The motion of the vibrating particle is, however, described in terms of the revolving particle. Thus, the angle at the center of the circle swept out by the radius as the revolving particle moves along the cir-

cumference a distance equal to the radius is called the **radian** (circular measure). The **angular velocity,** ω, of the particle is measured in radians per second. For every revolution of the particle, the angle swept out is equal to 2π radians. The **frequency** f, of the motion of the particle, is the number of times per second it passes through the same point. Hence, the angular velocity is given by

$$\omega = 2\pi f$$

If the particle revolves at a frequency f, then the **periodic time,** or **period** of one revolution is

$$T = \frac{1}{f}$$

where T = time in seconds,
f = frequency per second.

The frequency can be found from the formula,

$$f = \frac{1}{T}$$

when the period is known. Also

$$\omega = \frac{2\pi}{T}$$

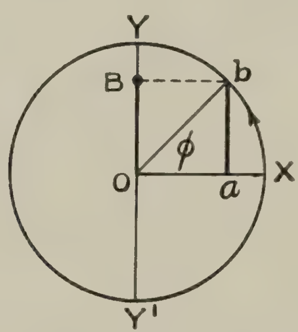


FIG. 213.—Showing the Phase of the Vibrating Particle.

It should be understood that it is the vibrating particle, and **not** the revolving particle that executes the simple harmonic motion. Thus, the period of the vibrating particle is the time elapsing between its passage in the same direction through any point, such as O, and the frequency is the number of times it passes through any point traveling in the same direction.

Referring to figure 213, the maximum distance that the particle travels along the line YOY', in either direction from the **median position** O, is the **amplitude, a,** of the vibration and, in this case, is equal to the radius OY of the **circle of reference.** When the particle is at B, its distance OB from O is the same as ab, which is the sine of the angle Φ. This angle is one of revolution, and is bounded by the line OX and the radius to the revolving particle as the terminal side. Thus, when the vibrating particle is at O the **phase** is zero, and when it is at Y, the phase is equal to the angle XOY.

The **displacement, x,** of the vibrating particle in terms of the amplitude a is given by the formula:

$$x = a \sin \phi$$

where ϕ is the phase measured from OX and, since the angular velocity is ω, the angle the particle has moved through from OX in any time t is ωt. Therefore, $\phi = \omega t$ and the displacement

$$x = a \sin \omega t.$$

This equation, therefore, represents the motion of a particle vibrating with a simple harmonic motion.

The harmonic or sine curve. Figure 214 shows a sine curve developed as explained in Section III, Trigonometry. Suppose that a

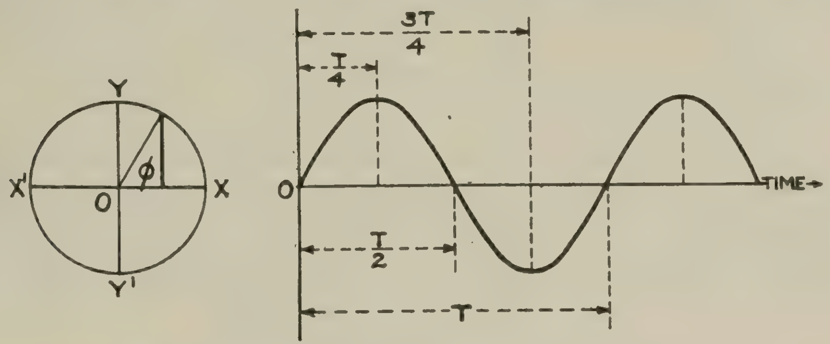


FIG. 214.—The Harmonic or Sine Curve.

particle is executing a simple harmonic motion along YOY' with a period T, and let $XYX'Y'$ be the circle of reference. Now, as the particle moves from its median position O toward Y, the sine curve shows its displacemtnt at any instant. Thus, when the particle is at O its displacement is zero; at time $t = \frac{T}{4}$ it is at a maximum positive displacement; at time $t = \frac{T}{2}$ the displacement again is zero; at time $t = \frac{3T}{4}$ the particle is at a maximum negative displacement and, after a time T, the displacement again is zero. During this interval, the particle has completed **one oscillation** or **one cycle** by moving from O to Y and back from Y through O to Y', and thence back to O.

Wave motion. The description of periodic and harmonic motion just given applies to a single particle. When particles are bound together more or less closely in a medium, such as water, the motion of any one particle sets the particles in its immediate vicinity in motion. If the first particle is executing a simple harmonic motion, this motion will be passed on to the next particle, which will perform a similar motion differing in phase from that of the first particle, by a definite amount. This last particle will pass on the motion to the particle next to it which, in turn, will go through the same motion differing in phase from that of the second particle by the same amount. The resultant motion of the medium is a **wave,** and is demonstrated in the following.

Suppose that there are a number of particles at rest and arranged at equal intervals along a line OX, figure 214, and then that all these particles execute simple harmonic motions of equal amplitude and period along lines at right angles to the median line OX. Also let the difference in phase between successive particles be 30°, which is $\frac{1}{12}$ of the period of each particle, or $\frac{T}{12}$.

This is shown in figure 215 at the instant when particle 1 is on OX, zero displacement, and particle 4 is at maximum displacement downwards. Curve A, drawn through the black dots, represents the position of the particles at this instant. After an interval of time equal to $\frac{T}{12}$, the circles will represent the position of the particles. Curve B, drawn through the new positions of the particles, is the same type of curve as curve A, and both are harmonic curves. Hence, the curve drawn through the particles at any instant is a harmonic curve. It is also apparent that, as the vibrating motion continues, the curve con-

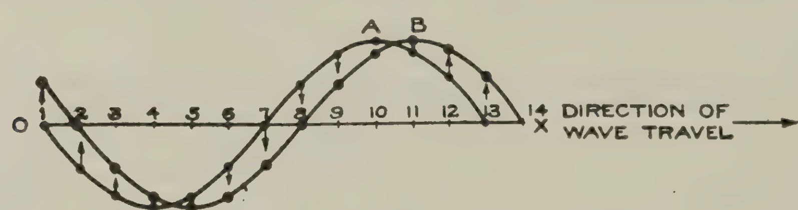


Fig. 215.—Wave Motion in a Medium.

necting the particles moves steadily to the right, although the particles themselves only oscillate about their individual mean positions.

The **crest** of the wave is the point of maximum positive displacement of the particles, and is at point 10 on curve A. Point 4 on the same curve is the point of maximum negative displacement of the particle, and is called the **trough.** The positions of the crests and troughs appear to move towards the right as the motion of the particles continues. Therefore, a **wave is a condition** or **form of disturbance** which travels through a medium from the source, and is due to the particles of the medium oscillating in succession about their mean positions.

The distance through which the wave travels during a complete period of one of the particles is called a **wave length,** λ, and is equal to the distance between two particles which are moving at every instant in the same direction and are equally displaced on the same side of their mean positions.

Ether waves or water waves are due to the **transverse vibrations** of the particles of the medium as just described, that is, the displacement of the particles is at **right angles** to the direction of wave travel.

Velocity of propagation of a wave. Frequency. Figure 216 is a conventional drawing of a wave. The speed with which the crest or trough appears to move through the medium is called the **velocity of propagation, v,** of the wave motion.

While any particle is making one complete oscillation, any given part of the wave will travel a distance of one wave length, λ, and, if T is the period of oscillation, the wave will travel one wave length in this period. Hence, the velocity of propagation is given by the formula:

$$v=\frac{\lambda}{T}$$

Also, the time interval between the arrival of successive crests at the same point is T and the **frequency, f,** with which the crests occur at the same point (per second) is,

$$f=\frac{1}{T}$$

Combining the two formulas:

$$v=f\lambda, \quad f=\frac{v}{\lambda} \text{ and } \lambda=\frac{v}{f}$$

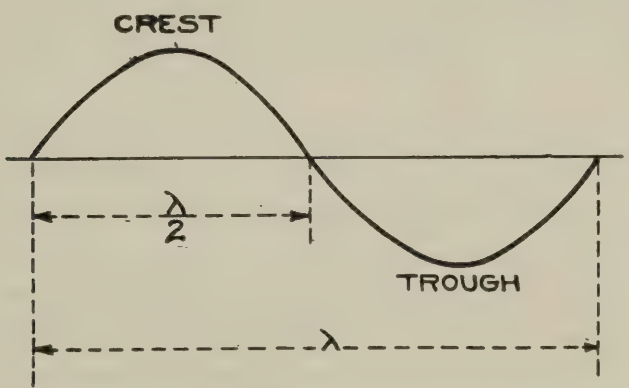


FIG. 216.—Conventional Drawing of a Wave.

These formulas, which show the relations between wave length, velocity and frequency, are **basic formulas** and are frequently employed.

In order that a wave may be formed in a medium it is necessary that the successive particles of the medium perform periodic motion. They cannot do this unless there is a **restoring force** which tends to return them to their original position. This restoring force, or **stress,** is called into play by the strain or displacement of the particles and is a measure of the elasticity of the medium. Ether possesses this property to a very high degree.

Wave front. Consider a medium through which wave motion is being propagated. If a curve is drawn in such a manner that it passes everywhere through the particles that are in the same phase of vibration, such a line, or surface, is called a **wave front.** When a disturbance is produced at a point, the wave front is spherical about the point as a center, and the direction of propagation is along any radius of the sphere,

that is, at right angles to the tangent to the wave front at any point. This is the **direction of motion** of the wave front, and the line drawn to indicate direction is called the **ray.** Wave front and **direction of wave propagation** are terms employed in connection with radio waves.

The ether. Ether is the name assigned to an all-pervading medium possessing elasticity to an extremely high degree and some inertia. The existence of this medium has never been proved, and in fact has been disputed, but the assumption of such a medium is very valuable in explaining the mechanics of the radiation of light, radiant heat and radio signals. The following will make clear why a medium such as the ether is supposed to exist.

It is known that the temperature of an incandescent metal is extremely high, and that it will give off both light and heat in air. Now, if a metal filament is sealed into a large glass container and the container pumped to the lowest pressure possible, the temperature of the glass walls of the container, when the filament is made incandescent, is much higher than would be due to conduction through the residual gas in the container. In addition, light from the incandescent filament passes readily through the evacuated container. Thus, it is evident that the air is not the medium through which light and radiant heat travel.

The existence of ether is further proved by the light and heat that the earth receives from the sun. The earth's atmosphere does not extend more than a few hundred miles above the earth's surface. This distance is infinitesimal when compared with the distance of the earth from the sun. It is certain, however, that light and heat in enormous quantities travel from the sun to the earth across the intervening space, which is devoid of air. It is also known that air is not the medium through which radio waves are propagated.

For these and other reasons, it is beleived that light, radiant heat and radio waves are propagated through the same medium, called the **ether.** It is also supposed that the ether is present everywhere, that no material is impervious to it and, further, that it can not be disturbed except by the electric fields associated with electrons.

Ether waves. It has been proved that a disturbance in the ether travels from the source in all directions with a wave motion due to the transverse vibrations of the ether particles and with a definite velocity. Recent measurements of the velocity of the propagation of light in vacuum give the value

$$C = 2.9982 \cdot 10^8 \text{ meters per second.}$$

Electromagnetic waves. Experiments have shown that not only do radiant heat and radio waves travel with the same velocity as light waves, but also that any disturbance of the ether results in the production of a wave motion by which electric and magnetic strains or displacements are conveyed from point to point. It is thought, therefore, that

the phenomena of light, radiant heat and electricity are due to the passage of electrical waves. Thus, disturbances of the ether are called **electromagnet waves.**

Effects produced by electromagnetic waves. Although the velocity of propagation of strains through the ether remains constant, the wave length and frequency are very different in magnitude and produce different effects. The following list of waves or rays include the most important. They are arranged beginning with the shortest and ending with the longest.

Number	Wave or Ray
1	Gamma rays
2	X-rays
3	Ultra-violet
4	Violet
5	Blue
6	Green
7	Yellow
8	Red
9	Infra-red
10	Electric waves

The first three rays are invisible, but can be detected by the photographic plate and fluoroscope. The **gamma rays** are given off by radium and are the shortest known.

$$\lambda = 1 \cdot 10^{-8} \text{ mm.}$$

The **X-rays** are produced electrically and are used in radiography,

$$\lambda = 1 \cdot 10^{-8} \text{ mm.}$$

The **ultra-violet** rays are still too short to be detected as light,

$$\lambda = 1 \cdot 10^{-7} \text{ mm.}$$

The waves from 4 to 8 inclusive are **visible** and, when combined, appear as white light. The range of wave length of this group is from

$$3.8 \cdot 10^{-4} \text{ mm. to } 7.5 \cdot 10^{-4} \text{ mm.}$$

Wave 9 in the table is detected as **heat.** The range of wave lengths of infra-red waves is from about

$$7.5 \cdot 10^{-4} \text{ mm. to } 1 \text{ mm.}$$

The **electric waves** are employed in radio signaling, and are detected by radio receiving equipment. The shortest electric wave which has been produced had a wave length of about 1 cm. The practical range of wave lengths used in radio communication is from 10m. to 30,000m. Wave lengths as low as 1m. are used for special applications of radio communication.

Characteristics of electromagnetic waves. An incandescent body emits both light and radiant heat. The electrons in the atoms of such a body are considered to be violently agitated, and the electric fields associated with the electrons give rise to electric displacements in **all directions** in the ether. These displacements travel outward, in the form of light and heat waves. Ordinary light, then, consists of transverse vibrations taking place in all directions at right angles to the direction of the ray. Figure 217 represents this condition. The page is to be

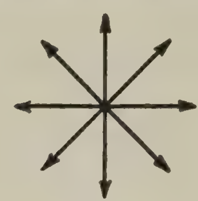


FIG. 217.—Transverse Vibrations of Light Waves.

taken as a plane tangent to the wave front and perpendicular to the ray, which can be considered to be traveling into the page. The arrows indicate the direction of the electric displacements for a single ray. These displacements are alternately positive and negative, that is, the ether particles at a given point are vibrating in straight lines but are continually reversing in direction with a definite period T, as just described.

Polarization and Reflection. If a ray of this ordinary light is passed through a tourmaline crystal at right angles to the axis of the crystal, the ray is **plane polarized,** that is to say, the transverse vibrations, which before were taking place in all directions, now occur parallel to some definite direction. It is assumed, therefore, that the vibrations of plane polarized light take place in the plane of the axis of the crystal, which is perpendicular to the direction of the ray. The **plane of polarization** coincides with the direction of the ray, and is at right angles to the direction of vibration of the ether particles. This is shown in figure

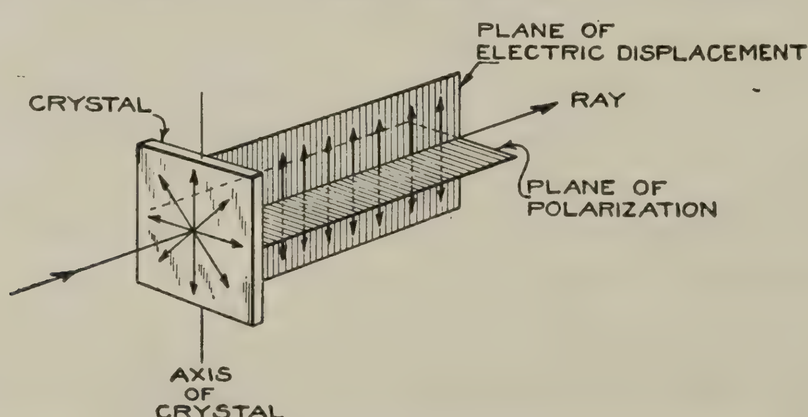


FIG. 218.—Polarization of Light Waves.

Electric waves, although very much longer than light waves, can also be reflected, polarized and refracted. This is shown in the following.

The **Righi oscillator** is an apparatus for the production of extremely short and polarized electric waves, and is connected across the secondary of an induction coil as the source of power. If such an oscillator is placed in the focal line of a parabolic reflector of proper dimensions in respect to the wave length of the oscillator, it will be found that the electric waves are **reflected** from the mirror and sent out in a parallel beam in the same manner as the rays of a searchlight are reflected. The

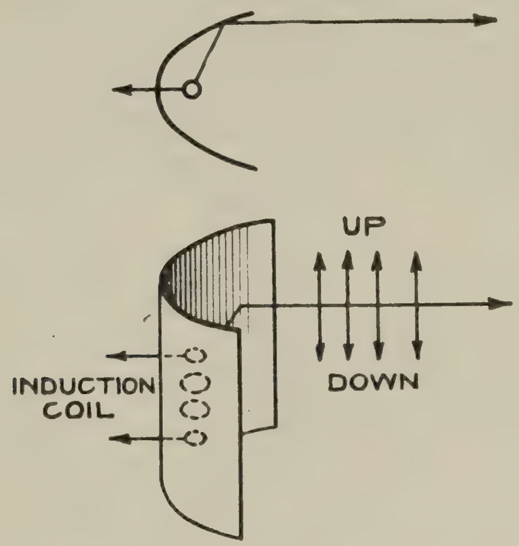


FIG. 219.—Righi Oscillator showing Polarization and Reflection of Electric Waves.

wave length of these electric waves should be not more than a few cms. Figure 219 shows the top and side view of such a mirror and oscillator. The waves reflected from the mirror are also **plane polarized** because the electric displacement is always parallel to the axis of the oscillator and at right angles to the direction of the wave propagation, that is, the wave front is parallel to the axis of the oscillator. The figure shows one ray of the beam and the electric displacement.

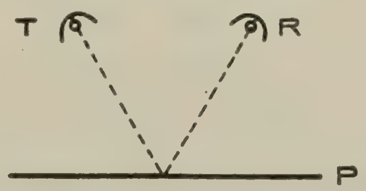
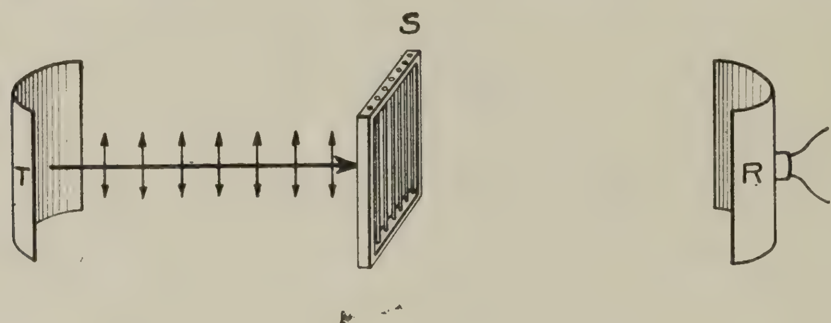


FIG. 220.—Reflection of Electric Waves.

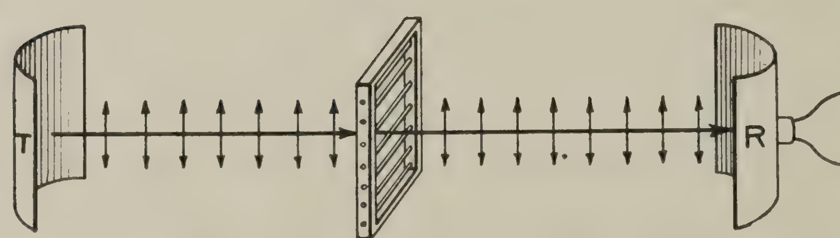
If a suitable detector of electric waves is arranged in the same manner as the oscillator in a similar reflector, the reflection of the electric waves may be studied. In this case the transmitter, *T*, is placed in respect to a large metal plate, *P*, so that the waves strike the plate at an acute angle. The receiver, *R*, will not respond until the angle that the rays perpendicular to its axis make with the plate equals that of the transmitter, that is, when the **angle of incidence equals the angle of reflection.** This is shown in figure 220.

The reflected waves are produced by the current induced in the surface of the metal plate by the incident waves. This is also shown by inserting a grating, consisting of parallel wires fastened in a frame and insulated from each other, between the transmitter and the receiver as shown in figures 221 (*a*) and (*b*). When the wires are parallel to the axis of the oscillator, figure 222 (*a*), no response will be given by the receiver. When the wires are at right angles to the axis of the oscillator, figure 223 (*b*), the receiver will respond.

The wire grating, if the wires are close together, acts like a metal plate when the wires are parallel with the displacement, the wave being completely reflected. In this case, the currents which are induced in the wires are sufficient to screen completely the receiver. However,



(*a*)



(*b*)

Fig. 221.—Screening of Electric Waves.

when the wires are at right angles to the electric displacement of the incident wave, no currents can be induced in the wires, and the screen allows the wave to pass through. The screen, therefore, acts on electric waves in exactly the same manner as the tourmaline crystal does on light rays, that is, the screen has a **polarizing effect** on the electric waves.

This polarizing effect is made clear in the following way. When the receiver is set with its axis at right angles to that of the transmitter, there will be no response. Now, if the screen is placed between the two, with its wires making an angle of 45° to both, the receiver will respond. Under these conditions, when the waves strike the screen, they are resolved into two **components,** which are at right angles to each other. The component in which the electric displacement is at right angles to

the length of the wires passes through, and these transmitted waves fall upon the receiver, where they are again resolved by the receiving mirror into components. The component in which the electric displacement is perpendicular to the axis of the receiver is reflected, while the component in which the displacement is parallel to the axis of the receiver causes the receiver to respond. This is shown in figure 222. The ray is shown

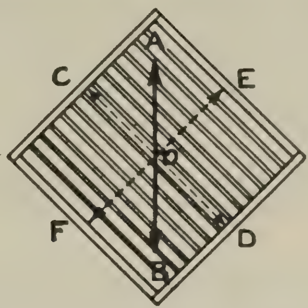


FIG. 222.—Resolution of an Electric Wave into Components.

entering the page as the plane of the screen at *O*. *AOB* is the electric displacement before the ray strikes the screen. The two components are *COD* and *EOF*. *COD* having a displacement parallel to the wires is reflected, while *EOF* passes through.

If the receiver is now placed with its axis parallel to *EOF*, this electric displacement will produce maximum effect in the receiver.

Refraction. Electric waves can be refracted similarly to light waves. Figure 223 shows how this is effected. *P* is a prism of pitch arranged with a metal screen *M* to prevent any waves from passing from the transmitter to the side on which the receiver is located except through the

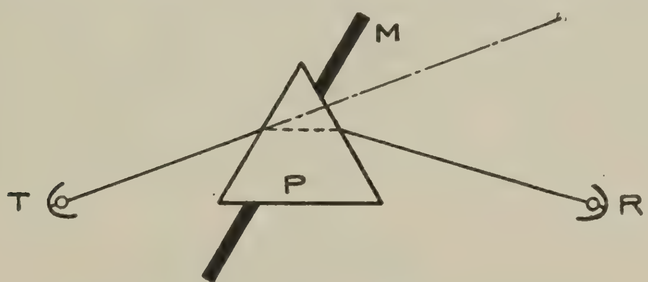


FIG. 223.—Refraction of Electric Waves.

prism. The receiver will respond most strongly when placed in a definite position in respect to the prism. This position will **not** be in line with the transmitter (dot-dash line). Therefore, the waves from the transmitter have been bent or refracted.

Magnetic force produced by motion of lines of electric force. It has been shown in Part 2 that magnetic lines of force are set up in and about a conductor carrying current, and that these magnetic lines constitute a magnetic field. It was also stated that the magnetic force acts in a direction at right angles to the direction of current flow, that the magnetic field varies in strength with every change in the current and, further, that the energy required to maintain the field is stored up

in it and is returned to the circuit by a collapse of the field on the circuit when the current flow ceases. Thus, it is seen that the magnetic field is tied to the circuit.

Now, current consists of a transfer of charges. Associated with he charges are lines of electric force. Hence, as the current flows through a conductor, an electric field is set up about the conductor. The lines of electric force constituting the field extend radially outwards from the conductor, the electric force acting in the direction of the displacement. Figure 224 shows two long open-ended conductors close together in

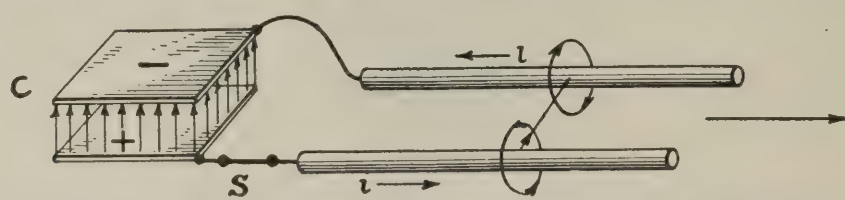


FIG. 224.—Motion of Lines of Electric Force Producing Lines of Magnetic Force.

air and a charged condenser at one end with the negative plate connected to one wire and the positive plate provided with a switch *S* to connect it to the other wire. Now, when the switch is closed, the charge in the condenser will move out from the condenser and distribute itself throughout the wire circuit and condenser. This operation can be considered to have been performed by some of the lines of force previously existing between the condenser plates, traveling out through the dielectric between the wires and guided by them. A momentary flow of conduction current will take place in the wires until the charge is equally distributed in the dielectric between them, the latter being the displacement current. At the instant the switch is closed, the flow of current will be heaviest and the magnetic field will be most intense. A little later the charge will be more evenly distributed and nearer its final and maximum value throughout the

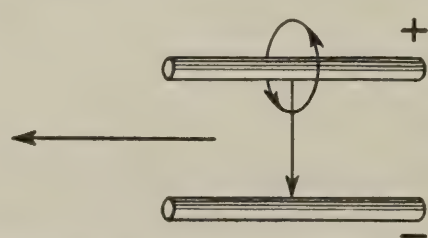


FIG. 225.—Moving Line of Electric Force with its Associated Line of Magnetic Force.

system. At this instant the flow of current will be less heavy and the magnetic field less intense. When the charge has become evenly distributed there will be no further movement of charges, hence no current and the magnetic field becomes zero. Figure 225 represents the condition while the displacement current is flowing. The electric displacement is upwards (from + to −) and the magnetic force is at right angles to the displacement, while the forward motion is perpendicular

to the two fields. When the system has reached a steady charged condition, only the electric field remains. This is shown in figure 226.

Thus, it may be said that the motion of the charges in a conductor, which is accompanied by a motion of lines of electric force, sets up magnetic forces in the dielectric surrounding the conductor. Likewise, magnetic forces will be set up in a dielectric simply by the motion of lines of electric force without any motion of charges. This is shown in the following.

If a condenser consisting of two metal plates in air is arranged so that it can be charged, and then rotated without changing the position

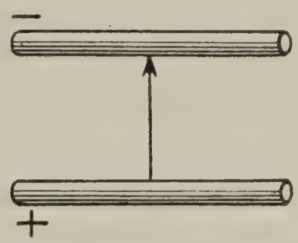


Fig. 226.—Disappearance of Magnetic Force with Association of Motion of Lines of Electric Force.

of one plate in respect to the other, the lines of electric force will not move with reference to the plates but will be rotated through the air. It is evident that the air between the plates will continually change, due to the rotation of the plates. There will be a motion of the lines of electric force without any motion of charges on the plates. It has been proved experimentally that magnetic forces are set up in this manner. The dielectric does not necessarily have to be air. The magnetic effect would result were any other dielectric employed. Hence, in every case the production of a magnetic field is due to the motion of the lines of electric force.

CHAPTER II. RADIATION OF RADIO WAVES.

Electric and magnetic fields. The production of electric and magnetic fields was discussed in the last chapter. An explanation of how a magnetic field is set up by a motion of lines of electric force in the dielectric surrounding a conductor was also given. It will also be recalled that the intensity of the electric field, $\mathcal{E}$, at a point, P, is measured by the force acting on a unit charge of electricity at that point and, that the intensity of the magnetic field, H, at a point, P, is measured by the force acting on a unit pole at that point. The direction in which either force acts at the point is indicated by a line or arrow, but it should be remembered that either force acts at every point throughout the space. The purpose of the lines or arrows in the figures is only to aid in visualizing what actions occur and what the directions are.

Displacement currents. A steady emf will produce only a momentary current in a circuit containing capacity. In this case, a displacement current is produced only so long as the voltage of the capacity is building up to its final and maximum value. An alternating emf is continually varying both in value and polarity and, consequently, a displacement current is continually flowing in the current. If the voltage across the condenser is varying sinusoidally, the strain in the dielectric will vary sinusoidally. The displacement current in the condenser depends upon the rate of change of this strain or displacement and will, therefore, be represented by a **cosine** curve.

It was shown in Part 3 that when a circuit containing inductance and capacity in series has the same period as that of the impressed alternating emf, that is, is resonant to the frequency of this emf, the inductive and capacitive reactances neutralize each other and only the resistance of the circuit for that frequency remains. This is the condition of series resonance, and the current I flowing in the circuit is found by Ohm's law

$$I=\frac{E}{R}$$

This resonant condition is desirable in radio transmission because the largest current is then obtained.

The voltage across the capacity or inductance, however, may rise to an extremely high value and cause insulation difficulties when resonance is obtained. It is important, therefore, to have sufficient capacity in the circuit to keep this voltage within practical limits Thus, the formula for the voltage across a capacity

$$V_C=\frac{I}{\omega C}$$

where VC = effective voltage in volts across capacity,
I = effective current in amperes,
C = capacity in farads,
$\omega = 2\pi f$

shows that, for a given current and frequency, the voltage varies **inversely** as the capacity. The formula for the voltage across an inductance is

$$V_L = I\omega L$$

where L = inductance in henries,

and the other quantities are as given for the preceding formula. This formula shows that, for a given current and frequency, the voltage across an inductance varies **directly** as the inductance. Since the product of L and C determines the frequency of a circuit, it is apparent that when C is increased, and L is decreased to keep the frequency constant, the voltage across both are reduced for the same current.

The antenna. The antenna is an elevated conducting structure which forms one plate of a condenser, while the ground forms the other. Sufficient inductance is inserted in series with the antenna and ground to tune the antenna circuit to resonance with the frequency of the impressed emf.

The antenna system can be a single vertical or inclined wire, or a very extensive network of wires, depending upon whether a small or a large amount of capacity is required. The type of antenna employed can be any one of the various types—single vertical wire, single inclined wire, **Γ** type, **T** type, umbrella, fan, triangular or quadrilateral flattop—and be symmetrical or nonsymmetrical. **The purpose of the antenna,** however, **is invariably the same—to provide sufficient capacity**—that is, **a place for the charges to go without raising the potential of the antenna to an excessive value.** Thus, the antenna capacity need not be large when the current is small, but must be large for large currents. Example:

The original antenna at the Annapolis station had a capacity $C = 0.016\mu f$. The capacity has recently been increased to $0.034\mu f$. Calculate the antenna voltage for each case at $\lambda = 17{,}145$ m. and for $I = 200$ amperes.

Solution: $C = 0.016\mu f = 1.6 \cdot 10^{-8}$ f
$\lambda = 17{,}145$ m; $\omega = 1.08 \cdot 10^5$

Formula $$V_C = \frac{I}{\omega C}$$

substituting $$= \frac{2 \cdot 10^2}{1.08 \cdot 10^5 \times 1.6 \cdot 10^{-8}} = \frac{2 \cdot 10^2}{1.728 \cdot 10^{-3}} = 1.157 \cdot 10^5$$

whence $V_C = 116{,}000$ volts (effective)

$$V_0 = V_C \sqrt{2}$$

whence $V_0 = 163{,}500$ volts (maximum)

This maximum voltage is excessively high and difficult to insulate. It represents very nearly the practical limit, because of difficulties of flash-over and corona; the latter results in serious loss of power.

For $C=0.034\mu f$, the wave length and current remaining the same, the voltage on the antenna will be decreased in the ratio

$$\frac{0.016}{0.034}=0.47$$

Hence, the voltage on the new antenna will be only 47% of that on the original antenna, or

$$V=0.47\times163{,}500$$

whence $V_0=77{,}000$ volts.

The antenna can be insulated more easily, and there should be no corona and consequent loss of power.

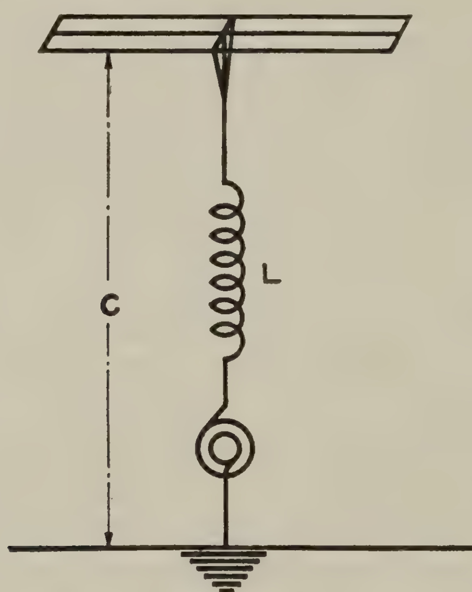


Fig. 227.—Loaded T Antenna.

Figure 227 shows the well-known **T** antenna with loading inductance L and alternator in series in the downlead. The alternator merely represents an alternating emf impressed in the antenna circuit and having the same frequency as that of the antenna circuit.

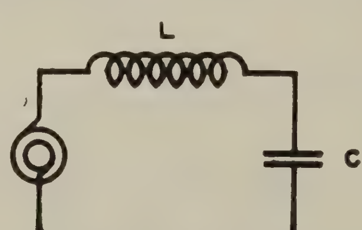


Fig. 228.—Equivalent Antenna Circuit.

The last arrangement of the circuit may be represented diagrammatically by figure 228 in which L and C are concentrated and the period of the oscillatory circuit made equal to that of the alternator.

The only difference in these two circuits is that the antenna top and ground form a condenser of physically large dimensions, but having the same capacity as the compact condenser. The reason for employing such a construction will be explained a little later.

The loop antenna. Instead of making the condenser of large dimensions, the inductance coil may be expanded in physical size. Figure 229 shows this arrangement. The circuit is quite the same as that shown in figure 228. Such a coil is commonly called a loop antenna, coil antenna, or simply a loop. It is important to remember that both the antenna circuit and the loop antenna circuit are essentially the same as the circuit in figure 228 and are **closed circuits.**

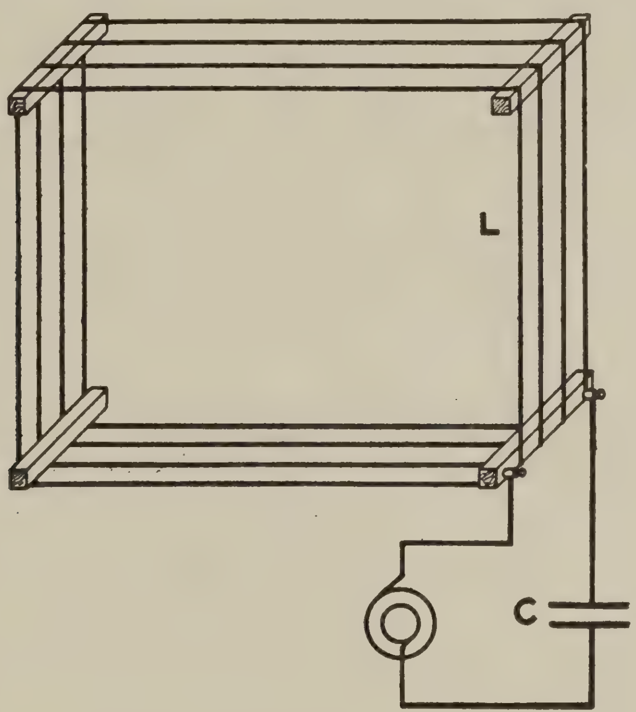


FIG. 229.—The Loop Antenna.

Radiation from an antenna. The theory of the different types of radio transmitters now in general use is given in Part 8. A physical conception of the process of radiation is very difficult, and figures drawn in an attempt to explain the phenomenon frequently lead to erroneous ideas. In the following paragraphs the assumptions are made that the antenna is of the **T** type, with negligible capacity between the downlead and ground, that is, the main capacity is between the flattop and ground, that the wave length employed is considerably greater than the fundamental wave length of the antenna—obtained by inserting inductance—and, further, that the current is practically uniform in all parts of the antenna downlead. Also, the alternator merely represents a source of alternating emf of high frequency in the antenna circuit, having a period corresponding to the resonant frequency of the antenna circuit.

When a radio transmitter is in action, the antenna becomes alternately charged positively and negatively in respect to the earth, and the

periodic motion of these charges constitutes an alternating current. Assume that the alternator has been supplying current to the antenna circuit long enough for conditions to become steady. Under these conditions the antenna current is in phase with the alternator voltage, the magnetic field about the downlead is in phase with the antenna current, the antenna voltage is 90° in advance of the antenna current and, as the electric field is in phase with the antenna voltage, it is, therefore, 90° in advance of the magnetic field. There is an oscillating exchange of energy between the magnetic and electric fields, the energy being stored first in one field and then in the other.

The relation between the periodically varying antenna voltage and current in the antenna circuit is shown in figure 230.

The cosine curve shows the changes in the voltage of the antenna capacity, while the sine curve shows the charging current in the antenna circuit lagging by 90°. The condition about the antenna is as follows.

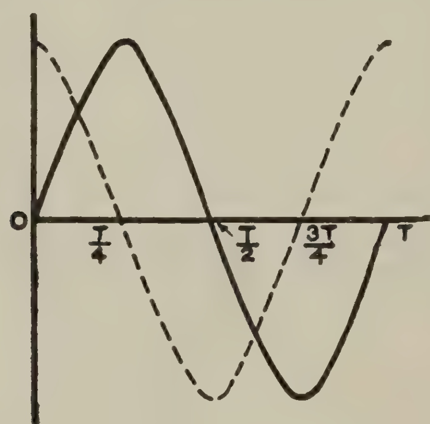


Fig. 230.—Relation Between Antenna Voltage and Antenna Charging Current.

Let T be the periodic time of one complete oscillation in the antenna circuit. Also, let it be assumed that at time $t=0$ the antenna is charged to a maximum positive potential. All the energy is then stored in the electric field $\mathcal{E}_1$ in the immediate vicinity of the antenna. At this instant there is no current flowing and, hence, there is no magnetic field. This condition is shown in figure 231. The electric displacement is downward, between the antenna and ground, that is, from positive to negative.

An instant later, the charge on the antenna begins to move, that is, electrons move up from the ground into the antenna, where there is a deficit of electrons. This movement is very rapid at first, and lasts until time $t=\frac{T}{2}$, when the antenna has become charged to a maximum negative value. At this instant, there is a surplus of electrons on the antenna and a deficit of them in the ground in the immediate vicinity of the antenna, the ground being charged positively.

The movement of the charge on the antenna constitutes a flow of current i in the downlead from antenna to ground, and produces a magnetic field H_1 about the downlead. The rate of change in this current is most rapid at the instant the charge starts to move, and the current reaches its maximum intensity at time $t=\frac{T}{4}$ when its rate of change is least. At this instant, also, the charge on the antenna has

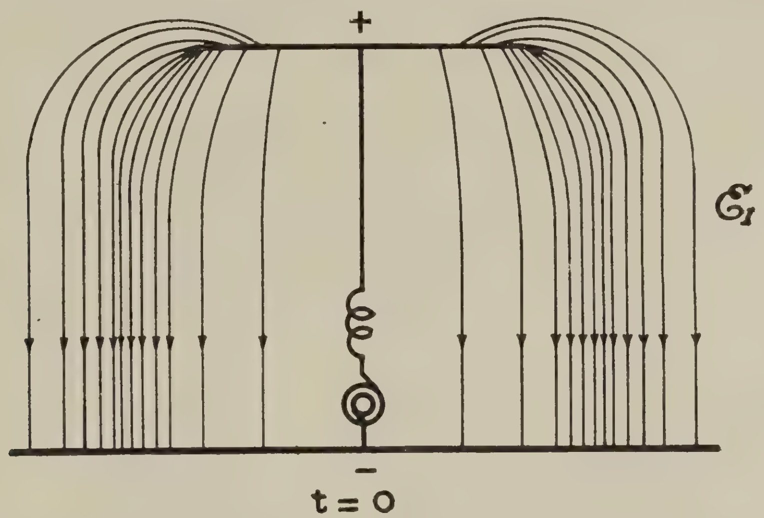


FIG. 231.—Electric Field about a Positively Charged Antenna.

become zero, and the energy previously stored in the electric field $\mathcal{E}_1$ is now stored in the magnetic field H_1 about the downlead. The magnetic field varies with every change in the current producing it. Hence, at time $t=\frac{T}{4}$ the magnetic field has reached its maximum strength and is

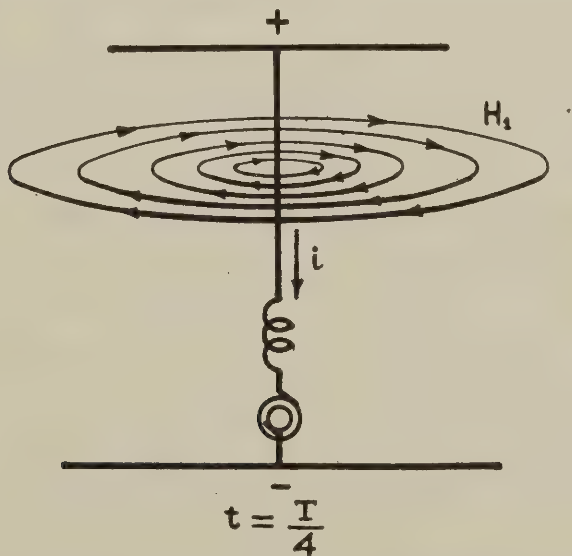


FIG. 232.—Magnetic Field about an Antenna Discharging from Positive to Zero Potential and then being Charged to a Negative Potential.

no longer varying. Figure 232 shows the magnetic field at this time.

Between times $t=\frac{T}{4}$ and $t=\frac{T}{2}$, the current i continues to flow

downwards in the vertical part of the antenna but with decreasing intensity until it ceases at time $t=\frac{T}{2}$, when the charge on the antenna has attained its maximum negative value. At this instant, the magnetic field has disappeared, and the energy is now stored in the electric field which was built up to its maximum intensity during this interval. The electric displacement is again maximum, but its direction is opposite to that existing at time $t=0$, that is, it is upwards from the ground to the antenna as shown in figure 233.

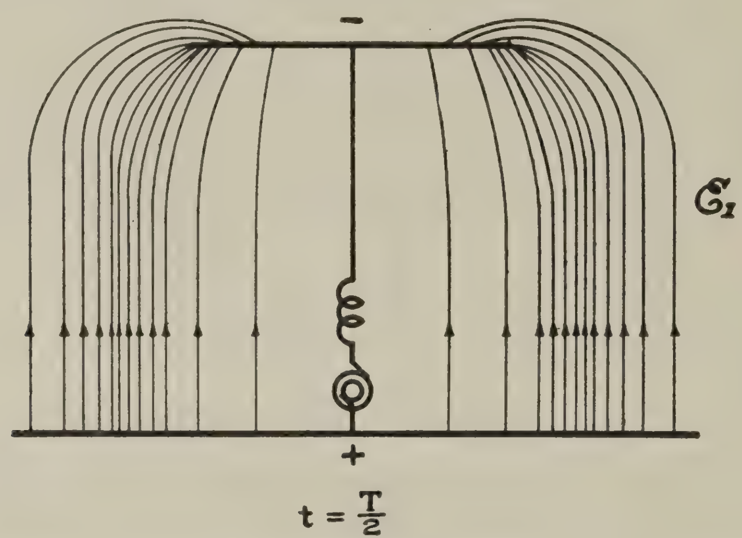


FIG. 233.—Electric Field about an Antenna Charged to a Negative Potential·

During the second half of the cycle, from time $t=\frac{T}{4}$ to $t=T$, the surplus of electrons on the antenna travel down the vertical part of the antenna to ground until the condition is the same as at time $t=0$. During this interval, the current i flows up the vertical part of the antenna. At time $t=\frac{3T}{4}$, the intensity of the current is maximum and the strength of the magnetic field is maximum, but its direction is opposite to that existing at time $t=\frac{T}{4}$. This is shown diagrammatically in figure 234.

During the interval between time $t=\frac{3T}{4}$ and time $t=T$, the current continues to flow upwards in the vertical part of the antenna, but with decreasing intensity, until it becomes zero, when the charge in the antenna has again reached its maximum positive value. The magnetic field H_1 produced by the current in the vertical part of the antenna varies in intensity with the current and finally disappears at time $t=T$ when the energy is again stored in the electric field $\mathcal{E}_1$ and the condition

about the antenna is similar to that existing at the beginning of the cycle. This completes one cycle. The exchange of energy between the electric and magnetic fields just described is periodically repeated.

Now, the field set up about a conductor carrying current is a stress in the ether, that is, the particles of the ether are strained out of their normal positions. Due to the elasticity and inertia of the ether, the strain, which is propagated by a wave motion consisting of transverse vibrations of the particles, tends to persist. The rapidly varying magnetic strain, produced about the antenna downlead by the rapidly

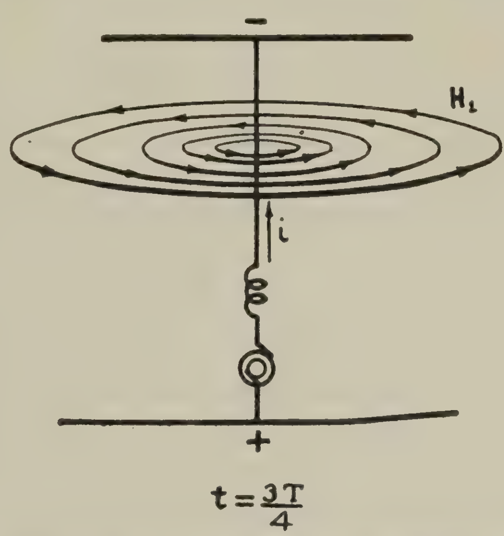


Fig. 234.—Magnetic Field about an Antenna Discharging from Negative to Zero Potential, and then being Charged to a Positive Potential.

varying current flowing in it, is propagated through the ether with the velocity of light. This moving magnetic field produces an electric displacement $\mathcal{E}_2$ in the ether, which is in **time phase** with it but in **space quadrature** and also at right angles to the direction of the propagation. Thus, the current i, the magnetic field H_1 produced by it and the electric displacement $\mathcal{E}_2$ associated with the magnetic field are in time phase, and constitute the primary effect of the movement of charges in the antenna circuit. They lag 90° behind the electric field $\mathcal{E}_1$, as shown in figure 235.

The magnetic field H_1 and its associated electric displacement $\mathcal{E}_2$ are the two components of the **induction field,** frequently called the **stationary field.** The energy in this field is periodically returned to the electric field $\mathcal{E}_1$. In other words, the induction field is tied to the antenna circuit and may be considered a part of it. The induction field is most intense at the surface of the downlead, and falls off in intensity proportionally to the reciprocal of the square of the distance r from the downlead, or

$$\mathcal{E}_2 \text{ and } H_1 \propto \frac{1}{r_2} \quad r^2$$

The electric field $\mathcal{E}_1$, however, diminishes in intensity according to the inverse cube law of the distance r, or

$$\mathcal{E}_1 \propto \frac{1}{r^3}$$

It is evident, therefore, that the intensities of the electric field $\mathcal{E}_1$ and of the induction field $\mathcal{E}_2$ H_1 decrease rapidly with the distance, and become vanishingly small at a distance of only one wave length. Radio signals, however, are transmitted over distances equal to many wave lengths, and it has been proved experimentally that the intensity of the field producing them decreases inversely as the distance r, or

$$\mathcal{E}_3 \text{ and } H_2 \propto \frac{1}{r}$$

This is the **radiation field,** and consists of a magnetic component H_2 and its associated electric displacement $\mathcal{E}_3$, which are in time phase and in space quadrature and both at right angles to the direction of the propagation of the radiation. The radiation field is 180° out of phase with the electric field $\mathcal{E}_1$ as shown in figure 235.

The **total field** about an antenna is very complex and can be shown mathematically to consist of **three components,** viz:

(1) **The electric field $\mathcal{E}_1$, due to the charges on the antenna.**

(2) **The induction field consisting of its magnetic component H_1 and its associated electric displacement $\mathcal{E}_2$, and**

(3) **The radiation field consisting of its magnetic component H_2 and its associated electric displacement $\mathcal{E}_3$.**

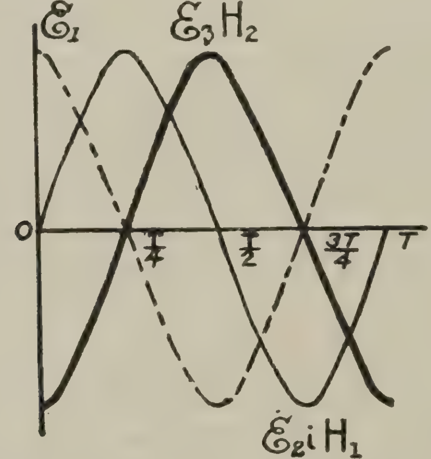


FIG. 235.—Phase Relations of the Electric Field, Induction Field and Radiation Field.

These three components produce confused electric conditions in the neighborhood of the antenna and are all equal in intensity at a distance equal to $\frac{\lambda}{2\pi}$ or $\frac{\lambda}{6.28}$ from the antenna, while beyond a distance of about one wave length, the radiation field, which falls off in intensity so much

less rapidly than the other two, is the only one which needs to be considered. The phase relations of the three component fields are shown in figure 235, and their relative intensities at varying distances from the antenna are given in figures 236 and 237. It will be seen from figure 237 that, when $r=\lambda$, the intensity of the induction field $\mathcal{E}_2$ H_1 is about 16 per cent of that of the radiation field $\mathcal{E}_3 H_2$ but, since these two components are 90° out of phase, the induction field adds only about 1 per cent to the intensity due solely to the radiation field. This increase is reduced practically to zero by the intensity of the electric field $\mathcal{E}_1$ at this distance, this field being 180° out of phase with the

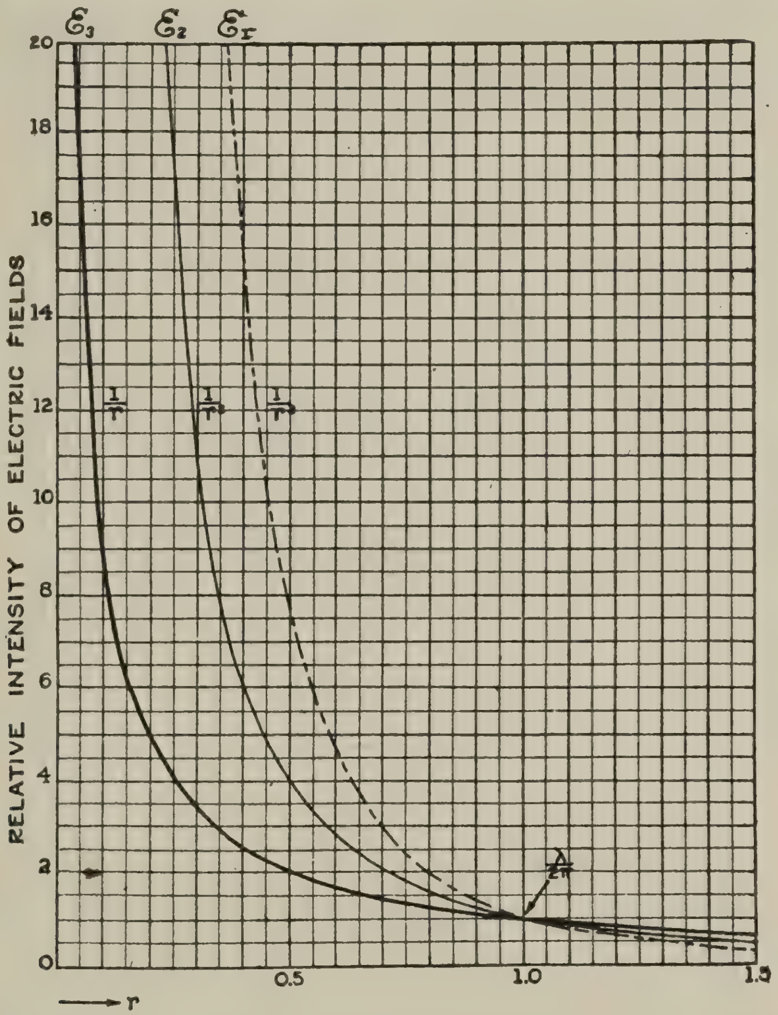


FIG. 236.—Relative Intensities of Component Fields within a Distance of 1.5 Wave Length for Antenna.

radiation field. From these considerations, it can be said that **the radiation field composes the total field at a distance of a few wave lengths from the antenna.**

The radiation field is a component of the total field about the antenna and **exists at all points.** Its presence close to the downlead of the antenna is masked by the induction field. It is unlike the induction field in that the energy in its electric and magnetic fields is **not** returned to the antenna circuit but is **free** and, therefore, cannot remain stationary

but travels outwards from the antenna with the speed of light. The **magnetic force and electric displacement are also at every instant in time phase and in space quadrature, and both are at right angles to the direction of wave propagation.**

Figure 238 shows the directions of the electric displacement and of the magnetic force at a given instant, relatively to the direction of the current producing them, and the direction of the wave propagation. Plane *ABPC* is at right angles to the plane of this page, which is repre-

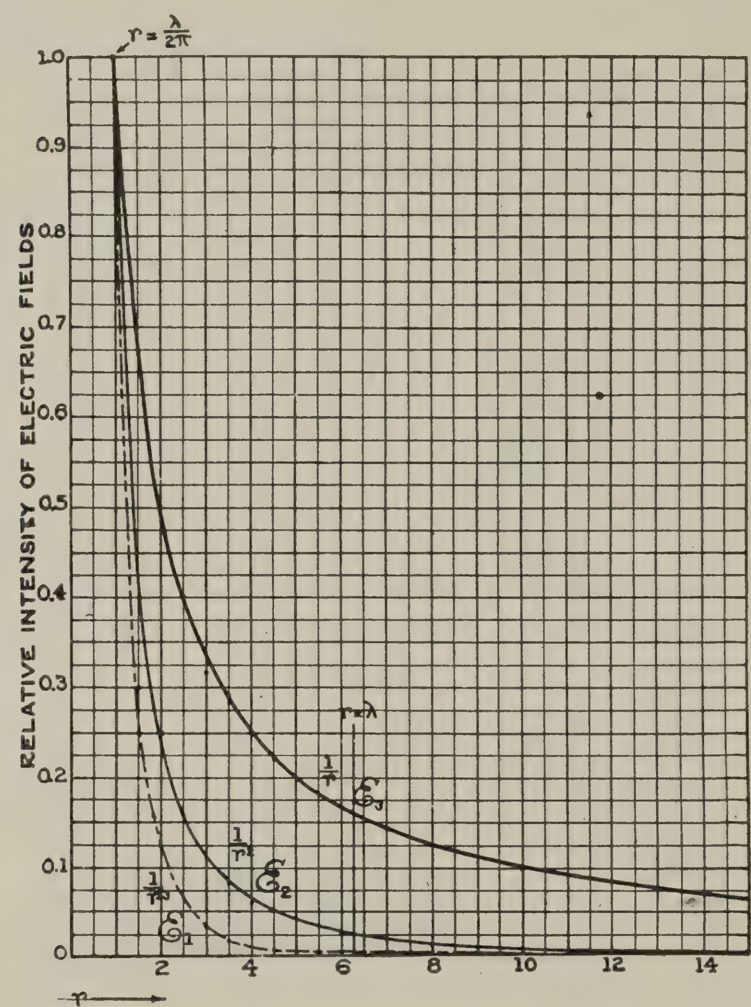


FIG. 237.—Relative Intensities of Component Fields at Varying Distances from Antenna.

sented by plane *DEFG*, and also to plane *GFJI* which is also at right angles to the page. The direction of wave travel is from left to right. The plane *ABPC* represents a section of the vertical wave front. The electric displacement acts vertically throughout this plane, while the magnetic force acts horizontally throughout the same plane. It should be remembered that lines of electric force and magnetic force do not exist, but are only used to show in what direction each force is acting and to help in visualizing what occurs.

Energy in the electric and magnetic fields. Definite and equal amounts of energy are stored in each of the fields. This condition is independent of the distance from the transmitter. Since the radiation field is moving and independent of the source, a difference in the energy of the two fields can not be maintained, because the motion of either field always produces the other field and, hence, were there an inequality, part of the surplus of one field would be transformed into energy of the other, and both fields would become equal. In this connection, it is very important always to remember that **neither field can exist without the other,** that **they cannot be separated by any transmitting or receiving system** and, further, that **they do not act separately.** Thus, it is incorrect to assume that the electric component alone affects an ordinary antenna and that only the magnetic component affects a receiving loop.

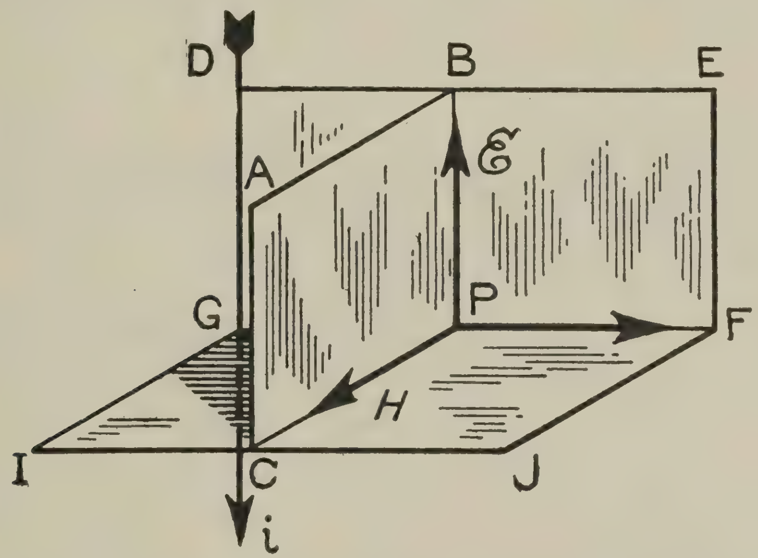


FIG. 238.—Relative Directions of Current in Antenna Downlead, Electric Displacement, Magnetic Force and Wave Propagation.

As a matter of fact, the two components may be considered as different aspects of the same thing.

Effective height of an antenna. The **effective height** of an antenna is defined as one-half the length of the Hertzian oscillator which will produce the same electric intensity at the same distance. The other half of the oscillator is supposed to be replaced by the earth or to be buried in the earth.

Figure 239 shows a Hertzian oscillator of length l. The distribution of the field at the **equatorial plane** is as shown in the figure. Now, if a perfectly conducting sheet is substituted for this imaginary plane, the distribution of the field about the oscillator will be the same as before, and the field of the lower half can be considered to be the image of that of the upper half. The lower half of the oscillator can now be removed without affecting the field above the conducting plane. Therefore, for points above the equatorial plane, the field of an oscillator

of length $\frac{l}{2}$ connected to such a perfectly conducting sheet is the same as that produced by an oscillator of length l acting by itself.

In the case of antennas, the surface of the earth is considered to be the conducting plane, and the distribution of the field above the earth's surface is the same as would be the case were the double of the antenna present instead of the earth.

The effective height of an antenna cannot be calculated from its physical dimensions, but in every case must be measured with the aid of special receiving apparatus. The method of making this measurement is given in Section II of this MANUAL. The presence of steel towers, metal guys, other antennas, trees, buildings, and the distance to permanently wet ground, affect the effective height, all operating to reduce it with the exception of the last mentioned. This is especially true in

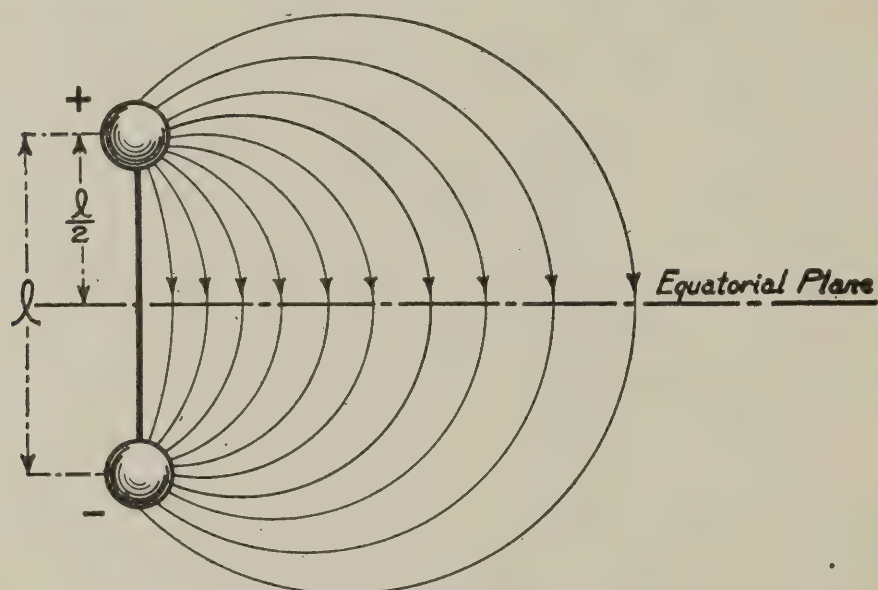


FIG. 239.—The Hertzian Oscillator.

the case of steel towers, because the current flow in them is opposite in direction to that in the downlead of the antenna.

Figure 240 shows the different heights frequently referred to in radio literature in connection with antennas. A T antenna is shown supported by steel towers of height $h_t = 100$ m. The height of the ends of the antenna is 95 m. and at the center the height is 80 m. The **mean height,** h_m, will be approximately 90 m. and the capacity of the downlead to ground can be neglected if the major part of the total capacity is that between the flattop and ground. Thus, the mean height will ordinarily be the **height to the center of capacity,** if the fan does not extend more than a few meters below the lowest point of the flattop. The effective height, however, in the case of such an antenna and steel towers may be **estimated** to be approximately 60 per cent of the mean height, as shown in the figure, but may actually vary between 40 and 90 per cent of

the mean height and must, therefore, be measured as stated above when accuracy is desired. Table 9 gives the mean and effective heights of several antennas.

Electric intensity at a distance from an antenna. The mathematical expression for the electric intensity at a point distant a few wave lengths from the transmitting antenna is:

$$\mathcal{E} = 120\pi \frac{I_s h_s}{\lambda d} = 377 \frac{I_s h_s}{\lambda d}$$

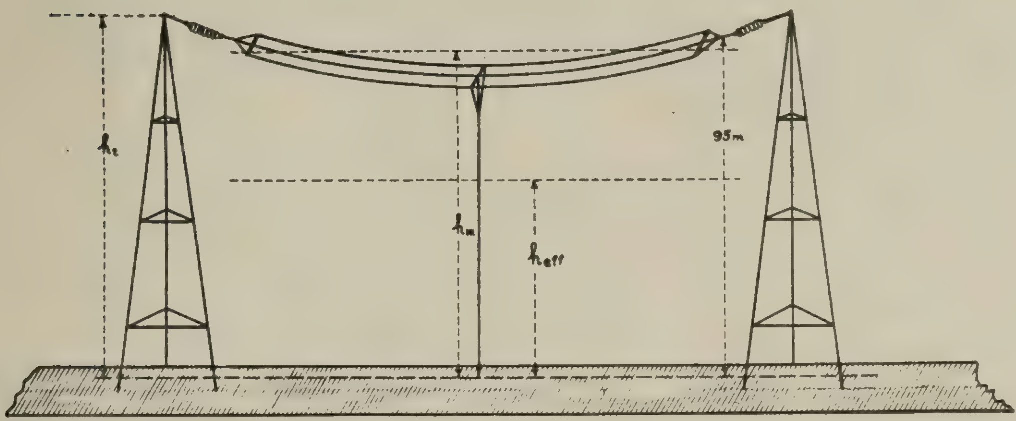


FIG. 240.—Diagram of an Antenna showing the Various Heights Referred to in Radio Literature.

where $\mathcal{E}$ = electric intensity in volts per meter,
I_s = rf current in transmitting antenna in amperes,
h_s = effective height of transmitting antenna in meters,
λ = wave length in meters,
d = distance from transmitting antenna to receiving point in meters.

The effective height h_s is generally about 60 per cent of the mean height of the flattop above ground.

The expression $I_s h_s$ in the numerator of the above equation shows that the electric intensity at a distance from the transmitting station is proportional to the product of the antenna current and antenna height. This product, $I_s h_s$, is sometimes called the **antenna current moment,** the unit of which is the **meter-ampere.**

The equation also shows that the electric intensity is inversely proportional to the distance d, as was mentioned above in speaking of the three fields which exist about a transmitting antenna, and also that it is inversely proportional to the wave length.

It should not be inferred from the foregoing that the radiation field from the transmitting station can be expressed only in terms of the electric intensity. The radiation field can also be expressed in terms of the magnetic component, since both components have equal amounts of energy stored in their fields at every instant and at all distances. It is generally considered, however, that the electric field can be pictured more easily and its action more readily understood than the magnetic

field. Consequently, the radiation field will be given in terms of the electric intensity in this MANUAL.

The electric intensity is generally expressed in **microvolts per meter** instead of in volts per meter. The formula just given then becomes:

$$\mathcal{E} = 3.77 \cdot 10^8 \frac{I_s h_s}{\lambda d}.$$

Example:

What is the electric intensity produced at a distance of 15 kilometers by an antenna current of 75 amperes in an antenna having an effective height of 50 meters, at a wave length of 5,000 meters? What is the antenna current moment?

Solution:

Formula $\mathcal{E} = 3.77 \cdot 10^8 \frac{Ih}{\lambda d}$

substituting $= 3.77 \cdot 10^8 \frac{75 \times 50}{5 \cdot 10^3 \times 1.5 \cdot 10^4}$

$$= \frac{3.77 \cdot 10^8 \times 3.75 \cdot 10^3}{7.5 \cdot 10^7} = 1.89 \cdot 10^6$$

whence $\mathcal{E} = 1.89 \cdot 10^6$ microvolts, or 1.89 volt.

The antenna current moment,

$$Ih = 75 \times 50$$

whence $Ih = 3{,}750$ meter-amperes.

Radiation from a loop antenna. It was formerly believed that a loop or coil would not radiate, for the reason that the field produced by the current flowing in one side is equal and opposite to that produced by the current flowing in the opposite side. The fields produced in this manner are **equal in intensity** but are **not exactly opposite in phase,** because a definite time is required for the field, due to either side, to be propagated to the other side.

Figure 241 shows a vertical, single-turn loop of rectangular cross-section, in which an alternating current of high frequency is flowing. Let h be the height of the vertical sides AD and BC, and l the horizontal length of the loop, both dimensions being expressed in meters. The axis of the loop is shown in the figure and is midway between the vertical sides, that is, at a distance of $\frac{l}{2}$ from both. Point P is at a distance d, which is great compared with l, from the far side AD of the loop, and at a distance $d - l$ from the near side BC of the loop and also is in the plane of the loop. The assumption is also made that the current distribution is uniform throughout the loop.

Under these conditions the vertical sides of the loop act like two vertical antennas, separated by a distance equal to the horizontal length

of the loop, in which the currents are 180° out of phase. Each of these sides radiates energy in the same manner as an antenna. **The radiation field produced by one side is equal and opposite to that produced by the other side only at points equidistant from both sides.** The two fields are practically equal at all points distant enough so that the length of the loop is only a very small part of the distance, but are exactly opposite

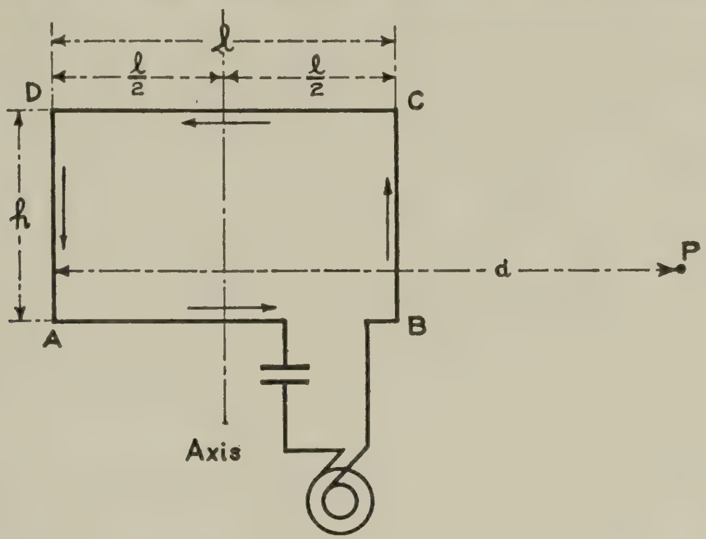


FIG. 241.—Single-turn Loop, showing the Dimensions Referred to in Text.

only when the distance each field has to travel is exactly the same. Hence, at all points lying in the plane of the axis of the loop, the radiation field due to both sides of the loop is zero. This plane is at **right angles** to the plane of the loop.

The superposition of the two radiation fields produces the resultant radiation field of the loop. The resultant field is shown in the polar diagram, figure 242. The distance along any radius from the center O to the point of intersection with the figure **8** represents the intensity of the resultant radiation field in that direction. Thus, the sector OA

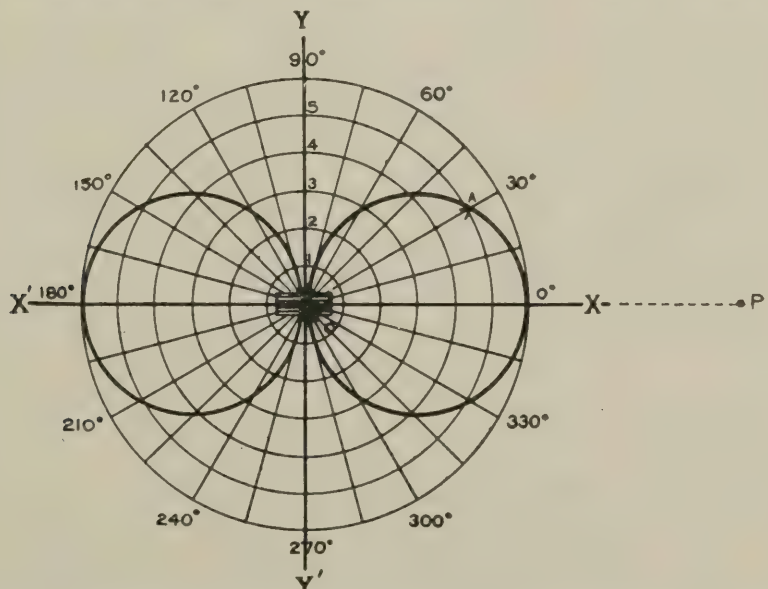


FIG. 242.—Polar Diagram of Resultant Radiation Field of a Loop Antenna.

represents the radiation in the 30° direction. The loop is shown at O, top view. The plane of the axis is YOY' and the plane of the loop is XOX'. The two heavy circles represent the resultant radiation field of the loop which, at all points, is equal to the vector sum of the radiation fields due to the vertical sides. Thus, the resultant radiation field is maximum for all points lying in the plane of the loop, that is, along line XOX'. If the loop is stationary and point P is rotated at a constant distance about the axis of the loop form 0° to 180°, the intensity of the resultant radiation field at P varies as shown in the figure, the intensity decreasing until at 90° it is zero and then increasing, again reaching a maximum at 180°. If point P is stationary and the loop is revolved about its axis, the intensity of the radiation field at P produced by the loop will vary in the same manner as just described. In the case that the loop is near the ground and the interest lies solely in the field near the ground, the two horizontal sides contribute nothing to the useful radiation field, because any point at a great distance from the loop is equidistant from each of the horizontal sides.

This is not the case, however, with the vertical sides. For example, point P figure 242, is at a distance d from the far side AD and at a

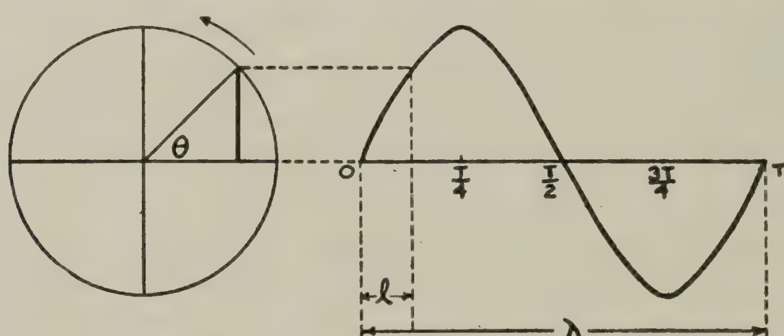


FIG. 243.—Showing Relation of Angle θ, length l of Loop, Wave Length λ, and Periodic Time T.

distance $d—l$ from the near side BC. Therefore, the equal radiation fields, produced by the current flowing in opposite directions in these sides, arrive at different times at point P, that from the far side arriving later than the one from the near side. It is evident, therefore, that the length of the loop plays an important part in the determination of the resultant radiation field. This is shown in the following.

The simple condition when point P is in the plane of the loop, but at a distance d great compared with the length l of the loop, will be considered. It was shown in the previous Chapter that radio waves are propagated through ether by a wave motion consisting of transverse vibrations of connected particles, each particle oscillating with a simple harmonic motion about its mean position in a periodic time T. It was also stated that these particles oscillated in succession, but different in phase by a constant and definite amount and, further, that the disturbance was propagated a distance of one wave length λ in the periodic

time T of one particle. The equivalent angle apparently turned through by such a particle during one period T equals 2π radians. Hence, if the disturbance travels one wave length λ while the particle turns through an angle of 2π radians, then while the disturbance travels a distance l equal to the length of the loop the particle will, in the meantime, turn through a certain angle θ which will be the same fractional part of 2π radians as l is of λ. This ratio is:

$$\frac{\theta}{2\pi}=\frac{l}{\lambda}$$

from which

$$\theta=\frac{2\pi l}{\lambda}$$

This relation is shown in figure 243.

Now, the radiation fields produced by the current flowing in opposite directions in the two vertical sides of equal height h of the loop are equal and would be 180° out of phase were these two sides not separated, and the resultant radiation field would also have been zero at all points. As the length l is increased, the two fields become less and less out of phase, angle θ increases, and the resultant field, therefore, increases in intensity. Thus, when $l=\frac{\lambda}{2}$, then $\theta=\pi$, or 180°, and the two radiation fields are in phase, and the electric intensity $\mathcal{E}$ of the resultant radiation field will be equal to twice that due to one side, or $\mathcal{E}=2\mathcal{E}_s$ where $\mathcal{E}_s$ is the electric field of the radiated wave due to one vertical side.

For any length l, the electric field is given by:

$$\mathcal{E}=2\,\mathcal{E}_s \sin\frac{\theta}{2}$$

where $\mathcal{E}_s$ = electric field of radiated wave due to one vertical side

θ = difference in phase of the two radiated fields.

The usual case is where length l is very small compared with the wave length λ. The angle θ is then small. Under these conditions:

$$\sin\frac{\theta}{2} \text{ becomes } \frac{\theta}{2} \text{ in radians,}$$

and then

$$\mathcal{E}=\mathcal{E}_s\theta \qquad \text{approximately.}$$

A vector diagram of the resultant radiation field of a loop is shown in figure 244. Angle θ is taken equal to 15°, corresponding to a length

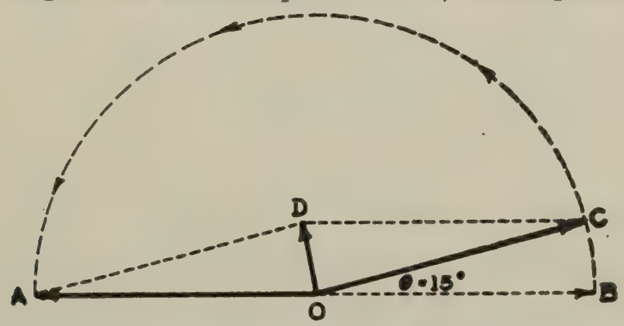


FIG. 244.—Vector Diagram of Resultant Radiation Field of a Loop.

$l=\frac{\lambda}{24}$. OA is the electric field due to the vertical side BC, figure 241; OC is the electric field due to the vertical side AD, which is out of phase with OA by an angle $180°-\theta$, instead of $180°$, due to the length of the loop (OA and OC are equal for the reason stated above). OD is the vector sum of the two radiation fields and is the resultant electric field of the loop. When $l=\frac{\lambda}{4}$, then OD is the diagonal of a square. When $l=\frac{\lambda}{2}$, OD is a straight line equal in length to $OA+OB$.

The second case to be considered is when l is given a definite value, and the loop is then rotated about its axis or, what is the same thing, when point P is anywhere except in the plane of the loop. Under these conditions, the **effective length** of the loop is reduced from its actual length, when point P is in the plane of the loop, to a length which finally becomes zero when point P is in a direction perpendicular to the plane of the loop. Figure 245 makes this clear. Point P is considered to be stationary, while the loop (top view) is swung about its axis at O in the direction indicated by the arrow. The actual length of the loop is represented by lines XOX' and AOB for the two positions. Point P is very distant and in the plane of the loop when the latter is in position XOX'. When the loop is in position AOB the effective length of the

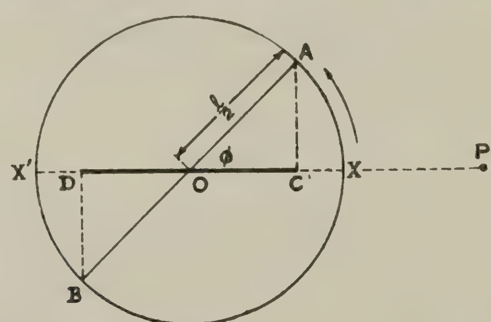


FIG. 245.—Change in the Effective Length of a Loop, due to its Rotation Relatively to a Point.

loop relatively to point P is reduced by $(XC+X'D)$, leaving an effective length of $(OC+OD)=CD$. But OC is the cosine of angle ϕ and, since

$$OA=\frac{l}{2},$$

then
$$OC=\frac{l}{2}\cos\phi$$

whence
$$CD=l\cos\phi$$

Therefore, whenever point P is not in the plane of the loop, the formula under consideration must be multiplied by the cosine of the angle ϕ formed between the plane of the loop and the line to point P. This always operates to reduce the effect at the point. The polar diagram

in figure 242 shows this. When the loop has been turned through 90° from the position XOX' the cosine of the angle is zero and the phases of the two fields are opposite at point P.

Effective height of loop antenna. It is assumed that the radiated field from a loop antenna grounds itself, taking the form of a field formed by the loop and its image as in the case of an antenna. This is probably true, at least for a loop whose dimensions are large compared with its distance from the ground. The **effective height** of a single-turn loop antenna is defined as the height of an equivalent antenna. By this is meant that the field due to the antenna is the same as the field due to the loop antenna when angle $\phi=0$, the current and frequency being the same.

The formula $\mathcal{E}=2\mathcal{E}_3 \sin\frac{\theta}{2}$ shows that the field of a loop antenna is $2\sin\frac{\theta}{2}$ times as strong as the field of an antenna of height h, which is the height of the loop antenna. Therefore, an antenna which has the same field as that of the loop antenna must have an effective height of $2\sin\frac{\theta}{2}$ times h, or

$$h_{\text{eff}}=2h\sin\frac{\theta}{2}$$

and, since

$$\frac{\theta}{2}=\frac{\pi l}{\lambda}$$

then

$$h_{\text{eff}}=2h\sin\frac{\pi l}{\lambda}$$

This formula is applicable in all cases. For small ratios of l to λ

$$\sin\frac{\pi l}{\lambda}=\frac{\pi l}{\lambda}$$

and

$$h_{\text{eff}}=\frac{2\pi lh}{\lambda}$$

If the loop has n turns, its equivalent effective height will be, when angle $\phi=0$,

$$h_{\text{eff}}=\frac{2\pi lhn}{\lambda}$$

where h_{eff} = effective height of loop in meters,
l = horizontal length of loop in meters,
h = vertical height of loop in meters,
n = number of turns,
λ = wave length in meters.

The product lhn is commonly called the **area-turns** of the loop.

An idea of the effective height of loop antennas may be gained from the following example.

Example:

Calculate the effective height of a transmitting loop antenna having the following dimensions, at 800 meters and at 8,000 meters:

$$l=50 \text{ meters,}$$
$$h=20 \text{ meters,}$$
$$n=1.$$

Solution:

Formula $\qquad h_{eff}=\dfrac{2\pi lhn}{\lambda}$

The area-turns are: $\qquad lhn=50\times20\times1=1\cdot10^3$

At $\lambda=800$ m. $\qquad h_{eff}=\dfrac{6.28\cdot10^3}{8\cdot10^2}=7.85$

whence $\qquad h_{eff}=7.85$ meters.

At $\lambda=8{,}000$ m. $\qquad h_{eff}=\dfrac{6.28\cdot10^3}{8\cdot10^3}=0.785$

whence $\qquad h_{eff}=78.5$ cms.

Table 8 gives the values of the equivalent antenna effective height at different wave lengths for loops having various area-turns.

Electric intensity at a distance from a loop antenna. The formula for the electric intensity at a point a few wave lengths distant from a transmitting loop antenna is derived from the formula for the electric intensity produced by an antenna, taking into account the phase relations due to the length of the loop and the angle which the plane of the loop makes with the line connecting its axis and point P. The complete formula is:

$$\mathcal{E}=120\pi\frac{Ihn}{\lambda d}\cdot2\sin\frac{\pi l}{\lambda}\cdot\cos\phi \qquad \text{(volts per meter)}$$

For the usual case, where the ratio $\frac{l}{\lambda}$ is small,

$$\mathcal{E}=120\pi\frac{Ihn}{\lambda d}\cdot\frac{2\pi l}{\lambda}\cdot\cos\phi \qquad \text{(volts per meter)}$$

whence $\qquad \mathcal{E}=2.369\cdot10^9\,\dfrac{Ilhn}{\lambda^2 d}\cos\phi$

where
$\mathcal{E}$ = electric intensity in microvolts per meter,
I = current in loop in amperes,
l = horizontal length of loop in meters,
h = vertical height of loop in meters,
n = number of turns,
λ = wave length in meters,
d = distance in meters.

Example:

Find the electric intensity in microvolts per meter produced at a

point which is 42° out of the plane of a transmitting loop antenna and distant 7,800 meters when:

$$I=1 \text{ ampere},$$
$$l=24.4 \text{ meters},$$
$$h=21.6 \text{ meters},$$
$$n=7,$$
$$\lambda=2{,}800 \text{ meters}.$$

Solution:

$\cos 42°=0.7431$ (Table 17)

Formula $\mathcal{E}=2.369 \cdot 10^9 \dfrac{Ilhn}{\lambda^2 d} \cos \phi$

substituting, $=2.369 \cdot 10^9 \left(\dfrac{1\times24.4\times21.6\times7}{7.84 \cdot 10^6\times7.8 \cdot 10^3}\right)0.7431$

$$=\frac{2.369 \cdot 10^9\times2.44 \cdot 10^1\times2.16 \cdot 10^1\times7\times7.43 \cdot 10^{-1}}{6.12 \cdot 10^{10}}$$

$$=\frac{6.50 \cdot 10^{12}}{6.12 \cdot 10^{10}}=1.06 \cdot 10^2$$

whence $\mathcal{E}=106$ microvolts per meter

If the loop is pointed toward the receiving station, the cosine of angle ϕ becomes unity, and may be dropped from the last formula, which then becomes:

$$\mathcal{E}=2.369 \cdot 10^9 \frac{Ilhn}{\lambda^2 d}$$

It should be noted that the square of the wave length appears in the denominator of the above formula, which indicates that the electric intensity of the radiation field falls off much more rapidly with increase of wave length than in the case of an antenna. Furthermore, it has been found that the directional characteristics of the radiation field, which is shown in figure 242, does not hold for transmission over great distances. It is thought that the radiation field from the horizontal sides of the loop, which attains a maximum value upward, gradually becomes grounded and travels off with a vertical wave front and, combining with the directed radiation field, thus destroys the directional characteristics of the loop antenna. Another possibility is that the resultant radiation field, which is directional in the vicinity of the loop, gradually forms a uniform field by a kind of **diffusion.**

Radiation resistance. The radiation field represents a loss of power, and is the important effect of the current flowing in the downlead of an antenna or in the sides of a loop antenna. It is a loss in that the energy stored in the radiation field does not return to the oscillatory circuit, as is the case with the energy in the induction fields, but travels away

with the velocity of light, half being stored in the electric field and half in the magnetic field of the radio wave.

Strictly speaking, the radiated power is not a loss because it is the only part of the total power supplied to the antenna or loop antenna that is available at a distance from the transmitting station and, therefore, makes possible radio transmission.

The **radiated power P_r** represents the power that leaves the antenna and travels outwards. If it is assumed that the antenna or loop is surrounded by an imaginary spherical surface a considerable distance away, then the radiated power is given by the power traveling outwards through this surface. This can be calculated and is found to be proportional to the square of the current in the antenna or loop. Hence, this loss of power is similar to the loss in power which would result if a resistance were inserted in the antenna or loop. The value of such a fictitious resistance which would give the same power loss as that caused by radiation is called the **radiation resistance R_r.** Thus, the radiated power equals the square of the effective current multiplied by the radiation resistance, or

$$P_r = I^2_s R_r$$

where P_r = average radiated power in watts,
I_s = current in amperes,
R_r = radiation resistance in ohms.

The **radiation resistance** of an oscillatory circuit is a measure of the ability of the circuit to radiate power; consequently, it should, if possible, constitute the major part of the total resistance of the antenna. All other power losses should be reduced to a minimum. The **radiation efficiency** of an antenna is measured by the ratio of the radiation resistance to the total antenna resistance at any wave length.

Only that part of the radiation field that is due to the current flowing in the vertical part of the antenna or loop antenna is useful. The radiation resistance due to this part of the radiation field may be approximately calculated in the case of an antenna by the following formula:

$$R_r = 160\pi^2\left(\frac{h_s}{\lambda}\right)^2$$

whence

$$R_r = 1.58 \cdot 10^3\left(\frac{h_s}{\lambda}\right)^2$$

where R_r = radiation resistance in ohms,
h_s = effective height of antenna in meters,
λ = wave length in meters.

It is important to note that **this formula holds only when the wave length is considerably greater than the fundamental wave length of the antenna,** in which case, the current distribution is practically uniform in the vertical part of the antenna. The effective height of any type of antenna is generally about 0.6 of the mean height of the antenna top.

Table 10 gives the radiation resistance of antennas of various heights at different wave lengths.

The formula shows that the radiation resistance varies **directly as the square of the effective height,** and **inversely as the square of the wave length** and, therefore, **directly as the square of the frequency.**

The formula for the radiation resistance may be substituted in the formula for radiated power, which then becomes:

$$P_r = 1.58 \cdot 10^3 \left(\frac{I_s h_s}{\lambda}\right)^2$$

in which P = average power radiated in watts,
I_s = current in amperes,
h_s = effective height of antenna in meters,
λ = wave length in meters.

The great change in radiated power with change in wave length is shown in the following example.

Example:
The antenna ammeter reads 2 amperes when transmitting with an antenna having an effective height of 12.2 meters (40 ft.) on a wave length of 200 meters. The same antenna is then loaded with additional inductance to give a wave length of 1,200 meters and the antenna current maintained at 2 amperes. Find the value of the radiated power in each case.

Solution:

Formula $$P_r = 1.58 \cdot 10^3 \left(\frac{I_s h_s}{\lambda}\right)^2$$

For 200 meters, $$P_r = 1.58 \cdot 10^3 \left(\frac{2 \times 1.22}{200}\right)^2 = 1.58 \cdot 10^3 \times 1.49 \cdot 10^{-2}$$

whence $P_r = 23.5$ watts.

For 1,200 meters, $$P_r = 1.58 \cdot 10^3 \left(\frac{2 \times 12.2}{1,200}\right)^2 = 1.58 \cdot 10^3 \times 4.13 \cdot 10^{-4}$$

whence $P_r = 0.65$ watt.

Thus, $\frac{1}{36}$ as much power will be radiated at 1,200 meters as at 200 meters, the radiation resistance of the antenna being only 0.16 ohm at 1,200 meters and 5.88 ohms at 200 meters.

In the case of a loop antenna, the problem of the radiated power, and hence, of the radiation resistance, is complicated. The reasons for this are: the radiation from a loop antenna is directional and the horizontal sides of the loop antenna radiate upwards, which is probably useless radiation in so far as transmission along the surface of the earth is concerned. On this account, it is not deemed advisable to give formulas for the radiated power and radiation resistance of loop antennas.

Attenuation factor. When the radiated wave has to travel a distance greater than a few wave lengths, an additional term must be introduced in the transmission formulas for the antenna and loop antenna. This term takes into account the fact that the earth component of the wave suffers absorption during its passage from the transmitting station to the receiving station. By long experiment it has been found that the average value of this factor for transmission **during daylight** and **over salt water** is:

$$\epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

where d = distance between transmitting and receiving stations in meters,
λ = wave length in meters,
$\epsilon = 2.7183$.

The value for the constant given as $4.75 \cdot 10^{-5}$ is only an average for daylight and salt water. The value will always be greater for transmission over land and, in general, varies on different days and hours of the day. **It does not hold for night transmission nor for the sky components of waves.**

Combining the attenuation factor with the transmission formula for the antenna and the loop antenna, they become:

For antenna $$\mathcal{E} = 3.77 \cdot 10^8 \frac{I_s h_s}{\lambda d} \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$ **(Austin-Cohen formula)**

For loop antenna, $$\mathcal{E} = 2.369 \cdot 10^9 \frac{Ilhn}{\lambda^2 d} \cdot \cos\phi\, \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

Values of ϵ^{-x} are given in Table 12, and examples are given showing how the attenuation factor is calculated. Table 11 gives the electric intensity produced at various wave lengths and distances by a given rf current flowing in an antenna of given effective height.

Propagation of radio waves from transmitting to receiving station. The velocity of propagation of radio waves through the earth's atmosphere is considered to be the same as that of light. In round numbers this velocity

$$v = 3 \cdot 10^8 \text{ meters per second.}$$

As shown in Chapter I of this Part, the velocity of propagation v, the period of one complete oscillation T, the frequency f, and the wave length λ are interrelated. The formulas showing their relations are:

$$v = \frac{\lambda}{T} \text{ and } f = \frac{1}{T}$$

Since v is a constant, λ and f are usually expressed in terms of v. Thus,

$$\lambda = \frac{v}{f} \text{ and } f = \frac{v}{\lambda}$$

In these formulas:

λ = wave length in meters,
$v = 3 \cdot 10^8$ meters per second,
f = cycles per second,
T = periodic time of one cycle in seconds.

The following examples show these relations.

Example:

Given $v = 3 \cdot 10^8$ m./sec. and $f = 300{,}000$. Find λ and T.

Solution:

Formula $\quad \lambda = \dfrac{v}{f}$

substituting $\quad = \dfrac{3 \cdot 10^8}{3 \cdot 10^5} = 1 \cdot 10^3$

whence $\quad \lambda = 1\,000$ meters

Formula $\quad T = \dfrac{1}{f}$

substituting $\quad = \dfrac{1}{3 \cdot 10^5} = 3.33 \cdot 10^{-6}$

whence $\quad T = 3.33$ micro-seconds.

Example:

Given $v = 3 \cdot 10^8$ m./sec. and $T = 2.5$ micro-seconds. Find λ and f.

Solution:

Formula $\quad \lambda = v\,T$

substituting $\quad = 3 \cdot 10^9 \times 2.5 \cdot 10^{-6} = 7.5 \cdot 10^2$

whence $\quad \lambda = 750$ meters.

Formula $\quad f = \dfrac{1}{T}$

substituting $\quad = \dfrac{1}{2.5 \cdot 10^{-6}} = 0.4 \cdot 10^6$

whence $\quad f = 400{,}000$.

Table 13 gives the frequency and angular velocity, $\omega = 2\pi f$, corresponding to any wave length.

Since the velocity of propagation is constant, it is evident that the distance a disturbance travels in a given interval of time is the **same** for all frequencies and, hence, for all wave lengths.* **The distance covered during one cycle or one period,** however, **is inversely proportional to the frequency and directly proportional to the period.**

It was stated in the first part of this Chapter that the radiation field was propagated by wave motion with the magnetic and electric components in the wave front at right angles to each other, and to the direction of propagation. **All radio waves are plane polarized at the transmitter,** because they are produced by the movement of charges in

a conductor which is fixed in space; that is, the direction of the electric displacement is parallel to the conductor in which the current producing the electric displacement is flowing.

The lines of force of the electric component of the radio wave produced by the usual type of antenna with ground connection are practically perpendicular to the surface of the earth, and are attached thereto and guided by the surface. In the case of an antenna with counterpoise, these lines attach themselves to the earth in the immediate vicinity of the radiator, and from then on act in the same manner as those in the radiation field of an antenna with ground connection. The same condition exists in the case of loop antennas. On the other hand, the radio wave from an airplane antenna does not become grounded until it has traveled a distance equal to the height of the airplane above ground. The wave then becomes grounded, and travels radially outward from that point guided by the surface of the earth. It is known that the airplane trailing wire antenna is quite directional within a few lengths, the maximum electric intensity of the radiation field being in the direction of flight, that is, the signal is stronger when the airplane is flying toward the receiving station than when the airplane is flying away from it. This effect disappears when the airplane is at a considerable distance. When the airplane is flying directly over the receiving station very curious directional effects are observed, the signal fading out completely during brief intervals, although the airplane is very near. When receiving from an airplane transmitter on a radio compass, and the airplane is quite near and at a considerable altitude, the direction indicated on the radio compass is to a point on the ground ahead of the airplane and in the line of flight. However, when the airplane is at a great distance and at any altitude, the direction indicated will be that of a point on the earth's surface practically underneath the airplane, regardless of its line of flight with respect to the receiving station. It is also evident that remarkably sharp directive effects should be obtained in transmission and reception between two airplanes with trailing wire antennas, provided that the distance separating the airplanes is less than the height of the lower one above ground. The directive effect would be especially marked if loops were used for both transmission and reception. The radio waves emitted by either the trailing wire antenna or loop antenna of an airplane would, under the above conditions, be **free waves.**

In passing from the transmitting to the receiving station, the earth component of the radio wave is subject to many losses which cause the energy reaching the receiving point to be much less than would be calculated from the equations for the electric intensity, which are valid for points only a few wave lengths distant. These losses are expressed mathematically in the exponential attenuation term:

$$\epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

which has already been discussed. The wave is further affected by the earth's magnetic field and the ionization of the earth's atmosphere.

TAYLOR-HULBERT THEORY.*

Propagation of Radio Waves over the Earth. There exists above the surface of the earth an ionized layer of atmosphere called the **Kennelly-Heaviside layer.** The electron density of the atmosphere increases gradually with the height up to a certain point. The distance above the earth where the rate of increase in electron density becomes less with the height is known as the height of the Kennelly-Heaviside layer. This is the height of maximum electron density.

This height varies as the electron density changes and the electron density is increased by the radiations from the sun. Thus in day and during the summer the height of the Kennelly-Heaviside layer is lower than at night and in winter.

The electrons in the atmosphere are still further affected by the earth's magnetic field which causes the free electrons to move in spiral paths at a specified frequency. The wave lengths of this motion of the electrons in the atmosphere has been calculated to be 214 meters from the formula $\lambda_0 = 2\pi cm/He$ wherein m is the mass of the electron, C the velocity of lighting vacuum, H the value of the earth's magnetic field, and e the charge on the electron.

One part of a radio emission will cling to the earth. This is called the "earth component". The other parts of this emission will travel upward to the Kennelly-Heaviside layer; this will be called the "sky component."

The absorption of the earth's component is very rapid and follows the Austin-Cohen formula which has been discussed earlier in this chapter. The absorption of the sky component, except in the neighborhood of the critical value of 214 meters, is negligible for the reason that no appreciable energy is lost in heat due to collisions between electrons and molecules. However, near the critical wave length of 214 meters there are many complexities that all tend to produce absorption in the Kennelly-Heaviside layer.

It has been shown that at the transmitter a radio wave is initially plane polarized. However, after leaving the source of emission a radio wave will be resolved, due to the effects of the earth's magnetic field, into components which are plane or circularly polarized according to the direction of propagation.

For North and South propagation the wave yields two vibrations circularly polarized in opposite directions; and if the wave remains coherent these components will add to give in general an elliptically

*From Dr. A. Hoyt Taylor and E. O. Hulbert's "Propagation of Radio Waves over the Earth."

polarized wave; but for equal intensities of the two components the resultant wave is plane polarized with the plane of polarization rotated from its original direction.

For East and West propagation the initially plane polarized wave is resolved into two plane polarized vibrations, one parallel and one perpendicular to the earth's magnetic field; and if the wave remains coherent the two components unite to produce in general an elliptically polarized wave.

In the general case of propagation in other than a North and South or East and West direction, all four vibrations will occur with the result that if the wave remains coherent it will become elliptically polarized, the orientation of the axes and the eccentricity of the ellipse changing as the wave proceeds.

In other words, the path of the ray of the coherent wave would be in something of the form of a cork screw. But if the components of the wave are separated by greater refraction or distance, each component will proceed of its own accord and the wave will not be coherent; this is the usual case.

As each of the four wave components enter the Kennelly-Heaviside layer we find that they are refracted or turned from their direction. Each component has a different refractive index. This index is the relation between the angle of incidence and the angle of refraction, and can be calculated as a function of the wave length, the critical wave length of the layer, the electron density, the charge on the electrons, their waves, and the intensity of the earth's magnetic field. The following four formulas are used to calculate the refractive index of the wave components:

$$\mu^2 = 1 - \frac{c\lambda^2}{1 - \lambda/\lambda_0}$$

For No. 1 circularly polarized component in N–S propagation.

$$\mu^2 = \frac{1 - c\lambda^2}{1 + \lambda/\lambda_0}$$

For No. 2 circularly polarized component in N–S propagation.

$$\mu^2 = 1 - c\lambda^2$$

For parallel plane polarized component in E–W propagation.

$$\mu^2 = \frac{1 - c\lambda^2}{1 - \lambda^2/\lambda_0^2(1 - c\lambda^2)}$$

For perpendicular plane polarized component in E–W propagation.

μ—refractive index.
λ—wave length used.
λ_0—$2\pi cm/He$
C—$Ne^2/\pi m$
c—velocity flight in vacuum.

m—mass of the electron.
e—charge of the electron.
N—number of electrons.
H—the value of the magnetic field.

All quantities are in C.G.S., C.M. units.

When the refractive index is zero the wave is totally reflected back to earth. When it is between zero and unity it is gradually bent back to earth. When the refractive index is unity the wave proceeds straight through the Heaviside layer and is not bent. When the index is greater than unity the wave is bent skyward.

A study of the refractive index formulas will show that the refractive index decreases as the wave length becomes longer and vice versa it increases as the wave length becomes shorter. The refractive index decreases as the electron density increases and vice versa the index increases as the density becomes less.

If refractive index curves should be plotted for a definite electron density we would find that for the critical wave length λ_0 the refractive index is $\sqrt{\pm\chi}$; for wave lengths greater than 50 meters the refractive index descends towards zero and that the index for the four components of the wave length 50 meters varies from 0.3 to an imaginary value. For wave lengths less than 50 meters the refractive index approaches unity and the indices for the four components of the wave do not vary greatly.

This means that in general, wave lengths in excess of 50 meters (except in the region of the critical frequency of 214 meters) will be totally reflected or refracted at all angles of incidence. However, one of the circularized components of waves greater than 214 meters will be refracted upward and lost.

For wave lengths less than 50 meters the electron density factor cannot become large enough to make the refractive index zero or imaginary because the number of electrons in the Kennelly-Heaviside layer does not in all probability exceed 1,000,000 per cubic centimeter. Therefore such short waves are not reflected at normal incidence but are refracted at any angle with the normal. Furthermore they will not be refracted back to earth at all angles of incidence. Above a certain "critical" angle they will either be refracted upward or refracted back at such a small angle as to be useless when they return.

This means that, except for the ground components which are rapidly absorbed, radio waves less than about 50 meters in length will not return to earth for a considerable horizontal distance from the transmitter and hence will not be heard between the transmitter and the spot of first return to the earth. This area of zero signal is called the "skip distance."

The mathematical explanation of the "skip distance" theory is as follows:

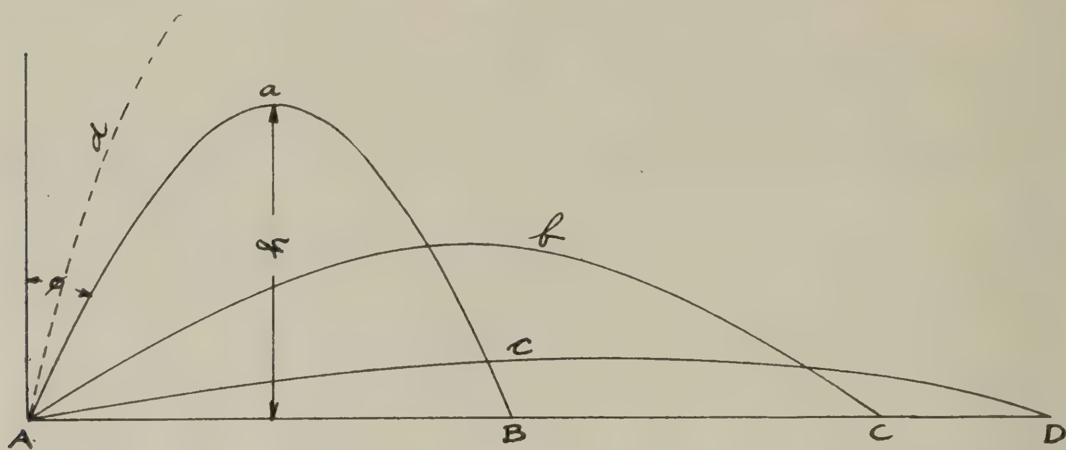


FIG. 246.

Factors:

a—constant of integration.
y—height of electron density above the earth.
μ_0—refractive index at
μ—refractive index at
i—angle with vertical caused by refraction of ray at height.
N—electron density at point Y.C.M. above the earth.
Y—variable which can be determined by identification with refractive index formulas previously mentioned in this chapter.
ϕ—angle of ray with vertical.
B—number of electrons at surface of earth.

$$\left.\begin{aligned} \alpha &= \mu_0 \sin \phi \\ \mu &= \frac{\mu_0 \sin \phi}{\sin i} \end{aligned}\right\} \text{From Snell's law.}$$

$$\frac{dy}{dx} = -\cot i = -\sqrt{\frac{\mu^2 - d^2}{d^2}} \quad \left\} \begin{array}{l} \text{Differential equation of} \\ \text{the path of the ray.} \end{array} \right.$$

$$N = By$$

$$\mu^2 = 1 - \gamma y \left\{ \begin{array}{l} \text{See refraction formulae} \\ \text{This is the refractive index stand at height} \end{array} \right.$$

Since at the surface the earth $y=0$ and therefore $\mu=1$. Therefore since $d=\mu_0 \sin \phi$ we find that $\alpha = \sin \phi$. Combining these values into the differential equation for the ray and integrating we find that

$$x^2 = \left\{ 4\alpha^2(1-\alpha^2-ry \right\} / r^2$$

This is a parabola. The maximum height above the earth reached by the ray is

$$h = (1-\alpha^2)/r$$

The ray comes down again at a distance from its starting point where

$$x_0 = \frac{2\alpha\sqrt{1-\alpha^2}}{2}$$

Combining we find that

$$x = 2h \tan \phi$$

The ray path for a specified wave transmitted from A, Figure 246 at increasing angles ϕ are illustrated by curves a, b and c, Figure 246. It is seen that as ϕ increases h decreases and $2\chi_0$ increases. AB is the **sky distance. This means that any other ray such as d whose angle of projection ϕ is less than that of ray a will not return to earth between points A and B.** This is because the maximum height that can be reached by a ray under the assumption of increasing electron density is h. In other words, if the maximum electron density is at height h no ray of the same wave length with an angle less than ϕ will return to earth between A and B which is the skip distance. The relation of the skip distance (S) to the angle (ϕ) and the height (h) of the Kennelly-Heaviside layer is

$$s = 2h \tan \phi$$

If the foregoing formulas are worked out for various wave lengths it would be seen that:

(1) The shorter the wave length the longer the skip distance.

(2) The higher the Kennelly-Heaviside layer the less the skip distance.

(3) The lower the wave length the greater theoretical angle ϕ.

Since the sky component of the wave after striking the center is reflected upward again and then refracted back to earth, the wave in traveling over the earth passes through a series of bouncing back and forth between the Kennelly-Heaviside layer and the earth's surface.

Furthermore, for these waves which have a skip distance there will be alternate regions of silence and reception, the areas of silence gradually decreasing due to the fact that the various components of the waves striking at different angles will, after a few reflections, spread out the areas of reception.

Since the absorption is negligible in the Kennelly-Heaviside layer, it can be seen that the shorter the wave (which means long skip distance) the stronger the signal will be at long distances. This is because the wave does not travel on the earth's surface when it is absorbed except when it returns after refraction; therefore the fewer the returns the stronger the received signal should be at a distance.

Combining the various factors we find that:

(1) Long waves are reflected at all angles of incidence but they carry well over the earth's surface due to low absorption.

(2) Very short waves are not altogether useful when the height of maximum electron density makes the skip distance too great. This

occurs on winter nights. The shortest wave that is useful for long distance communication is about 11 meters. This can be used in summer days when the height of the Kennelly-Heaviside layer is the lowest.

(3) Short waves between 11 and 50 meters are useful for long distance communication.

(4) Medium waves are useful for medium distance communication.

Two general types of fading occur, attributable to quite different causes. One type, which is common to all wave-lengths is an intensity fluctuation of relatively long period, of the order of a second or more. This is usually more noticeable at nearer distances than at greater distances from transmitter, and in general the longer the wave the slower the fluctuation. The other type is a fading at high speed, the signal intensity often varying from full strength to practically zero at a low audio frequency of the order of, say, 100 oscillations per second. This appears as a change in the quality of the heterodyne note when continuous wave signals are being received and as bad distortion in the case of speech signals. The audio frequency fading characteristic applies only to certain bands of wave-lengths at certain distances from the transmitter; it is, generally speaking, less observable in the day time and at longer distances. More specifically, for waves longer than 800 meters, roughly, the high speed fading rarely occurs; in the broadcast band, 300 to 600 meters, it is noticeable only at intermediate distances, at night especially, from about 100 to 1000 miles from the transmitter. In the region of shorter wave-lengths from 60 to 120 meters the high speed fading is violent at night for distances from roughly 5 to 300 miles from the transmitter. For the wave-lengths 16 to 40 meters the audio frequency fading is found in the flicker zones at the edge of the skip regions.

The fading at low frequency, of a few seconds in period is due to a distortion of the received wavefront by motions of large clouds of the refracting electron medium. On the assumption, which appears very reasonable, that the bodily motions of the electron clouds are of the same order of velocity and extent as the air movements and the ions and electrons will occur. The electron density therefore becomes less and the radio rays must search higher altitudes before being turned back to the earth.

Certain details of the dispersion equations (2), (3), (5) and (6) are of interest. From all the equations at any wave-length less than $\lambda_0 = 214$ meters, μ decreases to zero and becomes imaginary as N (or C) increases. This means that a ray of this wave-length directed normally upwards passes through regions of decreasing μ (and hence moves with increasing velocity) until it reaches that electron concentration for which $\mu = 0$ and there is totally reflected (or refracted) back to the earth. *Therefore wave-lengths from about 60 to 200 meters will be totally reflected from the layer at all angles of incidence.* The different component

modes of polarization will penetrate to different heights. For waves shorter than 50 meters, N can not become large enough to make μ imaginary, since the calculations fix the maximum value of N as 10^5, and hence these rays are not totally reflected at normal incidence, but at a greater angle; this of course is the essence of the fore-going skip distance theory.

For waves longer than 214 meters, μ decreases to zero and hence to imaginary values with increase of N in the cases of equations (3), (5) and (6). From Eq. (2) however, μ is always positive for these waves and increases from 1 as N grows larger. *Therefore for waves longer than 214 meters the modes of polarization corresponding to equations* (3) (5) *and* (6) *will be totally reflected from the layer at all angles of incidence, the mode of equation* (2), *however, always being refracted upward and being probably lost.* (Eq. (5) has further mathematical complexities which appear to be of no physical significance). The prediction that these waves suffer total reflection even at normal incidence should perhaps admit of direct experimental proof. For long waves, greater than 500 meters, (3) reduces approximately to $\mu^2=1-C\lambda\lambda_0$ and (5) and (6) to $\mu^2=1-C\lambda^2$. These are about the same for small values of C, and the latter is the refraction formula for zero magnetic field. Hence currents in the lower atmosphere, one would expect low frequency fluctuations in a wave refracted through such a medium. The phenomenon is a repetition on a larger and slower scale of the twinkling of the stars of the unsteadiness of a scene viewed over the surface of a hot road. For longer waves motions of larger electron clouds are necessary to modify the wave-front, and these on the average would be expected to be slower than in the case of smaller clouds, so that the fading would be slower. Further, for long distances for all wave-lengths the integrated cloud movements and hence the refraction effects would be expected to average out.

The audio frequency fading is attributed to shifting interference patterns. Thus, considering the band of waves from 300 to 600 meters, which exhibit occasionally audio frequency fading at distances from 100 to 1000 miles from the transmitter, it will be remembered that the various polarization components of the wave, except that of one of the circuited components, are totally reflected from the layer at all angles of incidence. Therefore, in this instance any receiver, no matter where situated, may expect to receive in general four possible rays, the ground wave and three overhead components which travel to the receiver by different paths. In the first 100 miles from the transmitter the ground wave is sufficiently intense to drown out any variations contributed by the overhead components. Beyond 100 miles the intensity of ground wave for these wave-lengths becomes comparable with or less than that of the overhead waves, with the result that a complicated interference pattern of various states of polariza-

tion and intensities is formed about the receiver. Movements of the electron layer will cause this pattern to shift to and fro, thereby causing the rapid fluctuation of signal intensity. In daylight the electrons gather into low lying clouds of relatively great density gradient, so that the paths of different overhead rays are relatively close together and the interference pattern becomes broad and hazy. Its movements therefore cause little change in the signals. At night, however, the electrons are more diffuse and their density gradient is much less, so that the ray paths are more widely separated. The interference bands are therefore narrower and sharper and the motion of the pattern will cause rapid and violent intensity variations. At distances greater than 1500 miles the ray paths are so long that the interference pattern becomes indistinct in both day and night and the average effects on it of electron cloud movements become less.

By similar reasoning fast fluctuations would not be expected to occur for long waves at any place, because in the near distances the ground wave is strong and in the far distances the interference pattern is diffuse. In the case of the 16 to 40 meter band it is easily seen that the possibilities for sharp interference exist only on the edge of the skip zones.

The discussion of the wave band from 60 to 120 meters is along similar lines. Here, however, the rapid fading occurs at distances as small as 5 miles from the transmitter. This means that for these waves at this distance the over-head components reach the receiver with an intensity comparable with that of the ground wave, even after traveling 100 miles or so up into the upper atmosphere and being reflected back at nearly normal incidence! There seems to be no escape from the conclusion that the ground wave in this case dies out rapidly and that the over-head components are very perfectly reflected.

The various components of the wave, besides interfering, also may arrive at a receiver at different times, because of the different paths which they traverse. This will have no effect on the interference patterns just discussed, if the wave trains are long, as in continuous wave signals, but will become an additional cause of distortion in the case of short wave trains, such as the modulated waves of speech signals. In order to estimate the magnitude of this it can be assumed that the electron density gradient is $N=0$ at h_0 of 30 miles and $N=3.95\times10^5$ at $h_0+h_1=91$ miles. From this and the refraction formulae it is found that $\mu=0$ at heights 69, 86, 78 and 64 miles, respectively. These are the heights where total reflection occurs for the respective waves. Assuming normal incidence the differences of the paths of the rays from the transmitter to the region of total reflection and back to the receiver are roughly 20 miles, or 10^{-4} seconds. The rays, moreover, pursue their respective paths with different speeds, which are slower in the longer paths. Calculation shows that this will increase the time

differences of the paths by two or three times, making them, say, 3×10^{-4} seconds. This would perhaps be barely perceptible in speech signals. At night, however, the ray-path differences would be increased roughly three times, and the time difference would be of the order of 10^{-3} seconds or more, which would cause strong distortion of the speech signals. This estimate of the time differences of the ray has depended on the electron distribution assumed for the calculation.

The observational data relevant to these matters are none too certain, but a recent program of tests with the 25.6 meter transmitter from this station (NKF) has permitted a few conclusions. A portion of the program involved the reception of the signals at every hour of a 24-hour day by some forty stations scattered to 7000 miles distance. In the first place, the first daylight skip zone for 25.6 meters was found to be between 500 and 600 miles. Secondly, no secondary skip zone appeared; the region from 700 to 1200 miles in daylight being unmistakably one of good signals. This meant merely that the lower transmitted rays were probably near to tangency with the earth. In this connection it might be possible to elevate the ray until the second skip zone was produced, by properly loading and exciting the transmitting antenna.[20] Thirdly, the signals in the region from 2000 to 3000 miles appeared more uncertain than in the region within the 2000 mile mark. The observations referred mainly to over-land waves and it would be of interest to repeat them with over-sea waves.

In general, for the shorter waves observations by many receiving stations in the United States indicate better signal reception in the region extending from the first skip zone to 2000 miles than in the region between 2000 and 4000 miles where at least one earth reflection is involved. At greater distances, however, from 5000 to 10,000 miles the short waves signals are very reliable, much more so than in the 2000 to 4000 mile zone. This is to be expected because, with increase of distance, there are a greater number of possible ray-paths connecting the transmitter and receiver, so that a local disturbance, such as a poor earth reflection, etc., of any one ray will have small influence on the signal. One might expect the inverse distance law of signal intensity to hold approximately in this region for the short waves.

[20] Van der Pol, Pro. Phys. Soc. London **29**, 269 (1916–1917).

CHAPTER III. RECEPTION OF RADIO WAVES.

The radio wave has been previously shown to have two components—the electric field and the magnetic field—which are in time phase and in space quadrature, both fields acting in directions at right angles to the direction of wave travel. The reception of radio waves can be explained using either of these components, and the results obtained will be the same in either case. For the purposes of this MANUAL, it seems best to use the electric component of the radiated field in showing how radio waves are received.

Electric gradient. The term **electric gradient** is quite frequently used in the same sense as, and also in place of, the term electric intensity. These two terms are synonymous, as has already been pointed out. The electric field intensity is defined as the force acting on a unit positive charge. The electric gradient is given by the rate of change of the electric potential in the direction in which this rate of change is a maximum. The work done in moving a unit positive charge from one point to another in the direction of the field intensity is the product of the intensity and the distance. This work gives the difference in potential between the two points. The rate of change of the potential, or gradient, is this difference in potential, or work done, divided by the distance. If, then, the product of intensity and distance is divided by distance, the intensity alone is left.

The electric gradient is thus seen to be the same quantity as the electric field intensity $\mathcal{E}$, and is usually measured in microvolts and meters, as for example:

$$\mathcal{E} = 50 \text{ microvolts per meter.}$$

The formulas used in calculating the electric gradient are the same as those given in the previous Chapter for the determination of the electric field intensity.

A determination of the value for the electric intensity produced at any distant point by a certain radio transmitter makes it possible to design the receiving system to give the required signal strength.

Action of the lines of electric force. The electric field produced at a distant point by a radio transmitter is practically a rapidly alternating vertical field. This is the case over sea water, where the electric force doubtless acts in a plane vertical to the surface of the sea. Over land that is a fairly good conductor, the wave front may incline forward about two degrees while over very deep, dry sand the inclination may be over thirty degrees. The last condition is exceptional, however, and is advantageous when it is desired to receive on ground wires.

Figure 247 shows in elevation the electric field near the earth's surface at a given instant in the vicinity of a point P, distant several

wave lengths from the radio transmitter. The wave is shown traveling from left to right. The curve in the lower part of the figure shows that the electric field is varying sinusoidally both in intensity and in direction. For example, point P is located where the electric field intensity is zero for the instant, but at the same time is changing most rapidly. An instant later the electric force will begin to act downwards at point P and will continue to increase in intensity until it reaches a maximum value at time $t=\frac{T}{4}$. From time $t=\frac{T}{4}$ to time $t=\frac{T}{2}$ the electric force will continue to act downwards but with decreasing intensity, finally becoming zero. From time $t=\frac{T}{2}$ to time $t=T$ the electric force will act upwards

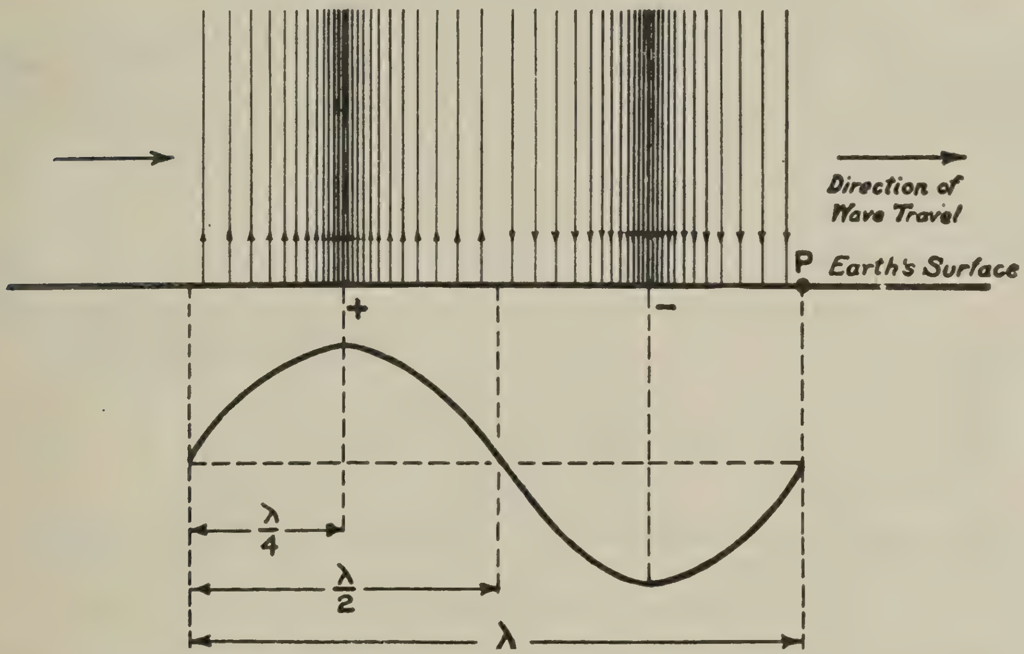


FIG. 247.—Electric Component of an Advancing Radio Wave.

and increase from zero intensity through a maximum and then decrease to zero. This occurs during each cycle or period, the strain being propagated a distance of one wave length λ during the interval. An idea of the sweeping and periodically varying electric field can be obtained by placing the finger on point P and moving the page under the finger in the direction shown by the arrow.

The receiving antenna. It will be remembered that an electric displacement can exist only in dielectrics, and is a strain in the dielectric because the electrons are not free to move from atom to atom as in a conductor. Hence, when a conductor is placed in a dielectric in which an electric force is acting, it affords a path for the charges, which are then able to move freely and the strain is relieved. This will occur when the conductor is **in any position other than at right angles** to the direction in which the electric displacement is acting. The difference in potential of the points between which the conductor lies, causes an emf in the

conductor and, as the potentials of the two points are varying both in value and sign due to the advancing wave, the emf acting in the conductor also varies, the variation being **in phase with the varying electric displacement.**

The intensity of the electric field of the radio wave at any point can be assumed to be uniform from the earth's surface to the heights ordinarily used in receiving antennas.

Thus, when an antenna is used for receiving, the emf induced in it by the advancing wave is equal to the electric field intensity at that point multiplied by the effective height of the antenna, or

$$E = \mathcal{E} h_r$$

where E = induced emf in receiving antenna in microvolts,
$\mathcal{E}$ = electric intensity in microvolts per meter,
h_r = effective height of receiving antenna in meters.

It will be seen that the emf induced in the receiving antenna is **independent of tuning,** and is equivalent to the emf that would be induced in

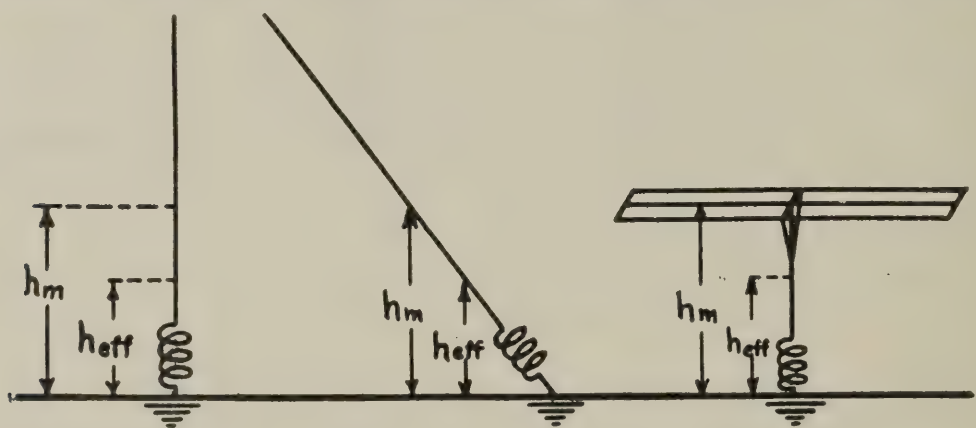


FIG. 248.—Various forms of Antennas used for Receiving Radio Waves.

a conductor parallel to the direction of the electric displacement and having a length equal to the effective height of the antenna.

Figure 248 shows three forms of antennas commonly employed for receiving purposes, namely: (*a*) the vertical wire, (*b*) the inclined wire and (*c*) the flattop antenna. Modern practice requires that the antenna be loaded, that is, the antenna is not used at its fundamental wave length. Assuming this condition, the height to the center of capacity in cases (*a*) and (*b*) will be the same as the mean height h_m Case (*c*) shows the advantage gained by the use of the flattop; there the height to the center of capacity is also equal to the mean height, but is further equal to the actual height to the flattop. The three antennas are drawn to scale, and a given electric field intensity with a vertical wave front should produce the same effect in all three antennas.

Example:

The calculated electric field intensity at a given point is 55 microvolts per meter. What emf will be induced in an antenna having an effective height of 25 meters?

Solution:

Formula $E = \mathcal{E} h_r$

substituting $= \frac{55\mu v}{m} \times 25m = 1{,}375\mu v$

whence $E = 1{,}375$ microvolts.

The advancing wave probably does not induce an emf in the horizontal part of the antenna, unless the wave front is considerably bent. Under these conditions the flattop will become effective, because it is no longer at right angles to the direction of the electric displacement. **The main purpose of a flattop in a receiving antenna is to increase the effective height and thereby increase the value of the induced emf.**

Thus, on the assumption that the electric displacement, due to the advancing wave, acts perpendicularly to the earth's surface, the resulting effect is that the difference of potential, which causes an emf in the receiving antenna, is that which exists between its highest and lowest points **measured vertically,** this actual vertical distance being further modified by certain factors which give the effective height. In other words, **all types of elevated receiving antennas** (not loop antennas) **are probably nondirectional when the wave front is vertical.**

Another condition exists, however, when the wave front is bulged forward, due to poorly conducting ground in the vicinity of the receiving antenna. An antenna having a flattop long in proportion to its vertical part will show some directional characteristics, receiving signals best which are traveling in the plane of the antenna and in the direction in which the open end of the antenna points. Signals are also poorest which come from the opposite direction. Figure 249 (*a*) shows the action of the electric displacement on such an antenna at a given instant. The emf e_1 induced in the flattop is acting in the same direction as the emf e_2 induced in the vertical part and maximum signal is received. In figure 249 (*b*) e_1 and e_2 are in opposition and minimum signal is received. When the wave comes from either side, the antenna acts the same as if the wave front were not tilted and the signal has a value intermediate between that obtained as just described.

Remembering that the wave front is tilted forward, perhaps as much as 45° when the wave is traveling over deep, dry sand, a long insulated wire having practically no vertical height, but laid directly on the sandy soil, or buried a few inches, becomes an efficient receiving antenna. In this case, the long wire is not at right angles to waves traveling in the direction of its length, and receives equally well waves coming from either direction along its length, and practically nothing from those crossing it at right angles. It is usual to place the tuning apparatus at the center of the wire. This apparatus can be placed at either end, and a connection run to ground water level; the antenna then approaches the

form given in figure 249. See figure 222 and text for the resolution of electric displacment into components.

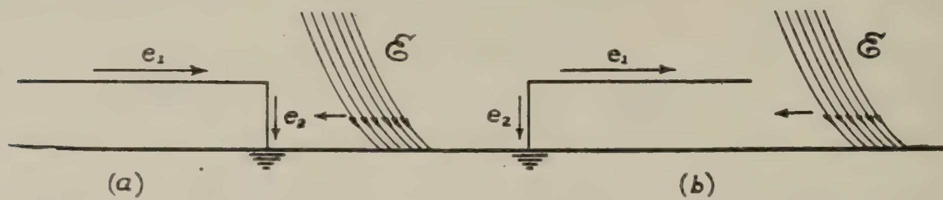


FIG. 249.—Directional Characteristics of a Long, Low Antenna.

Received current in antenna. Due to the alternating emf induced in the antenna there will be an alternating current. As in the case of the transmitting antenna, the current will be negligible in value unless the circuit is in resonance with the driving emf, because it is equal to the emf acting in the circuit divided by the impedance of the circuit, or

$$I_r = \frac{E}{Z}$$

For all conditions other than that of resonance, the impedance will be very large. However, when the antenna circuit is tuned to resonance with the wave to be received, the inductive reactance X_L and the capacity reactance X_C neutralize each other and only the radio-frequency resistance R remains. This is the condition of series resonance and

$$I_r = \frac{E}{R}$$

where I_r = received current in microamperes,
E = induced emf in microvolts,
R = radio-frequency resistance of receiving antenna circuit in ohms.

The current I_r is now in phase with the induced emf E and, therefore, in phase with the electric displacement of the advancing wave. The necessity for keeping the resistance of the antenna circuit as low as possible is at once apparent, because only in this manner can the received current be increased in a given antenna with a given electric field intensity.

The following example shows the necessity for tuning the antenna circuit to resonance with the wave to be received. It also shows why interference from transmitters sending on wave lengths other than that on which the desired signal is being received is greatly reduced.

Example:

Three continuous-wave transmitters, each producing an electric field intensity $\mathcal{E}$ of 55 microvolts per meter at a given receiving antenna having an effective height h_r = 25m., are transmitting on wave lengths $\lambda_1 = 725$m, $\lambda_2 = 750$m. and $\lambda_3 = 756.5$m. The desired signal is on λ_2 and the antenna is tuned to resonance to this wave length by using $L = 188.8\mu$h and $C = 838\mu\mu$f. The radio-frequency resistance of the antenna at $\lambda_2 = 750$m. is 7.9Ω. Calculate I_r for each wave length.

Solution:

Formula $E = \mathcal{E} h_r$

$= 55 \times 25 = 1{,}375$

whence $E = 1{,}375$ microvolts.

This is the emf induced in the antenna by each of the transmitters.

Formula $Z = \sqrt{(X_L - X_C)^2 + R^2}$

It will be necessary to calculate X_L and X_C for each wave length from the formulas

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

The values thus obtained are:

		X_L	X_C	Z
for	$\lambda_1 = 725$m	$4.91 \cdot 10^2 \Omega$	$4.59 \cdot 10^2 \Omega$	32.96Ω
	$\lambda_2 = 750$m	$4.75 \cdot 10^2 \Omega$	$4.75 \cdot 10^2 \Omega$	7.90Ω
	$\lambda_3 = 756.5$m	$4.70 \cdot 10^2 \Omega$	$4.79 \cdot 10^2 \Omega$	11.98Ω

Formula

$$I_r = \frac{E}{Z}$$

whence

		I_r
for	$\lambda_1 = 725$m	41.7 μa
	$\lambda_2 = 750$m	174.0 μa
	$\lambda_3 = 756.5$m	114.8 μa

The solution shows that, in the case of the continuous-wave transmitter operating on $\lambda_1 = 725$m. which is only 3.3% below the wave $\lambda_2 = 750$m. to which the receiving antenna is tuned, the received current is reduced to 24 per cent of that obtained from the desired signal. Also the wave $\lambda_3 = 756.5$m., which is but 0.9 per cent longer than λ_2, produces a received current that is only 70 per cent of that obtained from the resonant wave λ_2. The results given above are due to **wave selection by radio-frequency tuning,** which forms the basis of all modern radio reception. Greater selectivity and more freedom from interference can obviously be obtained by reducing the radio-frequency resistance of the receiving antenna or loop antenna to the lowest possible value. In this manner, the received current due to the wave to which the antenna is resonant will be increased in proportion to the decrease in the radio-frequency resistance as shown by the formula.

$$I_r = \frac{E}{R}$$

and, at the same time, there will be practically no change in the intensity of the interfering signals on neighboring wave lengths. Thus, if R in the above example is reduced to approximately one-half, or 4 ohms, the values of received current due to the three waves would be

		I_r
for	$\lambda_1 = 725\text{m}$	43.1 μa
	$\lambda_2 = 750\text{m}$	344.0 μa
	$\lambda_3 = 756.5\text{m}$	149.2 μa

and the desired wave would produce an overwhelming signal. Further selectivity can be obtained by the use of a circuit coupled to the antenna and containing the detector, and which is tuned to resonance with the desired signal. Should either of the interfering waves have a large effective decrement, the advantages gained by selective tuning would be nullified in part.

Antenna to antenna transmission. If the effective height of the receiving antenna and the electric field intensity are known, the received current can be calculated from the formula

$$I_r = \frac{\mathcal{E} h_r}{R}$$

Substituting in the above formula the value of $\mathcal{E}$, for antenna transmission, the formula for the received current when **antennas are used both for transmitting and receiving** is obtained. It is

$$I_r = 3.77 \cdot 10^8 \frac{I_s h_s}{\lambda d} \cdot \frac{h_r}{R} \cdot \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

where I_r = received current in microamperes,
I_s = current in transmitting antenna in amperes,
h_s = effective height of transmitting antenna in meters,
h_r = effective height of receiving antenna in meters,
R = radio-frequency resistance of receiving antenna circuit in ohms,
d = distance between stations in meters,
λ = wave length in meters,
$\epsilon = 2.7128$.

The resistance of the receiving antenna circuit includes that due to the inductance and capacity used to tune it to resonance with the signal to be received, and may be measured by the method given in Measurement No. 9, Part 1, Section III of this Manual. The measurement of the antenna circuit resistance should be made at the **same wave length as that of the received signal, and without the secondary circuit coupled to the antenna.** The value of the antenna resistance measured in this manner should be increased approximately 70 per cent to make up for the resistance added by coupling the resonant secondary circuit at the optimum coupling position when either the crystal detector or vacuum tube is used as a plain detector. It is also probably the same when the regenerative vacuum-tube detector or radio-frequency amplifier, with or without heterodyne, is used. It has been found that the resistance of the antenna is generally not increased when the autodyne

method of reception is employed without radio-frequency amplification. It is also probable that the same applies to heterodyne reception without radio-frequency amplification.

Received power. The received current is of value mainly because, when it has been determined, the received power can then be calculated from the formula

$$P = I_r^2 R$$

where P = received power in watts,
I_r = received current in amperes,
R = radio-frequency resistance of antenna circuit in ohms.

It is the usual practice to express signal response in terms of received power.

Loop antenna to antenna transmission. When a loop antenna is used for transmitting and for an antenna for receiving, the complete formula for received current is found by simply substituting for $\mathcal{E}$ in the formula

$$I_r = \frac{\mathcal{E} h_r}{R}$$

its value for loop antenna transmission. The formula is:

$$I_r = 2.369 \cdot 10^9 \frac{I_s l_s h_s n_s}{\lambda d} \cdot \cos\phi_s \frac{h_r}{R} \cdot \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

in which the units are the same as have previously been used. The attenuation term can be omitted from the above two formulas when the transmitting and receiving stations are separated a distance of only a few wave lengths. Cos ϕ_s can also be omitted from the last formula if the plane of the transmitting loop antenna coincides with the line joining the stations.

Loop antenna used for receiving. The loop antenna, used for receiving, shows directional characteristics to a much greater degree than when it is used for transmitting; in fact, its directional characteristics are independent of the distance from the transmitting station. The strength of the received signal, however, affects the accuracy of directional determination.

Practically everything given in Chapter II of this Part on the loop antenna used for transmitting is applicable to the receiving loop antenna. The effective height and effective length are calculated in the same manner, and the same nomenclature is used. Table 8 gives the values of the equivalent antenna effective height at different wave lengths for loop antennas having various area-turns. The formula for calculating the effective height of a loop antenna

$$h_{eff} = \frac{2\pi l h n}{\lambda}$$

shows that for a given loop the effective height varies inversely as the

wave length. It is apparent, therefore, that a loop should be used at short wave lengths in order to obtain an effective height comparable to that of even a small antenna. In other words, a loop antenna is as good a collector as an antenna, only when its dimensions are approximately equal to those of the antenna.

Let it be assumed that the single turn loop antenna shown in figure 250 has a horizontal length $l=\frac{\lambda}{8}$ and vertical height h and that its plane coincides with the line joining its vertical axis and the transmitting station. The effective length of the loop will then be equal to the actual length, or $l_{\text{eff}}=\frac{\lambda}{8}$. The vertical sides AD and BC are, in effect, two vertical antennas each having an effective height h. Emfs of equal amplitude but differing in phase will be induced in each of the vertical sides, that induced in the side nearer the transmitter leading the emf induced in the farther side by an angle

$$\theta=\frac{2\pi l}{\lambda}\text{ radian}$$

when the plane of the loop coincides with the line joining the two stations. For the loop given in figure 250, where $l=\frac{\lambda}{8}$,

$$\theta=0.7854\text{ radian or }45°.$$

For any other position of the loop the length l to be used will be the effective length of the loop, which is

$$l_{\text{eff}}=l\cos\phi$$

where l_{eff} = effective length of loop in meters,
l = horizontal length of loop in meters,
ϕ = angle that plane of loop makes with line joining its axis and transmitter.

An attempt to show the out-of-phase relation of the two emfs and the resultant emf is made in figure 250. The direction of wave travel is from left to right. Curves I to V represent the position of the advancing wave in respect to the loop at successive instants having a time interval $t=\frac{T}{16}$, and beginning at the instant when the node is at side AD. The curves give the direction and intensity of the electric displacement due to the advancing wave in the vertical sides of the loop. At time $t=0$, there is no electric displacement (node) at side AD and, hence, there is no emf e_1 being induced in that side, but a certain electric displacement is acting upwards at side BC and the emf e_2 is being induced in this side.

At time $t=\frac{T}{16}$, Curve II, the node is at the vertical axis of the loop

and the induced emfs are helping, that is, e_1 is acting downward and e_2 acting upward. Curves III, IV and V are similarly explained. Curve VI shows the variation in the emf e_1 induced in side AD while Curve VII shows the same for side BC. The resultant emf $e=(e_2-e_1)$ is given in Curve VIII and in amount is equal to the emf that would be induced in an antenna of equivalent effective height, as was explained in Chapter II of this Part. At time $t=\frac{T}{16}$, the resultant emf will be maximum.

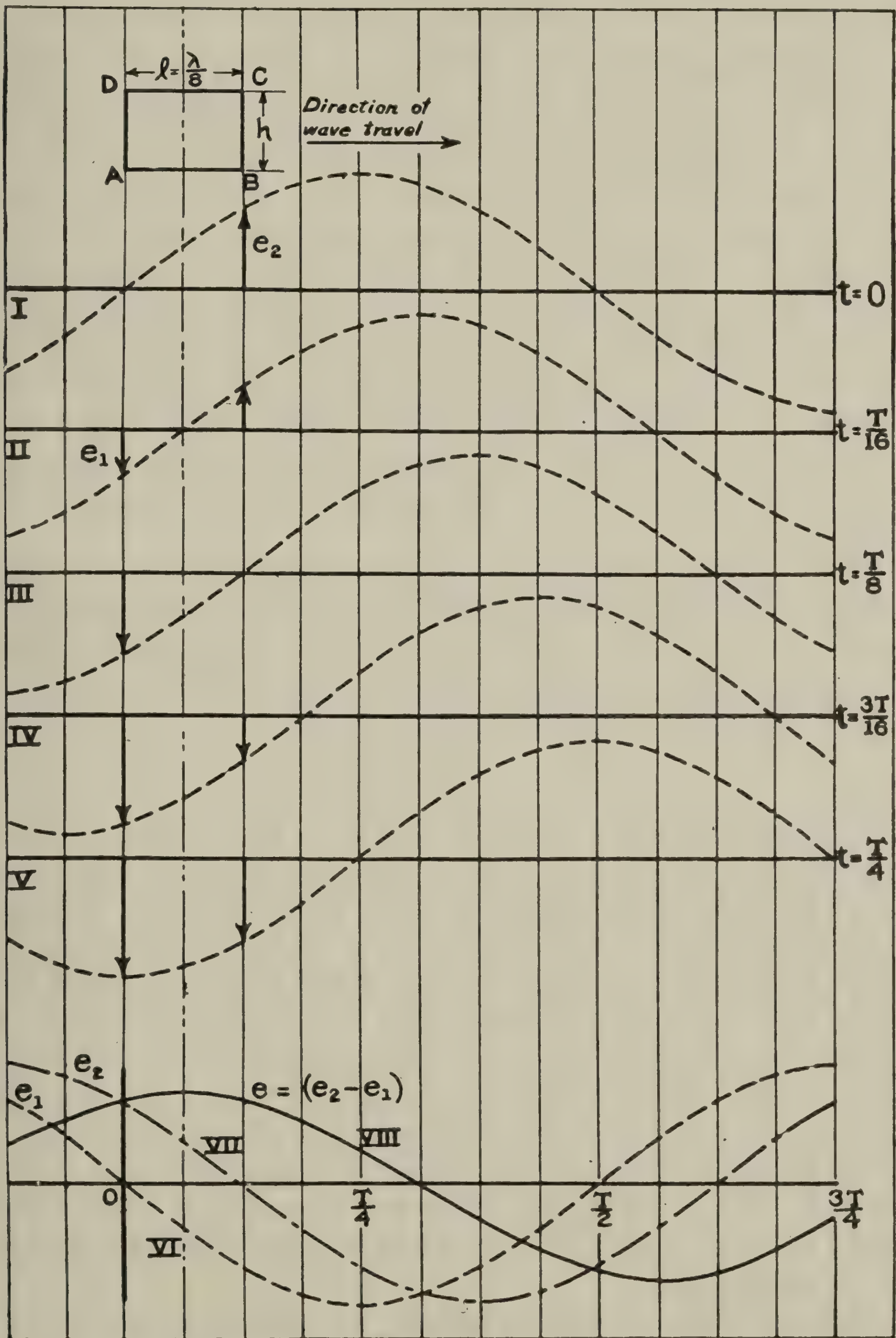


FIG. 250.—Induced Emfs and Resultant Emf in a Receiving Loop.

This is the emf that produces the received signal, and varies sinusoidally, being 90° out of phase with the electric displacement in the wave at a point midway between the vertical sides of the loop, that is, at the vertical axis of the loop. An important point to remember in this connection is that any bending of the wave front does not affect the values of the induced emfs.

Directional characteristics of receiving loop antennas. The resultant emf, produced in a loop antenna by an advancing wave, is a maximum when the plane of the loop coincides with the line joining the axis of the loop and the transmitter, that is, when $\phi=0°$ and $\cos\phi=1$. As the loop is rotated from this position, in either direction, its effective length is reduced and finally becomes zero at the instant when $\phi=90°$, or when the plane of the loop is at right angles to the direction of the advancing wave. The emfs induced in the vertical sides are always equal but, with the loop in this position, they are exactly 180° out of phase and the resultant emf is zero. Angle ϕ, due to the rotation of the loop about its axis, should not be confused with angle θ, which is determined solely by the length of the loop and the wave length. The latter fixes the maximum resultant emf that a given advancing wave can produce for any position of the plane of the loop in respect to the direction of the wave travel. The former always operates to produce a resultant emf having values between zero and the maximum just mentioned.

Thus, as a given loop is rotated through 360°, the resultant emf e produced by a fixed, nondirectional transmitter, passes through successive values which are equal to the maximum resultant emf E_0 multiplied by the cosine of the angle ϕ that the plane of the loop makes with the line joining its axis and the transmitter, or

$$e=E_0 \cos\phi$$

It is also apparent that if the receiving loop is fixed, and the nondirectional transmitter is moved in circle about the loop, the same results will be obtained. The resultant emf can be plotted in a polar diagram as shown in figure 242. Assuming that the transmitter is at point P, the distance along any radius from the center O to the point of intersection with the figure 8 represents the resultant received emf. For example, the vector OA represents the resultant emf when the loop is rotated to 30°. It will be seen that, as the loop is rotated in a counterclockwise direction from 0°, the resultant emf decreases from a maximum at 0° to zero at 90°, then increases to a maximum at 180°, again decreases to zero at 270° and then increases to a maximum as it approaches its original position. Figure 242 also indicates that the **maxima** are broad and the **minima** sharp.

Antenna effect of a loop antenna. In addition to the pure loop action described up to this point, there is the effect due to the loop and

its receiving apparatus acting as **an antenna.** As a result of this, a certain emf is produced in the loop receiving system by the advancing wave for any position of the loop antenna. The antenna effect causes a **residual signal** which destroys the sharp definition of the minima and thereby, reduces the accuracy of direction determination.

These errors and the methods used for reducing or correcting them are described in detail in Chapter VI, Part 1, Section II of this MANUAL.

Combination of antenna and loop antenna. The directional characteristics of the loop antenna, which have just been explained, may be combined with the nondirectional characteristic of the antenna. When the emfs induced by the advancing wave in these two collectors are properly combined by means of suitable circuits, a **blind** or **dark sector,** broad compared to that obtained in the minima of a loop alone, will be produced. Waves arriving through this sector will produce signals of minimum or zero strength.

This is based on the fact that, when an antenna is in resonance with the advancing electromagnetic wave, the emf induced in the antenna circuit produces a current which is **in phase** with the electric component of the wave while, in case of the resonant loop antenna circuit, the emf and current are both **90° out of phase** with the electric component of the wave. This means that the emf and resulting current in one circuit are 90° out of phase with those in the other circuit.

Now, if an inductance coil in series with the antenna circuit is coupled to one in series with the loop circuit, an emf will be induced in the loop circuit in addition to that induced in the loop by the advancing wave. The emf induced in the loop circuit by the current flowing in the antenna coil, however, is 90° out of phase with the current in the antenna, because this emf depends upon the time rate of change of the antenna current and, therefore, is either **in phase with** or **in opposition to** the emf induced in the loop antenna by the wave. If the magnitude of the emf induced in the loop circuit from the antenna circuit is **equal** to the emf acting in the loop circuit due to the wave, then, when these emfs are **in opposition** there will be no flow of current in the loop circuit and, hence, **zero signal.** Conversely, when these two emfs are **in phase and equal,** the total emf acting will be twice that of either, and the resulting current will be double that which would flow were the loop used alone. This causes a corresponding increase in signal strength. In practice, the emf induced in the loop circuit is made equal to the maximum value of the emf acting in the loop due to the wave.

The polar diagram, figure 251, shows the combined effect of both antenna and loop. The figure-of-eight characteristic of the loop and the circular characteristic of the antenna are shown in dash lines. Because of the equality in the emfs, the radius of the circle is equal to the maximum radius vector of the figure of eight. The combined effect of antenna and loop is obtained by adding the corresponding radius

vectors with due regard to sign, and is given by the **heart-shaped** or **cardioid** characteristic in solid lines either for the case of a fixed source and rotation of the loop, or for a fixed loop and rotation of the source about the system. For a given position of the source, when the plane of the loop is directed towards the source, the signal will be zero or maximum. The zero or the maximum signal can be selected at will, the change-over being effected by: (a) rotation of the loop through 180°, (b) reversing the connections of either of the coils which couple the antenna and the loop, (c) simply turning one of the coupling coils through 180°.

This peculiar characteristic of the antenna-loop system is made use of in reducing static. If static of a directional character is present and the direction of minimum reception is placed in the direction of the

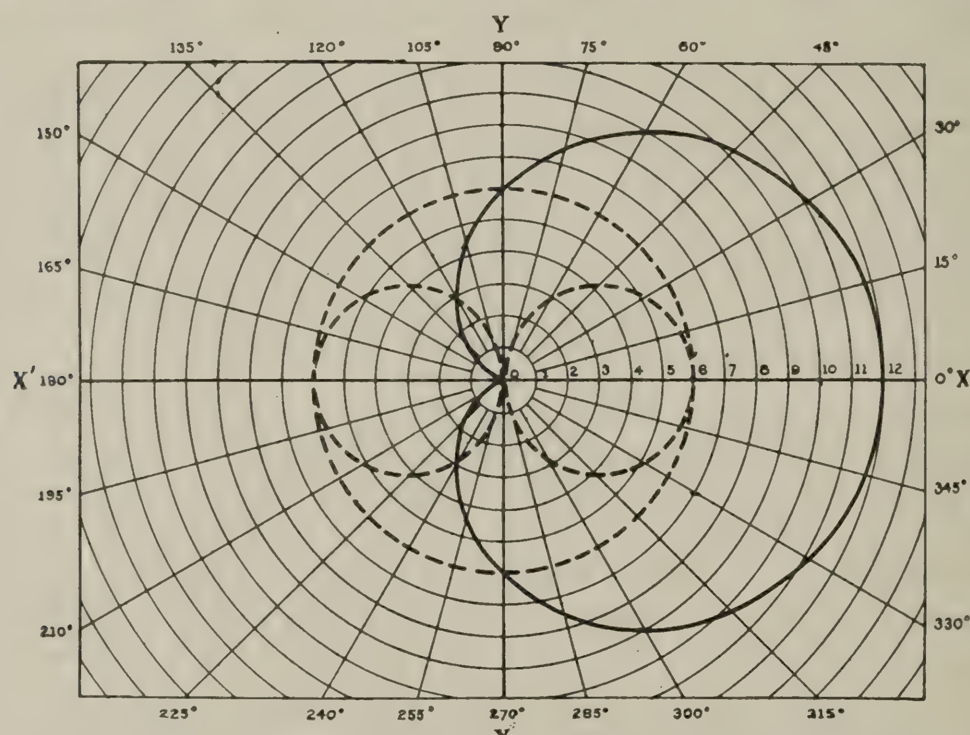


FIG. 251.—Polar Diagram showing Combined Effect of a Nondirectional Antenna and a Loop used in Receiving.

source of static, almost complete elimination of static is secured, and any signal coming from approximately the opposite direction is increased. Should the source of both static and signal lie in the same direction, it will readily be seen that a reduction of the static will result in a reduction of the signal strength. In other words, the ideal condition for static reduction would be a separation of 180° between the source of the static and that of the signal.

The antenna-loop system can be used for directional receiving, for which use it is superior to the loop alone in that the signal is increased in strength when the plane of the loop is pointed in the direction of the transmitter, with a reduction of static or of interfering signals, provided

that the approximate source of either is not in the same direction as that of the signal being received.

The antenna-loop system, when calibrated, may also be used to find the true direction of the source of static or of transmitters. Although the zone of silence is not nearly so sharply defined as it is when the loop is used alone, it has an important advantage over the loop in that it has only one zone of minimum signal, which is displaced 180° from the zone of maximum zero. There is, therefore, no ambiguity of 180°.

Antenna to loop antenna transmission. When an antenna is used as the transmitter and a loop antenna as the receiver, the formula for received current is obtained by substituting the expression for the electric field intensity $\mathcal{E}$ produced by an antenna

$$\mathcal{E}=3.77 \cdot 10^{8}\frac{I_s h_s}{\lambda d}\cdot \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

and the expression for the effective height h_r of a loop antenna

$$h_r=\frac{2\pi l_r h_r n_r}{\lambda}\cos\phi_r$$

in the formula for received current

$$I_r=\mathcal{E}\frac{h_r}{R}$$

The formula is

$$I_r=2.369 \cdot 10^{9}\frac{I_s h_s l_r h_r n_r}{\lambda^2 d R}\cdot \cos\phi_r \cdot \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

As before, $\cos\phi_r$ and the attenuation term can be omitted when angle $\phi_r=0°$ and the distance d is equal to one or two wave lengths. It will be noted that the **received current varies inversely at the square of the wave length.**

Loop antenna to loop antenna transmission. The formula for the received current when a loop antenna is used to receive from a transmitting loop antenna, is derived by substituting the expression for the electric field intensity $\mathcal{E}$ produced by a loop antenna, and the expression for effective height h_r of a loop antenna in the formula

$$I_r=\mathcal{E}\frac{h_r}{R}$$

It is

$$I_r=1.488 \cdot 10^{10}\frac{I_s l_s h_s n_s l_r h_r n_r}{\lambda^3 d R}\cos\phi_s\cos\phi_r \cdot \epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

The nomenclature is the same as has previously been used. Cos ϕ_s and $\cos\phi_r$ can be omitted when angle $\phi_s=\phi_r=0°$ and, when $d=\lambda$, the attenuation term can also be omitted. It will be seen that **the received current varies inversely as the cube of the wave length** and, therefore,

decreases very rapidly as the wave length is increased, other things being equal. On this account, transmission from a loop antenna received on a loop antenna is most satisfactory when short waves are used.

Atmospheric disturbances. Atmospheric disturbances, or **static,** form the chief obstacle to the progress of radiotelegraphy and radio telephony, giving rise to troublesome noises in the telephones which frequently make the reception of signals by ear impossible and, in the case of reception by the various recording devices, render the record unintelligible.

Atmospheric disturbances are extremely variable in intensity, depending upon:

(a) The time of day—generally being more pronounced during the late afternoon and night;

(b) The season of the year—being most frequent and violent in the summer;

(c) The locality—being most severe in the tropics and where the temperature differences produce ascending air currents;

(d) The wave length—generally increasing with increase in wave length.

Some localities are favored with a minimum of atmospheric disturbances the year round. Regions in the Pacific Ocean and the west coast of Europe are markedly free from these disturbances. Receiving conditions are also generally considered better out at sea than on land.

Atmospheric disturbances, in general, are probably produced in the upper atmosphere by readjustments of the electric potentials. The resulting electric waves expand in a more or less spherical manner until the lower portions of the wave front strike the earth, when the lines of electric force become grounded and travel outwards from the point of contact, guided by the earth, with a practically vertical wave front exactly like radio waves produced by an airplane radio transmitter. They probably act upon the receiving system by shock, and usually seem to have no definite wave length. However, they appear sometimes as short wave trains having a more or less definite wave length.

Careful observation of these disturbances indicates that they can be divided into two classes according to their characteristic sounds. The **first class** is characterized by a crashing sound followed by a rumbling of more or less duration. Disturbances of this character seem to come generally from definite disturbance centers which frequently appear to lie above mountainous regions. Some of these centers send out waves of tremendous energy, as for example, the one which is believed to lie in southern Mexico, and which appears to furnish most of the disturbances along the Atlantic Coast of the United States. Their intensity generally increases with increasing wave length. Secondary centers of disturbance, such as local clouds or nearby mountain peaks, also act upon the receiving system, causing additional difficulties in receiving.

The **second class** of atmospheric disturbances mentioned in the paragraph above has a crashing sound without rumbling. It is thought that possibly this type may be connected with sun spots and magnetic storms. It is possible that streams of electrons are emitted by the sun during these periods and strike the earth's upper atmosphere thereby giving rise to more or less violent disturbances. These do not seem to come from any definite direction.

Atmospheric disturbances, due to thunderstorms in the immediate vicinity of the receiving station, will obviously prevent reception except from nearby and powerful transmitters. In the case of a particularly violent local thunderstorm, it is best to ground the antenna as soon as sparking occurs at the safety gap on the receiver and thereby reduce the danger to personnel and prevent the apparatus from being burned out by lightning or heavy currents.

In addition to the atmospheric disturbances already described, there are the currents which are produced in antennas and loops located outside the receiving building by the direct discharge of electricity from particles of dust, water-vapor, rain or snow coming in contact with the wires. This type of disturbance usually has a hissing sound and will cause sparking between the plates of a condenser connected in series with the antenna. The periodic sparking can be eliminated by shunting the condenser with a high resistance, thus preventing the charge from accumulating on the antenna until the voltage is sufficient to cause sparking. Sometimes stray currents flowing either up or down in the antenna are present. These currents are frequently very troublesome when measurements are being made using a thermoelement and galvanometer in the antenna circuit. They are due to the electric gradient of the atmosphere in the vicinity of the antenna.

PART 8.

RADIO ACCESSORIES

CHAPTER I. INDUCTANCE COILS.

General. A coil of wire wound in such a manner as to possess the property of self-induction is called an **inductance coil.** It is apparent that such a coil must have all its turns insulated from each other and be wound in the same direction in order to obtain a maximum inductive effect. The properties of self-induction and mutual induction have been previously explained. Coils of wire can be wound in such a manner that no inductive properties are present, but this Chapter deals only with inductance coils.

Inductance coils are used in radio circuits in conjunction with suitable condensers to obtain desired wave lengths, also to induce an emf of a given frequency from one circuit into another by coupling one inductance coil to another. The self-induction, or inductance, of a coil depends upon the size of the coil, the size of the wire and the manner in which the coil is wound.

General Types. Inductance coils used in radio are divided into two groups, namely: **receiving inductance coils** and **transmitting inductance coils.** Receiving inductance coils are used in receiving circuits of low voltages, generally as inductive couplers or loading inductances. Transmitting inductance coils have a similar use in transmitting circuits, and are, therefore, built to withstand large currents and high voltages.

Receiving inductance coils. This type of coil usually consists of a number of turns of insulated wire wound closely together on a cylinder, or tube, of insulating material. Small magnet wire or litzendraht (finely stranded wire cable) is generally used. Nearly all receiving inductance coils are wound in one of the following four forms:

(a) Single-layer wound,
(b) Bank wound,
(c) Multiple-layer wound,
(d) Spiderweb winding.

These types of winding are shown in figure 252.

Figure 252, (*a*) shows the single-layer type, usually found in short wave receivers; (*b*) and (*c*) show the bank-wound and multiple-layer coils, respectively. The latter types of coils are used in long wave reception, where a large inductance is required in a limited space. The bank winding shown in (*b*) is often used in wavemeters, or other low resistance radio-frequency circuits, because of its small distributed capacity.

Transmitting Inductances. The transmitting inductance coil is generally wound in several different forms. Since there are different types of transmitters, each requires special forms of inductance. Inductance coils in spark transmitters are usually made of large copper or brass tubing, or strip, wound on an insulated form. The spacing between the windings should, of course, be large enough to withstand high radio-frequency voltages. Flat spiral wound inductance coils, figure 253 (*a*) are used in most of the modern transmitters. Edgewise-wound coils, or helices wound with very wide strips, are inefficient and should never

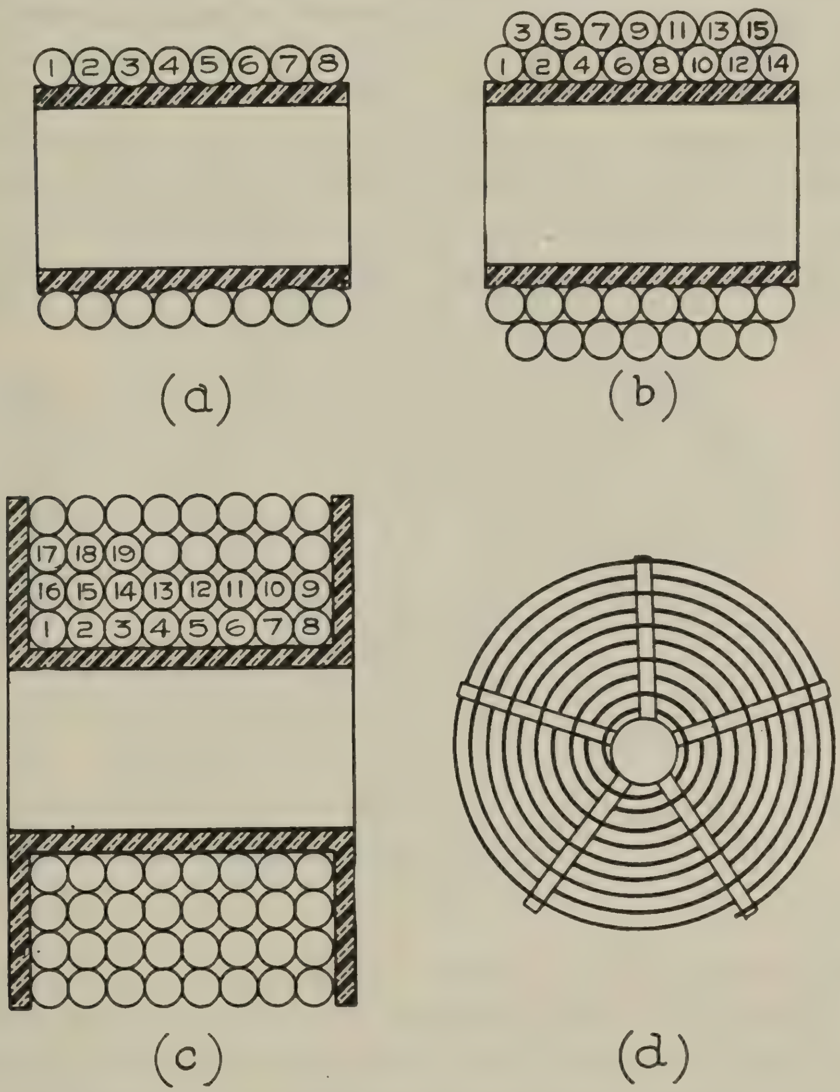


Fig. 252.—Various Types of Inductance Coils Used in Receiving Circuits.

be used. Figure 253 (*b*) shows an edgewise-wound helix. In this form of inductance, the current concentrates on the inner side of the windings, thus decreasing the inductance of the coil.

Inductance coils used in low and medium power vacuum-tube transmitters are often wound in a similar manner to the receiving inductance coils previously described, with the exception that they are wound with heavily insulated litzendraht capable of withstanding high radio-frequency voltages and of carrying the required current. Arc trans-

mitters use radio-frequency coils wound in helical or spiral form with heavy litzendraht, the turns being spaced several inches apart to prevent arcing or flashover between turns.

It has been found in practice that, in receivers designed to receive very short waves, solid wire gives better results, while litzendraht gives better results on the longer waves. This is also true in the case of low and medium power transmitting inductances.

Variable Inductances. It is often necessary in a radio circuit to use an inductance coil whose inductance can be varied in a continuous manner. Such a coil is called a **variometer** or **vario-inductor.** It consists generally of two concentric coils one of which is stationary, while the other rotates inside the stationary coil, as shown in figure 254. The two coils are connected in series, and the inductance is varied by rotating the movable coil, as is explained in Part 2, Chapter VII. By making the space between the windings of the two coils as small as possible, a large variation of inductance is possible. Care should be taken in the construction of variometers to reduce dielectric losses to a minimum.

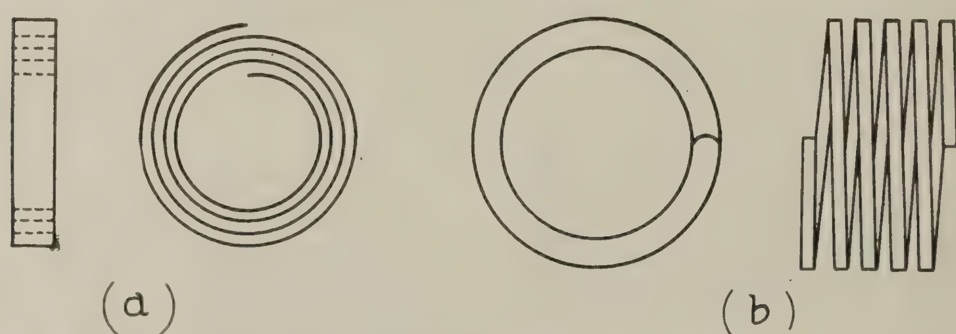


FIG. 253.—Types of Strip Inductances (a) Flat Spiral; (b) Edgewise-wound.

The type of winding, such as the basket-weave, the spiderweb, or other types, in which the amount of dielectric constituting the coil forms is reduced and where the coil capacity is largely through air, will have lowest dielectric losses.

Tapped Inductances. Where a continuous change in inductance is not necessary, but where a number of inductance values are required from the same inductance coil, resort should be made to the use of tapped inductances. A **tapped inductance** may be described as being an inductance coil of a certain number of total turns which has wire taps connected to intermediate turns: for example, on a coil having 50 turns, taps may be taken off at the 20th and 35th turns. Special switching arrangements are generally provided for use with tapped inductances, so that the unused turns of wire can be either short-circuited or open-circuited so as to have the least deleterious effect at the wave length to which the used portion is tuned.

Distributed Capacity of Inductance Coils. All types of inductance coils possess capacity in addition to the inductance of the coil. This

capacity is composed of a number of capacities which exist between turns of wire on the coil, between the ends, between taps or binding posts on the coil, or between the high-potential end of the coil and the walls of the room, or grounded bodies.

The capacity of an inductance coil is called the **distributed capacity** C_0 of the coil. This capacity almost always includes some poor dielectric in its electric field, the poor dielectric being the necessary insulation between the turns. The distributed capacity can, therefore, be considered as an **imperfect condenser,** and the same methods can be applied to reduce dielectric losses, as in the case of condensers. Using the same reasoning, the bad effect of distributed capacity is most noticeable when the capacity connected across the coil terminals for tuning

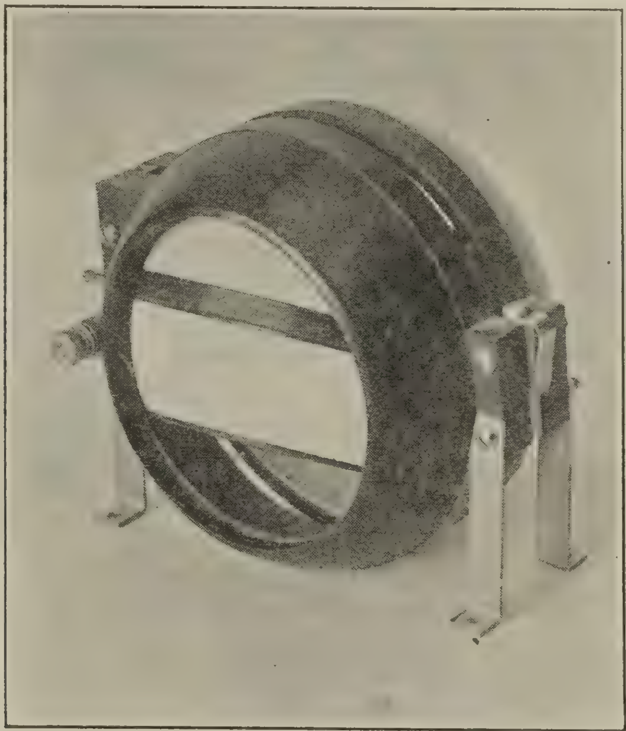


FIG. 254.—The Variometer.

purposes is small. The use of good dielectric winding forms, banked and special type multi-layer windings (honeycomb, etc.) produce inductances which have very small and low-loss distributed capacities.

The distributed capacity of a coil cannot be ignored when the coil is used in a radio-frequency circuit, nor can the coil be considered as a pure inductance, but must be treated as if it were an inductance with a small capacity across its terminals. Each coil, therefore, has a definite resonant frequency even when nothing is connected across its terminals, this resonant frequency being determined by the pure inductance L and the distributed capacity C_0. This is shown in figure 255, where the distributed capacity can be considered as a capacity C_0 across the whole coil.

The reactance and apparent inductance of such a combination will vary with the frequency, both increasing as the resonant frequency of the coil is approached. Therefore, in designing an inductance coil for use in a radio circuit over a wide range of frequencies, care should be taken to use a form of winding which has a small distributed capacity.

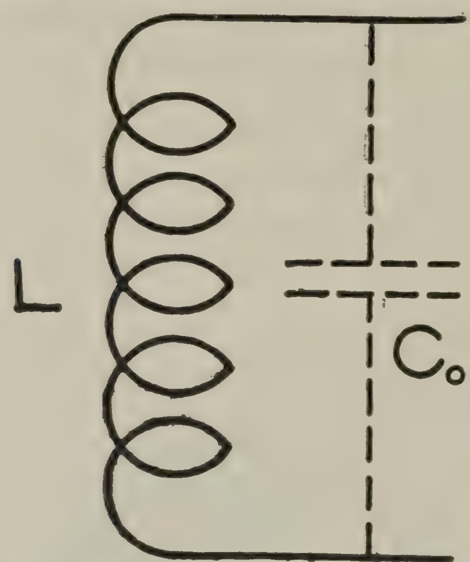


FIG. 255.—Distributed Capacity of Inductance.

Effect of Dead-End Turns. Difficulty will be encountered as a result of the distributed capacity if only a portion of an inductance coil is connected into circuit, with the remaining dead-end turns hanging on or, disconnected, still remaining in the field of the active portion of the coil. For example, in figure 256, the resonant frequency of the

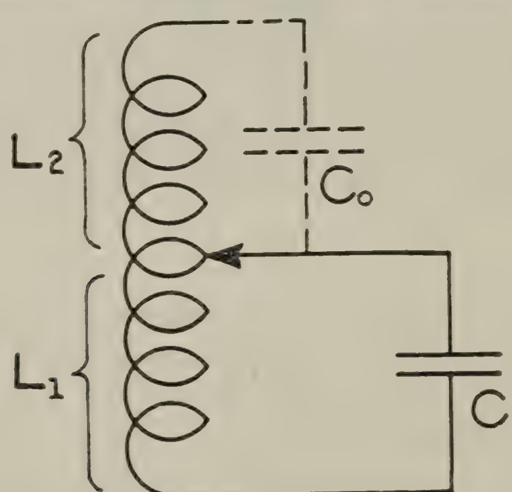


FIG. 256.—Effect of Distributed Capacity and Dead End Turns.

combination L_2C_0 may be so near the frequency of the operating circuit L_1C that an appreciable transfer of power will take place, and the apparent resistance of the operating circuit greatly increased. The remedy is to short-circuit the unused turns, which will destroy this resonant condition. Because of the difficulties mentioned above, it is

desirable, when convenient, to use separate and removable coils, when another value of inductance is needed, in such circuits as wavemeters, laboratory circuits, etc. The coils not in use should be removed entirely out of the field of the coil, or coils, in use.

Choice of material for coil forms. Since the insulation on the wire and the material used for the coil form constitute the imperfect dielectrics in the distributed capacity of a coil, it will be seen from the remarks under Condensers in Chapter II, that it is very important to use material which will introduce the lowest losses, and to reduce the material in the forms to a minimum. For this reason, winding a single-layer coil on a form built up of a number of narrow insulating strips is better than winding it on a cylindrical tube.

Choice of coil form and winding. The emf impressed on an inductance coil is divided among the turns, so that if a coil has n turns, each turn has $\frac{1}{n}$th of the total emf acting on it. Now, the distributed

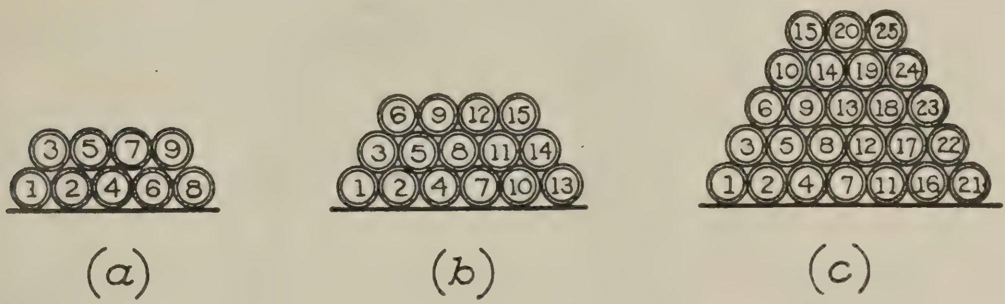


Fig. 257.—Sequence of Turns in Bank-Wound Coils of (a) 2 Layers, (b) 3 Layers and (c) 5 Layers.

capacity of a coil is increased by any method of winding in which turns, between which there is a large difference of potential, are brought close together. Thus, in figure 252, the single-layer coil (a) will have a small distributed capacity, because the potential difference between **adjacent turns** is $\frac{1}{n}$th of the total between the ends of the coil, which are widely separated.

The multi-layer coil shown in (c) has a large distributed capacity, because adjacent turns have a potential difference due to several turns. Thus, $\frac{15}{n}$ th's of the total potential difference exists between turns 1 16, which are adjacent.

The reduction in distributed capacity gained by the banked winding is apparent. For the two-layer, bank-wound coil shown in (b), adjacent turns never have a potential difference greater than that due to three turns. In a three-layer coil, this potential difference will be that due to four turns, etc.

The bank-wound coil is especially good when a large inductance in small volume is desired, together with small distributed capacity. For example, a coil having an inductance of 25 millihenries and a capacity of 12$\mu\mu$f can be wound on a 6-inch tube, six inches in length.

Figure 257 shows the sequence of turns for coils having (*a*) two layers, (*b*) three layers and (*c*) five layers. A greater number of layers is not recommended. The method of winding is shown in figure 258. Turns 1 and 2 are wound side by side. Turn 3 is kinked a little to pass from the form to between turns 1 and 2. Turn 3 is then carried up over and between 1 and 2 to the point at which it was kinked, when it is again kinked to pass down to the form, where it becomes turn 4. This process is then repeated to the end of the coil. Such windings can be made on threaded coil forms to prevent the lower layer from slipping.

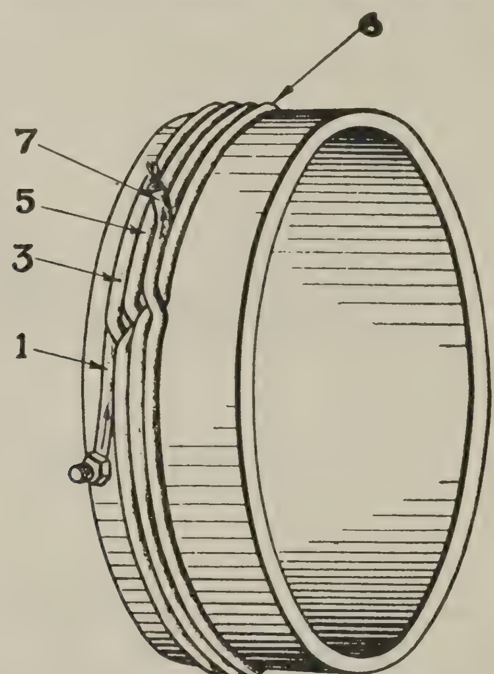


Fig. 258.—Method of Winding Bank-Wound Coils.

The distributed capacity and the losses due to poor dielectrics in the electric field of coil can be further reduced by the method of winding shown in figure 252 (*d*). This reduction is effected by spacing the turns in air. The same method of winding can be applied to the single-layer coil.

The reduction in distributed capacity and dielectric losses in a coil effected by careful design will be largely offset if such a coil is mounted very close to a poor dielectric, such as wood.

Design formulas. The formulas given below are intended for general use in radio work, where accuracy to within a few per cent is sufficient. If greater accuracy is desired, the formulas and tables given in the Bureau of Standards Circular No. 74 and in the Bureau of Standards Scientific Paper No. 169 should be employed.

Single-layer coil. The inductance of a single-layer coil, figure 259, is given by the following formula:

$$L=\frac{0.0395\ a^2n^2}{b}K$$

where L = inductance in microhenries,
n = number of turns of winding,
a = mean radius of the coil in cms.,
b = length of coil in cms. $= nD$
D = distance between centers of two adjacent turns,
K = shape factor of coil, as obtained from ratio $\frac{2a}{b}$ in Table 19.

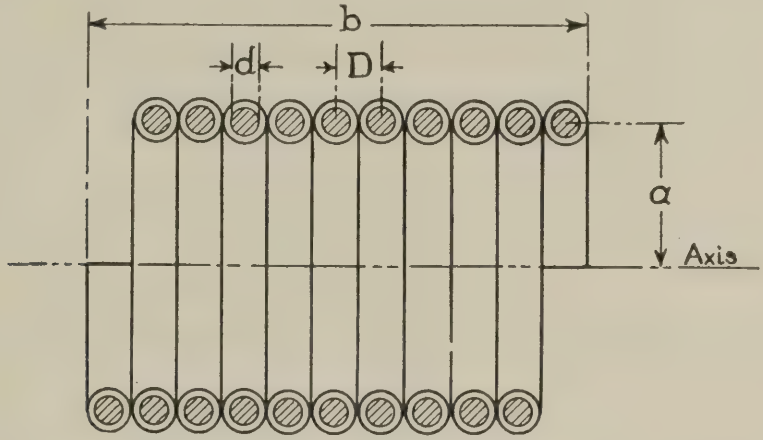


FIG. 259.—Dimension Used in Calculating the Inductance of Single-Layer Coils.

Example:

A coil has 300 turns of wire in a single layer, with a mean radius (radius of coil to center of wire) of 10 cms., and a winding pitch of 0.1 cm. Calculate the inductance.

Solution:

$a=10$; $b=nD=30$; $D=0.1$; $\frac{2a}{b}=0.666+$. The value of $K=\frac{2a}{b}=0.666+$ found in Table 19 is 0.7691.

Formula $L=\frac{0.0395\ a^2n^2}{b}\cdot K$

substituting $=\frac{3.95\cdot 10^{-2}\times 1\cdot 10^2\times 9\cdot 10^4}{3\cdot 10^1}\cdot 7.691\cdot 10^{-1}=9.114\cdot 10^3$

whence $L=9{,}114\ \mu h$

Helix, flat conductor, wound edgewise. See figure 260.

For such a coil the inductance in microhenries is given by

$$L=\left(\frac{0.0395\ a^2n^2}{b}\cdot K\right)-\left(\frac{0.0126\ acn^2}{b}\right)$$

where n = number of turns,
a = mean radius of coil in cms.,

D = distance between centers of two adjacent turns in cms.,
b = length of coil in cms.,
c = width of strip in cms.,
K = shape factor of coil, as obtained from ratio $\frac{2a}{b}$ in Table 19.

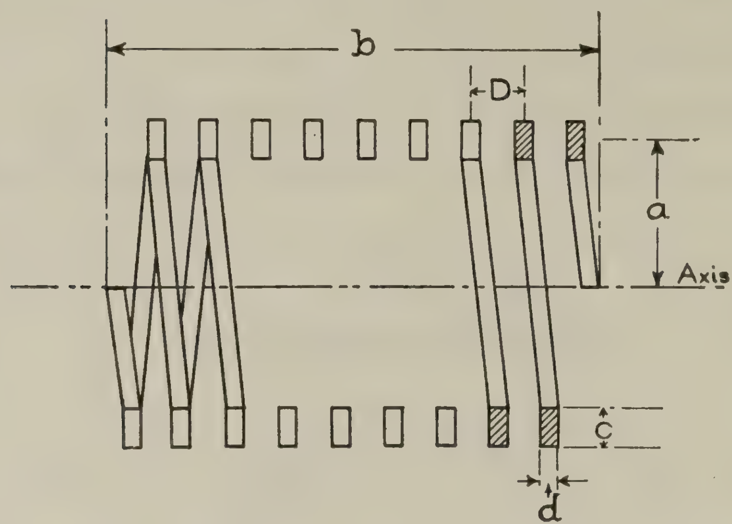


FIG. 260.—Dimensions Used in Calculating the Inductance of Edgewise-Wound Helices.

Example:

A helix is edgewise-wound with 30 turns of flat copper strip 0.635 cm. deep and 0.15 cm. thick. The winding pitch is 0.635 cm. and the mean radius of the coil is 12.7 cms. Calculate the inductance.

Solution:

$a=12.7$; $b=nD=30\times0.635=19.05$; $c=0.635$;

$n=30$; $D=0.635$; The value of $K=\frac{2a}{b}=\frac{25.4}{19.05}=1.333$ found in Table 19 is 0.623.

Formula $$L=\left(\frac{0.0395\ a^2n^2}{b}\cdot K\right)-\left(\frac{0.0126\ acn^2}{b}\right)$$

substituting $$\left(\frac{3.95\cdot10^{-2}\times1.614\cdot10^2\times9\cdot10^2}{1.905\cdot10^1}\cdot6.23\cdot10^{-1}\right)$$

$$-\left(\frac{1.26\cdot10^{-2}\times1.27\cdot10^1\times6.35\cdot10^{-1}\times9\cdot10^2}{1.905\cdot10^1}\right)$$

$$=(3.01\cdot10^2\times6.23\cdot10^{-1})-4.8=187.5-4.8=182.7$$

whence $L=182.7\ \mu h$

Multi-layer coils. For multi-layer coils made of closely wound insulated wire, where the length of winding b is greater than the depth of winding c, the following formula should be used. See figure 261. For **disk** or **pancake** coils, such as are used in some wavemeters, where b

is less than c, the formula for flat-spiral coils should be used, if very accurate results are desired.

$$L=\left(\frac{0.0395\ a^2n^2}{b}\cdot K\right)-\left(\frac{0.0126\ acn^2}{b}\right)(0.693\times B_s)$$

where n = number of turns,
a = mean radius of coil in cms.,
D = distance between centers of two adjacent turns, in cms.,
b = length of coil in cms. $=nD$,
c = radial depth of winding = distance between centers of wires times number of layers,
K = shape factor of coil from Table 19, based on ratio $\frac{2a}{b}$,
B_s = shape factor from Table 22, based on ratio $\frac{b}{c}$

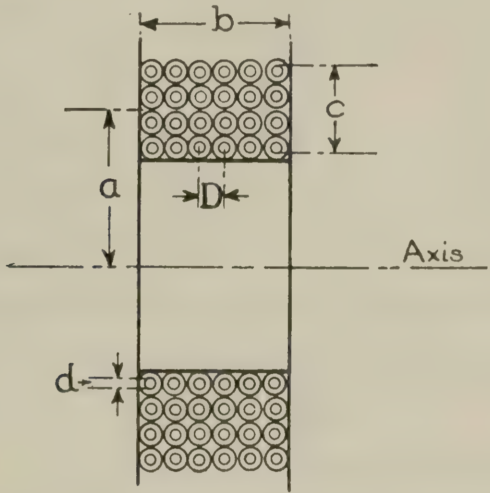


FIG. 261.—Dimensions Used in the Calculation of the Inductance of Multi-layerCoils.

Example:

A coil has 5 layers of insulated wire with 60 turns per layer; the mean radius of coil is 5 cms., the depth of winding is 0.5 cm., the length of winding is 6.0 cms. Calculate the inductance.

Solution:

$a=5$; $b=6.0$; $c=0.5$; $n=300$;

The value of $K=\frac{2a}{b}=\frac{10}{6.0}=1.666$ from Table 19 is 0.5702.

The value of $B_s=\frac{b}{c}=\frac{6.0}{0.5}=30$ from Table 22 is 0.2888.

Formula $$L=\left(\frac{0.0395\ a^2n^2}{b}\cdot K\right)-\left(\frac{0.0126\ acn^2}{b}\right)\left(0.693+B_s\right)$$

substituting $$=\left(\frac{3.95\cdot 10^{-2}\times 2.5\cdot 10^1\times 9.10^4}{6.0}\cdot 5.702\cdot 10^{-1}\right)$$

$$-\left(\frac{1.26 \cdot 10^{-2}\times 5\times 5 \cdot 10^{-1}\times 9 \cdot 10^{4}}{6.0}\right)\left(0.693+0.2888\right)$$
$$=8.446 \cdot 10^{3}-(4.73 \cdot 10^{2})(0.982)=8.446 \cdot 10^{3}-4.64 \cdot 10^{2}$$
$$=8{,}446-464=7{,}982$$

whence $L=7{,}982\mu h$.

Flat spiral coils. The formula given below is for use in obtaining the inductance of flat spiral coils, and also for disk or pancake coils wound with insulated wire, when accuracy is desired. See figure 262.

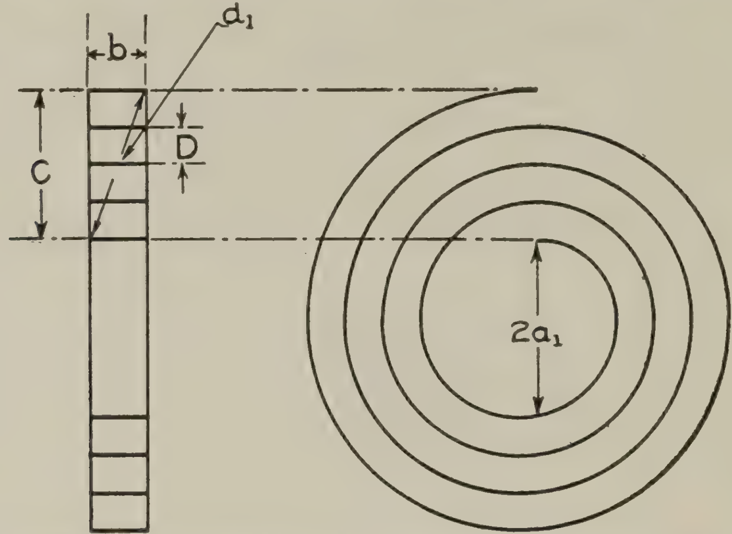


FIG. 262.—Dimensions to be Used in Calculating the Inductance of Flat Spiral Coils.

$$L=0.01257\, an^{2}\left\{2.303\left(1+\frac{b^{2}}{32a^{2}}+\frac{c^{2}}{96a^{2}}\right)\log_{10}\frac{8a}{d_1}-y_1+\frac{c^{2}}{16a^{2}}y_3\right\}$$

n = number of turns,
$2a_1$ = inner diameter of coil in cms.,
$a=a_1+\frac{1}{2}(n-1)D$ = mean radius of coil in cms.,
b = width of coil winding in cms.,
c = depth of coil winding in cms $=nD$,
D = distance between centers of two adjacent turns in cms.,
d_1 = diagonal of winding cross-section in cms. =

$\sqrt{b^{2}+c^{2}}$,

y_1 & y_3 = shape factors taken from Table 23, based upon ratio $\frac{b}{c}$.

Example:

Find the inductance of a flat spiral coil of 38 turns of copper ribbon, with an inner diameter of 10.3 cms., and a winding pitch of 0.4 cm. The copper ribbon is 0.953 cm. (3/8 in.) wide by 0.0794 cm. (1/32 in.) thick.

Solution:

$$a=\left(5.15+\frac{37}{2}0.4\right)=12.55;\ b=0.953;\ c=nD=38\times 0.4=15.2;$$

$$d_1=\sqrt{(0.953)^{2}+(15.2)^{2}}=15.23;\ D=0.4;\ n=38;\ 2a_1=10.3;\ \frac{b}{c}=0.0627.$$

Therefore, from Table 23, the values $y_1=0.5604$ and $y_3=0.599$ are obtained. Partial solutions:

$$\frac{b^2}{32a}=0.0002;\quad \frac{c^2}{96a}=0.0152;\quad \frac{c^2}{16a^2}=0.091;$$

$$\frac{8a}{d_1}=6.592, \text{ and } \log_{10} 6.592=0.819$$

Formula $$L=0.01257an^2\left\{2.303\left(1+\frac{b^2}{32a^2}+\frac{c^2}{96a^2}\right)\log_{10}\frac{8a}{d_1}-y_1+\frac{c^2}{16a^2}y_3\right\}$$

substituting $$=1.257\cdot 10^{-2}\times 1.255\cdot 10^1\times 1.444\cdot 10^3\left\{2.303(1.015)0.819-0.5604+(0.091\times 0.599)\right\}$$

$$=2.28\cdot 10^2\left\{1.915-.05604+0.055\right\}=2.28\cdot 10^2\times 1.41$$

$$=321.5$$

whence $L=321.5\ \mu\text{h}$

Single-layer square coil. The formula for calculating the inductance of a single-layer, square coil is given below. This type of coil is very generally used for small loop antennas and radio-compass coils. This formula is based upon the use of round wire and a winding in which the length b is small compared with the side a of the square. See figure 263.

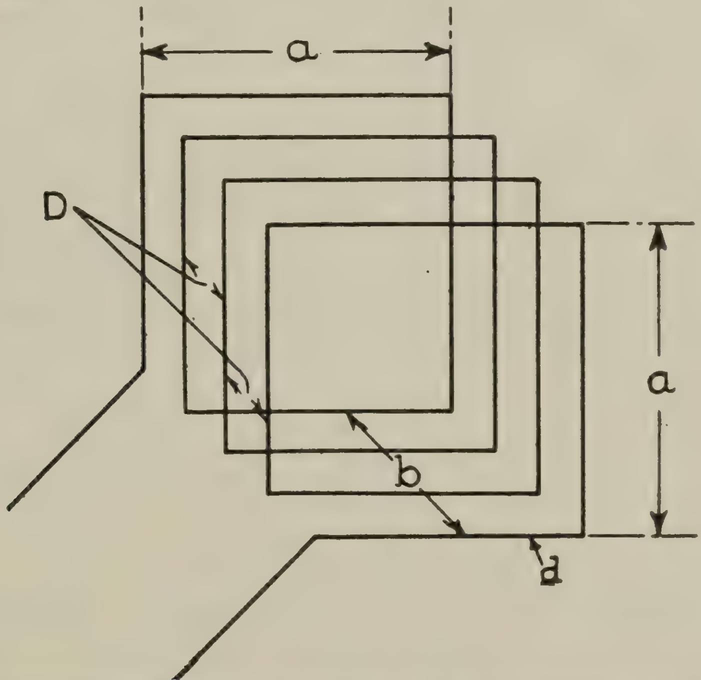


FIG. 263.—Dimensions Used in Calculating the Inductance of Single-layer Square Coils.

$$L=0.008an^2\left\{2.303\log_{10}\frac{a}{b}+0.726+0.2231\frac{b}{a}\right\}-0.008\,an(A+B)$$

where a = side of square in cms. measured to the center of the wire,

n = number of turns,

D = pitch of winding, that is, the distance between centers of adjacent wires in cms.,

$b = nD$ = length of winding in cms.,

A and B are form factors taken from Tables 20 and 21.

Example:

Find the inductance of a single-layer square coil having four turns of 0.1 cm. bare diameter, length of side = 100 cms., winding pitch = 0.5 cm.

Solution:

$$a=100;\ n=4;\ D=0.5;\ b=nD=2;\ d=0.1;\ \frac{d}{D}=0.2.$$

Entering Tables 20 and 21 with the last two quantities, the values $A=-1.053$ and $B=0.197$ are found.

Also $\frac{a}{b}=50$ and $\log_{10}\frac{a}{b}=1.699$ $\frac{b}{a}=0.02$.

Formula

$$L=0.008an^2\left\{2.303\log_{10}\frac{a}{b}+0.726+0.2231\frac{b}{a}\right\}-0.008an\,(A+B)$$

substituting

$$=8\cdot 10^{-3}\times 1\cdot 10^2\times 1.6\cdot 10^1\{2.303(1.699)+0.726+0.004\}-$$
$$8\cdot 10^{-3}\times 1\cdot 10^2\times 4(-1.053+0.197)$$
$$=12.8\{3.912+0.726+0.004\}-3.2(-0.856)$$
$$=12.8\{4.642\}+2.74=59.42+2.74=62.16$$

whence

$$L=62.16\mu\text{h}.$$

Additional coil shapes. Space is not available for inclusion of the formulas for calculating inductance of the other more special forms of inductance coils which may be occasionally required. These may be found in the Bureau of Standards Circular No. 74, or Bureau of Standards Scientific Paper No. 169.

CHAPTER II. CONDENSERS.

General. A **condenser** is a piece of apparatus arranged so as to have a large electrostatic capacity in a small volume. Its essential structure consists of two sets of metallic plates which are insulated from each other by a dielectric, or insulating substance.

The condenser is used in radio- and audio-frequency circuits. When used in conjunction with inductance, it serves to tune the circuit to resonance with some particular wave length. It is also used in circuits in which both alternating and direct currents are flowing to **by-pass** alternating currents around some portion of the circuit. It is frequently used to prevent direct current from flowing in a circuit where only alternating current is desired. When employed for this purpose it is called a **stopping condenser.**

Types of condensers. Condensers are divided into two classes, namely: **fixed** and **variable.**

The fixed condenser consists of two sets of plates, with one or more plates in each set, the two sets being insulated from each other and held rigidly in position, so that there will be no change in the capacity due to a change in the relative positions of the two sets of plates. If a gaseous or liquid dielectric is used, the plates themselves must have sufficient rigidity and strength, so that warping or bending will not occur. In this case, the two sets of plates are held in position by suitable solid insulation. To aid in obtaining rigidity, the plates are frequently placed on their edges instead of on their surfaces. The fixed condenser, with air or oil dielectric, is quite bulky when built to have a large capacity. It is mainly employed in high-voltage testing circuits on account of its low losses, and because the dielectric is self-restoring should a rupture occur.

The more common forms of fixed condensers use solid dielectrics throughout. They comprise the well-known **glass-plate condenser,** the **leyden jar** and the **mica-dielectric condenser,** the latter being the preferred form. In the first two types, the plates are formed by depositing copper electrolytically on both sides of the glass. A wide margin is left between the edge of each coating and the edge of the glass in order to prevent sparking from one coating to the other. The coatings do not need to be heavy, because they are supported by the glass. The glass-plate condenser and the leyden jar are obsolescent, being replaced by mica-dielectric condensers.

The mica-dielectric condenser is made up of alternate sheets of tin or copper foil and mica, the mica being considerably larger in area than the foil. Each sheet of foil is provided with a lug so that they can be connected together to form two sets of interconnected plates which are

insulated by the mica. The details of the construction of mica-dielectric condensers vary with the manufacturer. In order to obtain low losses and permanence, special precautions must be observed. The mica should be carefully selected and be free from scratches or other imperfections. If the condenser is to be subjected to high voltages, it is usually composed of two sets in series in order to reduce the voltage to the point where brushing will not occur. If brushing does occur, it will **etch** the mica and soon cause a break-down. The mica-dielectric condenser is usually treated in vacuum with a special wax, and then compressed so that no air bubbles or pockets will form between the sheets of mica and foil. The assembly is then placed in a metal container and the remaining space then filled with a suitable insulating compound.

Figure 264 shows a mica-dielectric condenser used for transmitting purposes. It has a capacity of 0.004μf. The connections to the two

FIG. 264.—Mica-Dielectric Fixed Condenser. Capacity 0.004 μf.

sets of plates are made one to the insulated terminal on top and the other to the case.

Mica-dielectric condensers have the following advantages:

(1) Large capacities in compact form,
(2) Rugged construction to withstand hard usage,
(3) Capacity remains very constant,
(4) Low losses.

Fixed condensers using a solid dielectric throughout are, in general, not self-restoring. A special fixed condenser for protective devices, figure 265, is constructed of mica and tin-foil. Should this condenser be punctured, the tin-foil around the puncture melts until the gap between the coatings becomes too long for the voltage, and the arcing stops.

Fixed condensers of the forms just described are used in both transmitting and receiving circuits whenever it is not necessary to vary the capacity in the circuit, or when the capacity does not need to be varied in a continuous manner, but only step by step. Very frequently a condenser consisting of several units having different or equal capacities, arranged so that they can be connected into circuit in series or in parallel, is employed when only large and abrupt changes in the value of the total capacity are required. Thus, the fixed condenser is used in the primary oscillatory circuit of a 500-cycle, quenched-spark transmitter, as a series capacity in the antenna circuit of a transmitter, in filter circuits, as a bridging or telephone condenser, as grid condenser, etc.

The **variable condenser** consists of two sets of plates, one set being stationary and the other movable. The two sets of plates are supported and insulated from each other by a solid dielectric, the dielectric between the plates themselves being either air or oil. The purpose of the variable condenser is to provide a capacity that can be varied in a continuous manner. Its construction differs only mechanically from

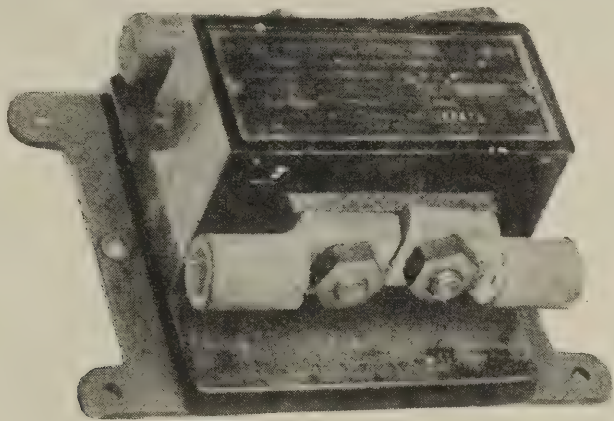


Fig. 265.—Special Type of Mica-Dielectric Condenser for Protective Devices.

that of the first type of fixed condenser described. On account of difficulties in construction and insulation, its application is confined mainly to low-voltage circuits, such as receiving circuits, wavemeters and low-power transmitters, where the voltage does not greatly exceed 500 volts.

Air-dielectric variable condensers are seldom built to have a capacity greater than 0.005μf at the maximum setting. They can be procured in sizes varying from 0.00025μf to 0.005μf at maximum setting. If the condenser is equipped with an oil-tight container, a petroleum-base oil can be used as the dielectric, and the capacity of the condenser will be doubled. In addition, if the separation of the plates is considerable and the solid dielectric is sufficient, the oil-dielectric variable condenser can be used in fairly high-voltage circuits without danger of break-down. Such a condenser affords a very satisfactory means of tuning a laboratory transmitter circuit to resonance, in cases where the capacity of the circuit must be varied in a continuous manner.

Variable condensers are built in various forms. One type has a set of fixed plates, which are usually set on edge and secured together at their corners by through-bolts and are additionally fastened at their tops and bottoms to insulating slabs. These two slabs project beyond one edge of the fixed plates and have grooves cut into them between the fixed plates. The movable section consists of a set of smaller plates that are secured together at one end and can slide back and forth in the grooves between the fixed plates.

Another type of condenser has two plates. One is a fixed circular plate and the other a movable circular plate. The latter is secured to a threaded shaft which screws through a collar that is rigidly supported but insulated from the fixed plate. The capacity of this condenser is

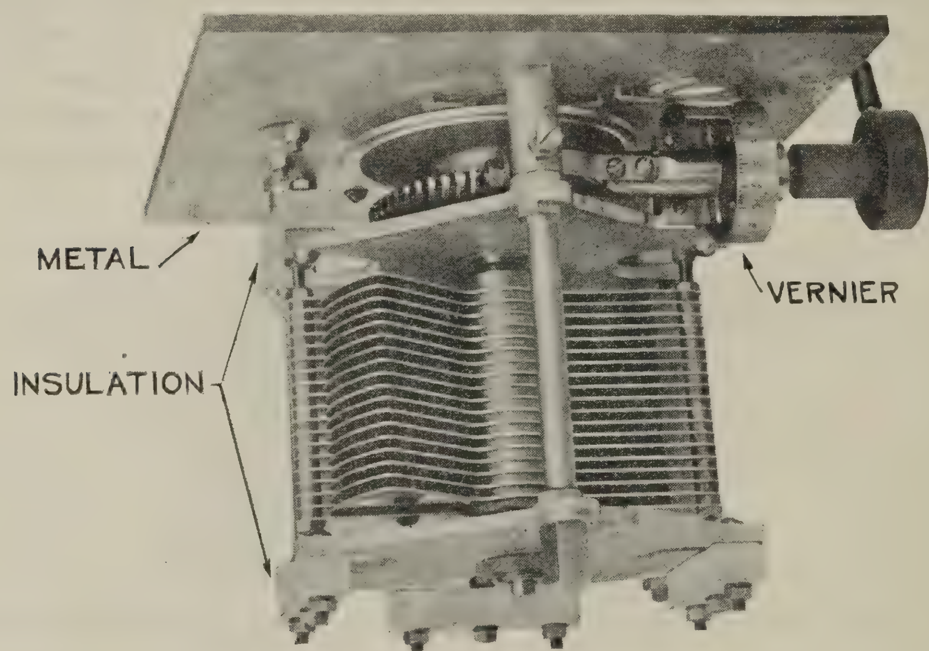


FIG. 266.—Air-Dielectric Variable Condenser.

varied in a continuous manner by screwing the movable plate in toward or away from the fixed plate. Such a condenser is frequently used in parallel with a large condenser as a **vernier condenser.**

The most common form of variable condenser employs two sets of semi-circular plates, one set of which is stationary, while the movable set is secured to a shaft so that it can be revolved between the fixed plates. The shaft is held in alignment by one or two bearings, depending upon the method of construction and the weight of the revolving part. The two sets of plates are supported and insulated from each other by suitable solid dielectric blocks, bushings or pillars. A pointer is keyed to the shaft. A semi-circular scale, which is marked in arbitrary divisions, is attached to the top of the condenser case. The pointer and scale indicate the angular position of the movable set of plates relatively to the fixed set of plates. The scale is marked either in degrees from

0° to 180°, or in divisions 0 to 100. The latter method of marking is preferable. The condenser has a minimum capacity when the pointer indicates 0 and a maximum when 180°, or 100, is indicated. A vernier may also be attached to the condenser, so that the condenser setting can be read accurately to a decimal part of one degree, or division. A further refinement in the adjustment of the capacity can be effected by the use of gears to actuate the rotating element. Some such device is practically necessary in some radio measurements. Figure 266 shows an air-dielectric, variable condenser of this type. The method of supporting and insulating the two sets of plates is clearly shown.

The capacity of a condenser is proportional to the area of its plates. Since the effective area of the plates in a semi-circular plate condenser is changed by rotating the movable plates, the change in the capacity is proportional to the angle of rotation. For this reason, the change in capacity is approximately proportional to the setting over a wide range. This is true in the case of a well-constructed condenser, provided that the separation between the two sets of plates remains constant during the rotation, that is, if there is no wobbling.

Figure 267 shows a typical capacity curve for such a condenser. It was plotted from the following data.

Degrees	Capacity $\mu\mu f$	Diff.	Degrees	Capacity $\mu\mu f$	Diff.
					131
0	68		100	697	
					130
5	85		120	827	
					128
10	115		140	955	
		64			128
20	179		160	1,083	
		128			64
40	307		170	1,147	
		130			
60	437		175	1,178	
		129			
80	566		180	1,197	

The column of differences permits the capacity of such a condenser to be accurately determined without the necessity of plotting a curve for the condenser.

Example:

Calculate the capacity of the condenser at a setting of 45.5°.

Solution:

Change in capacity between 40° and 60° = 130$\mu\mu f$. Hence, change

per degree $=\frac{130}{20}\mu\mu f=6.5\mu\mu f.$

change for 4.5° $4.5\times 6.5=29.25$
Capacity at 40° 307.
Adding 336.3
Hence, capacity at $45.5°=336.3\mu\mu f$.

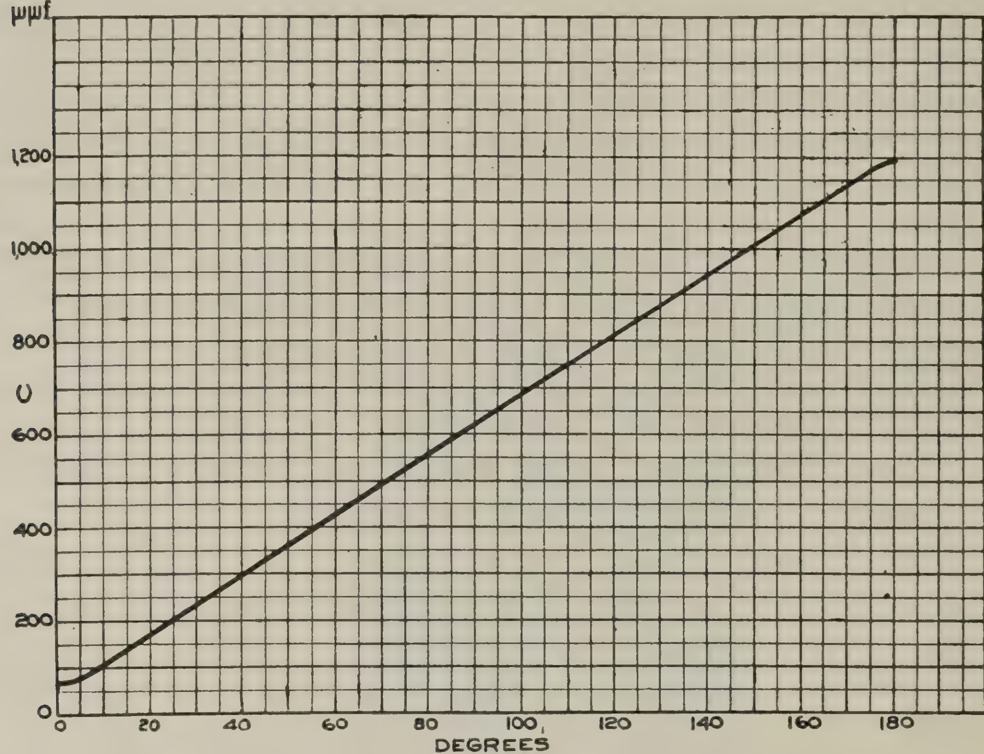


Fig. 267.—Capacity Curve of an Air-dielectric, Variable Condenser.

Special types of variable condensers. Variable condensers of the revolving plate type are also constructed to give a variation in capacity that is proportional to the square of the angular displacement of the rotating element. Such a condenser is sometimes used in wavemeters, when it is desired to have the pointer of the condenser indicate directly the wave length on a wave length scale. The wave length scale will be uniform if this type of condenser is used, while if the semicircular plate type were used, the scale would be non-uniform, and be either crowded together too closely at the lower end or too open at the upper end.

Another special type of variable condenser is used in decremeters. Here, the shape of the plates is such that the **per cent** change in capacity per degree is the same over the entire range of the condenser. Thus, if the capacity at 40° is $250\mu\mu f$, and a change of 1° in setting changes the capacity $2.5\mu\mu f$ or 1%, then a change of 1° at a setting where the capacity is $500\mu\mu f$ would produce a change of $5.0\mu\mu f$, or 1%.

Capacity of multi-plate condensers. The general formula for the capacity of a two-plate condenser has been given with an example of its application. It was also shown that the free capacity (so-called edge

effect) was a considerable part of the total capacity even when the plates were close together. This added capacity has not been allowed for in the following formulas, for the reasont hat such condensers are usually enclosed in containers, which introduce still more capacity. These formulas will, therefore, give only approximate values.

Parallel multi-plate condenser.

$$C = 8.85 \cdot 10^{-2} K \frac{(n-1)S}{\tau} \qquad (\mu\mu f)$$

where K = dielectric constant ($K = 1$ for air),
S = area of one plate (all similar) in cms.^{2},
n = number of plates,
τ = thickness of dielectric (separation between plates) in cms.

Example:

Calculate the approximate capacity in μf of a condenser having 40 metal plates, each 10 cms. square and separated by sheets of mica 1 mm. thick. $K = 8.0$.

Solution:

Formula $$C = 8.85 \cdot 10^{-2} K \frac{(n-1)S}{\tau} \qquad (\mu\mu f)$$

substituting $$= 8.85 \cdot 10^{-2} \times 8.0 \frac{(40-1)10^2}{1 \cdot 10^{-1}}$$

$$= 8.85 \cdot 10^{-2} \times 3.12 \cdot 10^5 = 2.76 \cdot 10^4$$

whence $$C = 2.76 \cdot 10^4 \mu\mu f = 0.0276 \mu f.$$

Variable condenser with semi-circular plates.

$$C = 0.139 K \frac{(n-1)(r_1^2 - r_2^2)}{\tau} \qquad (\mu\mu f)$$

where r_1 = outside radius of plates in cms.,
r_2 = inner radius of plates in cms.,

and the other quantities are the same as given above. All the plates are assumed to be similar in shape and size.

Example:

A condenser has 10 fixed plates and 9 movable plates; the separation is 2 mms. The outside radius is 7 cms., and the inner radius 2 cms. Air dielectric. Calculate the capacity in $\mu\mu f$.

Solution:

Formula $$C = 0.139 K \frac{(n-1)(r_1^2 - r_2^2)}{\tau}$$

substituting $$= 0.139 \times 1 \frac{(19-1)(7^2 - 2^2)}{2 \cdot 10^{-1}}$$

$$= 0.139 \frac{18(49-4)}{2 \cdot 10^{-1}}$$

$$=0.139\times 4.05 \cdot 10^3=5.63 \cdot 10^2$$

whence $C=563\ \mu\mu\text{f}.$

Points of design. Since air-dielectric variable condensers are most commonly used, the factors that enter into the design of such condensers will be considered. For use as a standard, the capacity must remain constant and be definite in amount. Constancy is gained by employing a rigid construction. Rigidity is gained by proper mechanical construction. The plates should be fairly heavy, strong and free from any inherent tendency to buckle or warp. The plates, shaft, and pointer of the variable section should be keyed together, so that they will remain in fixed relationship. No stops should be used to limit the motion of the movable section. The fixed and movable sections should also be rigidly supported and insulated from each other, so that the alignment of the plates will not vary, due to expansion or contraction of the insulation. For this purpose, an insulating substance having a negligible temperature coefficient should be employed. Constancy is further obtained by making the separation between the plates as great as possible. In this connection, it should be stated that the capacity should not exceed 0.002 μf using air dielectric, because the difficulties of mechanical design become too great. Compactness of design should be sacrificed to gain constancy.

In order to make the capacity definite, the condenser should be entirely enclosed in a metal container, and the **movable section** connected to the container, which should be provided with means for grounding.

Electrically, the construction should be as follows: The resistance of the plates, leads, and the contact resistance between the individual plates through the spacers should be kept very low in order to minimize the phase difference. The resistance due to poor contact at the bearings should be reduced by auxiliary bearing surfaces especially designed for electrical connection. The leads to the outside terminals should be as short and straight as possible to reduce their inductance. The fixed section should be connected electrically to the binding post through a mica window or opening in condenser case, but not insulated from it by a bushing.

The solid insulation between the fixed and movable section requires careful consideration as to: selection, design and location. Porcelain, quartz, high grade glass and hard rubber are excellent for this purpose, because they have low dielectric losses. The extreme importance of this point has already been shown elsewhere in the MANUAL. Figure 268 (*a*) shows one method of insulating the two sets of plates from each other. This is a good method of insulating. The capacity through the dielectric is made small by the construction, which uses as much air dielectric as is possible, without weakening the support. Figure 268 (*b*) shows a poor method of insulation. The capacity through the dielectric is large. A

condenser having extremely low losses is shown in figure 266. The method of supporting the plates of this condenser is quite different from that just shown in figure 268 (*a*), but in both instances the solid insulation is not located where the electric field is intense.

It is evident from the foregoing that not all air-dielectric condensers, whether of the fixed or variable type, are equally good.

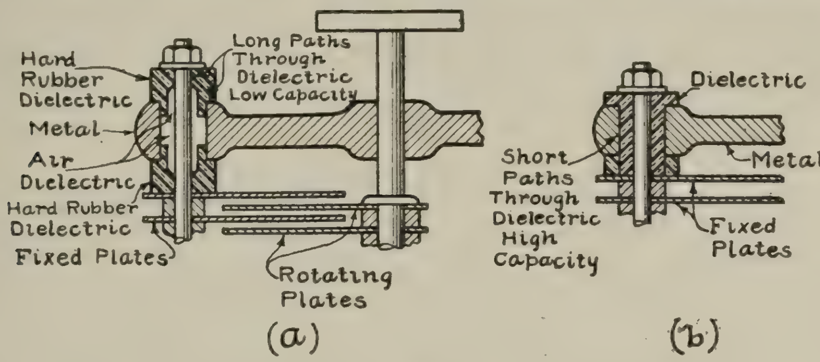


FIG. 268.—Methods of Insulating the Fixed and Movable Sections of a Condenser (a) Good; (b) Poor.

The purpose of the shielding is to minimize power losses, and to prevent varying capacity effects due to the presence of the operator's body. To prevent the latter effect, it is better to connect the movable section, rather than the fixed section, to the metal case. The addition of the case will increase the capacity of the condenser by a fixed amount if the movable section is connected to it. This is not the condition when the fixed plates are connected to the case.

CHAPTER III. CURRENT MEASURING INSTRUMENTS

The measurement of radio-frequency currents is of great importance in radio engineering, because upon it depends the measurement of the radio-frequency resistance and, therefore, the measurement of the efficiency of a radio circuit. In fact, very few radio-frequency measurements can be made without a knowledge of the value of the current flowing in the circuit.

Low-frequency, alternating-current ammeters cannot, in general be employed for measuring radio-frequency currents. Such ammeters are not designed to have extremely low capacity and inductance, because these are negligible in their effect on the accuracy of the instrument at low frequencies. However, within the range of radio-frequencies, the impedance and the capacity of an ammeter give rise to serious errors in current indication, the amount of the error varying with the frequency. This is caused by a division of the current in the instrument, part flowing through the conductor and part through the dielectric. For this reason, the simplest form of circuit is used in radio, frequency ammeters.

The thermal ammeter. When a current flows through a conductor, the conductor is heated. The power consumed in heat in the conductor is proportional to the square of the current and the resistance, or

$$P = I^2R$$

If the conductor is suitably chosen as to material, diameter and resistance, it will expand, or lengthen, when traversed by a definite current. The resulting change in its length can be utilized to actuate a pointer and move it across a scale marked either in amperes, or in arbitrary divisions.

Figure 269 shows a **thermal,** or **hot-wire ammeter.** The wire through which the radio-frequency current flows is called the hot wire. Attached to this wire is another wire which makes a turn around a light pulley, and is then secured through a spring to a pedestal. The purpose of the spring is to take up the slack in the hot wire. The pulley is fastened to a spindle, which is set in jewel bearings at each end. The pointer is attached to the spindle. A screw adjustment is provided to increase or decrease the tension of the hot wire, and thus permit the pointer to indicate zero when no current is flowing. The action is as follows:

When current flows through the hot wire, the wire is heated and lengthens. The spring takes up the slack, and as the wire attached to the spring moves, the spindle is turned, which results in a movement of the pointer.

In order that the same current at any frequency may give the same deflection, it is essential that the power P remain constant. It will, therefore, be necessary that the resistance remain constant over the range of radio-frequencies. Hence, the conductor in the ammeter must be either a fine wire, or a thin strip of suitable metal. A change in resistance of 1 per cent is not too great for radio-frequency ammeters, as a greater accuracy is hardly necessary for most radio measurements. An inspection of Table 6 shows that constantan wire must not have a diameter greater than 0.6 mm. if its radio-frequency resistance at 300 meters is to be not more than 1 per cent higher than its dc resistance. This size of wire is, therefore, suitable for any wave length longer than 300 meters, and will carry a few amperes without overheating, with a consequent increase in resistance. The hot wire should be long enough so that the distribution of current in it will not be affected by the proximity of the terminals. A length of 10 cms. or more is suitable.

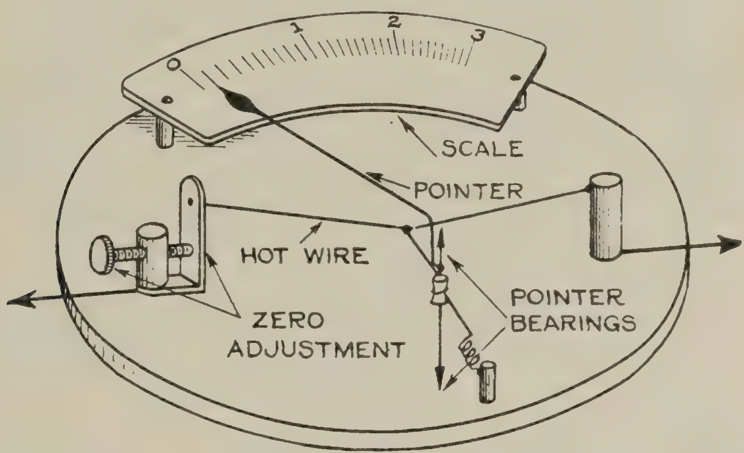


Fig. 269.—Typical Hot-Wire Ammeter.

The **calibration** of such an instrument can be made with either direct current or low-frequency alternating current, and the calibration considered correct for radio-frequencies. However, it is always good practice to compare the ammeter at radio-frequencies with some standard ammeter which is known to give correct current indications at these frequencies.

Thermal ammeters for small currents. Ammeters used to measure radio-frequency currents having values lying within the range—3 ma. to 5 amperes—employ a hot wire and some device to indicate the heat produced. The **hot-wire** ammeter, figure 269, in which the indication depends upon the expansion of the wire, is used extensively. A typical hot-wire ammeter for use in wavemeters will give a full scale deflection with 0.08 ampere, the resistance of the ammeter being approximately 5 ohms. Since the deflections of the pointer are, in general, proportional to the square of the radio-frequency current, the instrument is frequently called a **current-square meter.**

For measurement purposes, it is preferable to divide the scale into equal arbitrary divisions rather than to have the actual current in amperes indicated on the scale, because of the crowding of the divisions in the latter case. The difference in the two scales is shown in figure 270, which is illustrative only. The ammeter range is from 0 to 5 amperes, and the deflection of the pointer is proportional to the square of the current. Two positions of the pointer are shown in the figure. At position 1, the pointer indicates on scale *b* that 5 amperes are flowing. If the current is reduced to one-half of 5 amperes, or 2.5 amperes, the pointer will drop back to position 2. The distance from 0 to 2.5 is **one-quarter** of that from 0 to 5; hence, by halving the current the deflection is reduced to one-quarter. Now, the arbitrary scale *a* has 100 equal divisions. The current required to deflect the pointer 100

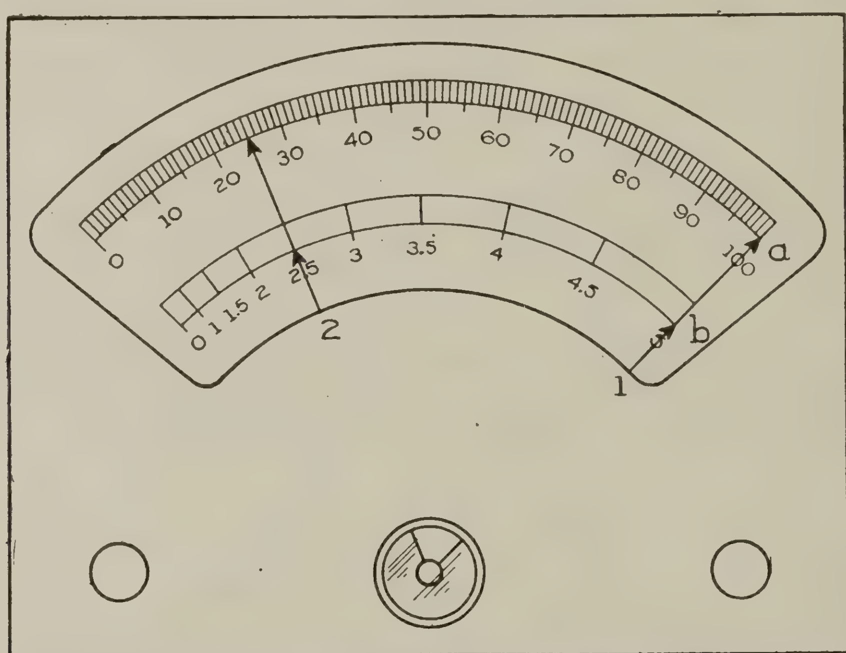


FIG. 270.—Difference between (a) Scale Indicating Amperes and (b) Scale Divided into Arbitrary Divisions.

divisions is 5 amperes, as shown in the figure, and one-half of this current would produce a deflection of 25 divisions, which is **one-quarter** of 100. In both cases, the deflection of the pointer is proportional to the square of the current; that is, one-half of any given value of current is represented by one-quarter of the deflection for the given current.

To illustrate further the current relations in a radio-frequency circuit, as applied to the deflections on the arbitrary scale, it is customary when measuring the radio-frequency resistance of a circuit to use the quarter-deflection method. In this method, a source of impressed emf is applied to the circuit being measured, and the deflection noted. Noninductive resistance is then inserted in the circuit until the deflection is reduced to one-quarter its original value. The value of the inserted resistance is then noted. It represents the resistance of the cir-

cuit, because when the quarter-deflection is obtained, the value of the current is reduced to one-half. Ohm's law shows that if the current in a circuit is to be reduced to one-half, it is necessary to insert a resistance equal to the resistance of the circuit; that is, the circuit resistance must be doubled.

Thermal ammeter for large currents. The single-wire type of hot-wire ammeter is limited to the measurement of small currents because, if larger currents are measured, the wire is heated to such an extent as to increase its resistance and impair the accuracy of the ammeter. To overcome the limitations connected with the use of the single wire in hot-wire ammeters for large currents, resort is made to the use of a number of parallel wires, or strips, placed equidistantly from each other as shown in figure 271. In this form, each wire or strip has the same self-inductance as the others and the same mutual inductance with respect to the others. The indicating system is usually connected to one

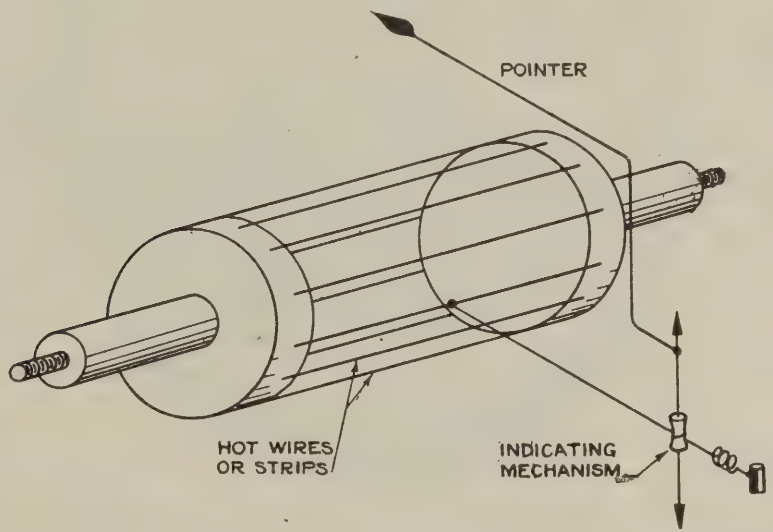


FIG. 271.—Heater Unit of an Ammeter Suitable for Measuring Large Radio-frequency Currents.

of the strips or wires, since equal currents will flow through each wire if wires of equal impedance are used. Figure 272 is a photograph of a commercial type of hot-strip ammeter.

Shunted ammeters. Ammeters which depend on shunting the heater wire to increase their current carrying capacity, or to reduce the resistance inserted in the circuit, should not be used for accurate current measurements at varying radio-frequencies. In such a combination, the current divides between the indicating wire and the shunt in inverse proportion to their impedances, as shown in the following formula:

$$\frac{I_1}{I_2}=\frac{\sqrt{R_2^2+(\omega L_2)^2}}{\sqrt{R_1^2+(\omega L_1)^2}}$$

where I_1 = current through ammeter in amperes,
I_2 = current through shunt in amperes,

L_1 = inductance of ammeter in henries,
L_2 = inductance of shunt in henries,
R_1 = resistance of ammeter in ohms,
R_2 = resistance of shunt in ohms.

Unless
$$\frac{L_2}{L_1}=\frac{R_2}{R_1},$$
the current ratio will vary with the frequency. When used in wavemeters, where low resistance is essential and only relative current values are desired, shunted ammeters will be satisfactory.

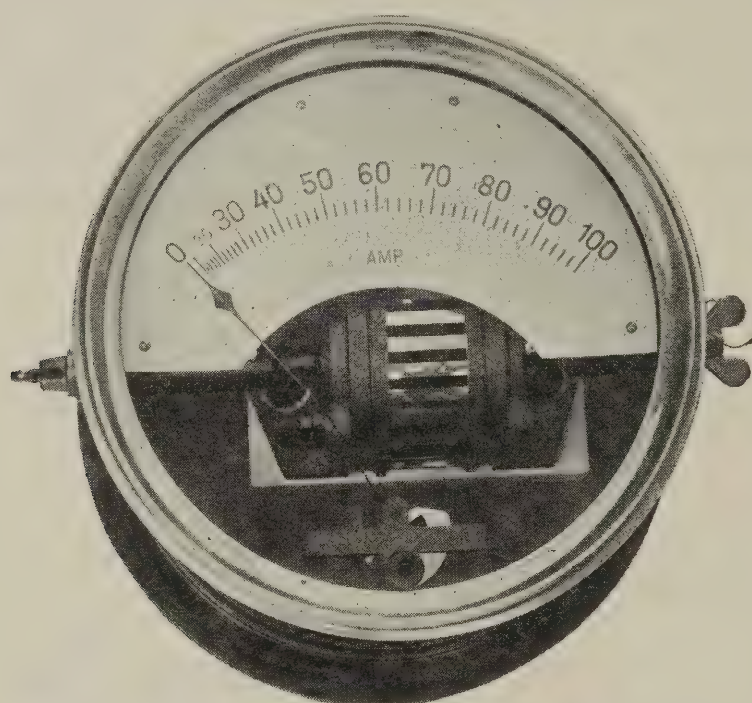


Fig. 272.—Hot-Strip Ammeter, Heavy Current Type.

Disadvantages of Hot-Wire Ammeters. The hot-wire type of ammeter, depending as it does on thermal expansion, is naturally rather sluggish in its response to current changes in the circuit. The zero shift is another disadvantage. After current has been flowing for a period, it will be found that the pointer will not return immediately to zero. This makes frequent adjustment of the zero position necessary for very accurate work. Care must also be taken when using such an ammeter not to induce electrostatic charges on the glass face of the instrument. Such charges are easily produced by rubbing dust off the face of the ammeter, and will introduce errors in readings by deflecting the pointer from the zero position. To remove an electrostatic charge, breathe on the glass.

Thermocouple Ammeters. This type of ammeter is generally preferred to the hot-wire type because of its inherent advantages over the hot-wire ammeters. It consists of a thermocouple, connected to which is a dc ammeter, generally a galvanometer. The use of thermocouple

ammeters, especially the type which employs a separate thermocouple, is desirable for radio measurement work where the thermocouple can be inserted in the circuit to be measured and the ammeter placed at a convenient place for noting the value of the current in the circuit.

Thermocouples. If two wires of different kinds of metal are placed in such a position that they cross and touch each other at one point in their lengths and this intersection is heated, a small emf will be generated, as explained in Chapter I of Part 2. This emf is proportional to the degree of heating of the intersection, or junction, and is unidirectional in nature. Such a combination of wires, which are capable of producing an emf by heating, is called a **thermocouple,** or **thermoelement.**

The heating of the thermocouple is accomplished by passing current through the wires of the thermocouple, as shown in figure 273 (*a*), where *A* and *B* are the terminals to which are connected the leads from the radio-frequency circuit, and *a* and *b* the terminals to which

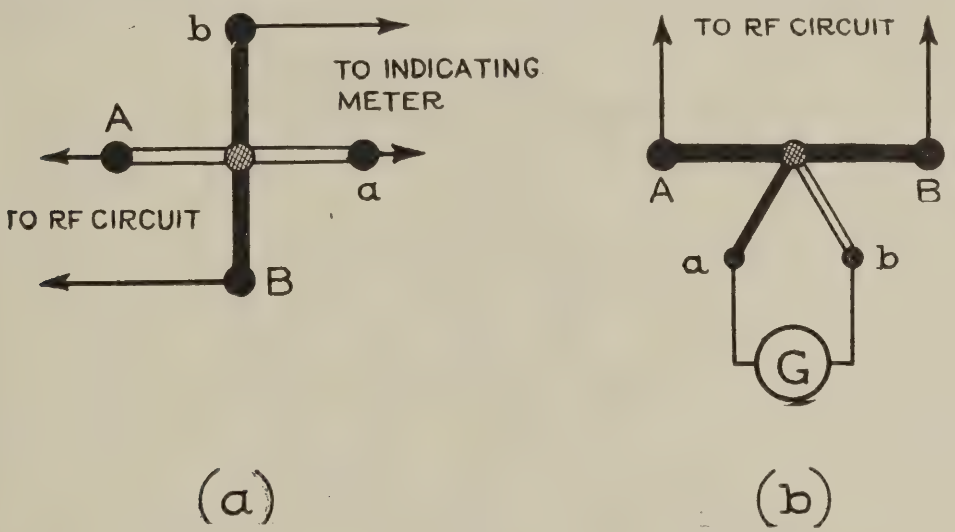


FIG. 273.—Common Types of Thermocouples.

the galvanometer is connected. The current flowing through the radio-frequency circuit heats the junction of the two wires of the thermocouple, and this heating may be represented by the term I^2Rt, where I is the radio-frequency current, R the resistance of the couple, and t the time. From this term, it will be noted that the heating is proportional to the square of the current. The emf, being a function of the heating effect, will consequently produce deflections on the galvanometer or indicating instrument, which are also proportional to the square of the radio-frequency current.

Types of Thermocouples. There are two types of thermocouples; one type is shown in (*a*) of figure 273, and the other type in (*b*) of the same figure. In (*a*) the heating current passes through the two wires of the thermocouple, while in (*b*) the heating current passes through only one wire and heats the attached thermocouple. The first type is applicable for use in measuring small currents, while the latter type is

suited for measurement of large currents. The combination of constantan and steel, or constantan and manganin, wires has been generally accepted for the construction of thermocouples.

The main precaution to be observed in the manufacture of thermocouples is to have both wires in constant and perfect contact with each other. The best way to accomplish this result is to solder the junction of the two wires, but to reduce the amount of solder used to a minimum.

For best results, fine wire should be used and the length of these wires reduced to a minimum. The thermocouple should be protected from air currents by a suitable container, preferably by placing it in an evacuated container.

A good thermocouple for laboratory use can be made of steel and constantan wires 0.02 mm. thick and 4 mm. long. Solder the intersection of the wires, using paste flux. A novel way to solder the intersection of the wires in the thermocouple is to place a little flux on the

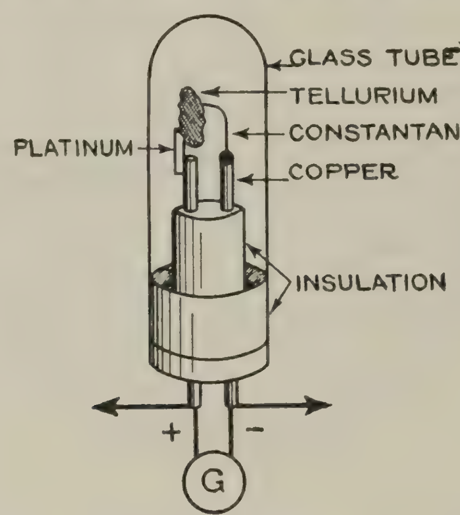


Fig. 274.—Tellurium-Constantan Thermocouple.

intersection and then lay a minute chip of solder on the flux. The joint is then soldered by the following method. Obtain a piece of nichrome wire No. 26 gauge, about ½-inch long, and secure it to two lengths of No. 14 copper wire. Pass sufficient current through the combination to heat the nichrome wire to a yellow heat, and then hold this heated wire close enough to the intersection of the thermocouple to melt the chip of solder around the intersection. It takes very little time to melt the solder and complete the work, thus eliminating the difficult method of applying the soldering iron. Wash the couple in gasoline and then with alcohol to clean off any trace of flux that may remain on the joint. Such a thermocouple should have a resistance of approximately one ohm, and will be capable of generating an emf of 40 microvolts with a heating current of 15 milliamperes of radio-frequency current. The resistance of the thermocouple should be measured between points A and B, and points a and b, figure 273 (a).

The sum of these two resistances will equal the sum of the resistances measured between *B* and *b*, and *A* and *a*, provided that the soldered contact between the wires is good.

A special type of thermocouple shown in figure 274 has been developed by Dr. L. W. Austin. The couple consists of a tellurium-constantan junction, and is extremely sensitive. It will generate 25 times the emf for the same temperature as is obtained with a constantan-platinum couple. For details on the construction of this type of thermocouple, reference should be made to Bureau of Standards Circular No. 74, page 160.

Current-transformer Ammeters. Thermal and thermocouple ammeters, because of their large size and high cost for use in measuring large currents, have not met with favor in radio engineering work. A new type of ammeter has been developed particularly for use in measuring large currents. This ammeter is referred to as the **Current-transformer ammeter,** and is considered to be the most economical and accurate instrument made for measuring currents of large values.

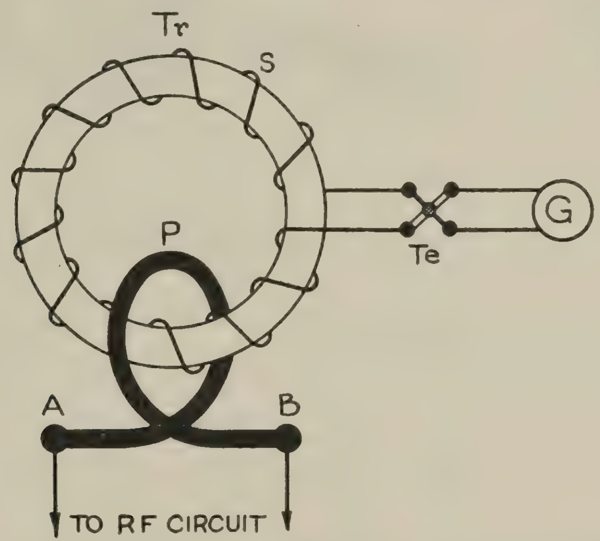


Fig. 275.—Current-Transformer Type of Ammeter.

This ammeter consists of a current step-down transformer, a thermocouple, and a galvanometer, figure 275.

The transformer *Tr* of figure 275 consists of a single-turn primary which is connected in the radio-frequency circuit and a toroidal core around which is wound the secondary. The transformers in general use in this country are of the iron-core type. The cores are made of very thin laminations, thus increasing their effective magnetic permeability, which results in greater accuracy. The thermocouple *Te* and the galvanometer *G* are identical with those described elsewhere.

Theory of the Current-transformer Ammeter. As previously stated, the current distribution in an inductance shunted ammeter is a function of the frequency. To overcome the inductive effect on the distribution of current, it is possible to use resistances which are of such

value that the resistance of the circuit is the controlling factor. It is also possible to reverse the case and have the resistance small and the inductance large, so that the inductive reactance is the predominant factor in the control of the current distribution. Such is the case in either the air- or iron-core, current-transformer ammeter.

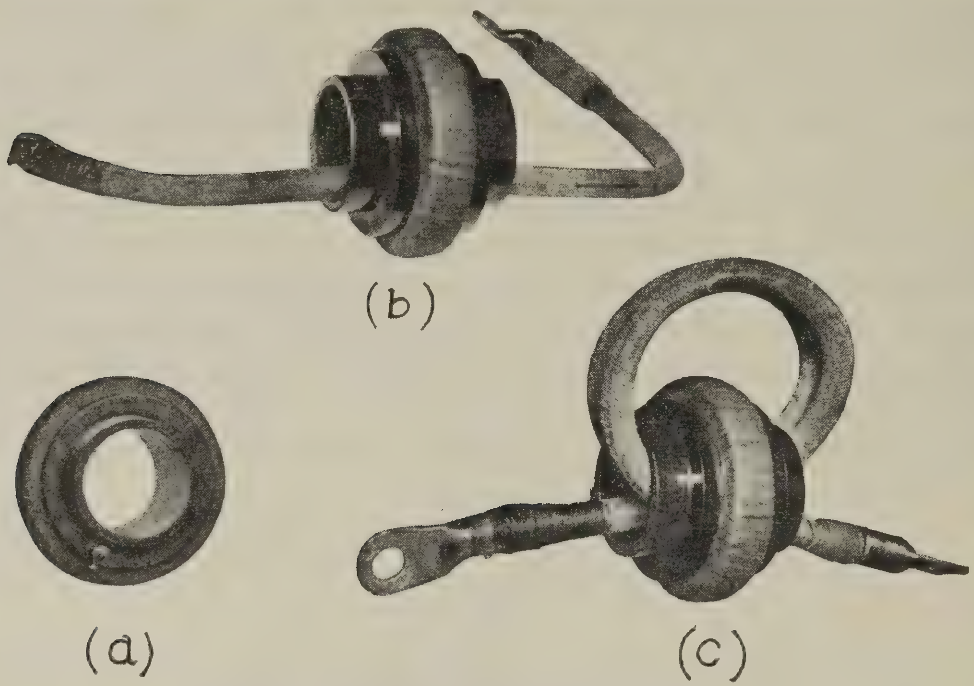


FIG. 276.—Current Transformer for Ammeters.

For this condition, the equation for the distribution of the current in an iron-core current transformer ammeter circuit may be given as follows:

$$\frac{I_1}{I_2}=\frac{n_2}{n_1}\left(1+\frac{aR_2}{\omega L_2}\right)$$

where I_1 = current in primary in amperes,
I_2 = current in secondary in amperes,
n_1 = number of turns in primary,
n_2 = number of turns in secondary,
R_2 = resistance of secondary and thermocouple in ohms,
L_2 = inductance of secondary in henries.

The term a in the formula represents the energy loss in the iron due to eddy currents and hysteresis, and is usually slightly less than unity. The term $\left(1+\frac{aR_2}{\omega L_2}\right)$ is the correction factor for the frequency range of the current-transformer ammeter. It can readily be seen that, for radio-frequencies, the correction factor is very small, especially when low resistance thermocouples are used. For low frequencies, the correction factor amounts to approximately 14 per cent, while for high radio-frequencies it is about 0.2 per cent.

Advantages of the Current-transformer Ammeter. By interchange of transformers, or by increasing the number of turns in the primary, it is possible to measure currents over a large range of values with the same thermocouple and galvanometer. Such is not the case with thermal or thermocouple ammeters. Thermal or thermocouple ammeters are much larger than the current transformer ammeters, and cost more to maintain, especially the thermocouple type, which in the past have given trouble due to overloading, corrosion, and destruction of the thermocouples.

Figure 276 shows a type of current transformer used in the U. S. Navy, where (*a*) is the iron toroidal core with secondary winding, (*b*) is the complete transformer with a single-turn primary, while (*c*) shows the transformer with a two-turn primary.

CHAPTER IV.

RESISTANCE UNITS FOR USE IN RADIO CIRCUITS.

Resistance units are used in radio circuits when it is desired either to by-pass part of the radio-frequency current around a portion of a circuit, or to increase the circuit resistance by placing them in series in the circuit. In either case, the radio-frequency resistance of the unit at the frequency employed should be known. Resistance units to be used in radio circuits should, therefore, fulfill all the requirements of a resistance unit that would be suitable for use in a direct-current or a low-frequency, alternating-current circuit. Among these are: ample current carrying capacity and negligible temperature coefficient. In addition, they should have low distributed capacity, should be non-inductive and should offer a constant resistance to currents within the range of radio frequencies. An ideal resistance unit would have all of these characteristics, and could be calibrated by means of direct current.

The ideal resistance unit would not change any of the constants of a circuit except the resistance when inserted in the circuit. It is impossible, however, to construct a resistance that is absolutely non-inductive and has no distributed capacity. The circuit in which a resistance unit is inserted will, therefore, be detuned from resonance by the amount of the inductance in the resistance. This change in circuit impedance can be neutralized by retuning the circuit to resonance by means of the capacity, and is, therefore, not very serious. The distributed capacity of the resistance unit acts, however, as a by-pass for radio-frequency currents and becomes serious at very high radio frequencies.

Radio-frequency resistance standards. The ideal resistance unit is very nearly attained by the use of a short length of very fine wire made of an alloy having a negligible temperature coefficient. The following is a description of the method of manufacture of a set of standard radio-frequency resistance units that can be used in precision mesurements.

These resistance units are so constructed that they may be interchanged in a circuit or substituted for a copper jumper. They are of uniform length and made up of very small wire of a material which has a temperature coefficient of practically zero.

A good set of these resistances for covering a range of from 0 to 40 ohms resistance is composed of from 18 to 20 units. This range of resistance is that which is ordinarily met with in radio-frequency circuits.

The resistance wire is soldered to two pieces of No. 18 or No. 20 bare copper wire, the ends of which have been sharpened. The resis-

tance wire should be of such a length that the direct-current resistance obtained on a bridge will be very close to a value which is easily handled, such as 0.5 or 2.0 ohms, and not 0.436 or 1.978 ohms. The proper length may be obtained by either of the following methods:

If the resistance per centimeter of wire is known, the copper wire may be soldered to one end of the resistance wire; the proper resistance may then be laid off with a centimeter scale and the second wire soldered on. It is convenient to attach the first copper wire and the resistance wire to a piece of paper with a few drops of wax or gummed paper, as is shown in figure 277.

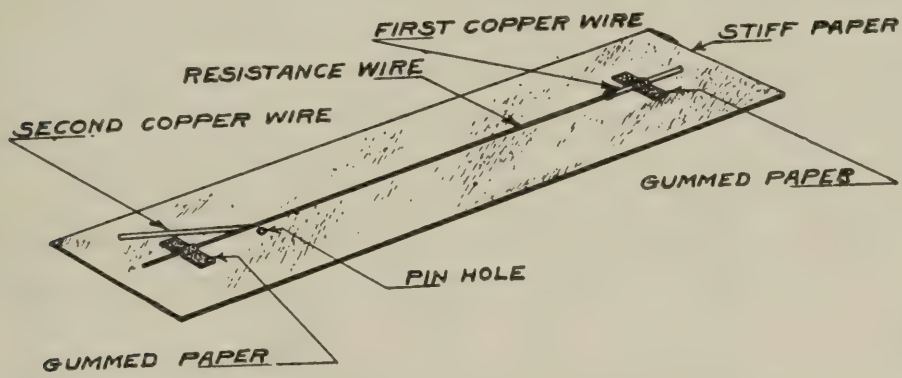


FIG. 277.—Method of Laying Off and Soldering Resistance Wire to Lead Wires.

The distance from the first copper wire to the pinhole is laid off according to the resistance desired; the second copper wire is then soldered to the resistance wire at the pinhole.

A second method is to construct a clamp which may be conveniently used with an ordinary Wheatstone bridge. Such a clamp is shown in figure 278.

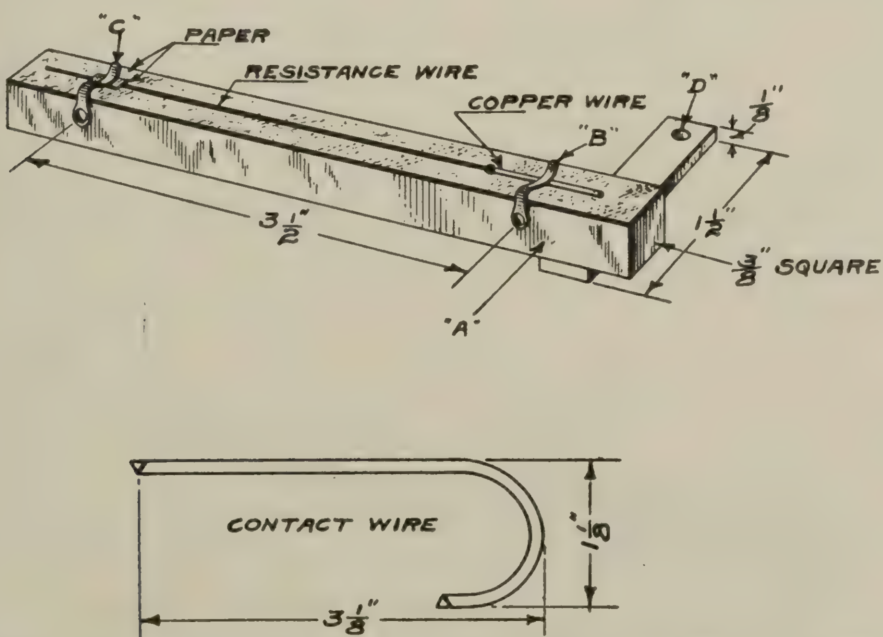


FIG. 278.—Alternate Method of Laying Off and Soldering Resistance Wire to Lead Wires.

The brass clamp bar A is secured to one of the bridge binding posts by means of the binding post set screw which is inserted through the hole D. A strip of paper, the width of the bar, is inserted under the two clips B and C. A copper wire, soldered to the resistance wire, is then placed under the clip B, and the resistance wire drawn tight under clip C. The resistance wire is insulated from clip C by means of a small piece of paper. The knife edge contact wire is then inserted in the free binding post of the bridge. The bridge is next set for any desired resistance which is obtainable within the limits of the resistance wire, and the knife edge moved along the resistance wire until a balance is obtained. The second copper wire is then soldered to the resistance wire at this point.

Either of the above methods will give very satisfactory results, the first being used where only a few resistances are to be made up, and the second where enough resistances are to be made up to justify the time necessary for constructing the clamp.

These resistances with copper ends are mounted in glass tubes, about 5 millimeters (0.196 inch) inside diameter, in the following manner:

The copper wires are inserted in, and held in place by, split corks which project about 1 millimeter (0.039 inch) beyond the tube ends,

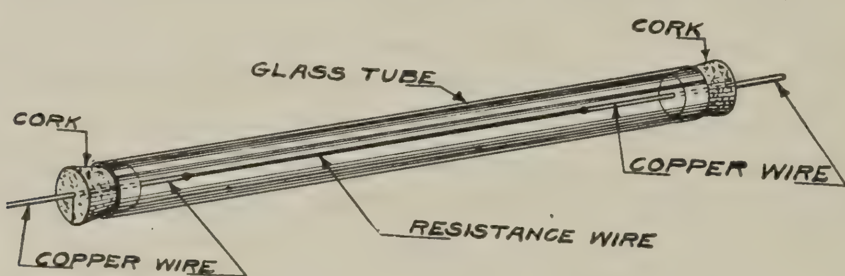


Fig. 279.—Method of Assembling Resistance Units.

figure 279. When the resistance wire has been centered in the tube, the copper wires are bent short over the corks, and the combination of wire, cork and tube secured together by ordinary sealing wax as shown in figure 280.

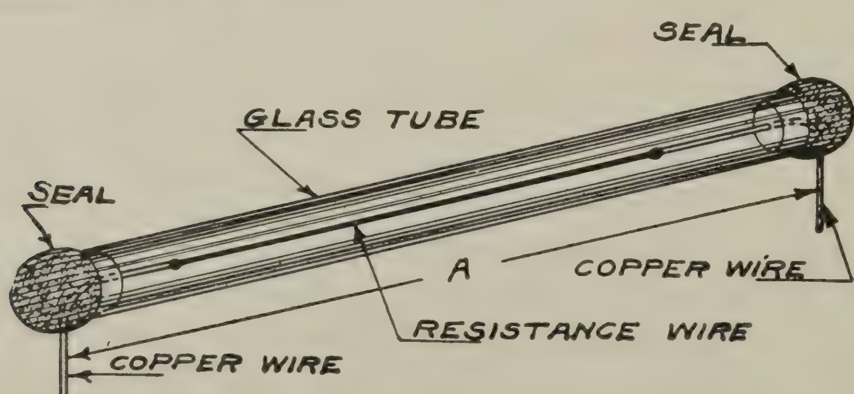


Fig. 280.—Completed Resistance Unit.

The distance A should be about 8 centimeters (3.12 inches). Convenient connectors, figure 281, for inserting these resistances into the radio circuit are made in the following manner:

Both holes in the copper blocks are made just large enough to admit the connecting wires and a little mercury. The rectangular holes are for receiving the resistance unit tips. These holes are made rectangular in order to allow for the variation in distance between the copper end wires. The blocks are made just large enough to give proper contact. The copper wires should be amalgamated for insertion in the mercury.

The following table gives the data of a set of such resistances. The glass tubes used were 8 cm. long and had an inside diameter of 5 mms.

Kind of wire.	Diameter of wire. mm.	Length of wire. mms.	Dc resistance. Ohms.
18 per cent German silver	0.25	38.5	0.2502
Do	.25	53.0	.3498
Advance	.128	21.0	1.027
Do	.128	42.5	2.002
Do	.128	63.5	3.003
Manganin	.06	29.0	4.03
Do	.06	43.0	6.02
Do	.06	58.5	8.00
Do	.05	43.0	10.00
Do	.05	52.0	12.03
Do	.05	59.5	13.50
Nichrome	.05	31.0	15.01
Do	.05	36.0	17.95
Do	.05	41.0	20.00
Do	.05	51.0	25.18
Do	.05	62.0	30.20
Manganin	.025	50.0	40.00
Constantan	.015	21.5	51.97
Do	.015	60.0	149.60

The inductances of these units vary from approximately 0.15 μh for the 40 ohm unit to 0.08 μh for the 0 ohm copper link. The difference, 0.07 μh, is negligible except in rare cases. The resistances have been found to remain constant for several years, but a non-corrosive flux should be used in soldering and the unit thoroughly cleaned with alcohol in order to obtain this degree of permanency.

On account of the small size of the wire, care must be exercised in measuring their resistance by means of a Wheatstone bridge, and also, when they are used in circuits, that less that 100 milliamperes is passed through them.

Decade resistance boxes. With such a set of standard resistance units, the suitability of commercial types of resistance boxes, and other

resistance units, for use in radio circuits, can be determined. **Decade resistance boxes** will, in general, be found eminently satisfactory for use in small current resistance work. They must, however, be properly made and should invariably be checked by the standard units at radio frequencies. When a decade resistance box is used in a circuit of low inductance, variation of the resistance setting may vary the inductance of the circuit. This can usually be compensated for by decreasing the capacity until resonance is again obtained. It has been found that a properly constructed decade resistance box can be used with very good results for wave lengths as short as 200 meters. Beyond this point, its use is likely to introduce a considerable error due to the distributed capacity.

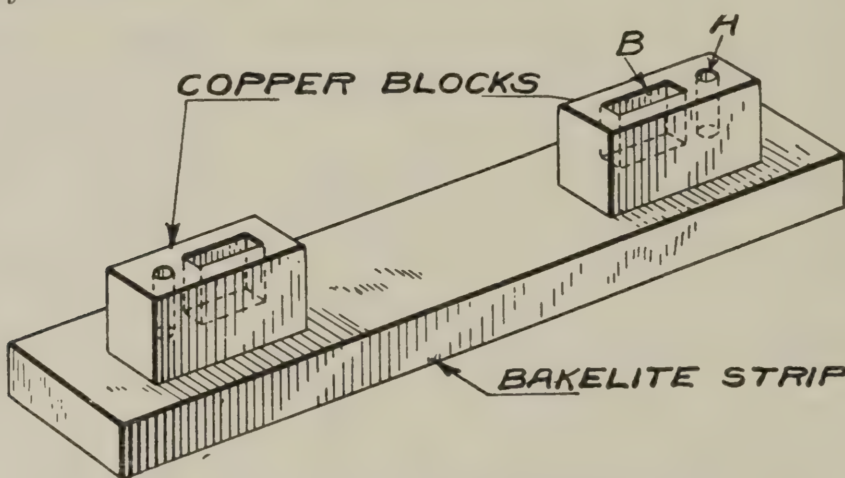


FIG. 281.—Blocks for Connecting Resistance Units into Circuit.

Resistances suitable for large currents. Resistance units that can carry considerable current are required for **dummy antenna** transmitter circuits, and infrequently as shunts for radio frequency ammeters. Only the former use will be considered, because the shunt method of increasing the range of an ammeter in order to measure large radio-frequency currents is no longer being employed, on account of the inherent difficulties and because very satisfactory unshunted ammeters are now available.

The design of resistance units that are capable of carrying large radio-frequency currents, and yet will maintain a constant resistance over a wide band of radio frequencies, is difficult and usually is not accomplished. Such units are made of manganin or nichrome wire, or ribbon. The wire is wound noninductively on an asbestos form, and the distributed capacity kept as low as possible. No metal pins, etc., are used to hold the wire on the form. The current capacity is obtained by using small units in series, parallel and series-parallel combinations as required.

The resistance of any such unit, or combination of units, for use in radio circuits must be determined by a comparison with the radio-frequency resistance standards, using a radio circuit in which a feeble oscillating current of the desired frequency is flowing.

CHAPTER V. INSULATORS.

Insulators are constructed of solid dielectrics, and are used to hold charged conductors in fixed positions relatively to other bodies. They prevent electrical contact from being made between the conductors and the adjacent bodies. Every insulator can be considered to be a **fixed condenser** with the insulating solid as the dielectric and the metal fittings of the insulator, or the conductor and the adjacent conducting bodies, as the two plates. Therefore, practically everything that appears in this MANUAL concerning the properties of dielectrics, insulation in general, insulation of condensers, power losses in condensers, etc., is applicable to insulators. If the similarity that exists between insulators and fixed condensers is kept clearly in mind whenever a question of insulation arises, losses due to poor insulation, whether arising from the selection of a poor dielectric or faulty design, will be less likely to assume serious proportions. A brief review of what constitutes an efficient insulator follows.

Electrical requirements of an efficient insulator. All dielectrics show some absorption, and this means a loss of a certain amount of electrical power in the form of heat. If the losses are great enough, the insulator will become overheated and, perhaps, fail.

All insulators have capacity, and since the dielectric constant of all substances used for insulators is greater than that of air, an insulator may have a considerable capacity. Consequently, the circuit of which the insulator forms a part, must supply a **charging current.** If the dielectric were perfect, no loss of power would result. No dielectric used in the manufacture of insulators is perfect; hence, there will be a phase difference and some loss of power will occur. The power loss in an insulator, as in an imperfect condenser, is given by

$$P = IE\psi$$

where E = the voltage on the insulator,

I = the current flowing through the insulator,

ψ = the phase difference.

If the capacity of the insulator is C, the current

$$I = E\omega C$$

Hence
$$P = E^2\omega(C\psi)$$

The product $C\psi$ is, therefore, a comparative measure of the power loss in insulators.

The losses in an insulator, due to the solid dielectric, can be reduced to a minimum by proper design. This is accomplished by transferring as much of the electric field as is possible from the solid dielectric to the air. Where the path is partly through the insulator and partly through the air, the length of the path through the insulator is shortened. The

voltage to which an insulator will be subjected, and hence the electric gradient through it, determines the length of the path which will prevent flash-over. The shape of the insulator and fittings and their separation have a large influence upon the minimum voltage at which corona and its attendant losses will occur.

Classes of insulators. Insulators may be divided into two classes: (a) **low-voltage** for receiving circuits, and (b) **high-voltage** for transmitting circuits. The question of providing suitable insulators for receiving circuits should receive as much attention as for transmitting circuits. A serious loss of receiving efficiency will result from the use of poor insulators. Although the loss of power in watts is extremely small, so too is the power available from the incoming signal; consequently the percentage of power lost is high. It is apparent from what has just been said that the difficulty in properly insulating a receiving circuit, including the antenna or loop, is not on account of the voltage, but rather in conserving the little power available for receiving purposes. The fallacy that any kind of an insulator will do for a receiving circuit is as plain as it is widespread.

A suitable insulator for high-voltage circuits must combine all the essential features of a suitable low-voltage insulator, and, in addition must be able to insulate the circuit for high voltages. High-voltage insulators are subjected to very intense electric fields and, if the losses are serious, will fail. The failure may occur at once or after a considerable length of service. Loss of power in the insulator will heat the insulator. The phase difference of dielectrics invariably increases with the temperature. Thus, the heating of the insulator tends toward a greater loss of power and still further heating—a cumulative effect which sometimes will actually cause an insulator to explode. In other cases, localized heating will be sufficient to char the material of the insulator and render it conducting. The charring will then occur in the insulator beyond the portion which has been made conducting, so that in a time, which may be hours or weeks, a conducting path will be eaten through the insulator, with resultant failure. The phase difference and power loss of high-voltage insulators can be accurately measured using low power by the method given in Measurement No. 10. Such a measurement, of course, does not give the corona losses, but only the dielectric losses at the temperature at which the measurement is made. From the latter can be calculated what power would be lost at a given operating voltage and frequency, and the possibility of failure of the insulator predicted.

Keeping in mind these electrical requirements, insulators should be designed so that they serve merely as a sufficiently strong mechanical support for a current-carrying conductor, with most of the lines of electric force passing through the air. This is accomplished by equipping high-voltage insulators with electrostatic shields, and by designing low-

voltage insulators so that a minimum amount of solid dielectric is included in the path of the lines of electric force. The bushing type of insulator, in which the conductor is held away from its surroundings by a sleeve which fills the entire space, is particularly bad and should, therefore, be avoided.

Material. The solid dielectric of the insulator can be porcelain, glass, or molded material, such as the phenolic or shellac base compounds. Any material that is used should be non-hygroscopic, to prevent losses due to moisture in the material. The voltage to which an insulator will be subjected, its location with respect to moisture, stack gases, soot, salt spray, etc., and its liability to mechanical damage should also be carefully considered when a choice of materials is being made.

Types of Insulators. Insulators used in radio can be further divided into three types: (a) **suspension** insulators, (b) **bulkhead,** or **wall,** insulators, and (c) **pillar,** or **pedestal,** insulators. These types are made in a variety of shapes and sizes and of various materials to meet the requirements.

Suspension insulators are used to support elevated portions of antenna structures and antenna leads. The conditions under which they must function, especially on naval vessels, are very severe. These conditions are: varying weather, temperature, mechanical and electrical load, sea air and frequently salt-water spray, stack gases, smoke and soot.

Due to inherent difficulties in design suspension insulators are now generally not provided with an internal locking arrangement to prevent the antenna from falling in case of mechanical failure of the insulator. Instead, suspension insulators are designed to have a large factor of safety mechanically.

Figure 282 shows a suspension insulator of the rod type (usually made of well-glazed porcelain) with large ring-shaped shields *R*. The

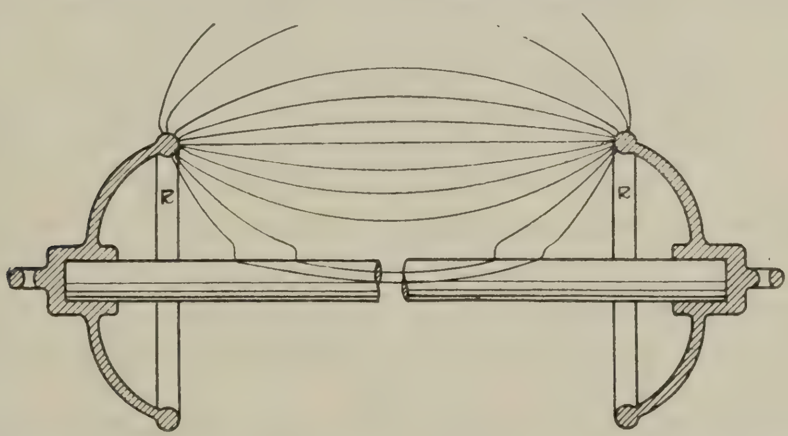


FIG. 282.—Suspension Type of Insulator Equipped with Shield Rings Showing Electric Field Mostly in the Air.

major part of the electric field is confined to the **air** between these rings, and the losses due to absorption in the porcelain are greatly reduced.

This is accomplished by placing the rounded shield rings well out from the rod and closer together than any other part of the metal end fittings.

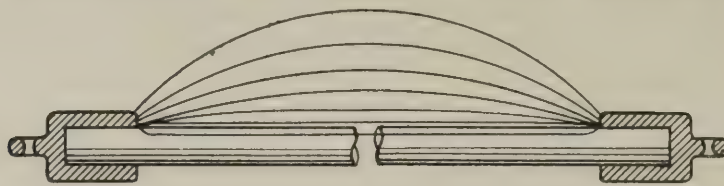


Fig. 283.—Suspension Type of Insulator without Shield Rings Showing Electric Field Concentrated at End Fittings.

The insulator shown in figure 283 is not so provided. The lines of electric force are crowded together and not kept away from the porcelain rod at the junction of the end fittings and the rod. This concentration of the electrical field causes serious losses, which result in heating at these points and eventual failure. In addition, the small radius of curvature of these end fittings is very favorable to the formation of corona at relatively low voltages, which leads to further losses.

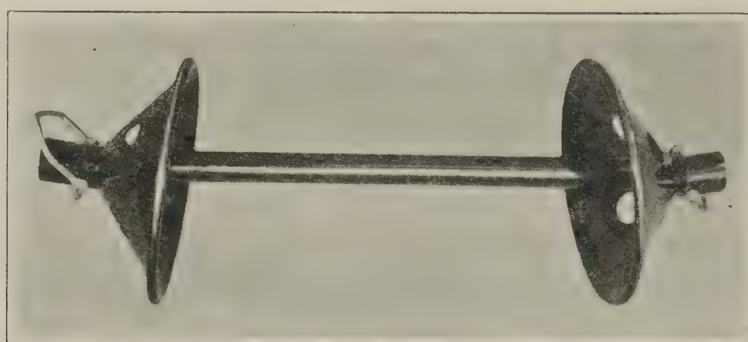


Fig. 284.—Photograph of Suspension Insulator Shown in Figure 282.

Figure 284 is a photograph of a porcelain rod insulator similar to that shown in figure 282. The arrangement of the shield rings is clearly shown. Such an insulator is used on shipboard to support the antenna from the lattice steel mast. Very frequently, at shore radio stations, the shield ring at the end of the insulator which is secured to the metal tower, is dispensed with. In this case, the adjacent metal of the tower serves to keep the electric field away from the insulator.

Bulkhead, or **wall,** insulators are used when a current-carrying conductor must pass through metal bulkheads, decks, or walls. It is of the utmost importance to keep the insulator out of the electric field between the conductor and the surrounding metal of the bulkhead. Figure 285 shows how this is effected.

The major part of the electric field is between the large ring R at the top and the metal of the clamping rings and bulkhead at the bottom. It will be seen that the capacity between the current-carrying parts and the metal of the bulkhead through the insulation is a minimum. This results from reducing the cross-section of the insulation as much as

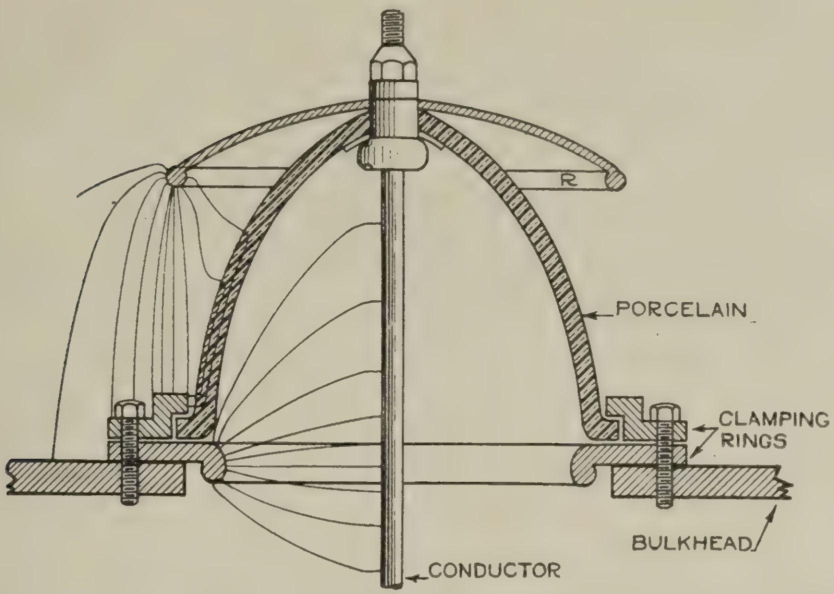


Fig. 285.—Bulkhead Insulator Shown in Cross-Section. Lines of Electric Force Kept in the Air by Means of the Shield Ring.

possible and by increasing the length of the path. Fortunately, such a shape for the insulating material is also mechanically strong. Figure 286 is a photograph of the insulator shown in figure 285.

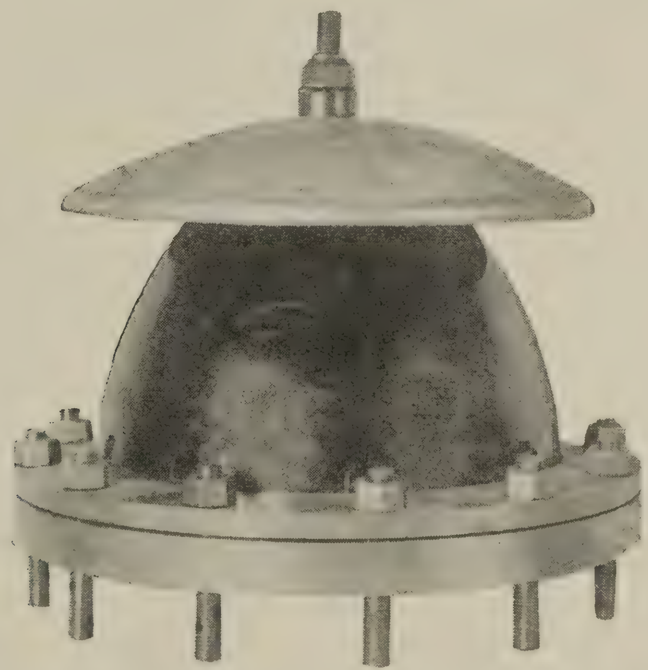


Fig. 286.—Photograph of Bulkhead Insulator Shown in Figure 285.

It has been found desirable, in the design of porcelain insulators, to keep the cross-section of the insulator uniform throughout, if this is not done, weak spots occur at the points of greater cross-section due to the drying and firing processes not acting uniformly. Failure generally occurs at these points.

Pillar, or **pedestal,** insulators do not offer so many troublesome points in design. The mechanical strains are usually relatively light.

Also, the general use of such insulators in protected locations removes the necessity for other than simple designs which will include the air rather than the insulator in the electric field. The voltage to which pedestal insulators are subjected is generally the same as in the case of suspension insulators at the antenna top, excepting when the antenna is operated at wave lengths near the fundamental, in which event there will be but a slight difference in the voltage. Consequently, they should be designed to have as high an electrical efficiency as either of the other types of insulators.

CHAPTER VI. MISCELLANEOUS INSTRUMENTS.

THE MICROPHONE TRANSMITTER.

The **microphone transmitter** is a device that is actuated by sound vibrations and produces corresponding variations in an electric current. Acoustics and electromechanics are involved in its operation. These will be considered separately, and then in combination.

Sound is produced by a body which is in a state of vibration. The sounding body, on account of its vibrating motion, sets up waves in the surrounding medium, such as air, water, etc. These sound waves are **longitudinal waves** in which the motion of each particle of the medium, through which the wave is traveling, is backwards and forwards along the line of the wave travel. These periodic movements of the particles produce alternate regions of compression and rarefaction. Energy is radiated in the form of sound waves.

Now, if one side of a diaphragm is acted upon by sound waves, it will be set into a vibratory motion by the alternating compressions and rarefactions of the medium in which it is located; the former exert a pressure against it, which tends to displace it from its normal position, while the latter permit it to return to its position of rest. The amplitude of the vibration of the diaphragm is dependent upon many factors which need not be discussed here. If the natural period of vibration of the diaphragm is the same as the frequency of the impressed sound waves, its response will be greatest. If it is nonresonant to any of the impressed waves, it will respond, but will be relatively insensitive. Resonance in a diaphragm used in telephony, whether line or radio, is not desirable. Also, sensitivity is more desirable in radiotelephony than in line telephony. The resonant condition produces an exaggerated effect, or distortion, of certain notes that have frequencies in the immediate vicinity of the natural period of the diaphragm.

The resulting vibratory motion of the diaphragm is practically in synchronism with the impressed sound. This motion may be imparted to another body in contact with the diaphragm, or the diaphragm itself may serve as a secondary source of sound. In the microphone transmitter, the motion of the diaphragm moves the elements composing the microphone.

The microphone. When carbon granules held in a suitable container are subjected to varying degrees of compression, the resistance through them will also vary; that is, when there is very little pressure on them, the resistance will be higher than when the pressure is increased. This arrangement, by which pieces of carbon are held in more or less close contact, is called a **microphone,** and constitutes a variable resis-

tance. Now, if a microphone is connected in series with a source of steady emf, any mechanical disturbance of the microphone will change the circuit resistance, and the current in the circuit will vary.

The combination of diaphragm and microphone is called a **microphone transmitter.** The action is best explained by referring to figure 287, which shows in cross-section the well-known **solid-back** transmitter used in line telephony. In the figure, *D* is the diaphragm, usually an aluminum disk 2½ inches in diameter. *T* is the solid back, on which is mounted the metal cup *B*, containing the carbon granules *C*. At the back of the cup is a small, hardened carbon plate *E*, which serves as one electrode of the carbon microphone. At the front is another very hard carbon plate *F*, which serves as a lid for the small metal cup. The diameter of this plate is a little less than the diameter of the inside of

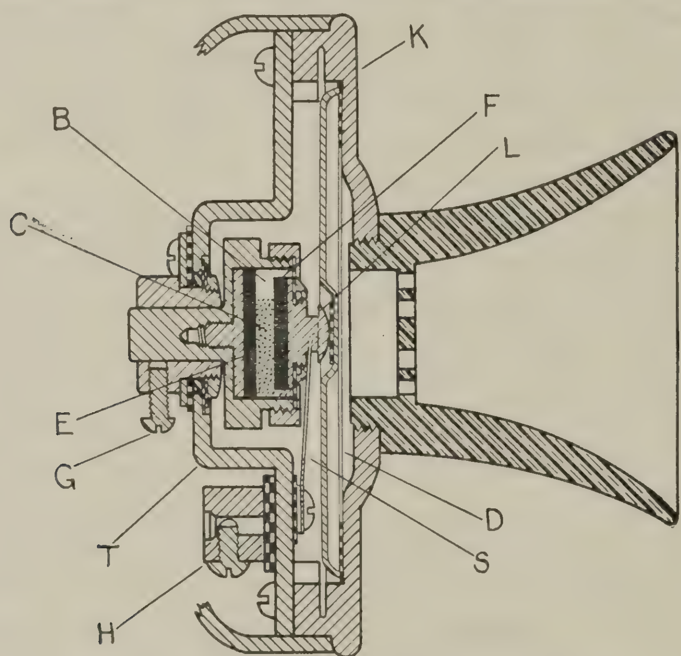


FIG. 287.—The Microphone Transmitter.

the cup. The cup is completely closed by a flexible mica disk, which is attached to the rim of the cup and to the carbon disk. This carbon lid, or cover, forms the second electrode of the transmitter. The button *L*, attached to the carbon plate *F*, is maintained in contact with the diaphragm by a metal spring *S*, which serves also to damp the vibrations of the diaphragm. The space between the carbon cover *F* and the back electrode *E* is nearly filled with carbon granules, and the electrodes *E* and *F* are so insulated that the electric current in the transmitter circuit, in passing from one electrode to the other, passes through the entire mass of carbon granules. The two wires leading to the transmitter are connected to the terminals *G* and *H*. The metal face *K* of the transmitter is made heavy to prevent excessive vibration, and the exposed metal parts are usually insulated from the current-carrying parts. The current through the usual type of microphone transmitter

is about 0.2 ampere, and the power consumed in the transmitter is about 2 watts.

When the sound waves are impressed on the diaphragm, the resulting motion of the diaphragm is imparted to the carbon plate *F*. The to-and-fro movement of this plate varies the pressure on the carbon granules *C*, and the resistance between *G* and *H* is varied. The action is best when the microphone transmitter is held in the position shown in the figure.

Microphone transmitters are classed according to their average resistance. A **high-resistance** transmitter has an average resistance of from 30 to 60 ohms, while the resistance of a **low-resistance** transmitter averages from 10 to 15 ohms. They are also classified as to their current-carrying capacity—whether it is small or large. The microphone transmitters in general use are of the former class. Serious difficulties are experienced in building a microphone that can carry a current much greater than the value specified above, due to arcing between contact points of the granules, which become redhot. Such an action destroys the usefulness of the device. Overloading is indicated by a frying noise.

The microphone transmitters used in radiotelephony at the present time are essentially the same as the one just described; in fact, the commercial type is very frequently used in radiotelephone transmitters. Many refinements have, however, been incorporated in the microphone transmitter and its associated circuits to reduce distortion of speech, and especially of music.

The microphone circuit. Figure 288 shows an elementary microphone circuit. *M* is the microphone transmitter, *B* is the source of steady emf and *P* is the primary winding of the step-up transformer *T*.

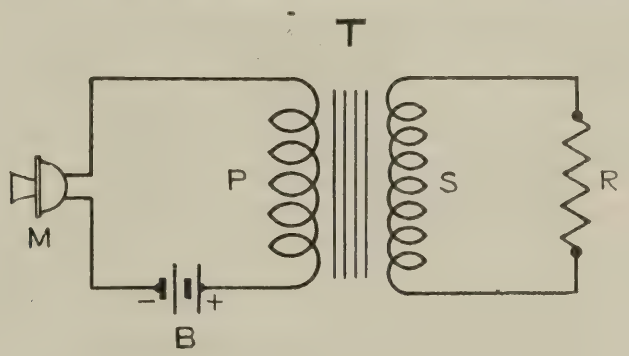


FIG. 288.—The Microphone Circuit.

The transformer is usually of the iron-core type with an air gap to reduce distortion. *S* is the secondary winding of the transformer and *R* is the load. The action is as follows:

As long as the microphone diaphragm is not disturbed, the resistance of the circuit is fairly high, and only a small steady current flows in the microphone circuit. Consequently, no emf is induced in the secondary winding. When the microphone is spoken into, its resistance varies, and the resulting variations in the direct current induce a

varying emf in the secondary. This varying emf causes an **alternating current** to flow in the secondary circuit. This alternating current varies in frequency, amplitude and wave form. If the distortion is not serious, the variations will be practically proportional to the pitch, intensity and quality, respectively, of the sound that is actuating the microphone diaphragm. This alternating emf can now be impressed into a line for line telephony, or into a radiotelephone transmitter. The output of the transmitter T can also be amplified by means of an audio-frequency amplifier, if desirable. Distortion due to electrical resonance in any of the circuits should, of course, be avoided as much as possible.

THE TELEPHONE RECEIVER.

The **telephone receiver** is a device by means of which variations of current are able to produce sound. It is the reverse of the telephone transmitter in that variations in an electric current are employed to give a vibratory motion to a diaphragm which, as the sounding body, gives forth sound.

Figure 289 shows in cross-section a **watch-case** telephone receiver, which is used in line telephony by switchboard operators and almost exclusively in radiotelegraphy and radiotelephony. Two such receivers, suitably fastened to an adjustable **headband** and connected in series by means of **telephone cords,** constitute a **headset,** or a **pair of telephones.**

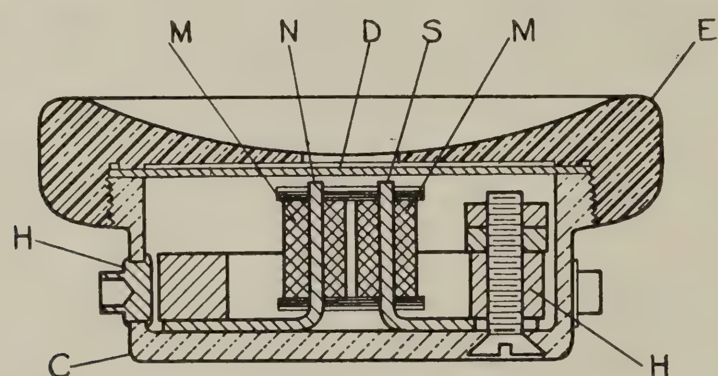


Fig. 289.—The Watch-case Type of Telephone Receiver.

Referring to figure 289, C is the case of the receiver. This case can be made of metal or hard rubber. When shielded telephone cords are used, the case should be mainly of metal. Attached to this case is a permanent magnet of the horseshoe type; the ends of this magnet are indicated by H H. Two bent pieces of soft iron are attached to the ends of the permanent magnet, and form the two pole pieces, N and S. The diaphragm, or armature D, is a thin circular disk of soft iron which is usually enameled on the side nearer the ear and varnished on the other. It is held securely around its edge against the case and very close to the pole pieces by the hard-rubber ear cap E, which is generally

screwed on to the case. The ear cap is fashioned so as to fit the ear comfortably and to exclude outside noises to a considerable extent.

The cap is also provided with a hole in its center, through which the sound produced by the vibrating diaphragm is heard. A coil *M*, consisting of a very large number of turns of fine wire (No. 40 B & S gauge), is wound on each of the pole pieces. These two coils are ordinarily connected in series in each telephone and the two telephones also connected in series. The dc resistance of a telephone receiver used in radio is high, usually being about 1,000 ohms for each receiver, or 2,000 ohms for a pair.

It will be remembered that the magnetizing effect of current flowing through a coil is dependent upon the ampere-turns. The required magnetizing effect in the case of telephones is obtained by a very feeble current which flows through a great many turns of wire. Thus, the larger the number of turns, the greater will be the magnetic effect of a given current. Other considerations, which will not be discussed here, limit the number of turns that can be used, so that telephones for radio purposes seldom have a dc resistance greater than 4,000 ohms.

The action of the type of telephones just described is as follows. Due to the pull exerted on the diaphragm by the adjacent *N* and *S* poles of the permanent magnet, the diaphragm is bowed in by an amount which is dependent upon its flexibility, but does not touch the pole pieces. Variations in the current in the telephone windings produce corresponding variations in the magnetic field of the pole pieces, and the diaphragm is moved. The movement of the diaphragm sets up a sound wave, which strikes the ear drum and is heard. For example, if the telephones are connected in place of *R* in figure 288, the sound that is impressed on the microphone transmitter will be reproduced quite faithfully by the telephones. The pitch and quality of the sound will suffer some distortion, due mainly to the mechanical resonance of the diaphragm to some particular frequency. The intensity of the sound is amplified by the microphone, the additional power being supplied by the local battery.

The mica-diaphragm telephone. Another type of telephone, developed by N. T. Baldwin, is shown in figure 290. *N S* is a ring-shaped permanent magnet to the poles of which are attached **U**-shaped soft-iron pole pieces, P_1 and P_2. A coil *M* is placed longitudinally between the two pole pieces. A pivoted and balanced soft-iron armature *A* lies between the pole pieces in a slot provided for it in the coil form. One end of this armature is connected by a bar *B* to a thin mica diaphragm *D*, which is held around its edge against a brass circular disk when the ear piece is screwed on the case.

An inspection of the figure will show that the armature, and therefore the diaphragm, is not subjected to a strain unless current is flowing in the coil, because equal amounts of flux flow through each end of the

armature between the tips of the **U**-shaped pole pieces. For a given direction of current through the coil, the magnetic effect of the permanent magnet is reduced at one end of the armature and increased at the other end. The balance of the armature is thereby disturbed, one end being drawn down while the other end is drawn up, and the diaphragm moves in accordance with this motion of the armature.

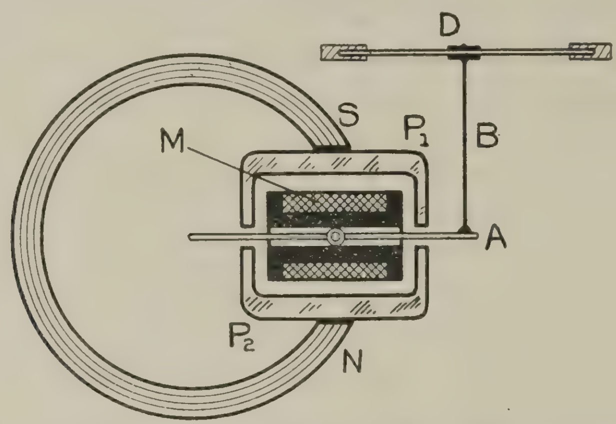


Fig. 290.—The Baldwin (Mica Diaphragm) Telephone.

A high sensitivity is claimed for this telephone, due to (a) the low reluctance of the magnetic circuit, (b) the balancing of the armature and its freedom from a steady strain which permits a greater displacement of the diaphragm, and (c) the armature being acted upon at each end in opposite directions which, in conjunction with the permanent magnet, doubles the magnetic effect of a given current flowing in the coil.

The **sensitiveness** of telephones is dependent upon many factors. It is essential that the moving parts be light, so that they will respond readily to rapid variations of current. The reluctance of the magnetic circuit should be low, so that very feeble variations in the current will be effective in varying the magnetic field. This requires that the diaphragm be set very close to the pole pieces. A large number of turns of wire should be used in order to obtain a relatively strong magnetic effect from a feeble current. The diaphragm should not be stressed abnormally by the permanent magnet. The main function of the latter is to keep the pole pieces partially magnetized, because less current is required to change their magnetism by a given amount when they are in this condition than would be required were they not magnetized.

The sensitiveness of telephones can be further increased by making the moving parts resonant to a certain frequency, such as to an alternating current of 800 to 1,000 cycles, passing through the telephone windings; in fact, most telephones used in radio have a **resonant frequency** that lies within this band. The amplitude of diaphragm vibration, and hence the sound produced, is greatest when the impressed frequency is the same as the resonant frequency. The ear is also most

sensitive to sounds having frequencies within this band. It has been found that an alternating current of approximately $1 \cdot 10^{-9}$ ampere, frequency 800 cycles, will produce an audible sound in the usual telephones that are resonant to this frequency. For higher or lower frequencies, the current required to produce audible sound is greatly increased; for example, nearly 1,000 times as much current is required at 60 cycles as at 800 cycles. This applies equally well when telephones are used to receive continuous-wave transmission by the beat method, and also in the straight detection of sinusoidally modulated continuous waves. In the case of spark transmission, **no great advantage** is gained by using 500 cycles instead of 60 cycles for spark transmission even though the telephones are resonant to 1,000 pulses per second. This is on account of the fact that the rectified pulses of current, when receiving from a 60-cycle transmitter, act by impact on the diaphragm and set it into oscillation at its natural period. Thus, the note heard in the telephones in the last case is not the same as would be heard directly from the spark, but is a mixture of the spark and diaphragm frequencies.

The increase in sensitiveness, caused by mechanical resonance, is a serious fault when the telephones are used for reproducing speech or music, because distortion will occur, as was explained in the case of the microphone transmitter.

Impedance of telephones. Since the windings of a telephone are highly inductive, they will offer a greater resistance to alternating current than to direct current. The resistance to alternating current is the **impedance,** which is sometimes called **ac resistance.** The impedance of one type of telephones having a dc resistance of 2,200 ohms is approximately 23,000 ohms at a resonant frequency of 1,000 cycles, while another type having a dc resistance of 2,000 ohms has an impedance of 30,000 ohms at a resonant frequency of 800 cycles. Telephones having impedance values between the two limits just given will, in general, be found suitable for radio purposes. For example, the telephone impedance should approximate the ac resistance of the plate circuit of a vacuum tube.

Telephone cords. The flexible wires that are used to connect the two ear pieces together and to the receiver are called the **telephone cords.** They are made of extremely flexible tinsel cord and are amply insulated with silk or cotton braided insulation. The covering is so chosen that noises due to the cords rubbing against the clothing are reduced to a minimum. The cords are provided with suitable tips for making the connections.

Shielded telephone cords are very frequently used in conjunction with amplifiers to reduce electric coupling between the output and input, which would cause the amplifier to self-generate oscillations, or howl. One type of shield consists of a wrapping of tinsel cord over the telephone leads, and the whole then covered with insulation. The

shield is connected to the metal part of the ear piece and to the shield of the amplifier, which may, or may not, be grounded.

THE TUNING-FORK OSCILLATOR.

The tuning-fork oscillator produces an alternating emf or current of audible frequency that is very nearly sinusoidal and, consequently, is practically free from harmonics. For this reason, it is well suited to alternating-current bridge measurements, when a balance is indicated by the null method with telephones. In this work, the presence of harmonics of very small magnitude will make it difficult to determine the balance, because when the bridge is balanced for the fundamental frequency, it will usually be out of balance for the harmonic frequencies, and these harmonics will give a residual sound in the telephones. The frequency must remain constant.

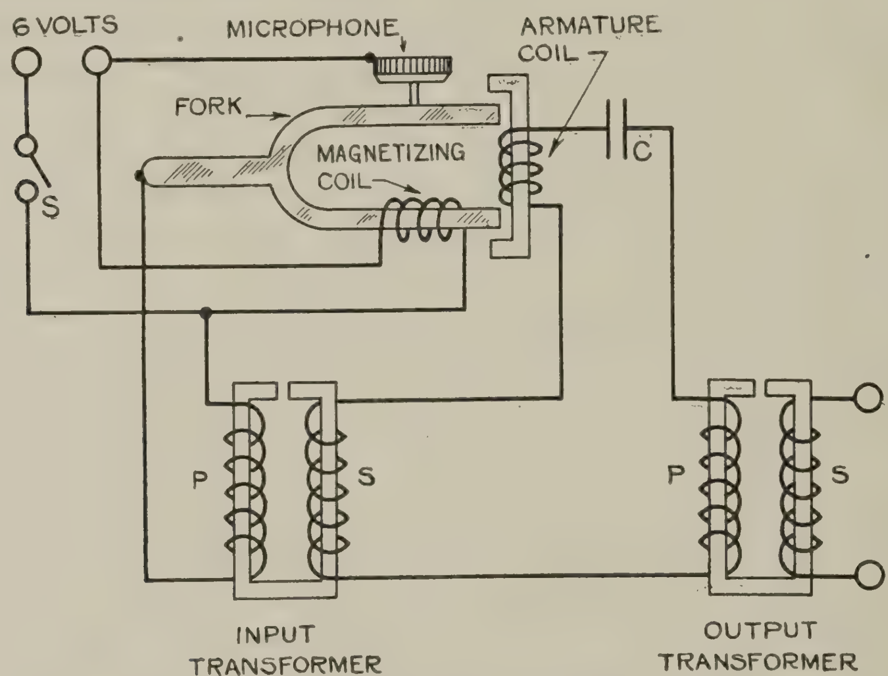


Fig. 291.—The Tuning-fork Oscillator Circuit.

Figure 291 shows the circuits of a low-power oscillator, the output of which is approximately 0.06 watt at a frequency of 1,000 cycles, the current being 12 ma.

When the switch S is closed, the magnetizing coil is connected across a 6-volt battery. The primary of the input transformer and the microphone button in series are also connected across the battery. The tuning fork has a period of 1,000 cycles. The circuit which includes the secondary of the input transformer, the primary of the output transformer, the armature coil and condenser C is tuned to be resonant to this frequency. Distortion is reduced by the construction of the transformers. The circuit to be excited is connected to the secondary terminals of the output transformer. The action of the tuning-fork oscillator is as follows:

When the switch S is closed, the fork becomes magnetized by the current flowing through the magnetizing coil, and the prongs are drawn away from their position of rest. This initial movement of the prongs disturbs the microphone button, and the resistance through the carbon granules is changed. Thus, the resistance of this circuit is varied and a variation in the current in this circuit results. Now, the motion of the prongs is most rapid at the instant they are passing through their position of rest. Likewise, the variation in the microphone resistance is most rapid at this instant. Consequently, the alternating emf in this circuit is a maximum for this condition. The primary of the input transformer, which is in this circuit, is highly inductive; hence, the current lags 90° behind the emf. The alternating emf induced in the secondary of the input transformer also lags 90° behind the inducing current. However, since the circuit in which the secondary is included is tuned to resonance, the current in this circuit will be in phase with the induced emf. This current is, therefore, either 180° out of phase, or in phase with the motion of the fork. Its effect on the armature coil, however, is made to be in phase with the fork, by connecting the coil in the proper sense, so that the magnetic effect of the current through the armature coil will assist the motion of the fork, and thereby increase its amplitude. The amplitude soon reaches a steady value, and the fork then continues to vibrate with this constant amplitude as long as the switch is closed. The operation is disturbed, however, if the oscillator is overloaded.

The tuning-fork oscillator is thus seen to be quite similar to an oscillating vacuum tube circuit.

SHIELDING OF RADIO APPARATUS AND THE ELECTRIC SCREEN.

The shielding of radio apparatus is generally for the purpose of eliminating electric coupling between the transmitter, or driver, and a receiver or measuring circuit. The magnetic coupling between circuits falls off very rapidly with the distance between the circuits, but electric coupling falls off very slowly. Thus, in attempting to measure the amplification of a sensitive amplifier, it will often be found that a loud signal is heard in the telephones even when the distance to the transmitter is so great that the magnetic coupling between the circuits is practically zero. Under these conditions, silence cannot be obtained by rotating the coupling coils, as would be the case if magnetic coupling alone were present.

In order to screen from electric effects successfully, it is essential to build a screen which completely surrounds the apparatus to be shielded. The screen is preferably of sheet copper, or copper screening. It is especially important also to put all batteries, etc., which supply the apparatus, within the screened enclosure. A wire running through the wall of the screened enclosure, and insulated from it, will destroy

almost completely the effectiveness of the screen. In some experiments, an amplifier was put inside a screened enclosure with a driver on the outside, some distance away. When no leads ran through the screen the amplifier was silent, but when an insulated wire not connected to anything was put through the screen from the outside so as to project a few inches inside the screen, a loud signal was heard, which continued to increase in intensity as the wire was pushed farther into the

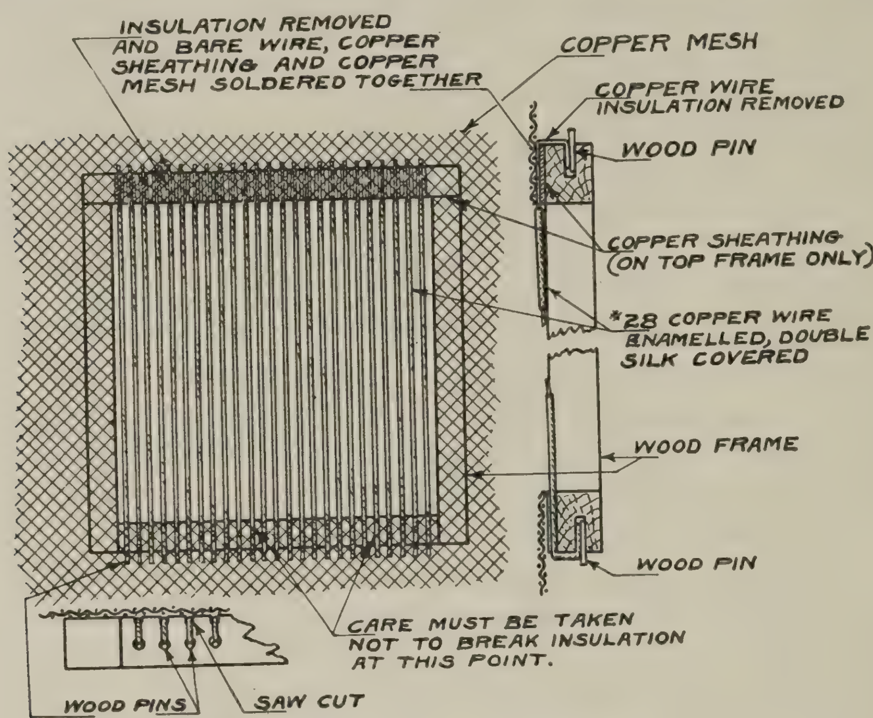


FIG. 292.—Construction of the Electric Coupling Screen.

cage. This experience has been met with on several occasions, and it has been found that the screen is a failure unless this precaution is taken.

It is possible to couple magnetically with apparatus within the cage very readily if a hole is cut in the screen and a window put in. The window is constructed as shown in figure 292. A wooden frame is made and a row of small brads driven close together across the top and bottom. Fine insulated wire is stretched between the top and bottom rows of brads so as to form a grid. The wires are connected together at the top end, and connected also to the screen. The wires are insulated from each other and from the cage at the other end. There are no closed paths for eddy currents to flow with this construction, so that a coil on the inside can readily be coupled magnetically to a coil on the outside, without much power loss in the screen. The lines of electric force, however, terminate on the wires of the window and there will be, in general, a complete disappearance of electric coupling.

It will be seen that if the window is made small, eddy currents will be induced in the sheet copper, or copper screen cage. As a result, the resistance of either the circuit inside the cage, or the one outside, or

both, will be increased. Consequently, if the window is made very large, the eddy current losses will be greatly reduced. Hence, the cage itself can be constructed in the same manner as the window. Thus, a cylindrical cage would be constructed of many insulated wires, all connected electrically at the center of the upper base, and extending radially to the side, down the cylinder, and then radially to the center of the lower base where they are brought close together, but kept insulated.

CHAPTER VII. QUARTZ CRYSTALS.

The importance of producing vacuum tube transmitters which generate a constant frequency has never been seriously considered until recently, when the advent of broadcasting stations and the increasing number of ship and shore radio stations have demanded that such transmitters be made available.

Various means have been employed to hold constant the frequency of transmitters but no absolutely satisfactory means have been devised which will maintain a constant frequency in vacuum tube transmitters which employ self-oscillating circuits.

When we consider that the beat note of a continuous wave transmitter has to remain within a certain tolerance, say 350 cycles, it can be realized that the constant frequency condition becomes harder to meet as the frequency of the transmitter is increased. This can be readily seen when we consider that the frequency of a 4000-kc. transmitter can not be changed more than one one-hundredth of 1 per cent before it has exceeded the specified frequency tolerance.

The best method for accurate maintenance of frequency is the piezo electric crystal controlled transmitter. Such a transmitter has been found to meet all requirements if suitable means are provided to keep the temperature of the crystal constant.

There are a number of crystalline substances such as quartz, tourmaline, and Rochelle salts which have excellent piezo-electric and pyroelectric properties. All these are from an optical standpoint doubly refracting and possess asymmetric atomic structure. Alpha quartz which is piezo electrically active has an unsymmetrical hexagonal atomic structure, while Beta quarts which has no piezo-electric properties is of a regular hexagonal atomic structure.

It is only natural to assume that any crystalline body which has double refracting properties and whose atomic structure is unsymmetrical should be piezo electrically active.

Considering the three commonly known piezo electric crystals, i. e., quartz, tourmaline, and Rochelle salts, we find that quartz is to be preferred. Rochelle salts, although it has ten times the piezo electric properties of quartz, is not reliable. It is fragile, extremely hard to manufacture, and its physical dimensions can be easily changed by handling, especially when subjected to contact with water. It also will not stand any electrical load, for instance, if used as a resonator in connection with the output of an oscillator of a few watts capacity it will break down. This breakdown will either be in the form of a series of mosaic cracks throughout the crystal or it will consist of a melting process wherein

the crystal suddenly flattens out and assumes an isotropic state. If the power is increased the salts will return to a liquid state.

Repeated attempts have been made to make Rochelle salt crystals function as well as quartz, for controlling the frequency of a vacuum tube transmitter, but no success has been obtained in this endeavor. The Rochelle salt crystal is not mechanically strong enough to withstand the vigorous vibration which is met with in the quartz crystal when controlling the frequency of a vacuum tube transmitter. It is also possible that the hysteresis losses in the Rochelle salt crystal are such that they tend to damp out any properties of the crystal for generating a return piezo electric voltage required for maintaining a vacuum tube circuit in an oscillating condition.

There is no literature available which shows the application of Rochelle salt crystals as a means for controlling the frequency of a vacuum tube circuit.

Tourmaline is too expensive to be considered as a commercial product.

Quartz can be obtained in reasonable quantities in Brazil, Madagascar, Japan, and the United States. Any quartz which has no flaws, intergrowths, or optical twinning can be so manufactured that it has excellent piezo electrical oscillating properties. By this we mean that such a crystal can be used to control the output of a vacuum tube oscillating circuit at one definite frequency and with maximum output.

Quartz will retain its physical dimensions if kept at a definite temperature. It will also stand considerable abuse, which accompanies its use in oscillation test circuits, where the crystal is heated momentarily to temperatures in excess of 45° centigrade and is subjected to frequent washing. Experience has demonstrated that crystals will hold the original oscillation frequency for periods in excess of ten months, when operated continuously in a high frequency transmitter system. Other exacting tests have proven that quartz is the only material known which is satisfactory for use in crystal controlled vacuum tube circuits.

The quartz crystal is hexagonal in shape and when in its true form has an apex at each end. The methods of mining and also the process of growing are such that the two apexes are rarely found on crystals which are purchased from the importers. In the majority of cases it is a rare case to obtain crystals having sides and one apex which are not chipped or cracked due to rough handling or poor mining methods.

The usual crystal when received is similar in shape to that shown in Figure 293. In this crystal the optical axis is parallel to an imaginary line Z which is drawn between two apexes. The electrical axes are of two types, one which is parallel to a line X drawn between the corners of the hexagonal sides and the other which is parallel to the line Y which is drawn between the opposite flat faces of the hexagonal sides. From this we note that there are three X electrical axes and three Y

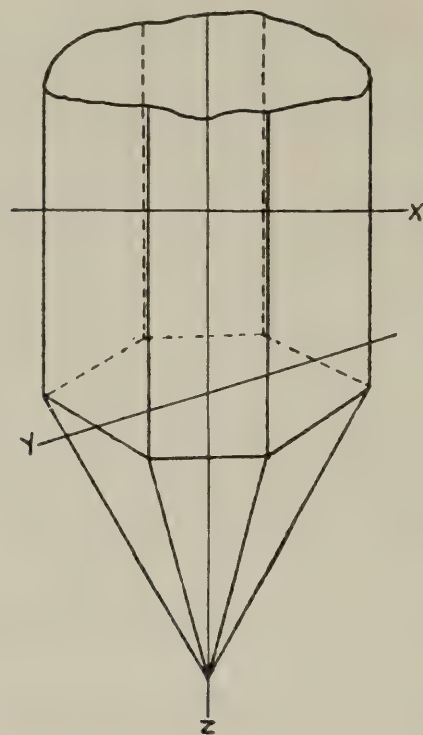


Fig. 293.

electrical axes and one optical axis. The optical axis is always at right angles or perpendicular to any of the electrical axes.

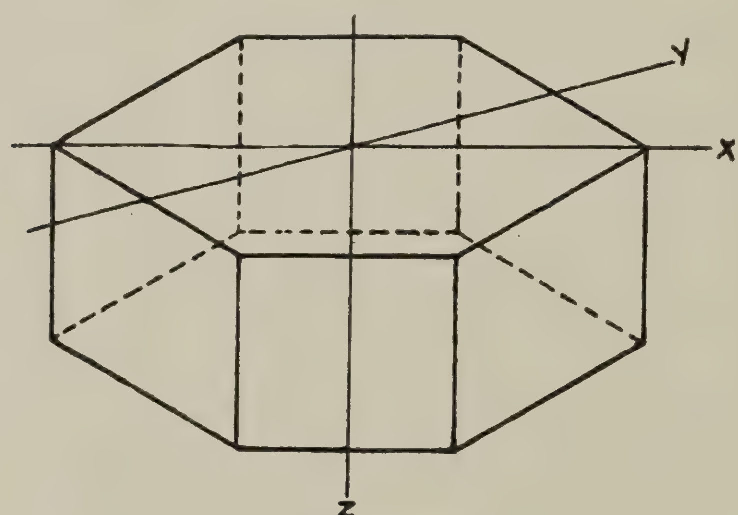


Fig. 294.

Now cut a slab of quartz from the crystal as shown in Figure 294, making this cut at right angles to the optical axis Z. Then in order to obtain a workable crystal we cut a slice from this slab as shown in Figure 295. This slice is so cut from the slab that the slicing produces a crystal which sides are parallel to one of the Y electrical axes and at right angles to one of the X electrical axes. We now have a crystal whose thickness represents an X axis, whose length a Y axis and the depth of a Z or optical axis.

Having completed the cutting of the crystal which we will term the "Curie" or "zero angle cut" we find that there are three frequencies to which crystal will resonate. One frequency corresponds to the X dimension, one to the Y dimension, and the other to a frequency which is between the X and Y axis frequency and is termed the coupling frequency. This coupling frequency depends on the dimensions of the X and Y axes. In round crystals the X dimension will produce 104.6 meters per mm., the Y dimension 110.5 meters per mm., and the coupling frequency is equal to 0.71 of the Y dimension wave length. In rectangular crystals the meters per mm. for the X dimension varies from 103.5 to 105 while for the Y dimension it varies from 110 to 117 meters per mm. The meters per mm. obtained for the coupling frequency can not be stated because it depends on the dimensions of the rectangular form which may be square or any shape which the require-

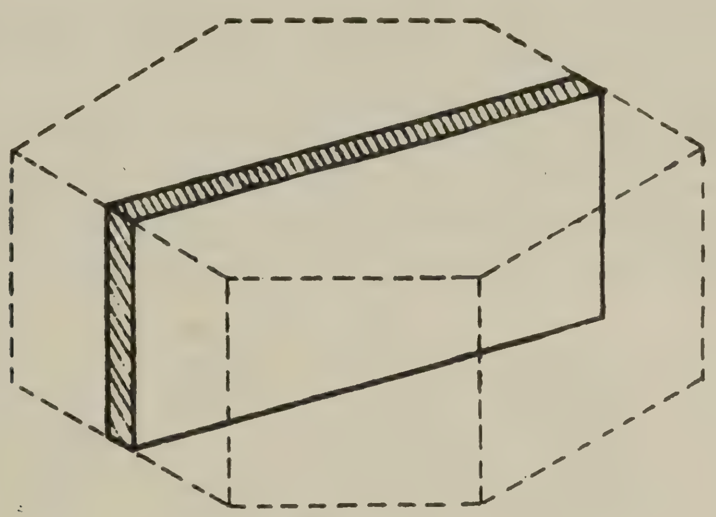


FIG. 295.

ments demand. These figures are based on the true Curie cut and on Crystals whose Y dimension is between 20 and 28 mm. If any cut is made which is at an angle from the Curie cut the meters per mm. will be greater, especially for the X dimension oscillation.

Rectangular crystals are to be preferred to round crystals, first because they are cheaper to make and second because they will control a greater radio-frequency output without cracking or chipping. The latter condition is probably explained by the uneven stress conditions present in round crystals when they are oscillating under influence of radio-frequency currents.

In Figure 296 the crystal is shown placed between grid and filament. This is the fundamental Navy circuit for crystal controlled vacuum tubes. The battery is such as to put a negative bias on the grid, thus greatly reducing the grid current, by keeping the grid negative throughout most of the cycle.

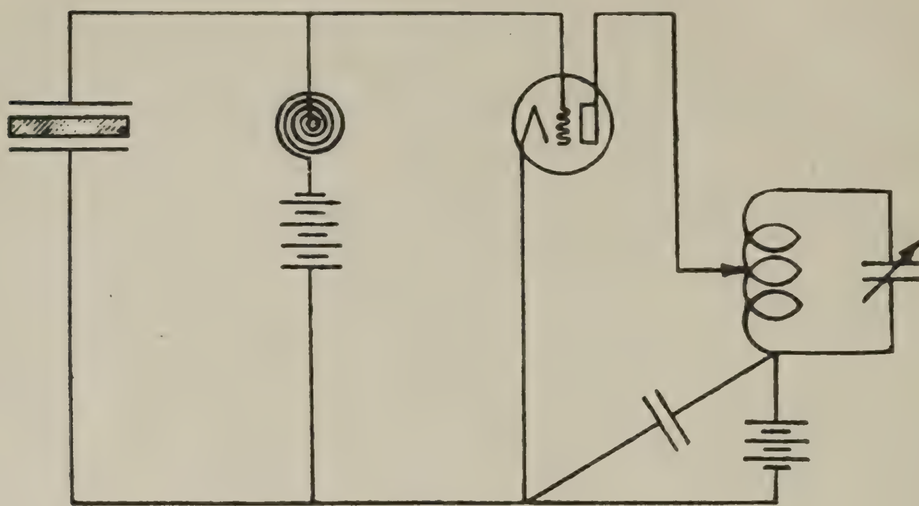


FIG. 296.

The greatest output can be obtained from the crystal in the 3000 to 4000 kilocycle band. Lesser outputs are possible on frequencies either higher or lower than this band.

Figure 297 shows an improvement on the circuit of figure 275, in which the direct current and radio frequency circuits to the tube plate are separated by means of the choke and stopping condenser shown.

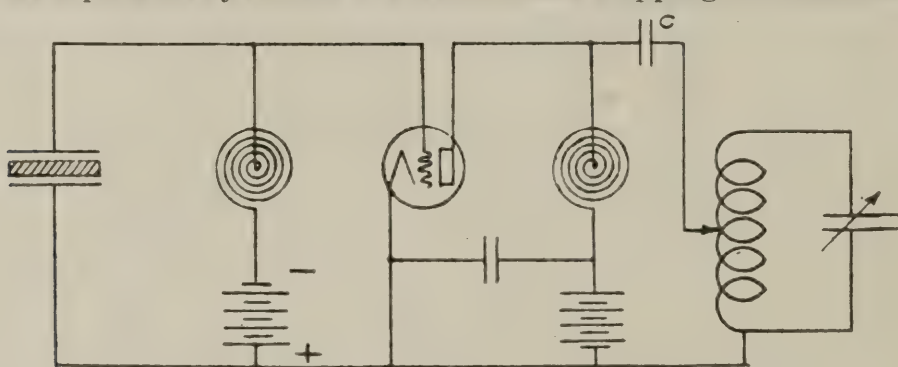


FIG. 297.

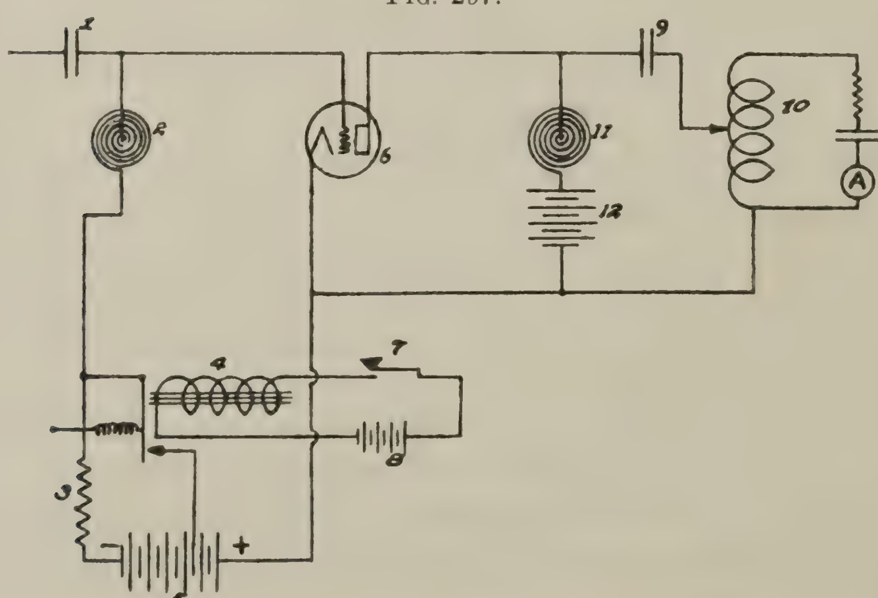


FIG. 298.

Figure 298 shows a method of keying suitable for use with crystal control transmitters. The crystal tube is, of course, allowed to oscillate continuously, and the keying is done in one of the stages of amplification between the crystal tube and the output of the set. It will be noted that keying is accomplished by changing the grid biasing voltage of the amplifier tube from an operating voltage to a high blocking voltage by means of the relay 4 and associated circuits.

CRYSTAL HOLDERS.

The subject of crystal holders is important. The crystal should be placed in a hermetically sealed container where no moisture or dirt can come into contact with the crystal.

It is necessary that capacities other than that between the crystal contact plates be kept as small as possible, thus eliminating the charging losses occasioned by extraneous shunt capacities. For reliable operation and maximum output the crystal contact plates should be intimately touching the surface of the crystal. Lapped surfaces on these plates are to be preferred, while the weight of the upper plate should be kept to a minimum. No restriction of up and down movement of the upper plate should be tolerated. Light spring pressure can be applied to this plate but for best results no pressure other than the weight of the plate is necessary.

Retaining rings of bakelite or other insulating material or brass retaining pegs can be employed to hold the crystal in one fixed position with respect to the sides of the container. A holder having all these features, together with means for restricting the tendency for the crystal to jump clear of the retaining pegs when being transported, is shown in Figure 299.

Experience has shown that any air gap between upper surface of crystal and the contact plate means a great reduction in output and when used in regular power circuit the air gap causes brushing between the surface of the crystal and the plate, which in turn causes the crystal to heat and crack. Crystals which have been subjected to the brushing effect show a discoloration on the surface of the crystal at the place where the brushing occurred.

The frequency of any crystal changes with temperature, and for absolute constancy of frequency it is necessary that some type of temperature control be applied either directly or indirectly to the crystal. One method is to place the crystal in a hermetically sealed container and by use of thermostat and heating unit in this container maintain the crystal at a predetermined temperature. The second method is to place the crystal in a crystal holder of similar type to that shown in Figure 299 and to secure this holder on a metal plate which can be maintained at a constant temperature. The heat from the plate will

be conducted through the lower crystal contact plate direct to the crystal.

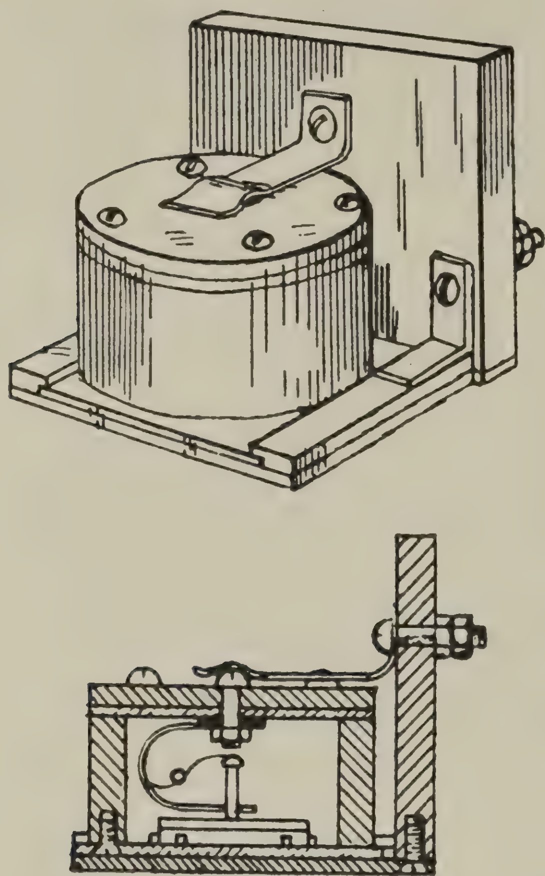


FIG. 299.

The metal-heating plate can be kept at a constant temperature by circulating water through it, or a subcompartment with suitable heating unit and thermostat can be attached to this plate. A thermostat can be employed with the water circulating system to turn on or off the current in a heating coil which is placed in the water intake line to the plate.

The importance of constant temperature control is appreciated when operating high-frequency crystals, as a change of 10 degrees centigrade will change the frequency as much as 1 kc. in the 4,000-kc. range. Extreme changes in temperature met with on board naval vessels when cruising can change the crystal frequency as much as 3 kcs., which change is very detrimental to perfect communication conditions. A remedy for this is to provide a thermostatic control which will maintain the crystal temperature above that which is ever encountered throughout the year. This is identical with the practice now in force with reference to the Navy standard 25-kc. crystal calibrator, which is used as a standard of frequency for the Navy.

SECTION II.

APPLICATION.

PART 1.—RADIO TRANSMISSION.

CHAPTER I. GENERAL.

All radio transmission is dependent on the simple series circuit containing capacity and inductance so connected as to form a complete path for alternating currents.

Simple oscillatory circuit. The frequency of such alternating, or as it is generally called, oscillating current, is dependent upon the total values of inductance L and capacity C in the circuit. If there were no losses, the oscillating current once started would keep on flowing at the same amplitude indefinitely. Energy is, however, dissipated in several ways, chief of which is through the resistance of the circuit. Unless these losses are made up from cycle to cycle, the energy in the oscillating current is rapidly dissipated.

Damped and undamped waves. When energy is supplied at intervals so that a group of oscillations is set up by each such energy pulse, the amplitude of each successive oscillation is decreased by uncompensated losses and damped waves result. Spark transmitters are typical of this class. Where energy is supplied coincidently with each oscillation to make up the losses incurred during that cycle so that the amplitude of successive oscillations remains constant, undamped waves result. Arc transmitters, high-frequency alternators and vacuum-tube transmitters produce undamped waves.

The antenna. In actual application, the simple circuit is modified by replacing the plate condenser by the antenna and ground. These form a condenser, the capacity of which can be readily measured by the usual methods and instruments. This antenna capacity varies roughly with height and area in a manner similar to that which applies to the spacing and area of the usual built-up condenser.

Wave length grouping. Very high frequencies, as compared with those of the usual commercial power circuits, are employed for radio communication. A range from 20,000 to 10 kilocycles is at present used for practical communication. This corresponds to wave lengths from 15 to 30,000 meters respectively.

The following represents the grouping and general use of wave lengths in the United States:

Kilocycles	Meters	Use
95 to 120	3156 to 2499	Government only.
120 to 153	2499 to 1960	Marine and aircraft only.
125	2399	Government (non exclusive).
153 to 165	1960 to 1817	Point to point, Marine and Aircraft only.
155	1934	Government (non exclusive).
165 to 190	1817 to 1578	Point to point and Marine only.
175	1713	Government (non exclusive-Ice Patrol, broadcast, etc.).
190 to 230	1578 to 1304	Government only.
230 to 235	1304 to 1276	University and College experimental only.
235 to 285	1276 to 1052	Marine only (phone).
245	1224	Government (non-exclusive).
275	1090	Government (non-exclusive).
285 to 500	1052 to 600	Marine and Coastal only.
300	1000	Beacons only.
315	952	Government only.
343	874	Marine only.
375	800	Radio compass only.
410	731	Marine only.
425	706	Marine only.
445	674	Government (non-exclusive).
454	660	Marine only.
500	600	Calling and distress and messages relating thereto, only.
500 to 550	600 to 545	Aircraft and Fixed Safety of Life Stations (non-exclusive).
550 to 1500	545 to 200	Broadcasting only (phone).
1500 to 2000	200 to 150	Amateur only.
2000 to 2250	150 to 133	Point to point (non-exclusive).
2250 to 2300	133 to 130	Aircraft only.
2300 to 2750	130 to 109	Mobile and Government mobile only.
2750 to 2850	109 to 105	Relay broadcasting only.
2850 to 3500	105 to 85.7	Public toll service, Government mobile, and point to point communication by electric power supply utilities and point to point and multiple address message service by press organizations, only.
3500 to 4000	85.7 to 75	Amateur, Army mobile, Naval aircraft, and Naval vessels working aircraft, only.
4000 to 4525	75 to 66.3	Public toll service, mobile, Government point to point, and point to point public utilities (non-exclusive).
4525 to 5000	66.3 to 60.0	Relay broadcasting only.
5000 to 5500	60.0 to 54.5	Public toll service only.
5500 to 5700	54.5 to 52.6	Relay broadcasting only.
5700 to 7000	52.6 to 42.8	Point to point only.

Kilocycles	Meters	Use
7000 to 8000	42.8 to 37.5	Amateur and Army mobile only.
8000 to 9050	37.5 to 33.1	Public toll service, mobile, Government point to point, and point to point public utilities (non-exclusive).
9050 to 10,000	33.1 to 30.0	Relay broadcasting only.
10,000 to 11,000	30.0 to 27.3	Public toll service only.
11,000 to 11,400	27.3 to 26.3	Relay broadcasting only.
11,400 to 14,800	26.3 to 21.4	Public service mobile, and Government point to point (non-exclusive).
14,800 to 16,000	21.4 to 18.7	Amateur only.
16,000 to 18,100	18.7 to 16.6	Public toll service, mobile and Government point to point (non-exclusive).
18,100 to 56,000	16.6 to 5.35	Experimental.
56,000 to 64,000	5.35 to 4.69	Amateur.
64,000 to 400,000	4.69 to 0.7496	Experimental.
400,000 to 401,000	0.7496 to 0.7477	Amateur.

Power input to antenna. Large antennas are required to permit the efficient use of long wave lengths and high powers. From the current formula

$$I = V\omega C$$

where C = antenna capacity in farads,
I = antenna current in amperes,
V = antenna voltage in volts,
$\omega = 2\pi f$,

it will be seen that there are three limiting factors to the antenna current input, namely: (a) antenna capacity, (b) antenna voltage, and (c) frequency. At short waves, a given antenna can receive a greater charge than at long waves. Since the economical antenna voltage limit is fairly definite, there remains only the one course for long wave stations, namely: to provide a large antenna capacity. With it must be combined a reasonable effective height. This is the theoretical height at which a concentrated capacity equal to the antenna capacity would be located. The common term for combining radiation and effective height is to rate a station in meter-amperes as regards its transmitting qualities, the effective height being given in meters.

Two methods are possible, to secure a given signal strength at a distant point, namely; (a) moderate radiation with a high effective height or, (b) a greater radiation with a lesser effective height. The first method requires a highly elevated capacity of moderate extent; and the second a capacity at a moderate elevation but of much greater value. Thus, the antenna for the Lafayette Station in France is supported by

820-foot towers, while the new Long Island station of the Radio Corporation employs towers which are approximately 400 feet in height.

Since the antenna is essentially a condenser, it follows that its capacity decreases as the height of the flattop is increased, and for stations equipped with high antennas of moderate capacity, the limiting factor in operation may become the antenna voltage. 120,000 volts is considered the maximum desirable voltage, due to formation of corona. The approximate antenna voltage under any conditions may be calculated from the relationship

$$V=\frac{I}{\omega C}$$

where C = capacity of the antenna in farads,
I = antenna current in amperes,
$\omega = 2\pi f$

Methods of generating radio-frequency current. Three principal methods are employed to generate currents of the frequencies used for radio communication:

(1) Excitation of the antenna by means of a resonant circuit containing the proper values of inductance and capacity to give the desired frequency.

(2) High-frequency alternators which generate directly the frequency delivered to the antenna or a definite submultiple of the antenna frequency.

(3) Vacuum tubes.

All the early types of apparatus are generally found under (1) and include the induction coil, buzzer and spark. These all use the damped wave form. The various types of apparatus for the generation of undamped waves were later developed. The Poulsen arc, the Goldschmidt, Alexanderson and Latour high-frequency alternators, the frequency doubler system, and vacuum tubes constitute the principal methods. Continuous waves are used almost exclusively, long distance transmission because of the advantage found to exist over spark operation in distance covered for a given antenna current and the greater ease to handle high powers.

Advantages of undamped waves. The use of undamped waves for radio telegraph transmission is gradually displacing damped waves. Several important advantages of undamped over damped waves are causing the change, viz:

Reception is much sharper, thereby permitting the use of a greater number of channels of communication in the ether. The resonance curve shows a considerably sharper slope than for the undamped wave.

Due to the greater purity of the wave, less interference is caused.

Beat reception is possible with its attendant advantages of amplified signals and note control. By the latter feature, a note can be ob-

tained in the receivers to suit the individual operator and the conditions of reception.

Better response is obtained from automatic recording devices.

With small antennas and small powers greater distances can be covered.

Summary. The general trend, as above discussed, is towards the use of undamped waves for all transmission because of (a) greater purity of wave resulting in closer wave bands and less interference, (b) greater range for the same antenna current, and (c) greater ease in using the large powers required for long distances.

CHAPTER II. SPARK TRANSMITTERS.

Spark transmitters have formed the most widely used type in the past. Their qualities of comparative simplicity, of ruggedness, and reliability assisted materially toward the rapid expansion of the radio field.

Circuit and action. The standard circuit for a modern 500-cycle, quenched-spark transmitter, is shown in figure 300. Power is delivered by the generator *G* to the transformer *Tr* and by it to the condenser *C*. When the voltage across the condenser *C* becomes sufficiently high to break down the quenched-gap *Q.G.*, an oscillating current flows in the circuit composed of the primary of the oscillation transformer *O.T.*, the gap, and the condenser *C*. This current is rapidly damped out to a point where it can no longer break down the gap. The circuit then remains open until a potential is again built up on the condenser which is sufficient to cause gap failure, when the action above described

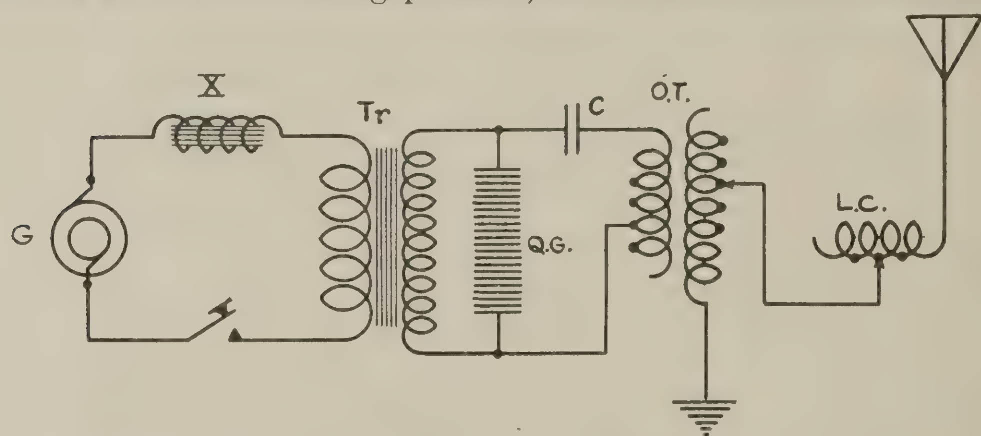


FIG. 300.—Circuit Diagram of a Quenched-Spark Transmitter.

is repeated. In order to produce radiation, a part of the power in the primary oscillatory circuit must be transferred to the antenna circuit. It is accomplished through the medium of the magnetic field which is set up by the current flow above described in the primary of the oscillation transformer. This flux cuts or interlinks the secondary turns of the oscillation transformer in the antenna circuit and thus sets up the potentials which, in turn, produce the actual current flow in the antenna. As shown in figure 300, the antenna circuit consists of the coupling coil, loading coil, and the antenna-ground system.

Going back now to the source of power for the primary oscillatory circuit, there is found the transformer *Tr*. It serves to transform the low-potential energy from the generator *G* into high-potential energy suitable to charge the condenser *C*. The cycle of operation is:

(1) Production of alternating current at about 220 volts and 500 cycles,

(2) Conversion of this low voltage into a high voltage which charges the condensers,
(3) Discharge of the condensers through the gap and primary of the oscillation transformer,
(4) Transfer of power from the primary oscillatory to the antenna circuit, resulting in a flow of antenna current.

Primary oscillatory circuit. The circuit consisting of the gap, condenser, and primary of the oscillation transformer as shown, figure 301 constitutes the primary oscillatory circuit, or, as commonly called, the closed circuit. Its general function is to convert the low-frequency energy delivered from the generator by the transformer into radio-frequency energy at the particular frequency desired. Energy stored in the condenser, as previously explained, is expended as an oscillatory

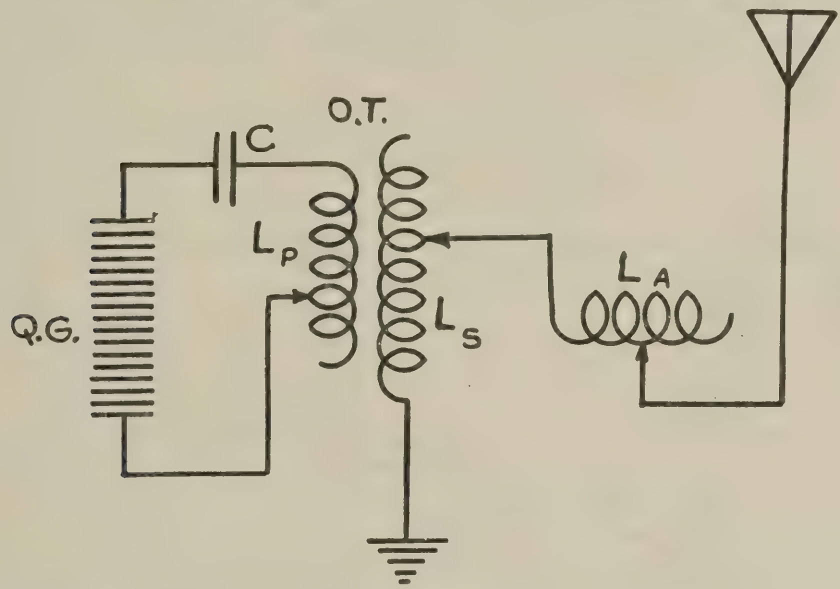


FIG. 301.—Primary and Secondary Oscillatory Circuits.

current in the primary oscillatory circuit, the time period or frequency of which is determined by the LC value of the circuit. Under average conditions approximately 50% of the energy from the primary circuit is transferred to the antenna circuit where it becomes apparent as antenna current, the remaining energy being dissipated chiefly in resistance and dielectric losses.

Energy transfer to antenna circuit. Regardless of whether the coupling between the primary and secondary oscillatory circuits is inductive or direct, the energy transfer from one to the other is effected through the medium of the magnetic field. It is well known that any current flowing in a conductor sets up around that conductor a magnetic field, the formation of which takes energy and the collapse of which delivers energy. If only the conductor or system of conductors active in the formation of the field is affected by its collapse, the return of energy is equal to the original expenditure and the net energy expenditure is zero. But if this field in its formation links with a second electric

system capable of absorbing electric energy of the characteristics involved, a transfer of energy will take place through the field to the second circuit. By varying the relative interlinkage of flux between the two systems it is possible to exercise a control over the energy transfer. For ordinary power circuits the closest possible coupling is sought, but conditions are widely different for the oscillation transformer in a radio circuit and best conditions frequently require a small mutual flux linkage, or **loose coupling.**

Referring to the antenna circuit, the component parts are found to be the simple oscillatory circuit, except that the familiar condenser is replaced by a special condenser, namely the antenna and earth. By transfer of energy from the primary circuit through the magnetic field, a current is produced in the antenna having the same frequency as that flowing in the primary circuit. This antenna current flows until all its energy is dissipated by resistance, dielectric, and radiation losses. The

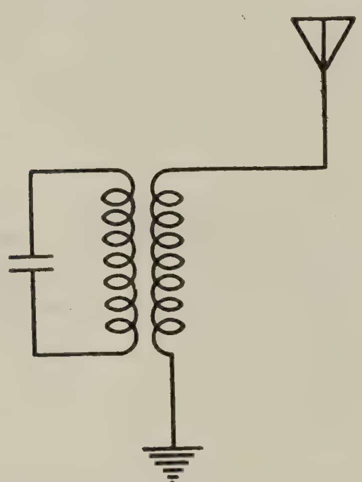


Fig. 302.—Inductive Coupling.

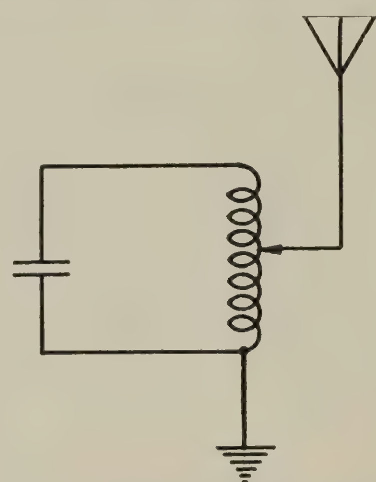


Fig. 303.—Direct Coupling.

latter only is of value as the signal carrying agent. For the ordinary transmitter it constitutes a very small percentage of the total energy delivered to the antenna, in the neighborhood of one to two per cent.

Coupling. A most important feature in the operation of all spark transmitters is the coupling. It may be inductive, figure 302, or direct, figure 303. The action is largely the same in either case being through the magnetic field, as previously explained. A greater flexibility is found in the inductive than in the direct arrangement, which has resulted in its general use for Navy type transmitters.

If the characteristics of the primary oscillatory circuit were such that only a single discharge took place through the gap from the positive to the negative side of the condenser, there would be impact excitation and the frequency of the secondary oscillatory circuit would be independent of the constants of the primary oscillatory circuit. The problem would then be to secure the greatest possible energy transfer from this single current pulse for use to build up radiation, and the coupling would

be made as close as possible. After each such energy transfer, the current resulting in the antenna circuit would flow in the period determined by the antenna LC value until reduced to zero by the various losses.

Such would be the ideal way, but the actual equipment used permits a series of surges through the gap as indicated figure 304. The resulting induced antenna current is shown figure 305. If the time periods or frequencies of the two circuits are not the same under these conditions, a very small energy transfer takes place. It, therefore, becomes of first importance that the primary oscillatory and the antenna circuits be adjusted to identical time periods. For this it is necessary that the L_1C_1 value of one circuit equal the L_2C_2 value of the other. This does

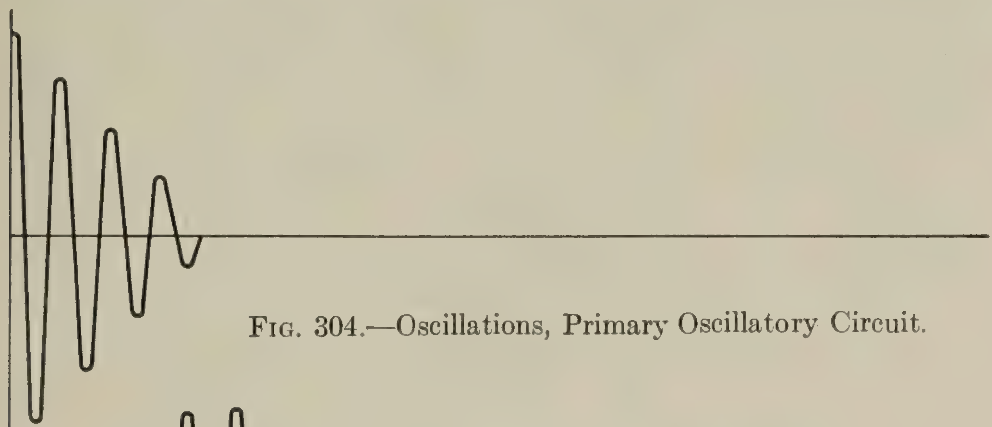


FIG. 304.—Oscillations, Primary Oscillatory Circuit.

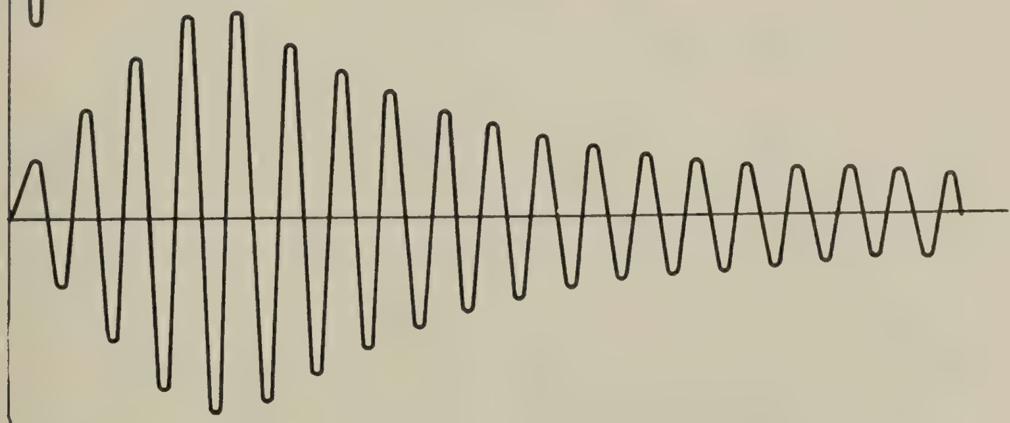


FIG. 305.—Oscillations, Secondary Oscillatory Circuit.

not mean that L_1 must equal L_2 and C_1 equal C_2, but it does mean that the product of L_1 and C_1 expressed in henries and farads, or convenient subdivisions, must equal the product of L_2 and C_2 expressed in the same units.

Figures 304 and 305 indicate the absence of any further interaction between the two circuits once the transfer of energy has taken place from the primary oscillatory to the antenna circuit. There is, however, a tendency for the antenna circuit to return energy to the primary oscillatory circuit as indicated figures 306 and 307, provided the gap does not permanently open the latter. The original process is then repeated and the exchange between the two circuits continues until the oscillations die out. This condition is aggravated by close coupling, and

lessened by loose coupling. Since efficient energy transfer cannot be obtained with very loose coupling, it is of great value to employ a type of spark gap which will quickly stop the flow of current in the primary oscillatory circuit and prevent its reestablishment by induction from the antenna current flow. The multiple-disc quenched gap has been found most suitable for this purpose.

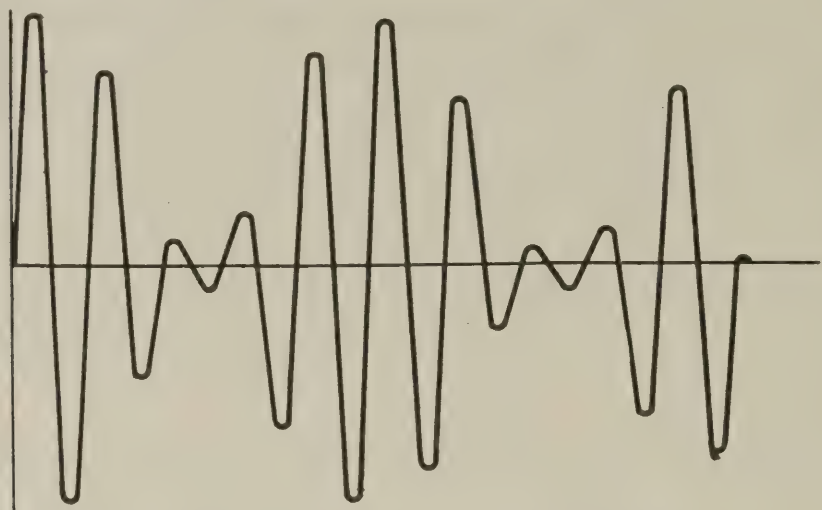


FIG. 306.—Oscillations, Primary Oscillatory Circuit.

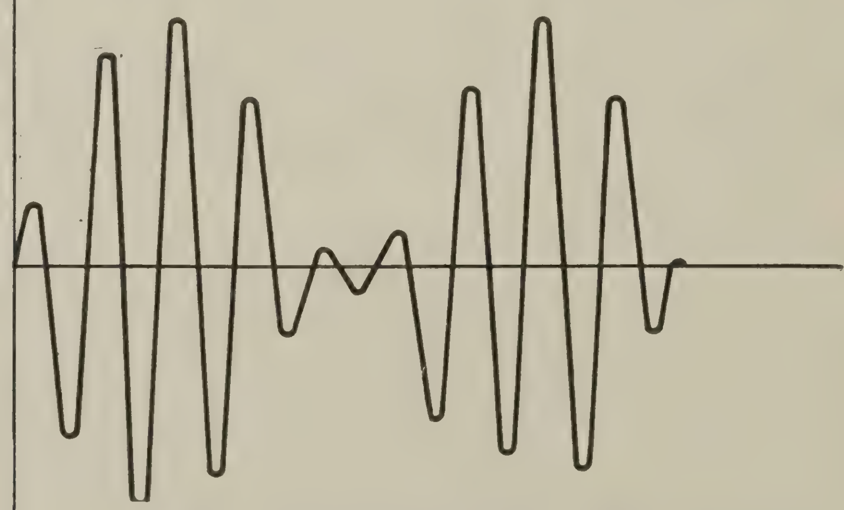


FIG. 307.—Oscillations, Secondary Oscillatory Circuit.

Of the general types of coupling used, inductive, figure 302, and direct, figure 303, the inductive coupling has the advantage of greater flexibility and a less tendency to pass on any objectionable frequencies which may be present in the primary oscillatory circuit. On the other hand, the circuit and apparatus is considerably simplified by use of direct coupling. For the former, the degree of coupling is governed largely by the actual separation between the two coils, while for the

latter it is determined by the number of turns common to the two circuits.

Resonance. Mention has been made several times of resonance and its importance in radio-frequency circuits. The condition of resonance is obtained for any alternating current circuit when

$$2\pi fL = \frac{1}{2\pi fC}$$

This holds equally well whether it be for radio frequency or for 500 cycles or for any lower frequency. An application of resonance principles to the 500-cycle circuit serves to reduce the turn ratio necessary for the transformer to develop the required potential to charge the condenser, and it also serves to limit the transformer short circuit current when the gap breaks down. Figure 308 may be resolved into the equivalent of figure 309, which is the simple oscillatory circuit. Here X_g is

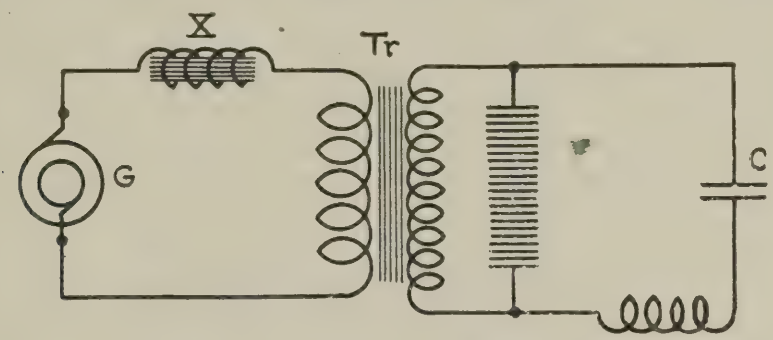


FIG. 308.—Generator and Oscillating Circuits.

the leakage inductance of the generator, X_t that of the transformer, and X_r the external adjusting reactance. For purposes of calculation the transformer can be considered as eliminated by making suitable changes in the value of C. To transfer values of capacity from the high-voltage to the low-voltage side, multiply by the square of the transformer turn ratio, and to transfer from the low to the high voltage side, divide by the square of the ratio. To transfer values of inductance, or resistance from the high voltage to the low side, divide by the square of the transformer turn ratio, and to transfer from the low to the high-voltage side, multiply by the square of the turn ratio.

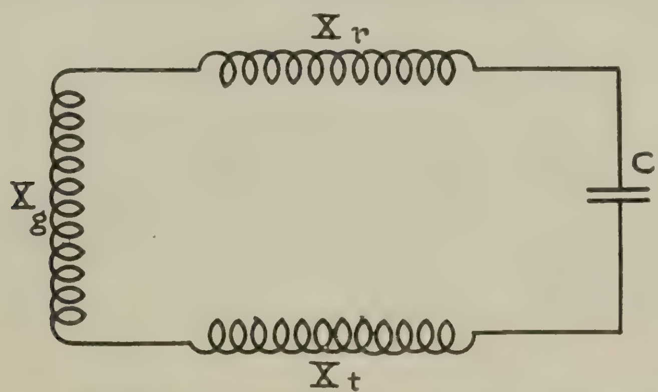


FIG. 309.—Showing Leakage Inductances of Generator, Transformer and Externa Reactance.

It will be noted that the process is reversed for inductance and resistance as compared with capacity. Since the value of C is generally fixed by the power to be absorbed and the voltage which the condensers can stand, it follows that resonance adjustments must be made through changes of inductance or reactance. In later Navy sets, adjustment to resonance of the generator, transformer, capacity circuit at 440 cycles has been standardized as most satisfactory. By means of this resonance effect, it is practicable to increase the secondary voltage developed by the transformer far above that due to the turn ratio alone. Thus, the turn ratio for the standard transformer used with Navy spark sets will give a secondary voltage of about 8,000 but by the resonance effect it is built up to a working value of 12,500 volts with normal

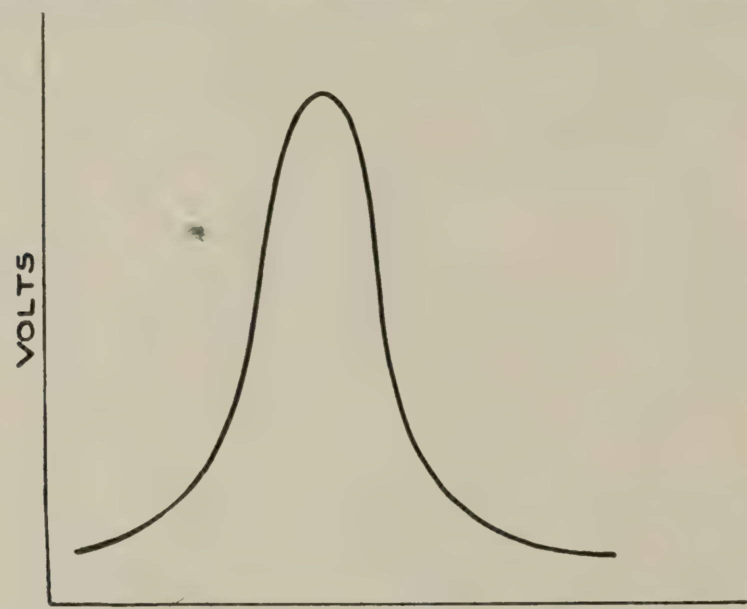


FIG. 310.—Variation in Voltage with Variation of Frequency Due to Resonance.

generator voltage. Figure 310 shows the building-up characteristic with variation of frequency. Were adjustment made for resonance at the operating frequency of 500 cycles, a ragged note would result; but by detuning to 440 cycles for the resonance adjustment, sufficient building up effect through resonance is retained to properly charge the condensers, and a good note is obtained as well.

A 250-cycle note or 500 pitch can be obtained in such a circuit by reduction of generator voltage so that the voltage on the condenser builds up to the point of gap break down only every second half cycle.

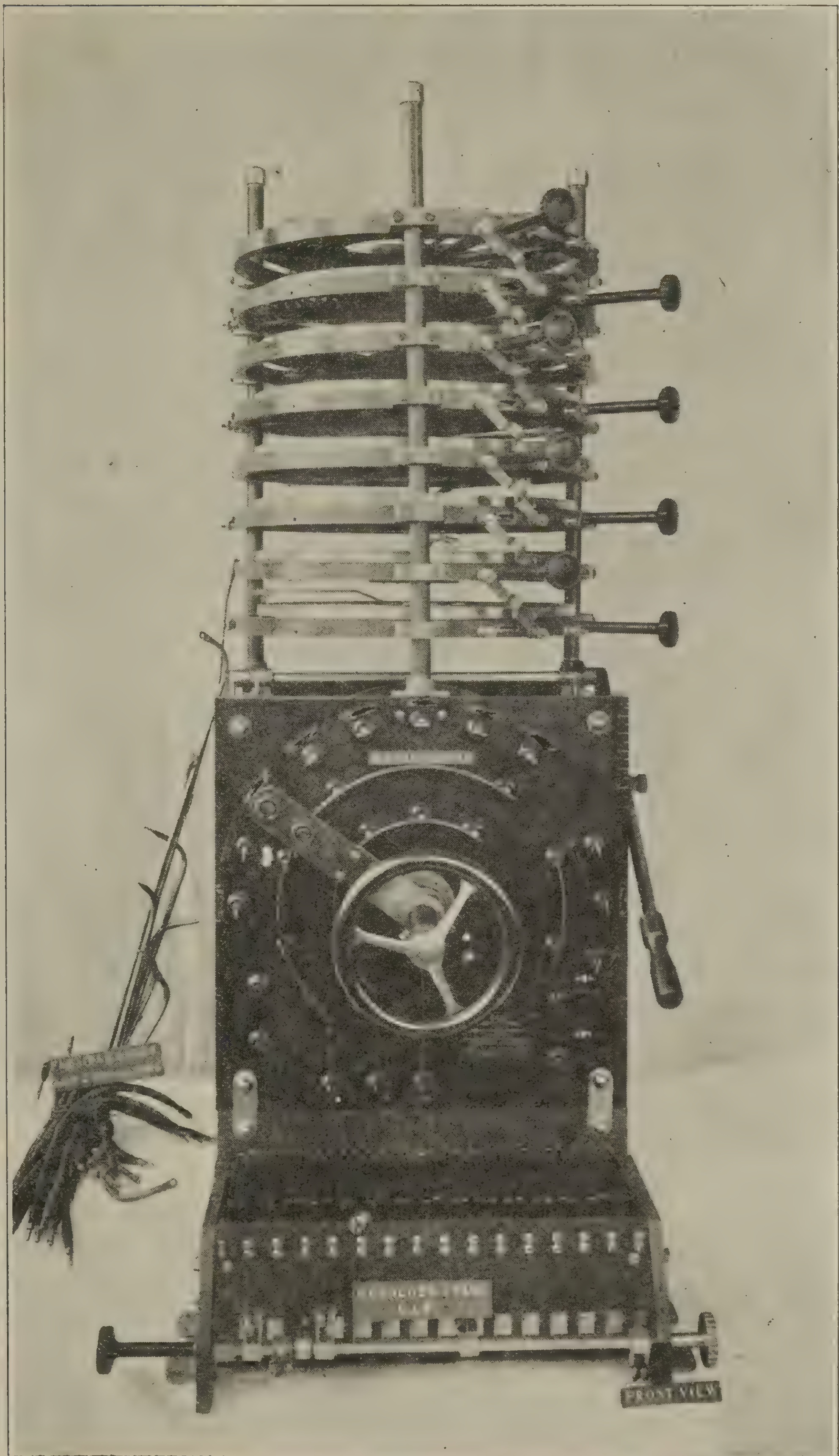


FIG. 311.—5-Kw, 500-Cycle, Quenched-Spark Transmitter. Front View.

CHAPTER III. ARC TRANSMITTERS.

The arc transmitter is used for some medium and high-power continuous-wave-transmission. It is used on shipboard as well as at shore stations. Practically all the transmission effected by means of the arc is of the *CW* telegraph type. The 2-kilowatt and 5-kilowatt arcs are additionally equipped with the chopper for *ICW* transmission. The use of the chopper, however, materially decreases the range of the arc transmitter.

The arc transmitter supplies current to the antenna continuously (except when the chopper is used). The decrement of the transmitted wave is practically zero except when signaling is done at high speed. For this reason, *CW* signals will tune in very sharply on the receiver; in fact, when *CW* signals tune in broadly, the trouble may be assumed to be in the receiving system. The antenna circuit of the receiver may have a high decrement and this, coupled with the action of the secondary circuit of the receiver, when used for autodyne reception (beat method), causes most of the interference phenomena. In the autodyne method of reception, the secondary circuit of the receiver is purposely detuned from resonance with the incoming signal and the antenna circuit of the receiver in order to obtain enough beats per second to render the signal audible. This detuning is most marked at the longer wave lengths, and is eliminated by the use of a separate radio-frequency driver as a part of receiving equipment.

As soon as signaling is done with *CW* transmission, and especially at very high speeds, the continuous wave is given an **effective decrement** which depends upon the speed at which the signaling is done. This is because the oscillations in the antenna gradually build up to their maximum amplitude when the key is pressed, and then decrease when the key is opened. Signaling accomplished by hand does not have sufficient speed to give the continuous-wave transmission an appreciable decrement.

Circuit. Figure 312 gives a schematic diagram of the circuit commonly used in arc transmitters and indicates the principle component parts and their relationship, and figure 313 shows the actual construction.

Function and operation. The arc converter, or arc, as it is commonly called, is a device to convert direct-current energy into the radio-frequency energy required for radiotelegraphy using undamped or continuous waves. Current from the generator G maintains the arc, the positive side being connected to the antenna and the negative side to ground.

The resistance of an electric arc decreases as the current flow through it increases, giving the voltage-current curve of figure 314.

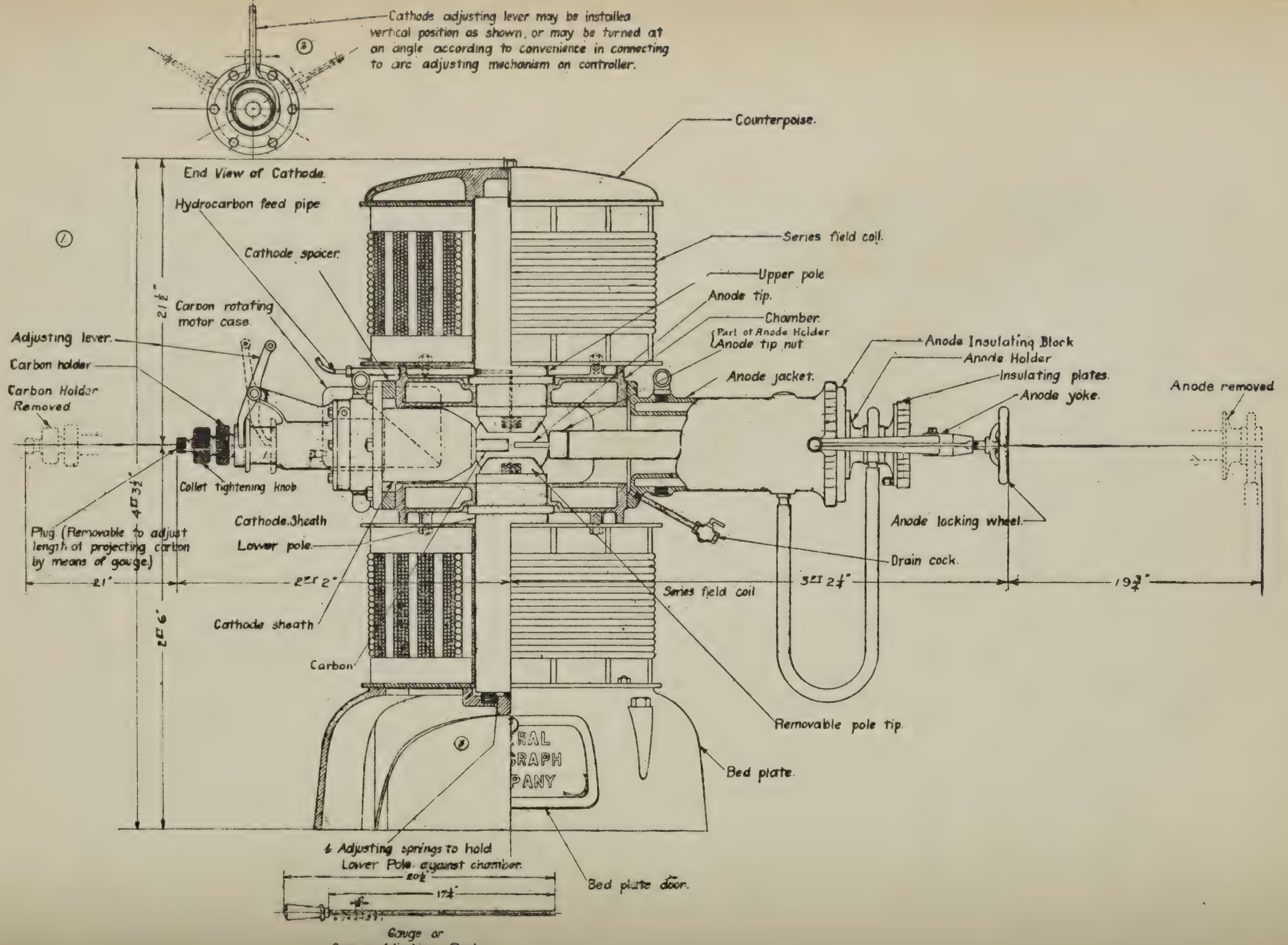


FIG. 313.—Construction of Arc Transmitter.

Since $R = E/I$, it will be seen that the resistance of the arc increases rapidly beyond a certain point as the current diminishes. This is because ionized gases given off by the electrodes, form the conducting means for the current flow between the electrodes, and when the current flow diminishes below a critical value, these ions are not given off in sufficient quantities to maintain a low resistance path. Suppose now the

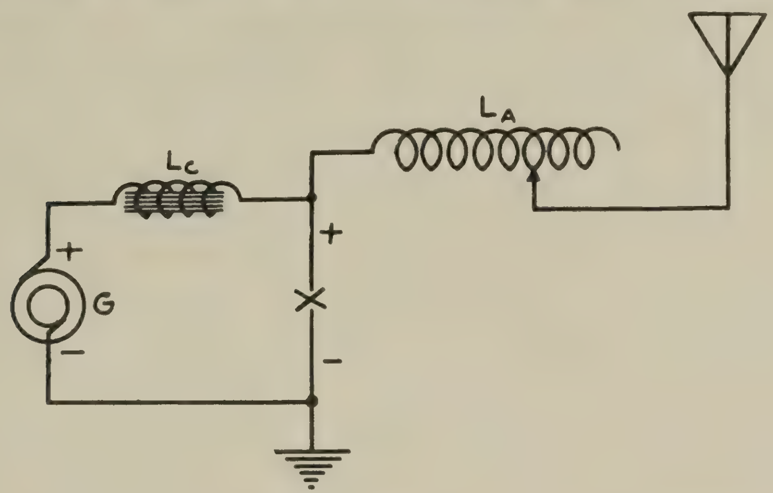


FIG. 312.—Schematic Diagram of Arc Transmitter.

arc has been struck by bringing the electrodes together and drawing them apart slightly. A small charge is given the antenna. When the maximum charge has been acquired for the conditions existing, a reverse flow or discharge of the antenna charging current starts and flows through the arc to the ground plate. This, in turn, reduces the resistance across the arc, facilitating discharge of the antenna current. A cumulative effect is thus caused in that as the antenna current discharges, the resistance of the path is lowered through the arc and the

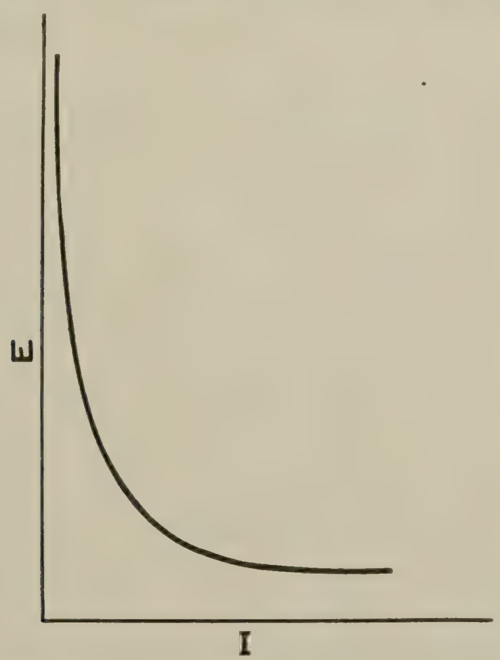


FIG. 314.—Current-Voltage Curve for an Arc.

antenna discharge assisted. This antenna discharge continues until completed and the condenser is charged in the opposite direction. When this point is reached, the flow from the ground plate to the aerial plate takes place, in its turn, through the arc by virtue of the energy stored in the inductance. But such flow of the oscillating current is opposed to the generator flow, and results in decrease of current through the arc and its eventual extinction when the amplitude of the oscillating current equals that of the generator current. During the period when the arc is open, diversion of the generator current into the antenna takes place until the latter is again fully charged and the arc is reestablished. The cycle of discharge and charge then resumes, as described, and oscillations continue to flow so long as energy losses in the antenna system are supplied from the generator.

Dc and oscillating current relations. Best operating conditions are obtained when the oscillating wave has a sinusoidal form and the arc is just extinguished once each cycle. The arc efficiency is then at a maximum and 50 per cent of the energy of the arc is delivered to the antenna. With other less favorable conditions the efficiency runs much lower.

Based on the general assumption of a sinusoidal oscillating wave having a peak just equal to the steady dc flow, the value of antenna current is equal to the dc value divided by $\sqrt{2}$. This relation is found to hold with fair accuracy for most arc installations and, in particular, for the larger ones. Figure 315 shows the two currents which are

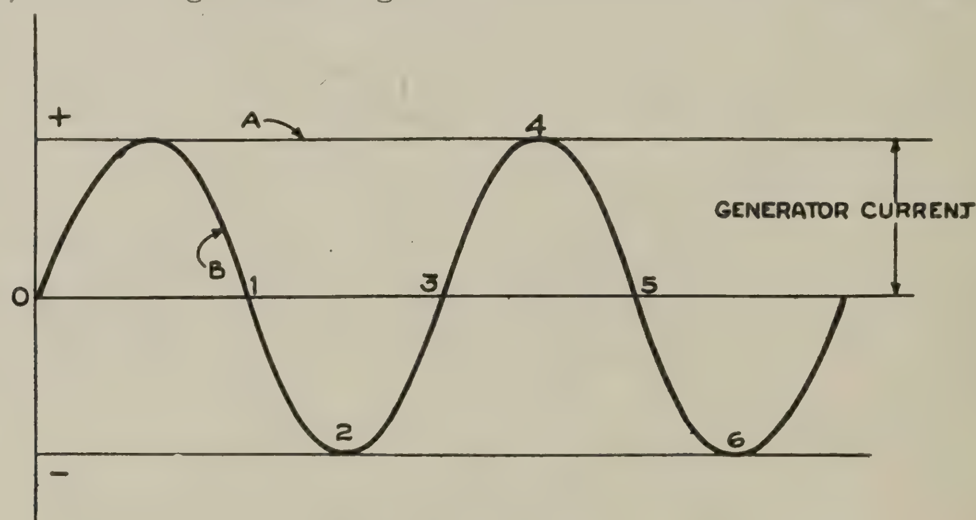


FIG. 315.—Arc Oscillating and Direct-Current Relations.
A = Generator Current. B = Antenna Current.

acting through the arc simultaneously. Curve B is the sine wave of the radio frequency when the maximum current amplitude is equal to the value of the generator current A. Knowing that the effectiᴧe value of a sine wave current is its maximum value divided by the $\sqrt{2}$ it follows that this relation is held in the value which a meter in the radio-frequency circuit will indicate.

Figure 315 also indicates the relationship between the radio frequency current and the generator current. It is self-evident that during one half of the oscillation they are adding and that during the other half they must be opposing. If we assume the two are adding during the period above the zero line for the generator current, they will be opposing during the period below that line. Since the amplitude of the radio-frequency current is the same in both directions and equal to the generator current, it follows that at the point 2 the arc will be extinguished because the two currents are of equal value and opposed. At point 4 the current flow through the arc will be at a maximum and equal to twice the generator current.

The maximum efficiency of the arc as a radio-frequency converter is 50% based on the following considerations:

$$I_{\mathrm{rf}}=\frac{I_{\mathrm{dc}}}{\sqrt{2}} \tag{1}$$

$$E_{\mathrm{rf}}=\frac{E_{\mathrm{dc}}}{\sqrt{2}} \tag{2}$$

$$(IE)_{\mathrm{rf}}=\frac{(IE)_{\mathrm{dc}}}{2} \tag{3}$$

If the equation (1) is true, it follows that equation (2) must be true, since the voltage curve must be in phase with and have the same form as the current curve. Then combining the two, equation (3) gives the power relationship as stated.

Two types of oscillations. Two types or forms of oscillations are possible: (A) Where the maximum amplitude of the oscillating current does not equal the value of the dc supply so that the arc is not actually extinguished at any time; (B) Where the maximum amplitude of the oscillating current equals the value of the dc supply so that the arc is extinguished. Case (A) gives a very smooth curve of oscillating current flow closely approximating a sine wave.

Operation under these conditions is critical, and a good power output difficult to obtain. It is therefore customary to use Case (B), which is less critical and permits greater power input to the antenna. The best wave form and most satisfactory conditions, in general, obtain when the peak of the oscillating wave is only very slightly greater than or just equal to the direct current flow.

Conditions of operation. Two primary means are employed to secure oscillations: (A) Surrounding the arc by a gas of high diffusion, as hydrogen or hydro-carbons, and (B) a transverse magnetic field acting on the arc. Both are to assist in rapid deionization of the space between the electrodes.

Experiments carried on by Pedersen demonstrate clearly the importance of a proper strength for the transverse field. Too weak a

field permits the formation of multiple arcs, while too strong a field causes improper operation through too rapid lengthening of the arc. As the wave length is increased, a weaker field is permissible and, indeed, is necessary for good operation, as will be apparent, since the longer wave permits a greater time for the magnetic field to act and also for the natural diffusion of the ions to take place. In practice, the range of wave lengths used for medium power installations has proven reasonably satisfactory without special adjustments of the transverse field strength being made. Provision is made for change of field strength in the larger sets by switches which vary the number of field turns.

Two other features are of practical importance in operation of the arc converter: a water-cooled copper anode, and a carbon or graphite cathode, the latter being slowly rotated as the arc burns. Copper is the most satisfactory material for the anode. Water-cooling is necessary to keep in from melting and also to assist in deionization of the arc. A constant rotation of the cathode is essential to prevent pitting, with consequent unsteady burning due to the arc jumping from point to point. Rotation causes the end of the electrode to burn away evenly so that a smooth surface is always maintained.

Signaling. Signaling is accomplished for spark sets by control of the alternator output. A similar method cannot be used with the arc transmitter, since the output of the arc must be maintained as nearly constant as possible. It therefore becomes necessary to signal by change of the radiated wave length or by diversion of the antenna current into a dummy circuit. The first is the **compensation method,** and the latter is the **uniwave method.**

In figure 316 is shown the compensation method. *A* and *B* are schematic, *C* and *D* the actual connections showing the subdivision of the total current among a number of breaks. Signalling is based on a slight change of antenna effective inductance as the sending key is open or closed, with corresponding change in the emitted wave length. Case *A* directly short-circuits the connected section of the loading inductance. The effect is to reduce the total antenna inductance and decrease the emitted wave length by a corresponding amount. Case *B* accomplishes the same thing through the inductive coupling of key coils which are then short-circuited. When the key is closed, induced circulating currents are set up in the short-circuited loops, the field from which reacts on the field of the main loading coil and decreases its effect. The result is a weakening of the main field and lessening of the antenna inductance during the period of key action. Such changes of wave length are of course effective only for the period during which the connection is closed by the sending key. Immediately upon release, the original adjustment is restored.

It will be noted that the keys, *A* and *B*, are shown connected to use the long wave for signalling. This is customary, although not

necessary, in order to gain the advantage of constant adjustment. With the other way, using the short wave to signal, bad contacts or

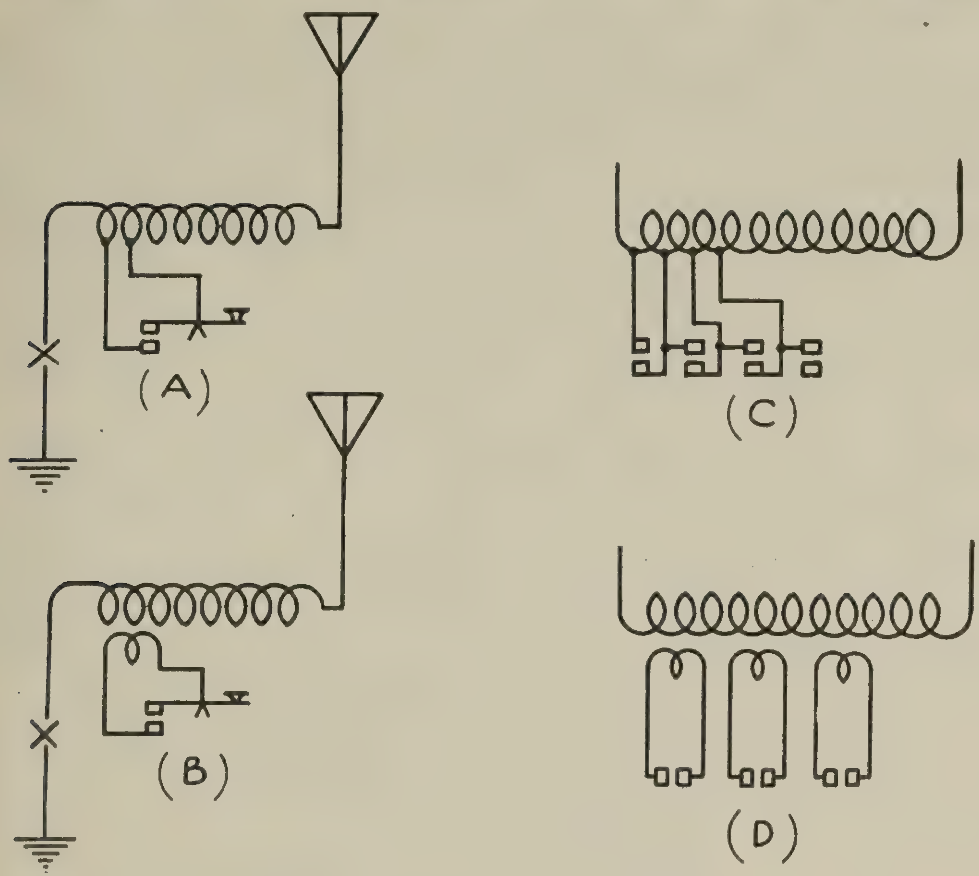


FIG. 316.—Compensation Signaling Circuits.

poor key action is more noticeable at the receiving end. Sending or **telegraph wave** and **compensation wave** are the names commonly used to distinguish these two. For satisfactory reception, the difference

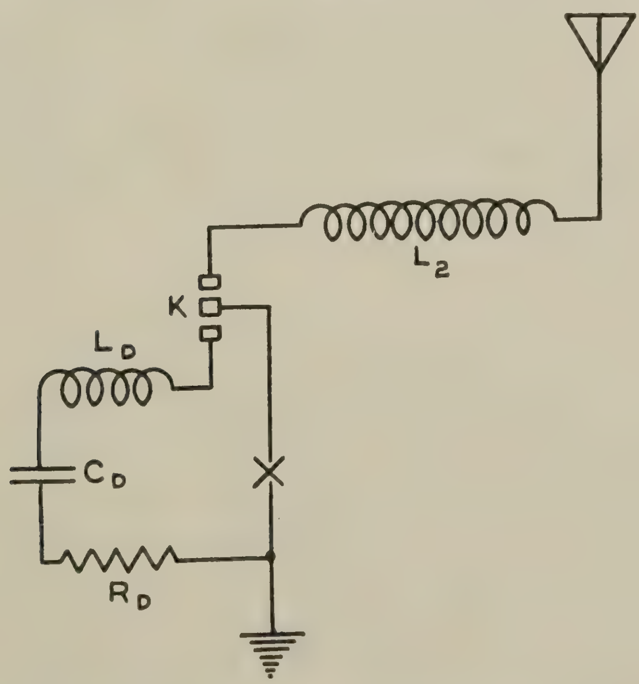


FIG. 317.—Uniwave Signaling Circuit.

between the telegraph and compensation waves must be not less than 1 per cent of the transmitting wave. Closer reception can be carried on, but interference between the two waves becomes troublesome.

A very objectionable feature of the compensation method is the presence of two waves. In order to eliminate one, the uniwave method is being developed. It is shown diagrammatically in figure 317. A dummy antenna $L_D C_D R_D$ absorbs the arc output during the periods of nontransmission, transfer from one circuit to the other being by means of the key K. In general the wave length of the dummy circuit does not need to be the same as the antenna wave, although too wide a divergence is not advisable. Satisfactory keys have been developed for 30-kw and lower rated arcs generally. The larger sets present difficult problems which, however, are well on the way to solution.

Coupled Circuits. The simple arc circuit of figure 312 may be replaced by the coupled circuit of figure 318. The latter has the advantage of permitting the arc circuit to be adjusted so as to give the best working conditions or highest efficiency. It also practically eliminates every frequency from the antenna except that to which it is adjusted. Keying is by short-circuiting the coupling inductance.

Current-transformer circuit. A circuit which has proved efficient with antennas of low resistance is shown in figure 319. It is known as the current-transformer circuit.

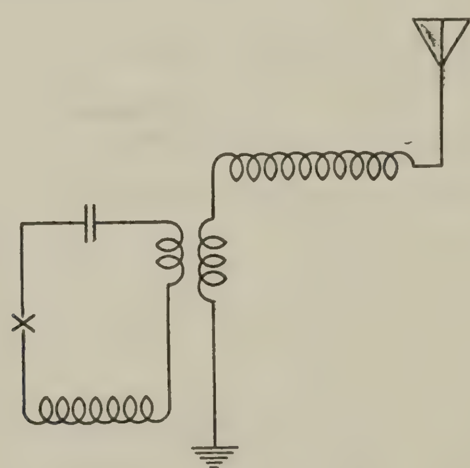


FIG. 318.—Coupled Circuit.

This circuit results in an increase of total radiation, the sum of the ammeter readings A_1 and A_2, by an increase of arc voltage without increase of generator current. All losses to maintain the radio-frequency current must be supplied from the arc while only part of it, A_1, flows through the arc. The energy required to maintain the radio frequency current through A_2 must then be supplied by an increase of generator voltage across the arc since the direct current through it cannot exceed the value $A_1\sqrt{2}$. Where the radiation limitation is determined by the current capacity of the generator rather than by the

voltage it can develop, this circuit is applicable with an improvement of efficiency for the system.

The division of current between A_1 and A_2 is in inverse relation to the values of L_1 and L_2. In general, this ratio of L_2 to L_1 requires to be made greater in proportion as the antenna resistance becomes greater, due to the fact that a higher voltage is required across the arc to maintain the radio-frequency current flow through L_1 with increased antenna resistance and, hence, leaving a smaller proportion available to maintain the current flow through L_2.

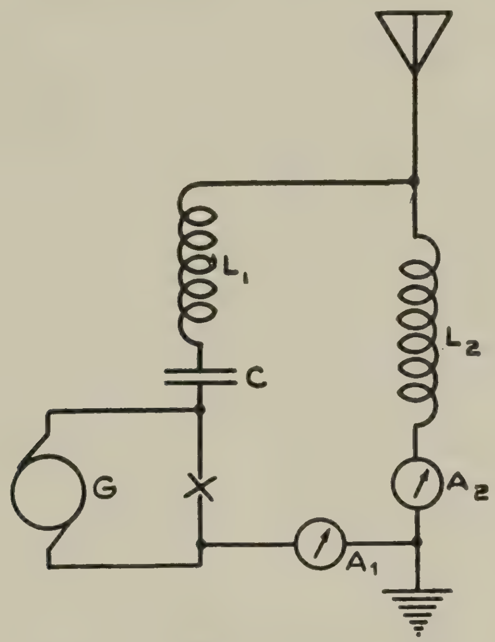


FIG. 319.—Current-Transformer Circuit.

A condenser C is necessary to prevent a short circuit on the generator through L_1 and L_2. It should be of a large capacity, so as to offer low resistance to the antenna current.

Similar results are obtained from the circuit of figure 319. $L_1C_1L_3$ is adjusted to set the wave length desired in the antenna circuit, and the point N is made a nodal point for this frequency, but not for any other frequency. Hence, by connecting a resistance from N to ground,

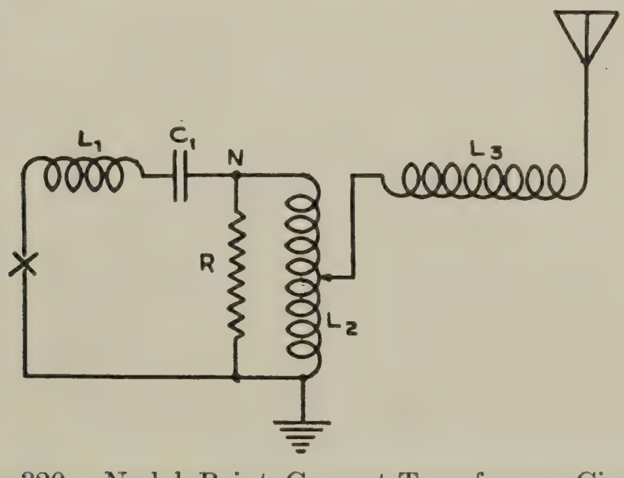


FIG. 320.—Nodal Point Current-Transformer Circuit.

other frequencies than that of the fundamental wave are shunted out. Keying is accomplished by short-circuiting R so that no current passes through L_2.

Uniwave operation is obtained with the circuits of both figures 318 and 320.

Arc equipment. The principal parts of the arc transmitter consist of the generator, arc converter, and loading inductor with signalling system. A direct current is required, the usual voltage being between 300 and 1000 as determined by the antenna characteristics and the antenna current. A fair average of the voltage may be taken as 500. The converter itself consists of three principal parts: electrodes, arc chamber and magnetic system. There remains the loading inductor with key system, either compensating or uniwave, and the antenna and ground to complete the transmitter.

The coil L_c, figure 312, serves a double function. First, it produces the transverse magnetic field. Second, it prevents the high-frequency antenna current from getting back into the generator. Shunt excitation of the transverse field may be used instead of the series excitation as shown, and has the advantage of flexibility but lacks the simplicity of the series connection. A special choke coil must then be used in the positive generator lead to hold back the radio-frequency currents.

In the United States, most arc transmitters adhere closely to the series system of field magnet excitation. For sets up to and including 100-kw input, an open magnetic circuit is used, while for sets of greater rating a closed type of magnetic circuit has been utilized.

On the sets of late manufacture, sectionalizing switches or taps are provided to vary the field excitation approximately according to the circuit needs. This is particularly the case for the large sets. As previously stated, it is in general true that the transverse field strength necessary for best operation decreases as the wave length increases. To put it in other words, the transverse field strength required varies with the frequency. The closed magnetic circuit is equally applicable to the small sizes, but has not been used owing to the simpler construction and the relative ease of providing a sufficient number of ampere-turns for good operation with the open-core type.

Figure 321 shows the 500-kw duplicate arc converters at the Annapolis Naval Radio Station. The tank contains the transverse field coil, which is immersed in oil for insulating and cooling purposes. Above it is seen the arc chamber with the control mechanism for the negative or carbon electrode. Pole tips to complete the magnetic circuit project though the top and bottom of the chamber. It will be noted that a double yoke form of magnetic circuit is used. This is a matter of mechanical rather than magnetic design, since for the latter purpose a sufficient cross-section to obtain low reluctance for the flux

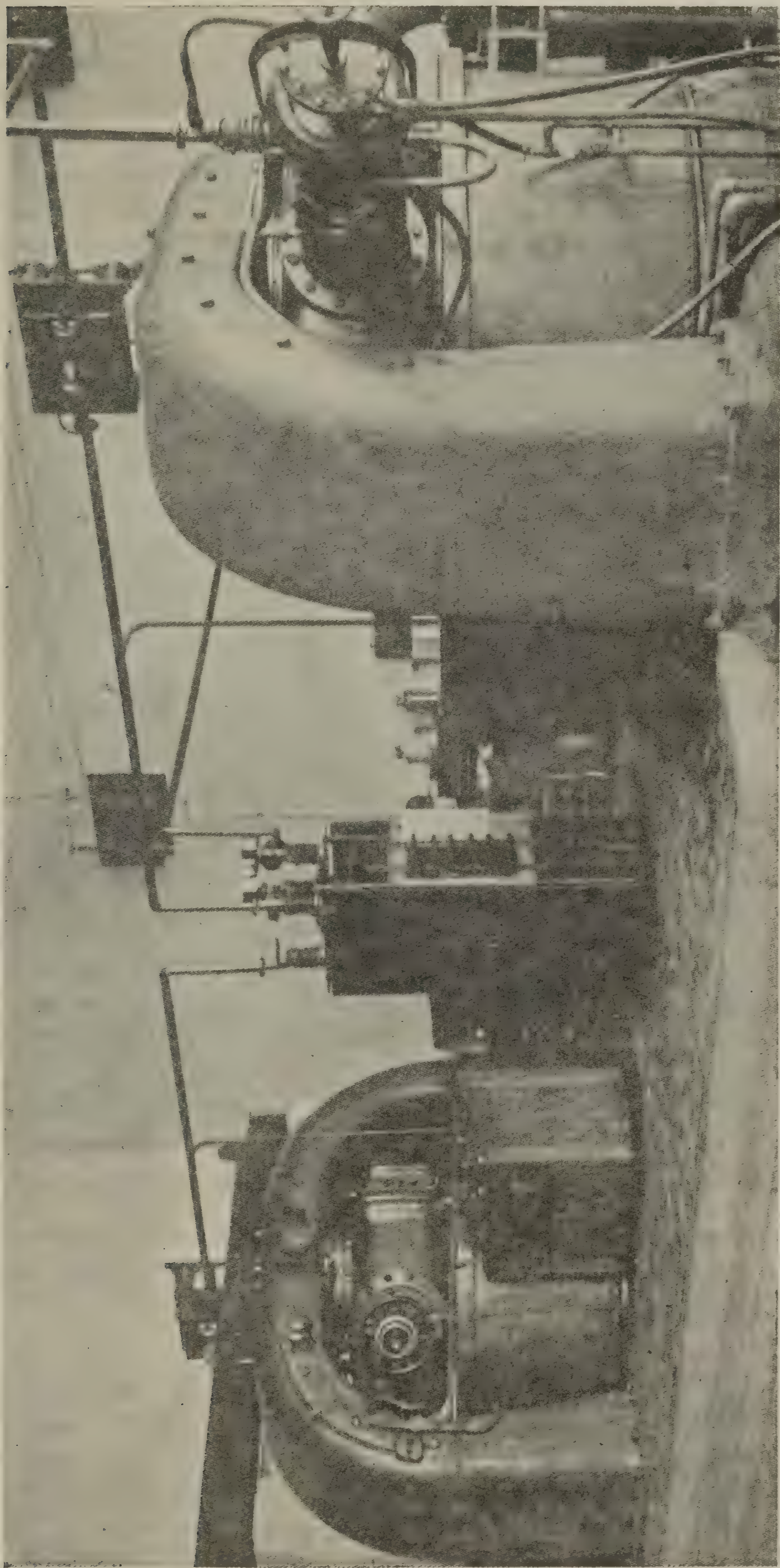


FIG. 321.—The 500-Kw Duplicate Arc Converters at the U. S. Navy High-Power Radio Station, Annapolis, Md.

only is required. Figure 322 illustrates a 5-kw arc, and shows one type of the open magnetic circuit construction.

The arc chamber is made of a nonmagnetic material, and is cored for water cooling. When it is realized that approximately half of the generator output is dissipated in the arc as heat and that all this energy must be taken up by the arc chamber, the need of water cooling becomes apparent. In addition to carrying away this waste heat, the arc chamber serves to contain the hydro-carbon atmosphere. As is to be expected, the size of the arc chamber increases with the larger arc ratings and is determined considerably by the heat to be dissipated.

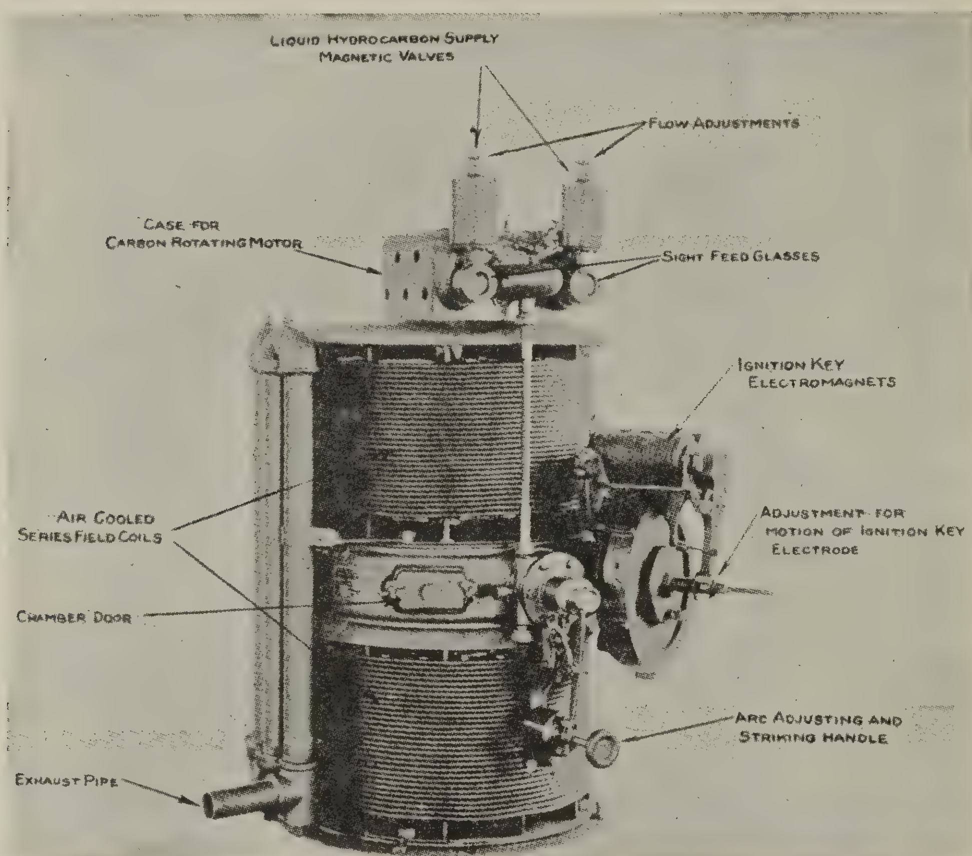


Fig. 322.—5-Kw Arc Converter. Federal Telegraph Company.

The arc chamber also serves to line up and support the anode and cathode holders. Exhaust gases are conducted away through an opening in the rear wall of the chamber, while a door is fitted in the front side to permit access for cleaning and inspection.

The antenna connection and positive generator lead are made through the anode or positive electrode. This is copper, and must be water-cooled.

Two insulations are required for the anode, first for the direct current generator voltage, and second for the high-frequency antenna current. The latter, while of no greater potential than the direct current from the generator, requires the greater consideration because of its charac-

teristics, since capacity and dielectric effects will cause the gradual deterioration of insulating material unaffected by the direct current and finally provide a conducting path for the direct current, resulting in grounds and a short circuit on the generator through the grounded chamber. The ground connection and negative generator lead are made through the cathode or negative electrode. In the earlier arcs, the cathode was insulated from the chamber in a manner similar to the anode, but in those of recent manufacture no insulation is used, the cathode holder being fastened directly to the body of the arc chamber. A carbon or graphite rod forms the negative electrode. It is held in a water-cooled sheath and is slowly rotated whenever the arc is in operation, to avoid the formation of objectionable pits in the carbon. Such pitting causes unsatisfactory operation through uneven burning of the arc. In late years in the Naval service, carbon has been largely replaced by graphite as the cathode terminal for all arcs. A more uniform product and steadier burning of the arc are obtained.

The direct-current generator *G*, figure 312, is of standard commercial construction without any special features. Voltage requirements differ with the antenna characteristics. Minimum and maximum requirements very seldom exceed 300 volts and 1,000 volts, with an average between 500 and 600 volts. Voltage regulation by shunt field control alone is found more satisfactory than either a flat or a cumulative compound field. A protective device to carry off radio-frequency leaking back to the generator is essential and is provided in the form of a condenser, connected to the generator positive terminal. This affords a low-resistance path for the radio-frequency current to earth, but prevents any flow whatever of the direct current.

Faults. The faults usually experienced in the operation of arc transmitters, their causes and remedies are given in the following table:

Fault.	Cause.	Remedy.
1. Rapid burning away or melting of the anode.	(1) Poor circulation of water due to failure of pump or closed valve, collapsed rubber hose.	(1) Inspect water-cooling system throughout. Make such repairs as necessary to insure strong water circulation.
	(2) Reversed polarity of arc supply circuit, thus making the anode the cathode.	(2) Reverse arc supply circuit so that positive line is connected to the anode (copper).
	(3) Tip incorrectly lined up causing arc to burn to one side rather than along center line.	(3) Line up the tip.
2. Explosion in arc chamber.	(1) Air leaks in air chamber.	(1) Close all air leaks and inspect the seal to the exhaust pipe.
3. Small antenna current	(1) Field strength not properly adjusted.	(1) Increase or decrease field strength until antenna current is increased.
	(2) Insufficient hydrocarbon feed.	(2) Supply more feed.
	(3) Poor adjustment of arc length (too short).	(3) Open out arc until antenna ammeter shows maximum reading.[1]
	(4) Decreased dc voltage.	(4) Inspect dc supply and make necessary repairs.
4. Excessive power input.	(1) Poor insulation anywhere in oscillatory circuit.	(1) Poor insulators will usually be detected by local heating after a run. Insulators should be kept clean and dry. Very little trouble will be experienced with porcelain insulators.
	(2) High resistance anywhere in oscillatory circuit.	(2) Remove sharp bends wherever possible. Clean and tighten all connections. Look for excessive heating in the loading coil. If heating is excessive, determine cause and if injuries to coil are due to arcing across banks, injured section should be cut out of circuit (rare). See if antenna is grounded.

[1]As the arc is lengthened, the antenna current will increase and then decrease. Set the arc length at the point to give maximum reading on the antenna ammeter.

The arc has one objectionable fault which can only be overcome by a change in design. This fault is that it generates mush and harmonics and interferes considerably over the wave length spectrum. The usual method of reducing the interference is a better design of the arc chamber, the use of filter circuits and screens.

CHAPTER IV. HIGH FREQUENCY GENERATION BY ALTERNATORS AND FREQUENCY MULTIPLYING TRANSFORMERS

There have been described two methods of generating radio-frequency currents, both using resonant circuits for determination of the time periods. A third method generates the high frequency directly, and a fourth, of associated characteristics, employs static transformer frequency multipliers.

General characteristics. Problems of mechanical detail assume equal importance with electrical problems in the construction of alternators for the direct generation of radio frequencies. The maximum practicable periphery speeds and the minimum clearance between the rotor and stator must be used.

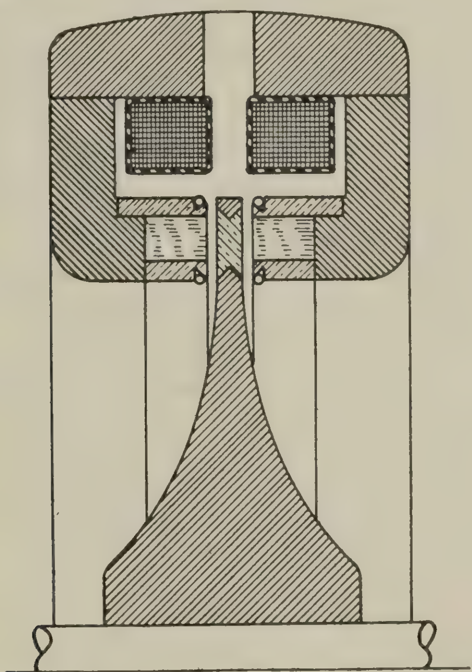


Fig. 323.—Construction of Alexanderson Alternator.

Special designs are necessary to reduce the losses due to the high frequencies to a minimum, and even then these losses are so large that special cooling means are required in machines of any appreciable size.

Such a generator may be connected directly into the antenna or it may operate into a local circuit, the latter being coupled to the antenna system. A coupled arrangement is generally preferred.

The Alexanderson alternator. The Alexanderson high-frequency alternator has been manufactured in commercial sizes up to 200 kw. This machine is of the inductor type and has the general form of construction shown in figure 323, which is a cross-section through rotor and stator. The high speed at which the rotor must run requires the heavy hub with tapering cross-section indicated. The 200-kw alter-

nator at New Brunswick operates at about 22,100 cycles and 2,170 rpm. The rotor alone weighs 5,500 pounds.

Equally spaced slots are cut near the circumference of the rotor and filled with nonmagnetic material, the whole being finished off smooth to reduce windage losses. As the rotor revolves it varies the reluctance of the magnet circuit, thereby causing a varying flux in the armature windings which are set into the face of the armature laminations. An emf is developed thereby in each armature winding which has a frequency dependent on the number of poles and the speed of rotation and has a value dependent on the usual consideration of rate of flux change.

In the machine noted above and shown in figure 323, there are 64 separate armature windings, each of which carries 35 amperes and develops 130 volts on open circuit. These 64 windings are paralleled in the primary of an air-core transformer, the combined output being delivered to the antenna by a single secondary winding. Under normal conditions of operation, the transformer output voltage is around 2,000.

Losses from eddy currents, hysteresis and dielectric absorption are large owing to the high frequency. Consequently, it is necessary to use very thin laminations and to water cool the armature magnetic path. An excellent grade of workmanship in manufacture is essential, as will be realized when it is known that the usual air gap is .015 inch.

A somewhat novel method of speed regulation is employed when driving from an ac source. It is well known that the speed of an induction motor is responsive to the voltage supply. Therefore, by placing a variable series reactance in the supply line and controlling the magnetic saturation from a dc source in accordance with speed variations, the voltage impressed on the motor terminals is varied in a manner to oppose such speed change and results in close regulation.

Signaling is by a magnetic amplifier placed across the output of the high-frequency transformer. It consists of an iron-core inductance specially wound to carry both the high-frequency current and a dc exciting current. By control of the dc exciting current a variation is effected in the antenna current. This system is suitable for both ordinary and high speed signalling and for telephony.

French alternator. High-frequency alternators have also been successfully developed by French engineers, and are being extensively used in the French radio stations of both high and low power. A 200-kw output alternator is installed at the Lyons Station. It is driven by a 375-kw direct-current motor at approximately 3,000 rpm, and develops a frequency of 20,000 cycles. The generator is of the inductor type. A system of fully enclosing the rotating parts and partial exhaustion has been adopted to reduce windage and friction losses.

Goldschmidt alternator. A second type of high-frequency generator, which uses the reflection principle to increase the frequency, is the Goldschmidt alternator. Bureau of Standards Circular No. 74, states the principle of operation as follows:

"If an alternator is excited with alternating current of frequency f, it will generate two frequencies f_1+f_2 and f_1-f_2 where f_2 is the frequency which would be generated with direct-current excitation. If $f_1=f_2=f$, then the frequencies would be $2f$ and 0. If the current of frequency $2f$ is used to excite the field of another similar generator running at the same speed, generated frequencies of $2f+f$ and $2f-f$, that is, $3f$ and f, would result. Thus, a series of generators running at a moderately high speed could be used for generating high-frequency currents. In the Goldschmidt generator, this frequency multiplication is attained in one machine. The stator is excited with direct current, and current of frequency f is generated in the wound rotor. Since the induction of currents depends only upon the relative motion of rotor and stator, we may consider that the rotor field is excited with current of frequency f

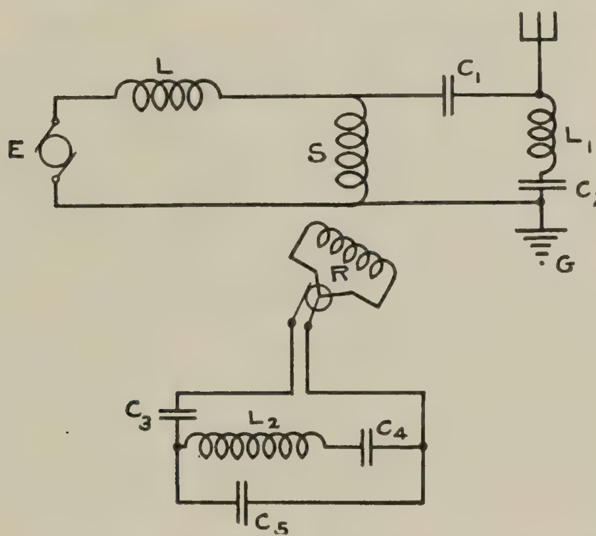


FIG. 324.—Circuit of Goldschmidt Alternator.

and that the stator is rotating in this field. Consequently, currents of frequency $2f$ and 0 will be generated in the stator, superimposed on the direct current used for excitation. The fields of these currents in turn react upon the rotor, producing in it currents of frequency $3f+f$, and in this manner the frequency is successively stepped up, the frequencies in the rotor being odd multiples of f and those in the stator even multiples. In order that the flow of current of these frequencies may not be prevented by reactance of the circuits, the principle of resonance is utilized and tuned circuits are provided for each frequency up to that which is to be used. The flow of current corresponding to the lower frequencies is suppressed to a great extent; for, as we have seen, starting with the fundamental frequency f, after two **reflections** we again have an induced frequency f in company with $3f$. It may be shown that these

two currents of frequency f will be opposite in phase and hence tend to neutralize each other."

Referring to the foregoing and figure 324, direct current is supplied the stator winding S from the exciter E. An alternating current of frequency f is generated in the rotor winding R. A tuned circuit for this frequency is provided $C_3L_2C_4$ and R. By reflex action, current of frequency $2f$ is induced in S superimposed on the dc excitation. The choke L prevents it from flowing into E. Circuit $C_1L_1C_2$ with S is tuned to frequency $2f$. By a second reflection current of frequency $3f$ is produced in R. This is tuned by circuit C_3C_5 with R to frequency $3f$. By still another reflection a frequency of $4f$ is set up in the stator winding to which frequency the antenna circuit AC_1SG is tuned.

The Tuckerton alternator was driven by a 250-hp motor, ran at 4,000 rpm, and generated 12,000 cycles which was multiplied four times to 48,000 cycles. Keying was by field control.

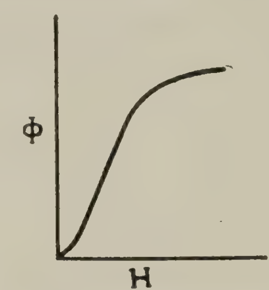


FIG. 325.—Iron Magnetization Curve.

It is possible to multiply the frequency of an ac current by utilizing the magnetization characteristics of iron. Figure 325 shows the form of this curve. It will be noted how the flux increase reaches a definite limit beyond which increase of magnetizing force has little effect. Utilizing this fact, it is possible to double or triple an initial frequency.

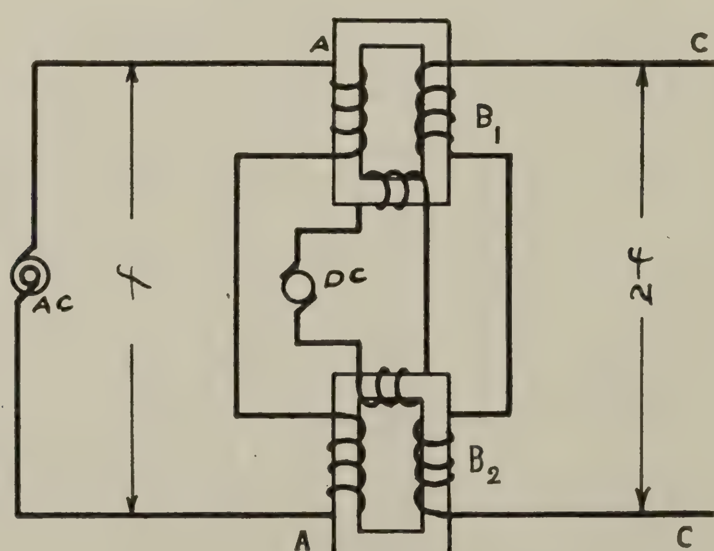


FIG. 326.—Frequency Doubler Circuit.

Figure 326 shows the transformer arrangements, and figure 327 the wave forms for doubling the frequency. The windings must be connected so as to give the opposed flattening effect indicated.

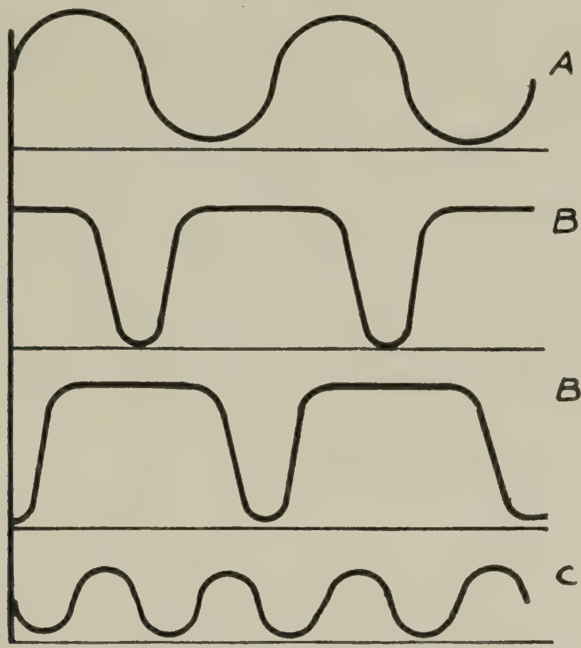


FIG. 327.—Frequency Doubler Curves.

An arrangement to triple the frequency is shown in figure 328, using two transformers and so proportioning the windings as to give the desired resultant wave form.

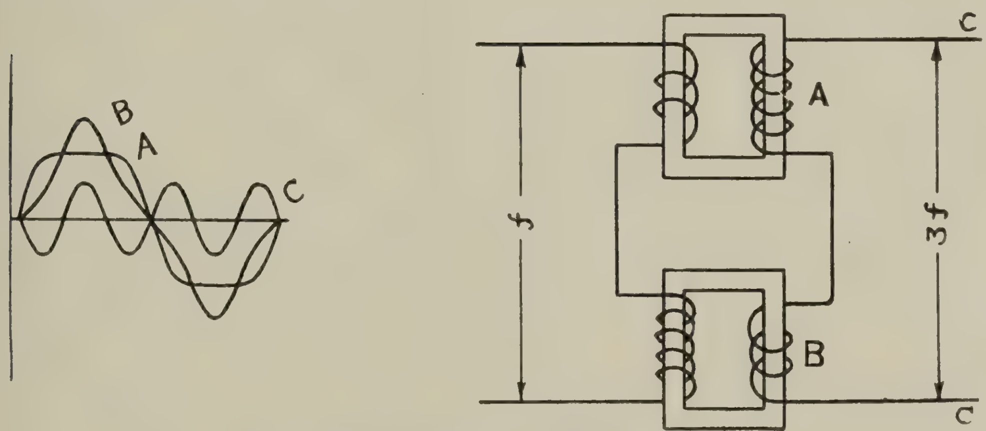


FIG. 328.—Frequency Tripler Circuit.

CHAPTER V. VACUUM-TUBE TRANSMITTERS

General. Vacuum-tube transmitters are essentially continuous wave apparatus, being similar to the arc transmitter in this respect. The vacuum-tube transmitter is superior to the arc in purity of emitted wave form, in the ease with which it can be keyed, in its adaptability to modulation by speech for radiotelephony or to modulation by other means for the production of interrupted (modulated) continuous waves, and in its successful operation at extremely short wave lengths.

Types of transmission. Vacuum-tube transmitters are very flexible. A single tube equipment may be designed for three types of transmission:

(a) Continuous-wave telegraph,

(b) Telephone,

(c) Interrupted continuous wave telegraph.

The first type of transmission (*CW*) is used for telegraphy, and is received in the same manner as in the reception of signals from an arc (autodyne or heterodyne reception). This is the most efficient of three types.

The second type of transmission (telephone) consists of continuous waves of radio frequency modulated by the voice (audio frequency). Straight detection is used when receiving telephone transmission. In other words, when used for telephony, the vacuum-tube transmitter can be received in the same manner as any of the damped wave transmitters. The reliable telephone range of any given tube transmitter equipment is approximately one-quarter of that obtained with the same equipment on continuous-wave transmission.

The third type of transmission (*ICW*) is effected by modulating the continuous waves at an audio frequency. This is done in one instance by using 500-cycle alternating current for the plate supply of the tubes. Another method of interrupting the continuous waves is to open and close the grid leak circuit periodically. The device now in general use for *ICW* (telegraphy), however, is the insertion of a motor drum "chopper" in the grid circuit. Either straight detection or the autodyne or heterodyne method of reception can be used for receiving any *ICW* signals. The note received by straight detection will be distinctive of the type of modulation used in the transmitter; but when the autodyne or heterodyne method is used the received note will be mushy except in the case of the transmitter using a sinusoidal modulation.

(A) CW VACUUM TUBE RADIO TELEGRAPH SETS.

The theoretical considerations of the application of vacuum tubes as generators of radio-frequency currents are given in Part 6, Chapter

V. Various circuits are described and their action explained. In view of the fact that numerous types of circuits are employed to produce radio-frequency current, it is not possible in this MANUAL to consider the description, design, and operation of more than a few types of circuits.

Simple circuit. The circuit considered in the following is a modified Meissner circuit, commonly termed the **Sure-Fire** circuit. This circuit is an excellent generator of radio-frequency current, and, as its name implies, is very positive in operation.

The Oscillatory circuit. Referring to figure 329, L_1 is an inductance coil which is common to both the antenna and the plate circuits. This coil can be wound with litzendraht or copper strip, preferably the latter, on a bakelite, hard rubber, or porcelain tube or skeleton frame. The litzendraht or the copper strip should be of sufficient size to carry the antenna current; edgewise-wound copper strip $\frac{1}{16}$-inch thick and $\frac{1}{4}$-inch wide is well suited for low-power, short-wave transmitters (5 to 100 watts output). Coupled to L_1 is the grid coil L' which feeds back the oscillations from the antenna-plate coil L_1, thereby giving rise to a continuous flow of oscillating current in the antenna. This coil is wound with insulated copper wire on suitable tubing, and is usually placed inside coil L_1. The wire on coil L' has about one-half the cross-sectional area of the conductor used in the antenna-plate coil.

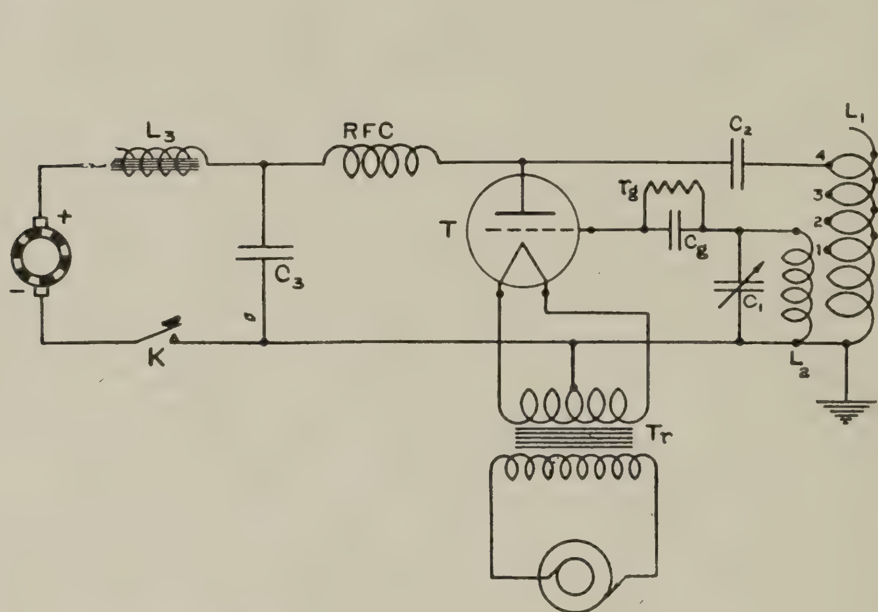
An air-dielectric variable condenser C_1 is connected in series with the grid coil L_2. This condenser permits the use of a smaller number of turns on the grid coil. If this condenser were not employed, the number of turns on the grid coil would have to be doubled, or even trebled. The condenser also permits the adjustment for maximum antenna current to be made rapidly and accurately, whereas without its use this adjustment could only be made by varying the number of turns in the grid coil until the correct number had been found by trial. The maximum capacity of this condenser for wave operation between 150–600 meters is approximately 0.0007 μf.

Connected in series with the grid is the condenser C_g which is shunted with a high resistance r_g. This condenser, when employed without the resistance r_g, allows the grid to be free and to accumulate as large a negative potential as is possible with the rated filament emission of the vacuum tube. The grid in such a condition can, by the use of a resistance of suitable value, be made to assume a definite potential at which the circuit will operate at its greatest efficiency. This resistance can be connected directly from the grid to the filament, or across the condenser C_g, as shown in figure 329.

Condenser C_2 is a **stopping condenser** which prevents the high-voltage direct current used for the plate circuit from flowing in the oscillatory circuit, but allows the generated radio-frequency current to pass through to the oscillatory circuit and flow in the antenna circuit.

A **radio-frequency choke coil,** *RFC*, is inserted between the plate of the vacuum tube *T* and the power source. This coil prevents the generated radio-frequency current from flowing into the power supply circuit. Without such a choke coil, high-voltage, radio-frequency surges would puncture the insulation, or otherwise injure the power circuit.

The choke coil is an inductance coil having a very small distributed capacity. The honeycomb, pancake or banked winding is suitable. For best results, the natural period (fundamental wave length) of the coil should be equal to the wave length at which the transmitter is to be operated.



Circuit Constants:

$C_1 = 0.0007\mu f$,	$L_1 = 200\mu h$,
$C_2 = 0.002\mu f$,	$L_2 = 50\mu h$,
$C_3 = 8.0\mu f$,	$L_3 = 10h$,
$C_g = 0.002\mu f$,	$r_g = 5,000\Omega$,
	RFC = 300-turn honeycomb coil.

FIG. 329.—Circuit Diagram of a CW Transmitter (Range 250–500 meters).

Direct current or alternating current can be used to heat the filament of the vacuum tube. Alternating current is preferred for filament heating, because it permits a more even current distribution throughout the entire length of the filament, while if direct current is used, one end of the filament will carry more current than the other, thus shortening the life of the filament.

The transformer *Tr* in figure 329 is a step-down transformer and is usually supplied with 60-cycle alternating current. It will be noted that the power and oscillatory circuit leads are connected to a mid-point tap on the secondary of the transformer. This connection keeps the grid of the vacuum tube at the same potential as the mid-point of the filament, thus preventing the generation of a 60-cycle modulation current in the transmitter.

More or less rapid fluctuations of voltage occur in practically all high-voltage dc circuits. This fluctuating voltage will modulate the radio-frequency current in the antenna circuit. This is objectionable, especially in radio-telephony. These fluctuations may be due to imperfect commutation in the dc generator, or to the varying voltage supplied by vacuum tube or chemical rectifiers, and can be greatly reduced in amplitude by employing the proper type of filters.

The type of filter in most general use in vacuum-tube power circuits is the **low-pass** filter. This filter will prevent fluctuations of high or medium frequencies from reaching the plate circuit of the vacuum tube, but will allow variations of low frequency to pass. Filters can also be designed to pass all frequencies except those below the desired audible range (voice frequencies).

The low-pass filter consists of the iron-core choke coil L_3 and the condenser C_3 connected as shown in figure 329. This filter will allow currents having frequencies of 36 cycles or less to pass, and will attenuate currents of frequencies above this value. Should it be necessary to construct a low-pass filter for other frequencies, the following formula should be used for finding the current values of inductance and capacity:

$$= \frac{1}{\pi\sqrt{LC}}$$

where f = **cut-off** frequency,
C = capacity in farads,
L = inductance in henries.

For further information on filter systems, reference should be made to the patents of Dr. G. A. Campbell, of the American Telephone and Telegraph Company, on this subject.

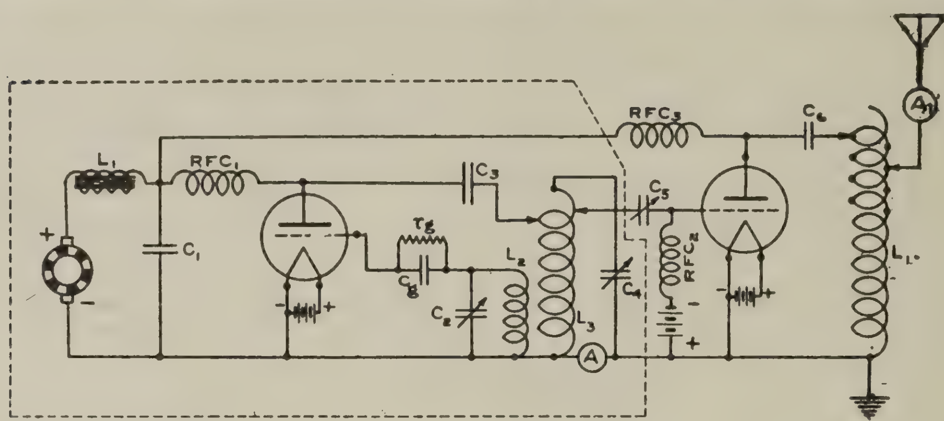
The current at high voltage for the plate circuit of the vacuum tubes may be obtained from a dc generator, or by rectifying alternating current.

The master-oscillator, power-amplifier system. Vacuum-tube transmitters, which have their oscillation generating circuit connected directly to the antenna system, are known to radiate waves additional to the fundamental wave. These waves are the harmonics of the fundamental wave and are the source of additional interference to receiving stations and, therefore, should be eliminated.

A means for eliminating these harmonics has been perfected in the so-called **master-oscillator, power-amplifier system.** This system employs the usual oscillation generating circuit and an amplifier which is coupled by capacity or inductively. This amplifier amplifies the output of the oscillation circuit in such a manner that maximum amplification of currents of the fundamental frequency is obtained with little or no amplification of currents of the harmonic frequencies. This result is obtained by using a stiff circuit—one having a large inductance to

capacity ratio—in the oscillation generator circuit, and by tuning the antenna or output circuit of the amplifier to the fundamental frequency of the oscillation circuit.

A simplified diagram of the master-oscillator, power-amplifier circuit for a short-wave (300—500 meter) transmitter is shown in figure 330. The part of the diagram enclosed within the dotted lines is the oscillation generating circuit, and is similar in construction to that shown in figure 329. The only difference in the two circuits is the substitution of condenser C_2 in figure 330 for the antenna system of figure 329. It will be noted that C_4 has a small value, because the large inductance L_3 is necessary to produce a stiff circuit for reducing harmonics.



Circuit Constants:

$C_1 = 8.0\mu f$,
$C_2 = 0.00075\mu f$ (max.),
$C_3 = 0.002\mu f$,
$C_4 = 0.0003\mu f$, (max.),
$C_5 = 0.0003\mu f$,
$C_6 = 0.002\mu f$,
$C_g = 0.002\mu f$,
C = Grid battery,
$L_1 = 10h$,
$L_2 = 50\mu h$,
$L_3 = 250\mu h$,
$L_4 = 200\mu h$,
$r_g = 5{,}000\Omega$
$RFC_1 = RFC_2 = RFC_3 = 300\mu h$ (honeycomb).

FIG. 330.—The Master-Oscillator, Power-Amplifier Circuit (range 300–500 meters)

The part of the circuit outside of the dotted lines is the amplifier circuit, which is coupled capacitively to the oscillation generator by the condenser C_5.

When coupling by capacity, using the method shown in figure 330, it is essential to use a negative potential on the grid to reduce the amplifier tube plate current to one-half the rated current capacity when no radio-frequency controlling voltage is being received from the oscillation generator. The use of this negative potential makes for greater efficiency in the amplifier. To place this negative potential on the grid, and at the same time prevent the short-circuiting of radio-frequency current through the battery, a radio-frequency choke coil is inserted in the circuit, as shown in figure 330. This choke coil is identical with the one previously described.

The other parts of the amplifier circuit are the vacuum tube, the radio-frequency by-pass condenser C_6, the antenna inductance L_4, the

antenna and the ground. The amplifier vacuum tube is always of greater power than the vacuum tube used in the oscillation generating circuit. A 5-watt vacuum tube will be sufficient to control a 50-watt amplifier tube. The condenser C_6 should be constructed to withstand voltages up to 2,000 volts for use with 50-watt tubes and should have an air or mica dielectric. The inductance L_4 can be identical with L_1 in figure 329. If the transmitter is to be used for wave lengths longer than 600 meters, litzendraht made up of many strands of No. 38 or No. 40 B, and S enamelled wire, laid up with a very slow twist, can be employed. Should litzendraht be used, it is advisable to use a variometer in the antenna circuit in order to obtain the required closeness of tuning, which is so readily obtained by clips when the coil is wound with bare copper wire or strip.

Operation. The operation of the **master-oscillator, power-amplifier** is not as complicated as the circuits indicate. The oscillation generator, or master oscillator, is first adjusted to the desired wave length with a wavemeter by varying C_2 and C_4. The plate coupling L_1 of the generator is then changed from one turn to another until maximum current is obtained, as shown by ammeter A. Having obtained maximum output from the oscillation generator, it only remains to adjust either C_5 or the inductance L_3 taps until maximum voltage is transferred to the grid of the amplifier tube. Maximum voltage on the grid of the amplifier tube will produce maximum antenna current, provided that the proper adjustment of inductance L_4 is made.

It is often the case that the wave length is changed slightly during the process of tuning the transmitter. To correct this condition, readjust C_4 and inductance L_4 until the correct wave length is indicated on wavemeter. If the antenna current is considerably reduced by this adjustment, vary C_2 and change the amplifier plate coupling until maximum antenna current is obtained.

When the wave length of the **master oscillator** is set, it will be noted that the swinging of the antenna due to wind or rolling of a ship will have no effect on the wave length of the radiated wave. The master oscillator maintains the wave length at a fixed value, and all the amplifier does is to amplify the output of the master oscillator, thereby increasing the power supplied to the antenna. For this reason, the application of the master-oscillator power-amplifier to all types of stations is preferred when constancy and purity of emitted wave are essential.

Figure 329 shows the schematic wiring diagram and Figures 332 and 333 show the appearance of a modern vacuum tube transmitter of the master oscillator power amplifier type used in the U. S. Navy. This transmitter can be used either for *CW* or *ICW* emissions.

Crystal controlled master oscillator power amplifier circuits.

In order to still further hold the emitted frequency of a transmitter constant in a very large degree of accuracy the latest practice is to utilize

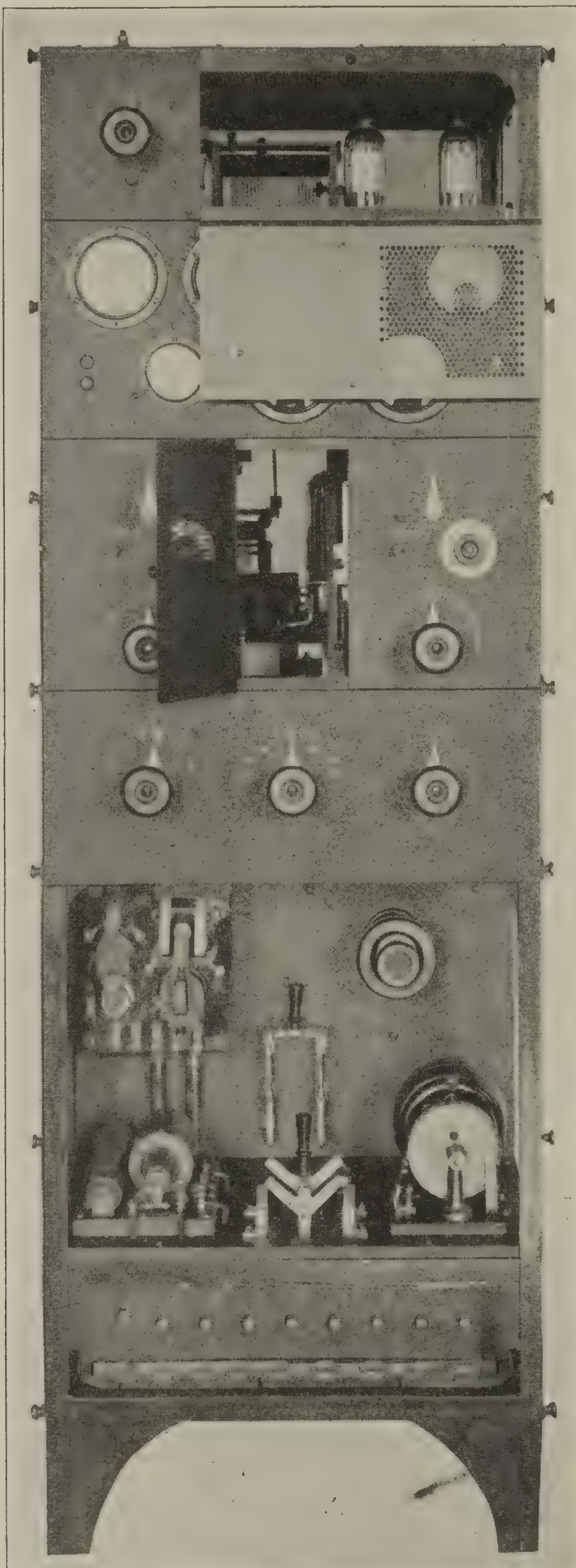


FIG. 331.—Tube Radio Transmitter for Ships.

the Piezo electric effect of quartz crystal to control the frequency of the master oscillator. For convenience on very short wave lengths this crystal control feature becomes a necessity. Figure 332 is a schematic diagram of such a transmitter.

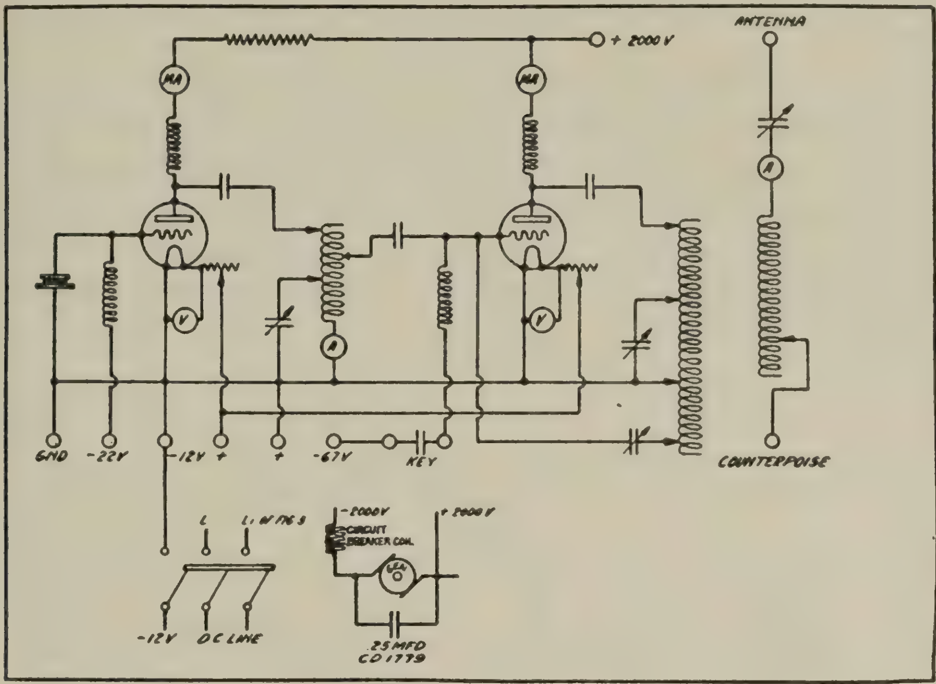


FIG. 332.

(B) RADIOTELEPHONE BUZZER AND CHOPPER MODULATED SETS.

The elements of the design of a continuous-wave telegraph transmitter were given in detail in section *A* of this chapter and, although this transmitter is an integral part of a radio-telephone transmitter, no further mention will be made of the characteristics of this transmitter.

A **radiotelephone transmitter** is a continuous-wave transmitter whose output is modulated, or varied, at telephonic or voice frequencies. There are various means of controlling or modulating continuous wave transmitters, among which are: (a) the grid-voltage variation method, (b) the plate-voltage variation method (Heising) and (c) the antenna current absorption method. Of these three methods, the plate-voltage variation method is considered the best, and this method will be employed in the design of the radiotelephone transmitter which follows.

A simplified diagram of a radiotelephone transmitter is given in figure 333. The part of the diagram enclosed within the dotted line is the oscillation generator, or continuous-wave transmitter (see figure 329), while the part outside this line is the **modulating circuit,** or **system.** This circuit includes a six-volt battery *B* which furnishes power for the circuit through the microphone *M* and the primary of the transformer *Tr.* The variations in the current in this circuit, due to the action of the microphone under the influence of sound waves, cause large voltage variations in the secondary of the step-up transformer *Tr* which are

impressed on the grid of the vacuum tube *T*. These voltage variations correspond to the vibrations of the microphone diaphragm, and these voltage variations, in turn, control the voltages impressed on the plates of the vacuum tubes in the modulating and oscillating circuits through the action of the choke coil *AFC*, as described in Part 10, Chapter V.

The grid battery *C* functions in keeping the grid voltage at the middle part of the straight line portion of the grid voltage—plate current characteristic curve when the tube is not operating, thus producing a condition whereby the impressed grid voltage will produce undistorted speech currents in the plate circuit. For best operation with 5-watt tubes, a negative voltage of 22 volts is required; for 50-watt tubes, 60 to 80 volts are necessary.

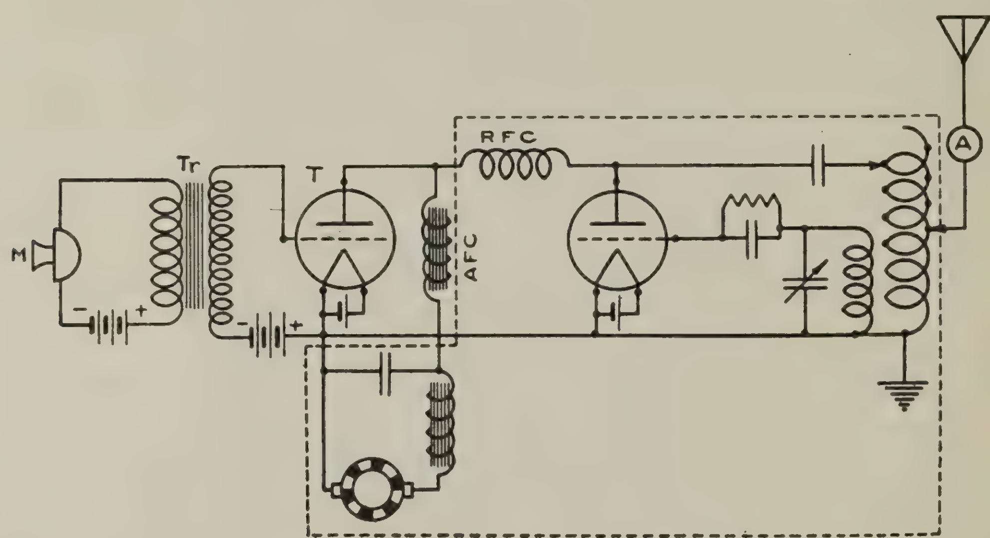


FIG. 333.—The Radiotelephone Transmitter (Heising Circuit).
B = 6-volt battery, *Tr* = Step-up Transformer,
M = microphone transmitter, *AFC* = 2-henry, choke Coil.
C = Grid battery,

For best operation, both the modulating and the oscillating tubes should be of the same size and characteristics. The choke coil *AFC* should be so designed that maximum inductance is obtained with minimum resistance. Coated filament vacuum tubes are preferred for use in radiotelephone transmitters. This preference is due principally to the large filament emission of the coated filament tube, which permits greater ease in variation of the current in the plate circuit of the tubes.

The foregoing statements are based on the fact that, with equal filament heating power, it is possible to obtain a much greater filament emission from the coated filament than from a tungsten filament. This fact has long been recognized, and at the present time the General Electric Company is introducing a new type of filament made of thori-

ated tungsten. The thorium in the tungsten renders it equal to the coated filament in filament emission qualities.

When operating a radiotelephone transmitter, similar to that shown in figure 333, a good indication that the transmitter is functioning properly is the slight increase in current that occurs in the antenna circuit when the microphone is operating.

Speech amplifier system. In most radiotelephone transmitters, especially those used for broadcasting purposes, the voltage variations in the secondary of the modulation transformer Tr are not sufficient to produce the required modulation of the output of the oscillation generator. This is due to various causes, such as insensitive microphones small step-up ratio of the transformer Tr, or to the distance at which the speaker or artist is from the microphone. To overcome this difficulty, one or more amplifiers, commonly termed **speech input amplifiers,** are employed.

A single-stage **speech amplifier** system is shown in figure 334 where M, B_1 and the primary of the transformer Tr constitute the

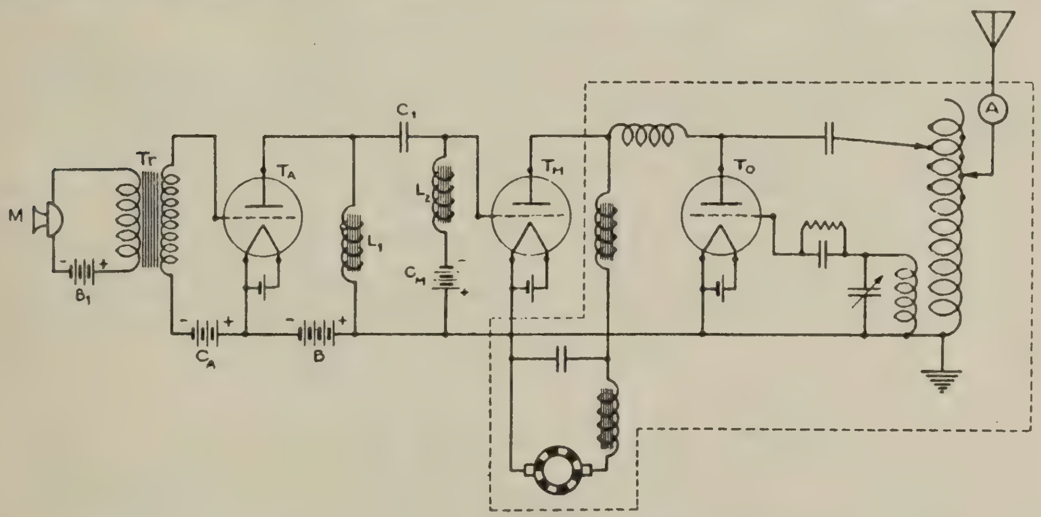


FIG. 334.—The Speech Amplifier System.

Circuit Constants:

B = Plate battery, 300 volts,	L_1 = choke coil, 4 h,
B_1 = 6-volt battery,—	L_2 = choke coil, 10–50h,
C_a = Grid battery, 22 volts,	T_A = vacuum tube, amplifier,
C_M = grid battery, 20–40 volts,	T_M = vacuum tube, modulator,
C_1 = by-pass condenser, 0.5μf,	T_0 = vacuum tube, oscillator,
M = microphone transmitter,	Tr = step-up transformer.

microphone circuit; the secondary of Tr, grid of the amplifier tube T_a, and C_a, the amplifier input circuit; the filament-plate of T_a, L_1 and B the output circuit of the amplifier. The variations of current through the choke coil L_1 produce a varying voltage across this coil, and this voltage is impressed on the grid of the modulator tube T. through the condenser C_1. In order to operate T. at a point where best voice reproduction is obtained, a certain negative voltage should be placed on the grid of T_m. This is done as shown in the diagram. The choke

coil L_2 permits the negative dc voltage to be placed on the grid, but keeps the audio or amplified voice frequency voltages from being short-circuited through the battery C_m. The values of L_1 and L_2 will be different for each type of vacuum tube used. The constants given in figure 334 are for use with Western Electric Type E, 5-watt vacuum tubes.

The **buzzer method** is similar to the telephone method—and consists of substituting a key and buzzer for the microphone shown in figure 333. Figure 335 shows the method of connecting the buzzer *M*, or a tuning-fork oscillator. It will be seen from the figure that the buzzer serves the same purpose as the microphone; that is, it causes variations in the current in the primary of transformer *Tr*.

The buzzer method of modulation has not met with general favor as a substitute method of transmission for the spark transmitter.

Chopper modulation. The **chopper method** of modulation affords a means of obtaining 100 per cent modulation or, in other words, the

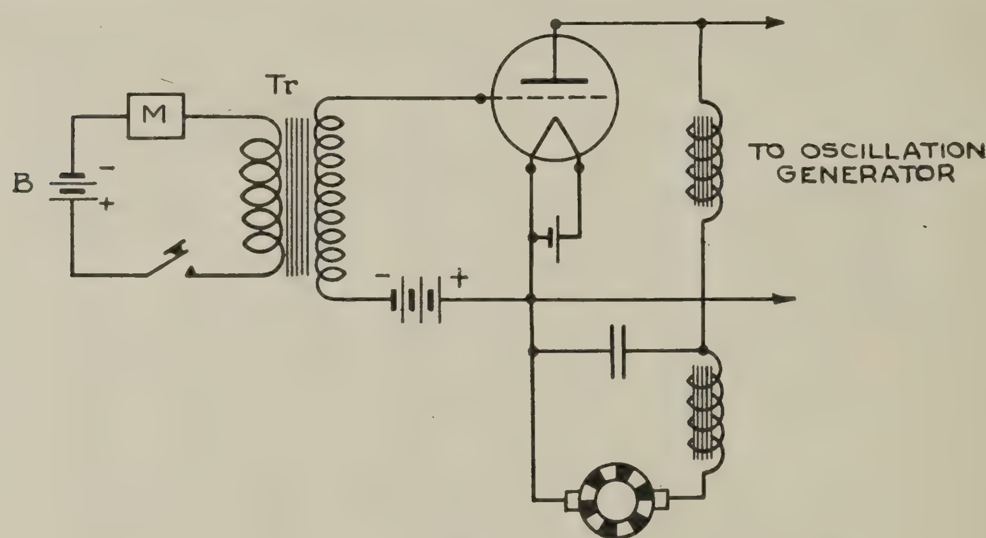


FIG. 335.—The Buzzer Method of Modulating Vacuum-Tube Transmitters.

complete variation of antenna current from a zero value to the maximum value. This is accomplished by inserting in the grid circuit of the oscillation generator a device for making and breaking the circuit. The usual way of accomplishing this is to use a metal disk with a brush bearing on the periphery of this disk. The periphery of the disk is slotted and the slots filled with insulating material as shown in figure 336.

The metal disk is rotated by a motor at a speed which will produce the required modulation or interruption frequency. Most choppers are made to rotate at a speed which interrupts the grid circuit one thousand times per second, and the received signal will have a note closely resembling the tone of signals received from a 500-cycle, quenched-spark transmitter.

The wave emitted by the chopper-modulated *CW* transmitter has a low **decrement**, but an extremely high **increment.** This high

increment (rate of charging antenna) produces an effect similar to that caused by the quenched-spark transmitter, which tends to shock local antennas into oscillation. For this reason, the chopper modulated *CW* transmitter is not recommended for use in districts where there are numerous radio stations. It is used in the Navy for emergency communications. The circuit is included in the set shown in Figure 336.

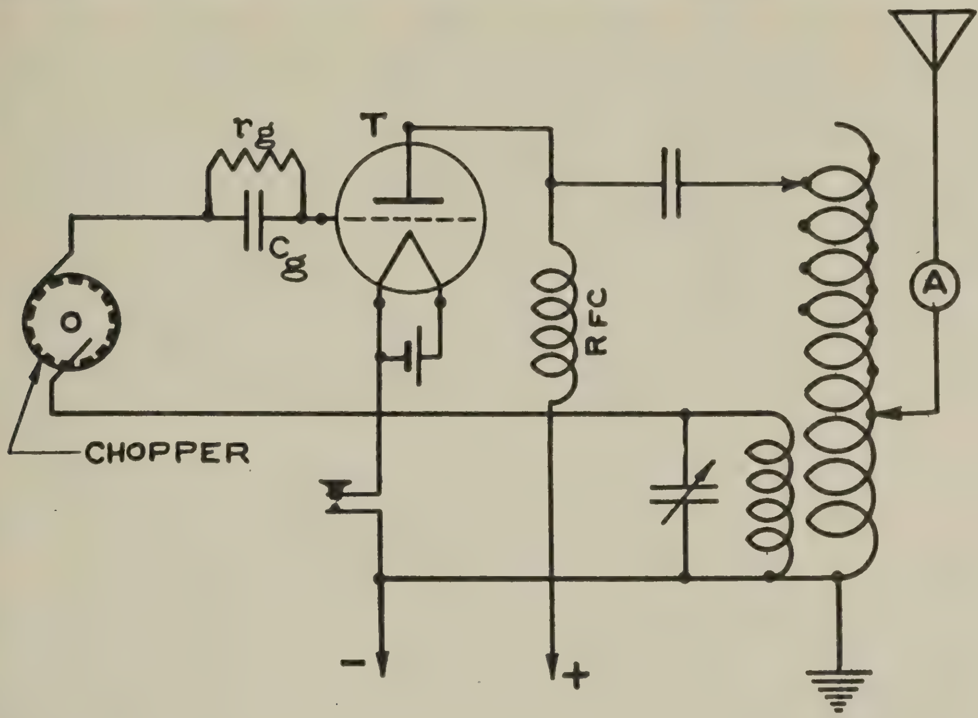


FIG. 336.—The Chopper System of Modulating Vacuum-Tube Transmitters.
Circuit constants:

$$C_g = 0.002\mu f; \quad r_g = 5,000+\Omega.$$

(C) THE AC VACUUM-TUBE TRANSMITTER.

The **AC vacuum-tube transmitter** is a type of vacuum-tube transmitter which employs alternating current for the plate supply. The principal reasons leading to its adoption were that the power equipment ordinarily supplied for spark transmitters can be used.

As previously stated, the power supply for the ac vacuum-tube transmitter is obtained from the 500-cycle power unit generally furnished with spark transmitters. This ac power supply circuit includes *G*, *K* and *Tr* of figure 337, with the exception that, instead of obtaining 12,000 volts from the secondary of *Tr*, a voltage of 3,000 volts is generally used. This lower voltage is obtained by paralleling a number of the pancake windings on the secondary. The tertiary winding for filament heating purposes can be placed on the core of the transformer, or another transformer can be supplied for this duty. A mid-point tap on the secondary is made to which one end of the filament of each tube is connected.

Both alternations of the cycle are used for supplying power to the plates of the tubes, as is shown by the connections in figure 337. Radio-

frequency choke coils of the pancake, honeycomb or banked winding type serve to keep the radio-frequency currents from entering the transformer windings.

The oscillatory circuit in figure 337 is practically the same as that shown in figure 329 with the exception of an additional condenser C_1

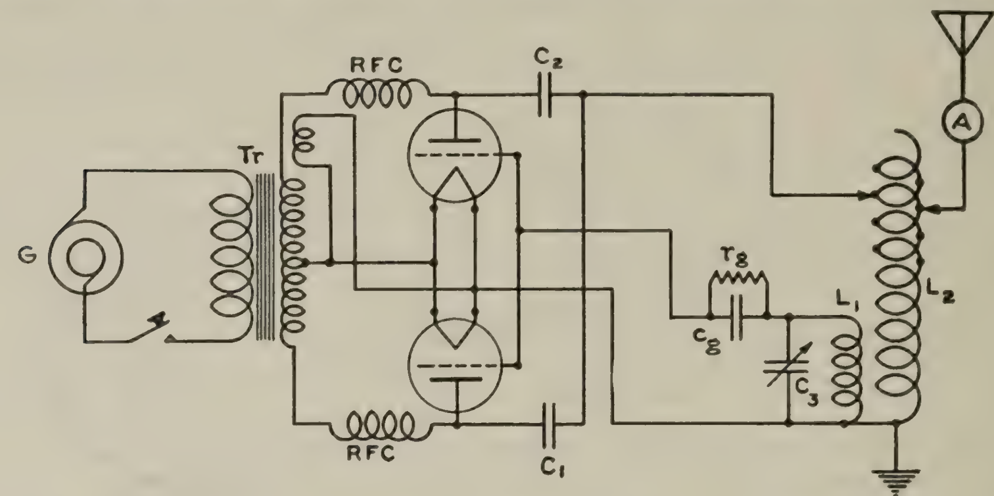


FIG. 337.—The Ac Vacuum-Tube Transmitter Circuit Diagram. Low-Power Short-Wave Type (200–500 meters).

Circuit constants:

$C_1 = C_2 = 0.002\mu f$ mica-dielectric condenser,
$C_3 = 0.00075\mu f$, air-dielectric variable condenser,
$C_g = 0.002\mu f$, 1,000 volt mica-dielectric condenser;
G = 500-cycle generator,
$L_1 = 50\mu h$,
$L_2 = 200\mu h$,
RFC = 300 turn honeycomb coil,
Tr = Transformer, 200/3,000 volts with 10-volt winding for filaments.
$r_g = 10{,}000\Omega$.

which allows the radio-frequency current component of the other tube to be added to that of the tube to which condenser C_2 is connected.

The radiated wave from an ac vacuum-tube transmitter has an audible note which is the equal of, or clearer than that of a 500-cycle spark transmitter, when being received with a crystal detector, or with a regenerative vacuum-tube receiver. When being received by an oscillating vacuum-tube receiver, chords are produced, and by adjusting an autodyne receiver suitably, these chords can be changed at will.

There is a general tendency in the design of ac tube transmitters to work the tube at a greater efficiency than 50 per cent. With such operation, the characteristics of the radiated wave are changed from a sinusoidal form to a distorted form, and currents of harmonic frequencies are generated. These currents should not be permitted to be radiated from the antenna and, therefore, filter circuits or coupled circuits should be resorted to in the design of an ac tube transmitter.

(D) PRECAUTIONS REQUIRED IN THE DESIGN AND OPERATION OF VACUUM-TUBE TRANSMITTERS.

Power supply. Power supply circuits for vacuum-tube transmitters should always be provided with fuses or, in the case of high-power tube equipment, with circuit breakers. These precautions are necessary because it is comparatively easy to stop vacuum tubes from oscillating, and, when such a condition exists, the extra power, which was being supplied to the antenna, is dissipated in the tube and overloads it. If the fuse or circuit breaker does not open the power circuit, the tube will be destroyed. The overload devices can also take care of any short circuits that may occur in the rectifying or filter circuits.

When tube (Kenetron) rectifiers are used, it is only necessary to heat the filament to the proper brilliancy, as specified by the tube manufacturer. The best practice in this respect is to use a voltmeter across the filament terminals and to keep the voltage supply at a definite working voltage throughout the entire operating life of the tube.

Chemical rectifiers need very little attention if care is taken to keep foreign matter out of the electrolyte.

The filter system should not prove to be a source of trouble, provided that the choke coils and the condenser have been designed to withstand the working voltages and currents. A slight generator hum may appear from time to time, due to a momentary reduction in the supply voltage to the filaments of the vacuum tubes. This voltage change permits the emission of a hum which sounds like a combined generator and a 120-cycle hum, assuming in this case that 60-cycle filament heating supply is used.

Oscillation generating circuit. If the plate of the vacuum tube in the oscillation generating circuit becomes too hot, as indicated by an increase in the coloring of the plate, this means that the plate or grid coupling is not correct. Increase or decrease the number of turns in the plate circuit until the coloring disappears. If this change does not remedy the trouble, change the amount of capacity in the grid tuning condenser until the coloring disappears. Finally, if these changes do not reduce the heating of the plate, inspect the antenna system to see if the antenna is grounded or open-circuited.

The radio-frequency choke coils should give no trouble if they are kept free from dirt and moisture. The same holds for the coil system, the condensers and the resistance in the grid circuit.

When designing the coil system for the oscillation generating circuit, special care should be taken to select insulating material which has low dielectric losses. The location of this material in the oscillatory circuit should also be carefully chosen. Porcelain, hard rubber, kiln-dried paraffined wood and the best grade of phenolic base insulating

material are recommended for use in the oscillating circuit, the preference of material being given in the order named.

It is possible to blister and burn up specimens of various insulating compounds by placing them in the field of a coil carrying radio-frequency currents. Present practice shows a preference for kiln-dried wood, boiled in paraffin, and porcelain for use as insulation in the coil system of the oscillatory circuit.

Do not use iron or steel screws or bolts for securing parts of the oscillation circuit; also make sure that no closed metallic loops are within coupling distance of the coil system. If such loops exist it is advisable to open-circuit the loop either by cutting out a portion of the metal or by inserting mica or bakelite bushings between parts which hold the loop circuit together.

The capacity effect of leads from the oscillatory circuit to metal parts of the transmitter panel and frame should be reduced to a minimum by supporting high-voltage leads at a maximum distance from grounded metal parts. If this precaution is not taken, losses will occur in the transmitter due to the distribution of output current between the capacity within the transmitter and that of the antenna. For this reason, the transmitter should be designed so that a minimum amount of capacity exists between the oscillatory circuit and the grounded parts of the transmitter.

The filament should be heated to the required brilliancy, as indicated by the voltmeter. Any increase in filament brilliancy over that specified greatly reduces the life of the filament, thereby increasing maintenance costs.

Modulation circuits. The main precaution to be observed in the modulation circuit is to keep the grid and microphone batteries in good condition at all times. It is also advisable to operate radiotelephone transmitters at reduced power rather than at increased power, because the extra power dissipated in the plate circuit of the modulation tubes greatly shortens the life of the tube. This is due to two causes, namely: the increased temperature of the plate may be sufficient to cause it to collapse, or to melt, with consequent destruction of the tube; the increase in power is accompanied by increased filament temperature, which results in shortening the life of the filament. The increase in radiated power obtained by overloading the tubes does not produce an appreciable increase in received signal strength and, in certain cases, such an increase may even cause a decided decrease in signal strength. The latter would most likely be due to excessive heating of the plates of the tubes, with a consequent reduction in the overall efficiency of the transmitter.

CHAPTER VI. THE TRANSMITTER ANTENNA.

The simple antenna. The simple antenna with its associated ground system is essentially a condenser where the overhead portion forms one plate, the ground under it forms the other plate, and the air space between forms the dielectric. Such a system contains not only capacity; it contains inductance as well. These two act together to reproduce the conditions of the simple oscillatory circuit composed of lumped capacity and inductance. The natural period of oscillation for the antenna is determined by the equivalent concentrated values of capacity and inductance in accordance with the usual formulas.

Figure 338 represents a simple antenna and shows the manner in which the capacity effect is distributed throughout the system in accordance with the distance of each part from the ground plate. Antenna capacity is generally spoken of as **distributed capacity** on account of this varying value.

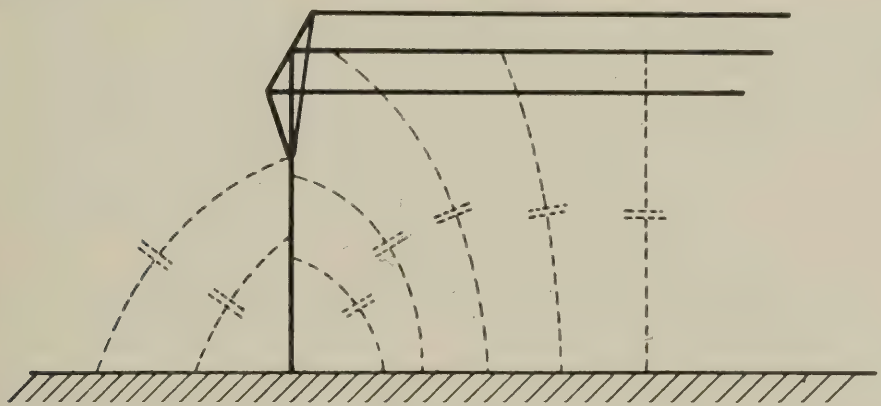


Fig. 338.—Distribution of Antenna Capacity.

The inductance contained in the simple antenna is very small, since it is only that due to straight wires. But it is easily possible to insert concentrated inductance in the form of coils and spirals, which adds to the natural inductance of the antenna. In this manner, the period of oscillation, or, as more commonly called, the wave length can be built up to any desired value.

Solid plates are customary for the built-up condenser, with close spacing between plates. Two purposes are served: first, mechanical rigidiny for the plates is secured, and second, maximum capacity values are obtained for the overlapping sections of plates. If some sort of coarse screening were used, the capacity would be less than for solid plates, because the flux lines would be distributed as in (*b*) rather than as in (*a*), figure 339. But as the separation between plates is increased the flux lines spread out so that the arrangement which in (*b*), figure 339, measures less capacity than two plates, measures as much capacity

as two plates in (*c*). It thus becomes possible with an antenna, where the separation between plates is large, to use wires for the overhead plate without loss of capacity.

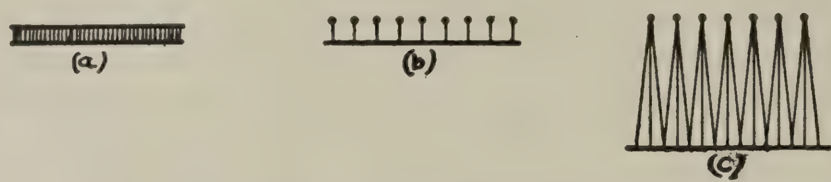


Fig. 339.—Effect of Spacing on Electric Field.

Another feature in which the antenna condenser differs from the usual condenser with small spacing between plates relates to change of capacity with variation of height. For close spacings between plates this is an inverse relationship, but for the large spacings used with the antenna, the capacity decreases much less slowly as the height is increased. A formula worked out by Dr. L. W. Austin for antenna capacity is:

$$C=0.4\sqrt{S}+8.85\cdot 10^{-2}\frac{KS}{\tau}$$

where C = antenna capacity in $\mu\mu$f,
K = dielectric constant = 0 for air,
S = area of flattop of antenna in cms.^{2},
τ = mean height of area S in cms. above ground,
π = 3.1416.

This formula does not take into account the capacity due to the downlead, but only that of the flattop.

It is the function of the antenna to provide a means for conversion of the high-frequency energy generated by the transmitter into the radio-frequency ether waves which travel out in all directions until their energy is dissipated. The phenomenon whereby these waves are emitted and propagated is described elsewhere and will not be taken up here.

Requirements for a good transmitting system. Two principal requirements for a good transmitting antenna are:

(1) High effective height,

(2) Low resistance losses.

Other conditions remaining the same, the strength of a received signal is directly proportional to (1) the effective height of the transmitting antenna and to (2) the antenna current. These two requirements can be set against each other to get a certain signal strength at a distant point, since the greater the effective height of the transmitting antenna the less the current required, and vice versa.

An important influence in the effective height of an antenna is exerted by the supports. If grounded towers or masts are employed, they must be considered as elevated grounds and, as such, the result is

a reduction in the effective height which would be obtained otherwise. This is one reason that guys are generally broken up by insulators.

A means is necessary to distribute the radio-frequency current into the ground plate of the antenna condenser. This means is commonly called the ground system or ground. Any connection made to earth will serve, but unless the connection is extensive and made in such a manner that the current finds a low-resistance path into the earth, much energy is wasted and the efficiency of the complete antenna-ground system as a radiator becomes very low.

Ground systems. The first systems connected the ground terminal of the transmitter to a plate of a few square feet area buried in permanently moist earth. This method was later developed by replacement of the plate with an interconnected net of buried wires radiating from the ground connection at the transmitter and extending under the entire overhead section. These have both proved satisfactory where the earth is moist and has a good conductivity so that the currents can pass off the metallic conductor and into and through the earth with little loss.

For the case where the earth is of high resistance, such a buried ground system is necessarily inefficient and the **counterpoise** method is substituted. It consists of a network connected to the ground terminal of the transmitter which extends under the antenna and is insulated from ground.

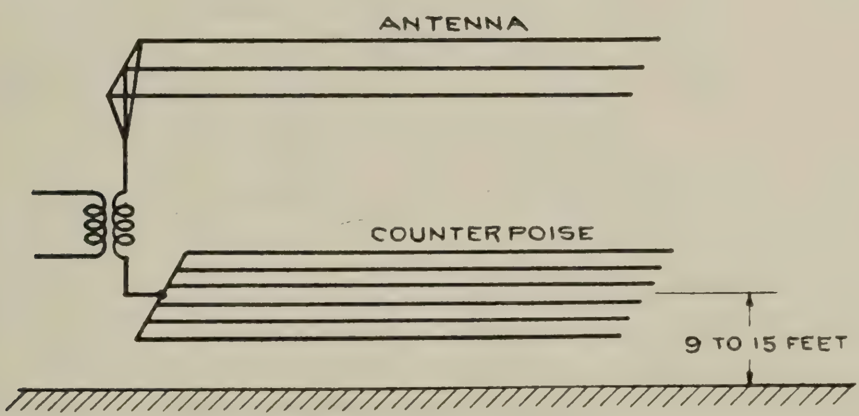


FIG. 340.—Counterpoise.

Figure 340 is a diagrammatic representation. The ground plate is then, through the medium of the field set up to earth from the current flow in this network, charged with little loss, due to the lack of concentration of flux at any one point.

It would be thought that the field would be set up directly between the counterpoise and the antenna. Actual investigation has proved, on the contrary, that the fields from both the antenna and the counterpoise act largely through the medium of the earth, and that only a minor part of the total is directly effective.

The counterpoise is essentially a second antenna of large capacity and low height. Consequently a voltage to ground will be developed in it determined by its capacity and the intensity and frequency of the current in accordance with the relation:

$$V = \frac{I}{\omega C}$$

Another slightly more complicated ground arrangement which has proven effective is the **tuned-counterpoise,** shown in figure 341. It utilizes both the buried ground connection and the counterpoise. Under these conditions, the point of attachment of the counterpoise must be at zero potential to earth. Hence, the counterpoise must be tuned to the frequency of the transmitted wave by inserting the inductance L_C. The point of attachment then becomes a nodal point, and the radio-frequency current can flow direct to ground, or into the counterpoise and to ground through the medium of the magnetic field set up.

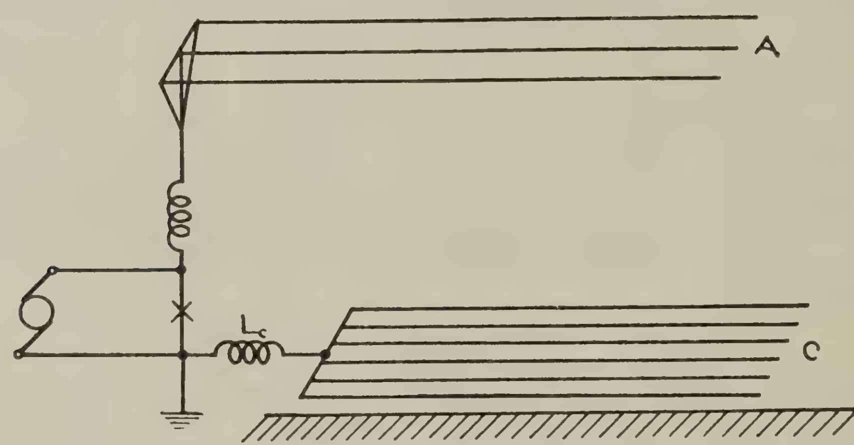


FIG. 341.—Tuned Counterpoise.

Distribution of the radio-frequency currents into the earth through the buried ground connection may be inefficient regardless of how extensive a system is used if for any reason the current does not follow the metal but flows to earth near the transmitter. On the basis of resistance alone, such a condition is unlikely to occur. But for the frequencies involved, any slight **inductive** effect in the ground conductors will greatly overbalance even a large resistance. Therefore, conditions may readily exist, difficult to detect or correct, which greatly influence the efficiency of the buried ground system. In order to overcome this difficulty and to prevent passage of the radio-frequency current into the earth until well out under the antenna, the **star-ground** system has been used. It utilizes insulated leads, usually carried overhead on poles, to conduct the current from the transmitter out to buried plates located at various places under the antenna, from which the current passes into the earth.

Another ground system proceeds on the basis that the distribution of the currents in the ground plate corresponds to the intensity of the

field from the antenna. For the usual antenna, this field is concentrated to a considerable degree around the edge. It therefore follows that most efficient results will be obtained by distribution of the ground current into the earth within this area of maximum field intensity. Insulated leads are used for the star-ground to conduct the current to suitable buried plates approximately under the edge of the antenna. For best results, plates should be located both inside and outside the antenna edge. Approximately uniform current distribution is desirable and may be obtained by small regulating inductances inserted in the leads to the plates.

Energy utilization. All energy delivered to the antenna system is used up in one of three ways:

A—Conductor losses,
B—Dielectric losses,
C—Radiated energy.

Conductor losses include those due to the ohmic resistance of the antenna and ground wires, loading inductors, etc. A slight change

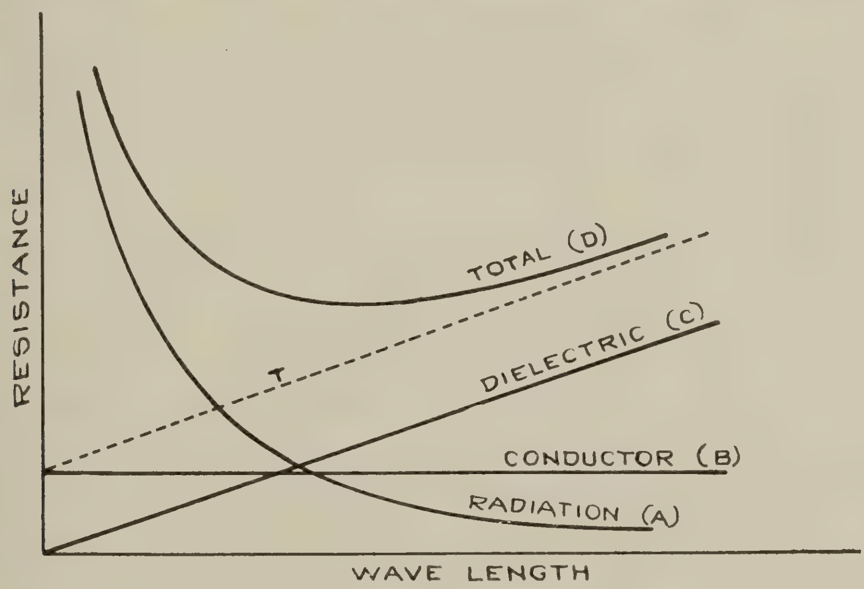


FIG. 342.—Antenna Resistance Analysis.

occurs (from the skin effect) with variation of wave length, but it is very small and these losses are considered as having a steady value. The straight line *B*, figure 342, so represents it.

Dielectric losses are due to the presence of various objects in the antenna field which absorb energy from it. The action is the same as for any ordinary condenser. The importance of these losses is dependent on the size and position of the absorbing bodies. Poor antenna insulators may be an important cause of the loss. Trees, buildings, etc., under the antenna may also be the causes of loss. It is known that the losses in an absorbing condenser increase in proportion to the wave length. The same may be assumed to be true of the antenna, and is so represented by *C*, figure 342.

There remains the radiated energy which actually goes to produce the signal at the distant station. The amount of power radiated depends upon the form of the antenna, is proportional to the square of the current at the ground connection, and is inversely proportional to the square of the wave length. The general form of curve is shown by A, figure 342.

Instead of speaking of conductor losses, dielectric losses, and radiated energy, it is customary to speak of the respective resistances and, in particular, of radiation resistance. This is convenient and permissible, because equivalent energy consumptions may be obtained by inserting the proper value of resistance in the antenna circuit for each of the three cases.

Actual measurement has to be made of the total resistance value for any given antenna, shown in curve D, figure 342. As the wave length is increased, this curve acquires a fixed slope. The radiated energy and

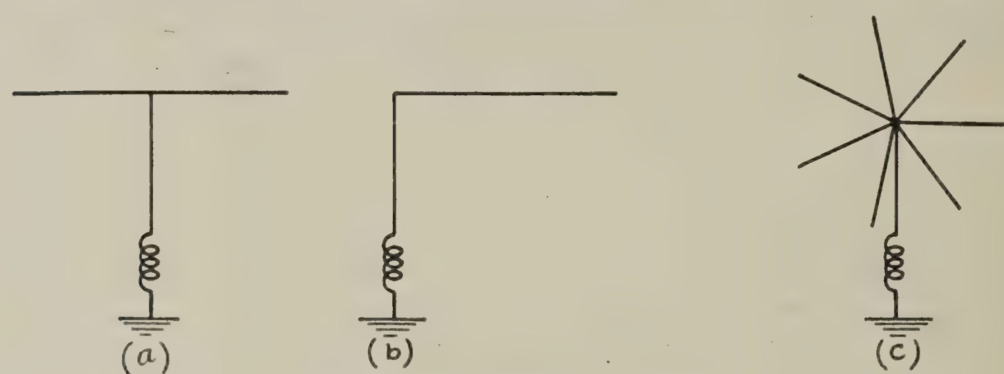


FIG. 343.—Forms of Antenna.

hence radiation resistance has then become very small and may be disregarded. The total resistances represented by the curve then consist only of the conductor and dielectric values. The dielectric resistance must become zero at zero wave length and, since the conductor resistance is constant and the dielectric resistance has a fixed slope, it is possible to draw T tangent to D. Where T cuts the vertical axis, gives the value of conductor resistance. Then knowing the values of total resistance, conductor resistance and dielectric resistance, the radiation resistance is easily obtained.

The maximum input to an antenna is limited by the voltage developed. About 120,000 volts is found to be the economical limit at the high-powered Naval Stations due to corona losses. With a known capacity, the permissible current is readily found from the relation

$$I = \frac{V}{2\pi f C}$$

Forms of antennas. Typical forms of antennas are shown in figure 343 where a represents the symmetrical **T** type; b represents the Γ type; and c represents the umbrella type.

Each type has its advantages. Using two towers, the **T** type provides maximum separation from the supporting masts or towers and the minimum distance of travel for the radio-frequency current. When the antenna is supported on three or more towers, the Γ antenna is convenient and satisfactory. Use is made of the umbrella type when only a single support is available. It is not as satisfactory as the others because of a neutralizing effect between the vertical and the radial arms, resulting in a low effective height for transmitting.

One particular type of recent development uses several earth connections for a single aerial. It is known as the multiple-tuned antenna and is shown in figure 343. Downlead A is excited from the power

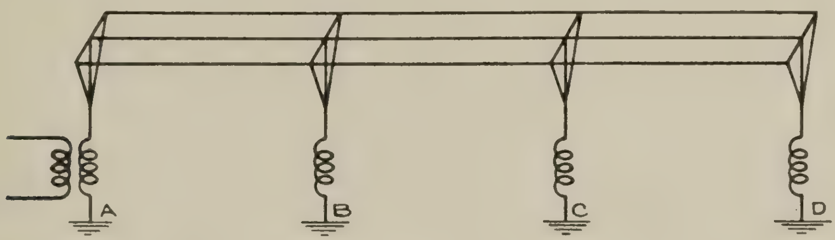


FIG. 344.—Multiple Tuned Antenna.

source, and, by capacity coupling, responsive currents produced are set up in B, C, and D, resulting in an increase of the overall transmitting efficiency.

Such an antenna may be considered as N simple antennas and the effective antenna current as the sum of the current in all N downleads. The effective power in the total antenna will then be NI^2R, where I is the current in any one downlead, on the assumption that the current is equal in all. Actually, there is a slight decrease due to losses as the downleads set farther from the transmitter. Since the total current is NI, the power absorbed by a simplex antenna would be N^2I^2R. For the same radiation the ratio of power is then

$$\frac{N^2I^2R}{NI^2R}=N$$

assuming that R has the same value in all cases. Viewed in another way, the antenna resistance is only $1/N$th as great for N downleads as for the simple antenna. But on the other hand, the resistance into which the transmitter must work is N times as great as for the simple antenna. This is apparent when it is considered that all energy losses must be supplied from the one source.

The multiple-tuned antenna cannot be used to advantage where the antenna capacity is low relative to the radiation used. Its effective field is in connection with an antenna of large capacity only slightly loaded. Under such conditions the aerial can be divided into sections constituting, in effect, antennas in parallel fed from a single source, without raising the antenna voltage to an undue value. But where the

antenna is high and of small capacity relative to the antenna current, so that the antenna voltage is high, no field exists for multiple tuning. A particularly favorable field for multiple tuning is in the case of a large capacity antenna which it is desired to use at short wave lengths. By dividing it into several sections, each section has a largely reduced natural period suitable for the short wave.

PART 2.

RADIO RECEPTION.

CHAPTER I. GENERAL.

It was shown in Part 6 of Section I that the received current in a receiving antenna is given by the formula

$$I_r = \frac{\mathcal{E} h_r}{R}$$

where $\mathcal{E}$ = the electric field intensity,
h_r = the effective height of the receiving antenna,
R = the rf resistance of the receiving antenna.

The received current will be larger, the greater the effective height of the antenna and the lower its resistance. Thus the conditions which are desirable for an efficient receiving antenna are the same as those which lead to an efficient transmitting antenna. The resistance of a receiving antenna should be made as low as possible; in fact, an ideal receiving antenna would have a resistance no greater than its radiation resistance. For transmission, an antenna with a large capacity is required in order that the voltage of the antenna will not be excessive. This is not a requirement for a receiving antenna. The capacity of the top should, however, be sufficient to keep the current distribution in the vertical portion very nearly uniform. In this case the effective height becomes more nearly equal to the actual height of the antenna. For receiving purposes, therefore, the length of the antenna is not of much importance unless a certain antenna capacity is required in order to make the antenna fit the rest of the receiving equipment.

The received current in the case of a loop antenna depends upon similar factors. The received current can be obtained from the expression for the effective height given in Part 6, and is given by

$$I_r = \frac{\mathcal{E} 2\pi S n}{R\lambda}$$

where S = the area of one turn,
n = the number of turns.

The received signal increases with increase in the area-turns and decrease in resistance and wave length. For a given wave length and area-turns the minimum resistance is obtained with a single-turn loop, since the minimum length of conductor is required. Such a loop is, however, very much larger and less convenient in many cases than the multiple-turn loop.

CHAPTER II. SINGLE CIRCUIT RECEPTION.

When the collector circuit, whether antenna or loop, is tuned to resonance for a particular wave length, the received current for signals of that particular wave length will be a maximum. The circuit will have a zero reactance at this wave length and will be characterized by resistance alone. For other wave lengths the reactance will be large and the response to signals on these waves will be very much reduced relative to those on the wave length to which the circuit is tuned. Thus, the tuning of the antenna or loop circuit results in selectivity.

The antenna, when operated at wave lengths longer than its fundamental, is a capacitive reactance and is tuned by the insertion of an

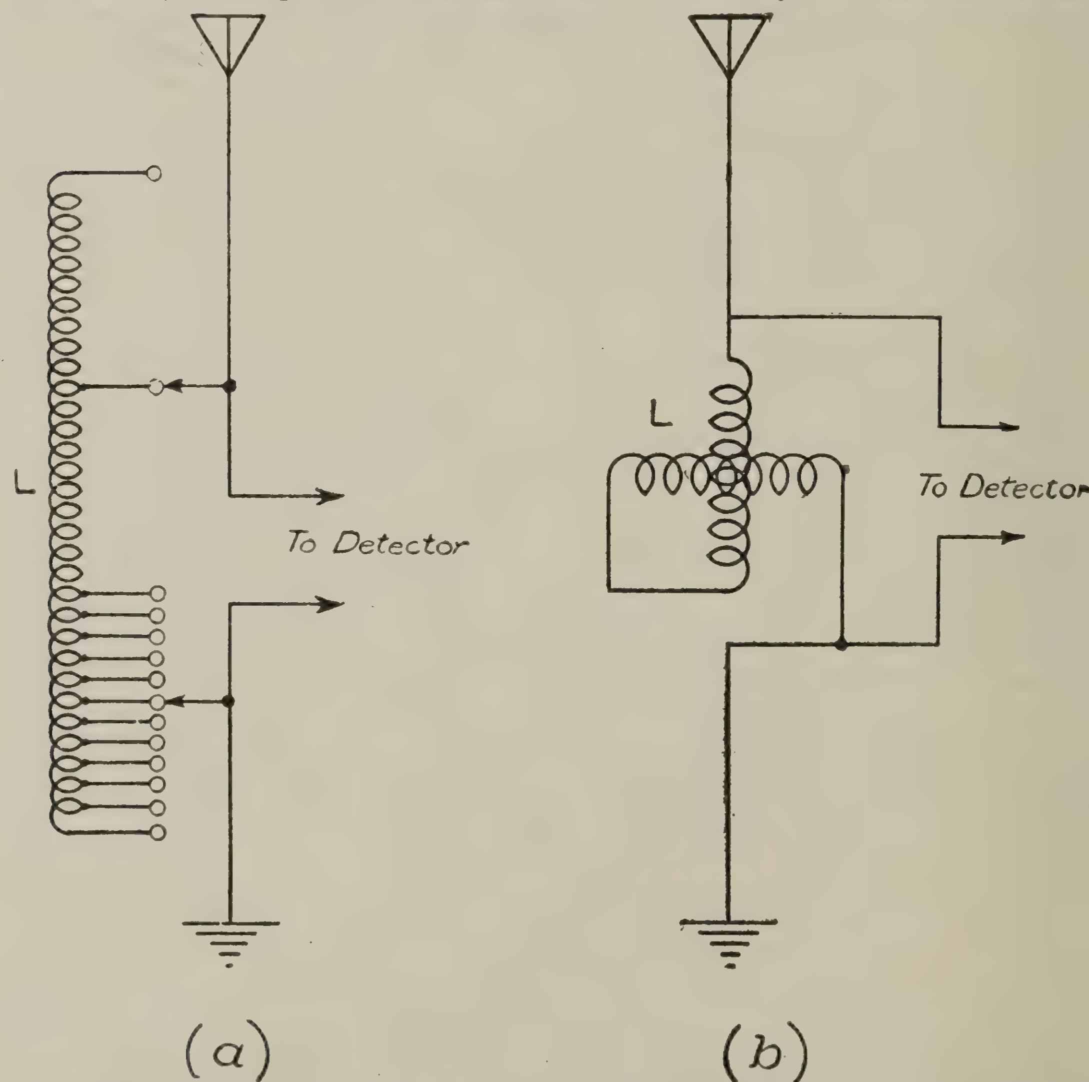


Fig. 345.—Tuning of Antenna Circuit to Resonance by Inductance Alone: (a) Tapped Inductance Method; (b) Variometer Method.

inductive reactance. Figure 345 (*a*) and (*b*) show two simple methods of tuning the antenna by inductance alone. In (*a*) the tuning is ac-

complished by varying the portion of tapped inductance L which is included in the circuit antenna to ground. The coil is tapped every turn for ten turns, and the remainder of the coil is tapped every ten turns. Thus, the inductance can be varied in large steps and a fine adjustment obtained by single-turn variations.

In figure 345 (*b*) the inductance is of the variometer type, permitting a continuous variation throughout the range of the instrument.

The method shown in (*a*) has the disadvantage that unused portions of the inductance are in the field of the used portion and can absorb energy, in particular, at or near wave lengths corresponding to natural periods of oscillation of the coil.

The variometer method is not very satisfactory at low settings for there, while the inductance is small, all of the wire is still in the circuit and the resistance is relatively large.

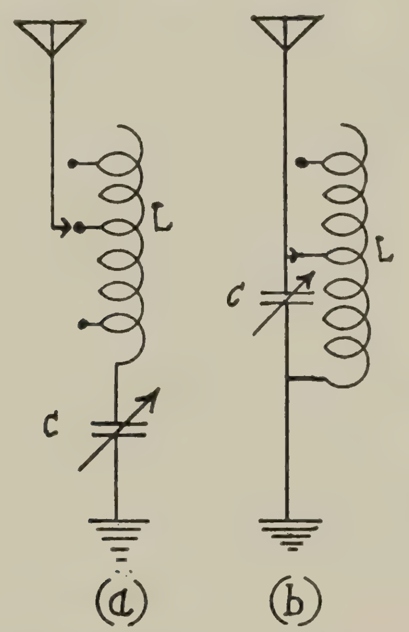


FIG. 346.—Tuning of Antenna Circuit to Resonance by Inductance and (a) Series Capacity, and (b) Parallel Capacity.

Other methods of tuning the antenna circuit are represented in figure 346 (*a*) and (*b*). These circuits utilize tapped inductances, but the taps are spaced a considerable distance apart and the fine variation is obtained by a variable condenser. In figure 346 (*a*) is the circuit which is used more generally, perhaps, than any other. The condenser C is here in series in the antenna circuit and reduces the total capacity. The lower its value, the more the total capacity is reduced and, hence, the shorter the wave length to which the antenna is tuned. In the case of two condensers in series, the resultant capacity is somewhat smaller than the smaller of the two condensers. Thus, when C is large the total capacity is somewhat less than the capacity of the antenna, while if C is very small, the total capacity is somewhat less than the value of C. It is evident that a considerable range of wave length can be

covered with a single value of inductance, using this circuit. It is also possible to receive at wave lengths shorter than the fundamental of the antenna.

The circuit of figure 346 (*b*) makes use of a condenser in parallel with the inductance and is equivalent to a capacity in parallel with the antenna. With this circuit, the total capacity cannot be less than that of the antenna, and is increased as C is increased. There is a sacrifice in signals when C is very large. The signal emf is impressed in the antenna, and relative to this emf the condenser and coil are in parallel. The resistance of this parallel combination becomes high when C is large. Usually the capacity of C is kept below half the antenna capacity.

The loop is an inductive reactance and is tuned by a variable condenser across the terminals of the loop. Resonance in this case is likewise series resonance. The impressed emf from the wave must be considered to be in series in the circuit, and the resonant voltage across the condenser or loop can be many times larger than the impressed emf.

Connection of the detector. The function of the detector circuit is to convert radio-frequency power into audio-frequency power which then becomes audible in the telephones. The way the detector performs this function will be considered later; at present, the connection of the detector to the tuned circuit will be considered.

If the detector circuit is connected across the whole of the inductance in the antenna circuit as indicated in figure 345 (*a*) and (*b*), the voltage applied to the detector circuit will be equal to the resonant voltage across this coil. Assuming the inductance of the antenna to be negligible, this will be the maximum voltage in the antenna circuit. If the detector circuit is extremely high in resistance for radio-frequency currents, the antenna current and the voltage across the coil will be the same after the detector is connected across the coil as it was before. The detector will take an inappreciable current and, hence, no power. Such a condition can be realized with a vacuum-tube detector. If the detector resistance is not extremely high, making the connection of the detector across the coil will reduce the antenna current and hence the resonant voltage on the detector circuit. The detector will in this case consume power. It may be that connecting the detector across the whole of the coil will actually reduce the antenna current and the voltage applied to the detector to such an extent that the power taken by the detector will be less than that which would be taken were the detector connected across only a portion of the coil. The response of the detector will then be greater if it is connected across a portion of the coil.

The detector circuit may be considered to increase the resistance of the antenna circuit by an amount R'. The total antenna resistance will be $R+R'$. The antenna current with the detector not connected

will be $I_1 = \frac{E}{R}$ where E is the voltage impressed upon the antenna by the wave. When the detector is connected the current will be $I_2 = \frac{E}{R+R'}$ which is less than I_1. The power dissipated in R will be I_2^2R, while the power taken by the detector will be I_2^2R'. As R' is increased from zero, the power I_2^2R' will increase, at first, despite the decrease in I_2. When $R = R'$ the power I_2^2R' will be a maximum, and a further increase in R' will reduce the power taken by the detector. The best signal will therefore be obtained when the detector takes half of the power. The value of R' will be a maximum when the detector is connected across the whole coil. If this does not exceed R, the best signal will be obtained; otherwise the detector should be connected across a portion of the coil, or less strongly coupled with the antenna circuit. The lower the resistance of the detector circuit within the usual practical limits, the higher will be the value of R'. Therefore, a low-resistance detector circuit should be connected across a smaller portion of the coil than would be the case for a high-resistance detector circuit. Detector circuits employing crystal detectors are usually low in resistance as compared to vacuum-tube detector circuits.

Besides the disadvantage of reduced signal strength when the detector circuit is too strongly coupled with the antenna, there is also the disadvantage of poor selectivity under these conditions. The higher the resistance of a tuned circuit, the lower in proportion is the current corresponding to an emf of the resonant frequency, while the currents produced by emfs of other frequencies are very little affected. Hence, for attaining selectivity it is advisable to reduce the coupling of the detector even below the point of strongest signal, as a decided increase in selectivity is thereby obtained with only a slight reduction in signal intensity.

CHAPTER III. COUPLED CIRCUIT RECEPTION.

In general, a much higher selectivity is required than is afforded by the single circuit. The usual antenna circuit is not a low-resistance circuit and in addition, its resistance is generally increased by the detector. **Coupled circuits** permit a very much higher selectivity to be obtained. The form of coupled circuit most generally used is that of figure 347. The antenna circuit or **primary circuit** is tuned but does not include the detector. Another tuned circuit, called the **secondary**, is

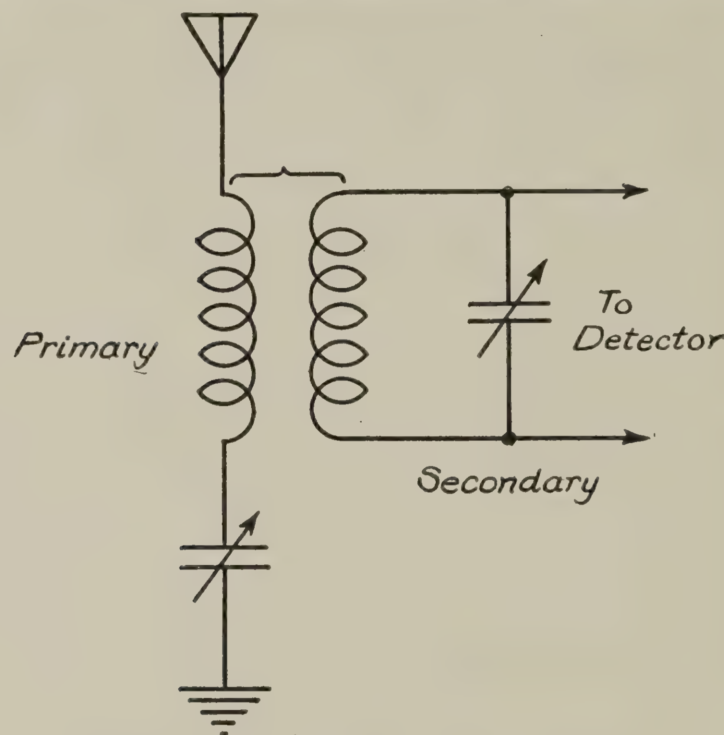


FIG. 347.—The Coupled Circuit for Receivers.

coupled to the antenna circuit by means of mutual iudnctance between coils in the primary and secondary circuits. The greater the mutual inductance between the coils, the stronger the coupling between the two circuits. When the mutual inductance is small, the coupling is **loose**; when the mutual is large the coupling is **tight.** By reason of the coupling between the circuits, a current in the primary induces an emf and current in the secondary. Suppose both circuits to be tuned to the wave length of the desired signal and loosely coupled with each other. Because of the tuning of the primary circuit, the current in that circuit for the signal frequency will be relatively much larger than currents produced by interfering signals. The emf induced in the secondary circuits will also be relatively greater. Similarly, the emf of signal frequency induced in the secondary will produce relatively larger currents and, hence, a larger resonant voltage across the secondary tuning

condenser than would be produced by interfering signals. The response of the detector, which is usually connected across the secondary condenser, will, therefore, be very great for the signal emf relative to the response for interfering emfs. Still higher selectivity can be obtained if three tuned circuits are used, each coupled to the one ahead. Because this latter arrangement is equivalent to the usual two-circuit arrangement with another circuit interposed between, it is frequently referred to as the **intermediate circuit.**

Optimum coupling. The secondary circuit takes power from the primary circuit just as the detector circuit takes power in the previous discussion. The power taken by the secondary circuit is the equivalent of the insertion of a resistance in the primary. The closer the coupling between the circuits, the larger the fraction of the power which is taken by the secondary. However, as the coupling is increased and the resistance of the primary is increased thereby, the power taken by the primary circuit from the signal wave is reduced; in fact, when the coupling is increased beyond a certain point, the received power in the primary circuit is reduced so rapidly that even the increased percentage of the power taken by the secondary circuit will amount to less than that or looser coupling. Again, the maximum power is obtained in the secondary circuit when it takes half of the power from the primary or when the increase in resistance of the primary due to coupling is equal to the resistance of the primary by itself. Very much higher selectivity is obtained, however, with slight reduction in signal strength, if a coupling somewhat looser than the optimum is used.

Method of tuning coupled circuit receivers. When the coupling between the primary and secondary circuits is not loose, there will be a great number of combinations of primary and secondary adjustments, which leads to an apparent maximum signal. Thus, for one setting of a primary, a setting of the secondary will be found which gives a maximum response in the phones, but for a different primary setting; likewise, a different setting of the secondary will give the maximum response. It is then very difficult to tell whether the optimum response is being attained. The following method of tuning gives uniform results as to signal intensity, good selectivity and also very nearly the maximum signal.

Having picked up the desired signal, the coupling is reduced to the minimum at which the signal can still be heard. Both primary and secondary circuits are then tuned for maximum response. The coupling is then gradually increased until the point of maximum response is found and then backed off to a slightly lower value. The secondary circuit is then again tuned for the best signal without changing the primary tuning.

CHAPTER IV. DETECTOR CIRCUIT.

A diagram of a complete radio receiving circuit utilizing a crystal detector is shown in figure 348. It consists of the primary, secondary and detector circuit, which latter is usually connected as shown across

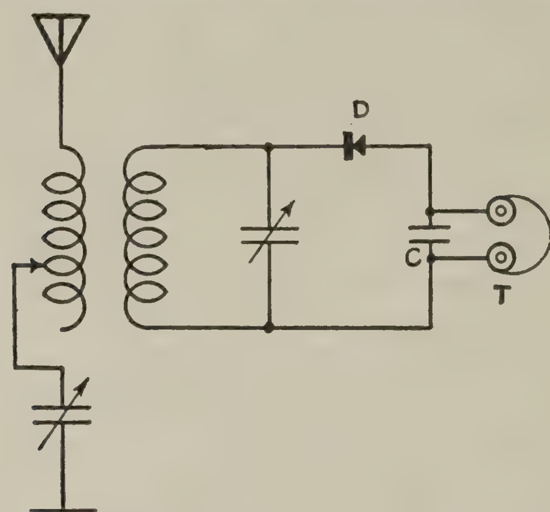


FIG. 348.—Inductively Coupled Receiver with Crystal Detector.

the tuning condenser of the secondary circuit. The detector circuit. as shown in the figure, consists of a crystal detector D in series with the parallel combination of telephones T and bypass condenser C.

When a vacuum-tube detector is used the circuit is usually that of figure 349 (a) or (b). In (a) connection is made from one side of the secondary condenser through a parallel combination of so-called grid condenser and grid leak to the grid of the vacuum tube. The other side of the condenser is connected to the positive terminal of the filament. The coil L in the plate circuit of the tube is called the **tickler** or **feed-back** coil, and is coupled to the coil in the secondary circuit. The plate circuit also includes the telephone T with a parallel condenser C and the plate battery B. The battery A serves to heat the filament. The operation of the tube as a detector with grid leak and grid condenser, and also the action of the tickler coil, is explained in Section I, Part 7.

Figure (b) shows a less usual method of connection of the vacuum-tube detector, which does not utilize the grid condenser and leak but does include some means, such as the battery C, of rendering the grid of the tube negative with respect to the negative terminal of the filament. This negative voltage is referred to as a negative bias, and this mode of connection is sometimes referred to as the **detector with grid bias.** The mode of operation of the vacuum tube with this connection is also treated in Section I, Part 7.

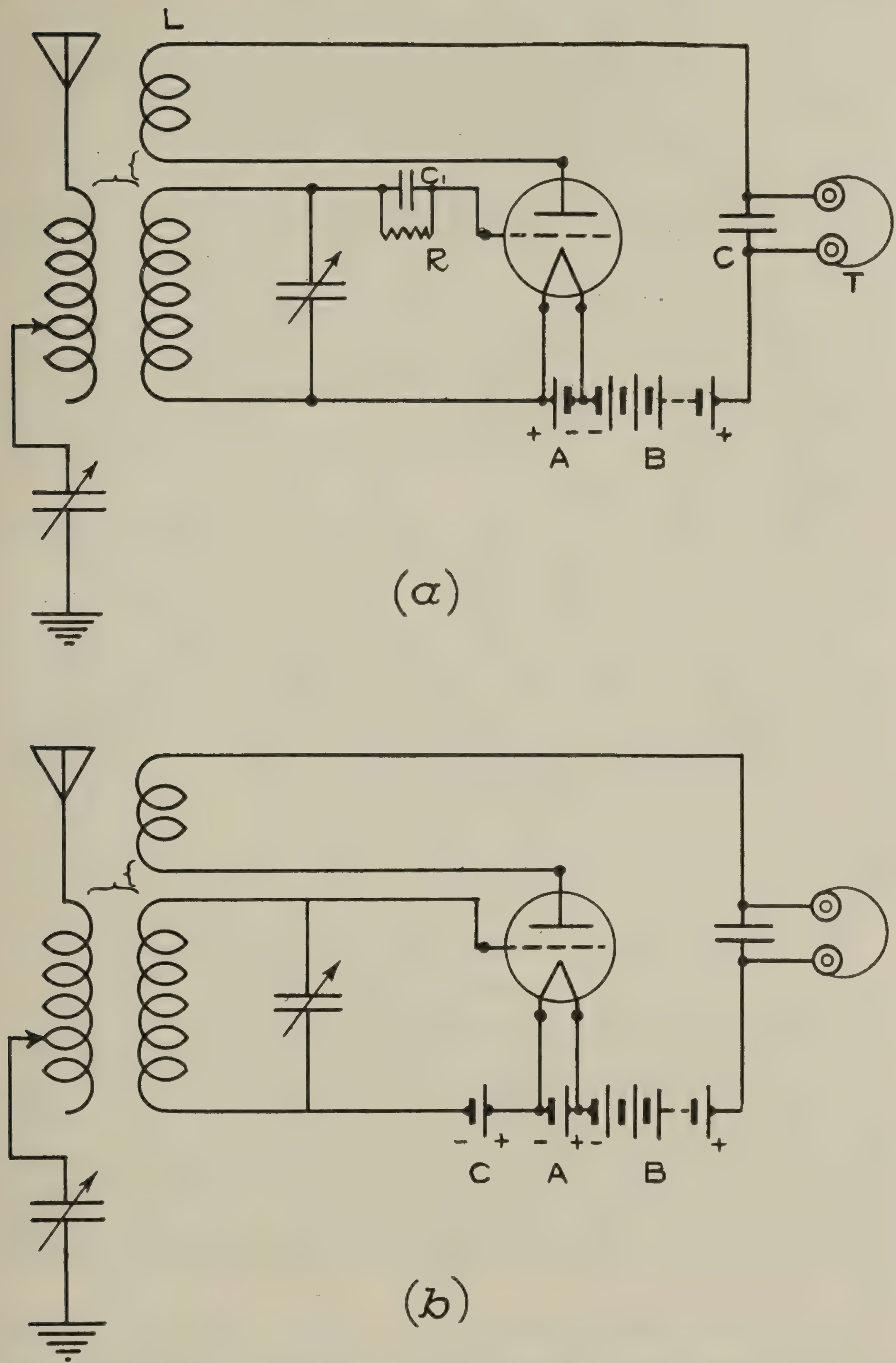


FIG. 349.—Inductively Coupled Receiver with Vacuum-Tube Detector Arranged for Regeneration and Oscillation with: (a) Grid Condenser and Grid Leak, (b) Grid Bias.

CHAPTER V. AMPLIFIERS, DRIVERS AND AUDIO TUNING.

A. THE AUDIO-FREQUENCY AMPLIFIER.

General. The audio-frequency amplifier is, as its name implies, an instrument for increasing the amplitude of the audio-frequency pulses, and may be considered as a telephone amplifier (highly sensitive telephone). This type of amplifier can be used on any wave length for spark, *ICW*, radio-telephone and heterodyned *CW* signals. The audio-frequency amplifier is not a detector, but will **amplify** the current passed on to it from the detector. The detector may be either the well-known crystal or a vacuum tube. (The vacuum tube is now in general use as a detector).

The first stage. The output of the detector (telephone terminals on receiver) passes through the primary winding of a step-up transformer in the amplifier, the secondary terminals of which are connected to the filament and grid of the first tube of the amplifier. The direct current pulses from the detector, passing through the primary winding of the transformer, induce voltage variations in the secondary winding. These voltage variations are impressed between the grid and filament of the first tube, and produce corresponding current variations in the plate circuit of the first tube. These current variations are many times greater than those in the plate circuit of the detector tube. There is included in the plate circuit of the first tube the primary of another step-up transformer.

The second stage. The amplified variations of current passing through the primary of the second transformer induce voltages in the secondary of the second transformer, which voltage is similarly impressed in the grid circuit (filament-grid) of the second tube. The amplified variations in current in the plate circuit of the second tube pass through a pair of telephone receivers, and a much louder signal is heard than when the telephones are plugged in either the detector circuit or after one stage (step) of amplification. The average two-step audio-frequency amplifier will amplify the signal passed on to it by the detector about four hundredfold (amplification per stage equals 20).

The vacuum-tube detector is not a part of the audio-frequency amplifier, but is either included in the receiver or is a separate unit. Connections are made between the **telephone** terminals of the detector and the **input** terminals of the amplifier.

A three-stage audio-frequency amplifier is, in general, of little use because it is extremely sensitive to induction and vibration (only the first two stages are used in practice).

Description. A diagram of a **two-step audio-frequency amplifier** is shown in figure 350. The complete amplifier is a unit by itself in the receiving system and is equipped with three sets of terminals, two terminals to a set. The **input** terminals connect the amplifier to the receiver, while the **filament battery** and **plate battery** terminals are used for connecting the filament and plate circuit batteries to the apparatus. The filament current for each tube can be adjusted to the

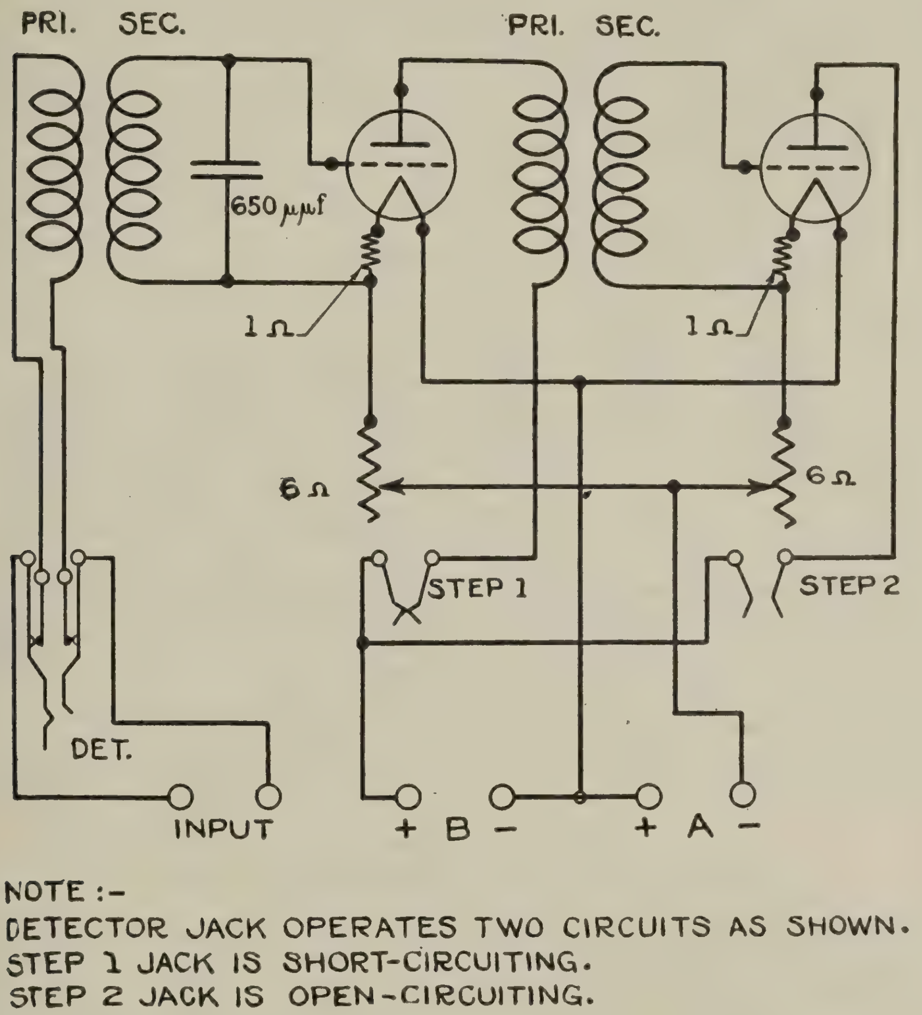


Fig. 350.—Diagram of a Two-step Audio-Frequency Amplifier.

proper value by means of a rheostat. The grid voltage is adjusted to the proper value by means of a grid bias resistance of one ohm.

Three **telephone jacks** are supplied, namely:

(*a*) Detector,

(*b*) First step,

(*c*) Second step.

When the telephones are plugged to the detector jack, the amplifier is cut out of circuit and the telephones are in the detector circuit of the receiver. When the telephones are plugged to the **step one** jack, the detector and the first stage of the amplifier can be used, while the second step remains open. When the telephones are plugged to the

step two jack, the first step jack short-circuits itself and the detector and the entire amplifier are in circuit.

B. THE RADIO-AUDIO-FREQUENCY AMPLIFIER.

The radio-audio-frequency amplifier, figure 351, permits a very much higher degree of amplification to be obtained than is possible with the audio-frequency amplifier alone. Its development has made possible the use of extremely small collectors, such as the short, single wire antenna and the radio-compass coil, without any sacrifice being made in the intensity of the received signal. The maximum amplificaton that can be obtained with the radio-audio-frequency amplifier is, in reality, so great that it is not suitable for use on an antenna of the ordinary size. This is mainly for the reason that the large antenna

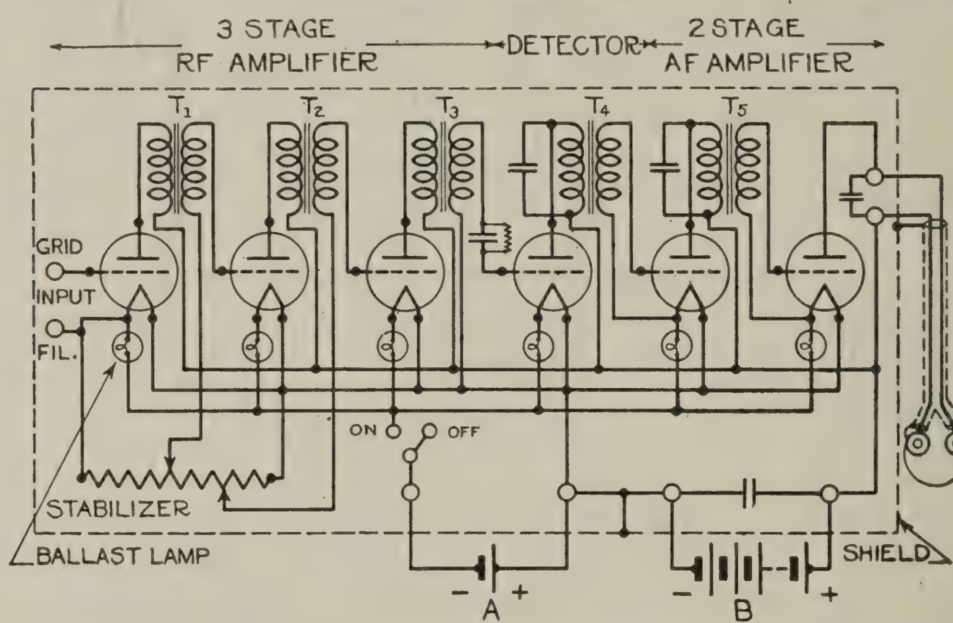


FIG. 351.—Radio-Audio-Frequency Amplifier.

collects so much static that the action of the amplifier is paralyzed. In the practical application of this type of amplifier, the apparatus is so designed and constructed that the usual antenna can be satisfactorily used as the collector.

The addition of **radio-frequency stages** to the audio-frequency amplifier is very advantageous. Such an amplifier is less noisy, because noises due to the tubes and induction from neighboring circuits are not amplified by the radio-frequency stages in the same proportion as the signal. In heterodyne reception of continuous-wave signals, a decided advantage is gained on all wave lengths. There is always a large amplification obtained on all wave lengths in the reception of spark signals with straight detection; in fact, the more radio-frequency stages used for spark reception the better will be the result.

The **detector** follows the square law; consequently, the greater the emf applied to the detector the more efficient it becomes. For this

reason, it is advisable to amplify before detection. An amplification of 10 times before detection will give 100 times the response in the telephones. For the same reason, a radio-frequency amplification of only 5 times is equal to an audio-frequency amplification of 25 times. The square law action of the detector is shown more clearly by the following:

Suppose a transmitting station is transmitting with 10 amperes in the antenna and that the signal received (detector only) at a given receiving station has an audibility of 10. Now, if the current in the transmitting antenna is reduced to one-tenth of its former value—1 ampere—the audibility of the received signal will be reduced to one one-hundredth of its former value—one-tenth of unit audibility. If two stages of audio-frequency amplification are added to the detector and if each stage amplifies 20 times (total amplification—$20 \times 20 = 400$), then the resulting audibility would be 40 and the signal would be heard. On the other hand, if two stages of radio-frequency amplification preceded the detector and each stage amplified five times, the total amplification would be 25 and the audibility of the received signal would be 62.5 after detection, and 10,000+ if two stages of audio-frequency amplification are used in addition.

C. RADIO-FREQUENCY DRIVERS.

General. The radio-frequency driver or heterodyne, figure 352, is a low-power generator of continuous waves (sustained oscillations). Its purpose is to supply local oscillations for modulating incoming continuous-wave signals and thereby render them audible by means of the well-known **beat principle.** This instrument is necessary when receiving continuous-wave signals, except when the detector is used in the oscillating condition (autodyne reception).

The radio-frequency drivers were developed primarily for use with the radio-audio-frequency amplifiers in which the detector tube is used in a nonoscillating condition, but can be used with any vacuum-tube receiving equipment in place of autodyne reception.

The advantages of using the radio-frequency driver instead of a vacuum-tube detector in the oscillating condition are:

(*a*) Gives more freedom from interference because detuning of secondary circuit is not necessary.

(*b*) Permits a very flexible adjustment of the strength of the local oscillations.

(*c*) Reduces interference between receivers.

Its main **disadvantages** are:

(*a*) Requires the tuning of an additional circuit, thereby complicating pick-up work.

(*b*) Makes determination of resonance between primary and secondary circuits somewhat more difficult.

Theory. Principle of beat reception. Incoming continuous waves occur at a radio-frequency rate and are, therefore, inaudible even after detection, because the average plate current is changed only at the beginning and end of the dots and dashes. If, however, local radio-frequency oscillations differing from the frequency of those being received by a small amount are impressed on the detector circuit, the two sets of radio-frequency oscillations will, on account of their difference in frequency, swing periodically in and out of phase. The amplitude of the resulting frequency at any given instant will be the sum (algebraic) of the amplitudes of the two radio-frequencies. As a consequence, a slow audio-frequency oscillation will result which can be heard in the telephones. The resultant audio frequency can be adjusted at will, the normal beat frequency employed being in the neighborhood of 1,000.

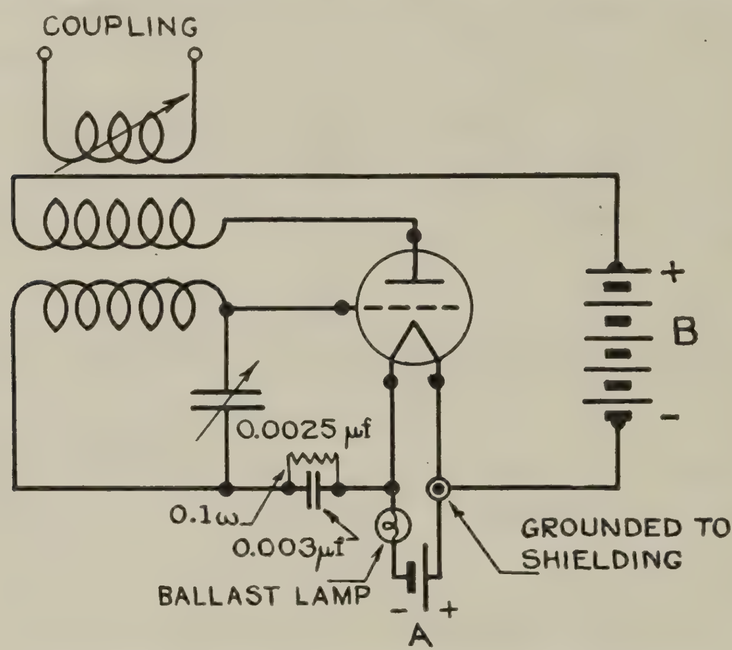


Fig. 352.—Radio-Frequency Driver or Heterodyne.

The note will be clear and steady as long as the frequency of both the incoming signal and the radio-frequency driver remain constant. Warbling and bubbling in the signal note is due, ordinarily, to a rapid change in the frequency of the transmitter (frequently the case with the arc on very short wave lengths, and with the vacuum tube transmitter due to swinging of the antenna or changing capacity effects). A slow and gradual change in the received note is obtained when the received signals are transmitted by a high-frequency alternator having poor speed control, and also when the filament or plate battery of the radio-frequency driver is nearing exhaustion.

D. THE TUNED TELEPHONE.

General. The tuned-telephone circuit provides a means for tuning at audio-frequencies, and should not be confused with the ordinary

radio-frequency, tuning provided by coupled circuits and the like. Its purpose is to prevent undesired signals, which vary in audio-frequency from that of the desired signal, from passing through the telephones. On this account, it is particularly adapted to the reception of continuous-wave signals by the beat method. It is of less value for the reception of spark signals, for the reason that the frequencies of practically all Navy spark transmitters lie within the 450-550 cycle zone. Further, the tuned telephone does not effectively separate signals whose frequencies vary only by a few cycles, because the tuning is quite broad on account of the resistance, etc., of the circuit.

Theory. The tuned telephone is a selector circuit in that it is actually tuned to resonance with the frequency desired. In the resonant condition it has no reactance, and the only resistance is that due to the circulating resistance; therefore, any signal having the same audio frequency as that to which the tuned-telephone circuit is resonant will be heard, and any other signals having an audio-frequency differing from that to which the circuit is resonant will be diminished and probably not be heard.

For example, suppose that it is desired to receive a station transmitting on 10,000 meters, frequency 30,000 cycles, with a beat note of 1,000. The secondary of the receiver would be tuned to either 29,000 cycles or 31,000 cycles. Assume that the secondary circuit is tuned to 29,000 cycles, and that another station transmitting on a wave length of 11,000 meters is interfering, with a signal of approximately the same intensity as that to be received. The signals from the interfering station beating with the local oscillations will result in the signal having a note of 1,728 cycles, which is within the audible range and would be disturbing. Now, if the tuned telephone is cut into circuit and adjusted to be resonant to 1,000 cycles, the interfering signal will either disappear, or be greatly reduced in intensity. The tuned telephone, therefore, gives selection in wave length by audio-frequency tuning.

The intensity of static can also be greatly reduced if the tuned telephone is used, and the note of the received signal increased to a frequency of approximately 1,500 cycles, or more.

Description. A simplified drawing of the tuned-telephone circuit is shown in figure 353. The tuned-telephone circuit consists of three parts:

(*a*) Telephone,

(*b*) Condenser,

(*c*) A large inductance.

This circuit is connected between the negative terminal of the filament and the plate of the receiving vacuum tube.

The telephones are changed from their usual position, in series with the plate battery, to a series connection with the tuned telephone circuit, and at the same time a high resistance is substituted in the first circuit in place of the telephones, to prevent the value of the plate

current from being changed due to the transfer of the telephones from that circuit.

The condenser is of the continuously variable, air-dielectric type, with a maximum capacity of 0.0025 μf.

The inductance consists of 22,000 turns of No. 34 B&S, DSC copper wire wound on an iron core. The frequency of the tuned telephone circuit can be adjusted, by means of the condenser, to practically any audio frequency.

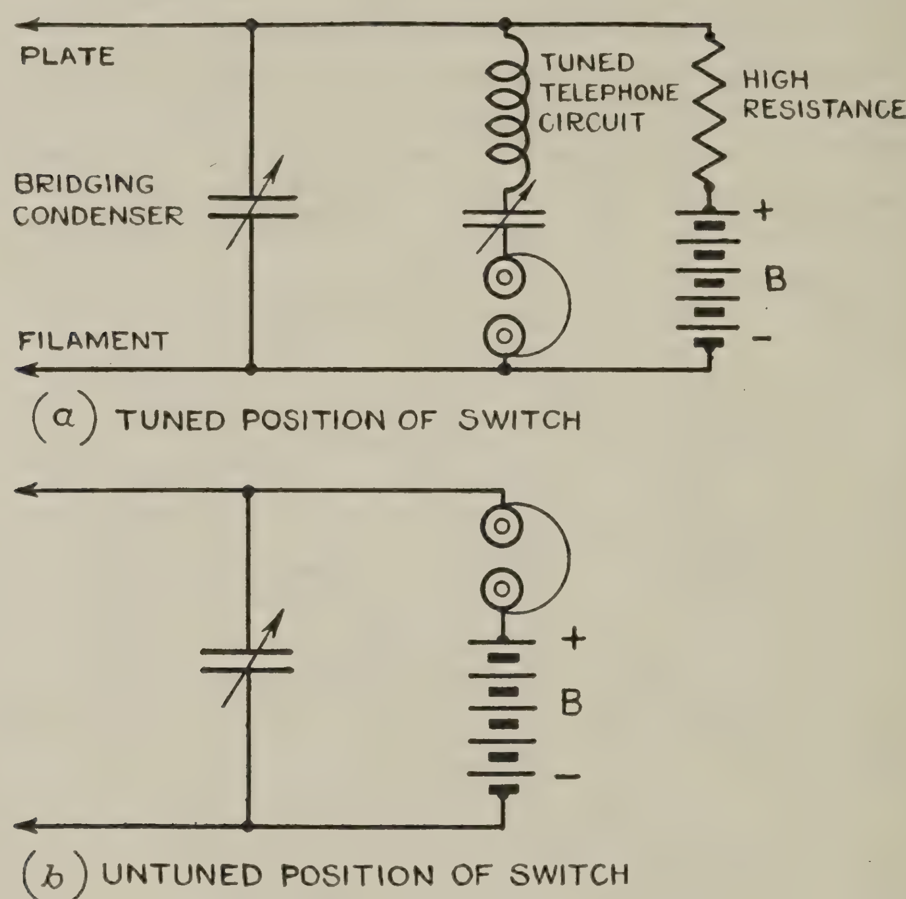


Fig. 353.—The Tuned-Telephone Circuit.

A transfer switch is provided for cutting the tuned telephone out of circuit. When the switch is thrown to the tuned position, the tuned telephone is cut into circuit; and when it is thrown to the untuned position, the tuned telephone is cut out of circuit and the regular receiving arrangement is used.

Operation. The tuned telephone should be used when interfering signals make the reading of the desired signal uncertain, and also when static is troublesome. **The tuned telephone should not be used in stand-by or pick-up work, for the reason that every change in the wave length of the secondary circuit, while receiving CW signals, will change the audio frequency of the note, thereby throwing the tuned telephone out of resonance and thus preventing the signal from being heard.**

The following procedure should be followed when using the tuned telephone:

Throw the switch to tuned position.

Vary the setting of the tuned-telephone condenser slightly until undesired signals are lost and the desired signal is brought to maximum intensity.

It should be remembered that the radio-frequency tuning is to be performed in the usual manner, and that the tuned telephone is solely an audio-frequency tuning device. For this reason, select the beat note by varying the secondary circuit condenser before changing over to the tuned telephone.

CHAPTER VI. PRACTICAL RECEIVER CIRCUITS.

The receivers in use today employ in their fundamentals the elementary circuits described in Chapters I to V inclusive of this Part 2 of Section II of the MANUAL. It will be noted that the receiver circuit when analyzed contain in their elements combinations and adaptions of the fundamental principles that have been described.

The principal receiver circuits in use today are:

1. Single circuit crystal.
2. Coupled circuit crystal.
3. Heterodyne.
4. Autodyne and regenerative.
5. Super regenerative.
6. Super heterodyne.
7. Tuned and untuned radio frequency amplification.
8. Reflex amplification.

The single and coupled circuit crystal receivers are used in most cases because of their low cost. While they are good receivers for radio telephone reception at short distances and when there is no interference, they cannot be used for receiving continuous waves nor are they preferred for any purpose when vacuum tube receivers can be obtained. The crystal circuits have been described previously and will not be repeated here.

The heterodyne receiver. The principle involved in this type of receiver is (as suggested by the name) that of detecting the beats between the frequency of an incoming signal and that of a locally generated alternating current. Theoretically, this local source may be of any type whatever. In practice it is nowadays, however, nearly universally a vacuum tube generator, the circuits of which are coupled to the tuned input circuit of the detector tube. The beat frequency is adjusted to lie within the audible range by variation of the frequency of the local generator, and is usually made to be about 1 kc as that is the frequency to which the response of the ear is most sensitive. Figure 354 shows a simple way in which the apparatus may be connected up.

Analysis of the resultant current in the plate circuit shows that the amplitude of what may be conveniently called the detector current is proportional to the product of the amplitudes of the signal and the locally generated voltages. (This is true within limits only; in fact, in practical operation, the problem resolves into means for decreasing the amplitude of the local oscillating potential impressed on the detector grid to the point of maximum response). Maximum signal intensity is the main advantage claimed for this type of CW

reception since the detector tube input circuit may be tuned to **exact resonance** with the incoming signal, and the strength of the impressed voltage on the detector grid from the local oscillation source may be readily adjusted down to the optimum value by a simple variation in coupling without effect on the stability of the oscillator. This

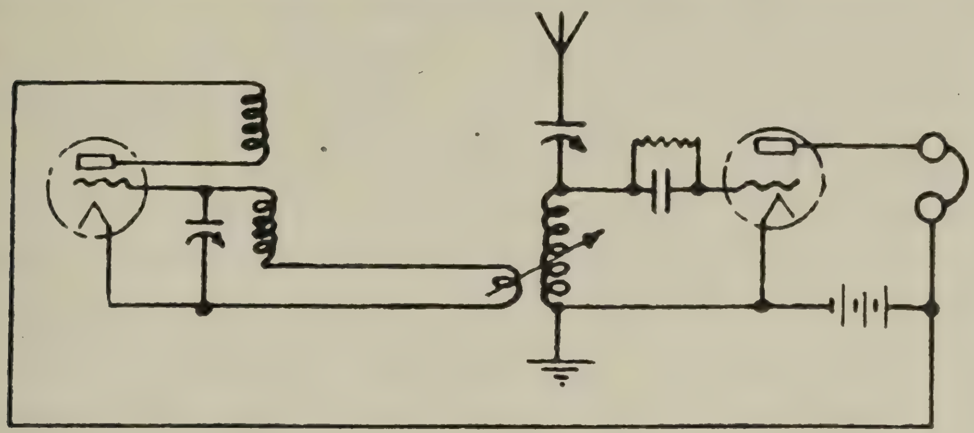


FIG. 354.

system has been employed in some Navy receivers, (plus radio and audio amplifying stages of course). Its principle disadvantage lies in its requirement of an additional vacuum tube with associated circuits and tuning controls. It is also believed to be inferior electrically to the autodyne system since its selectivity is poor due to the inherent resistance of the tuned detector circuit, and no amplifying means are provided for the case of reception of damped and modulated waves.

The autodyne and regenerative receiver. The principle involved in the autodyne type of receiver is identical with that just described. The only difference is that instead of using a separate oscillator tube for the generation of the local oscillations the detector tube with its tuned input circuit is made to act as the local generator as well. A simplified sketch of the necessary connections is shown in Figure 355.

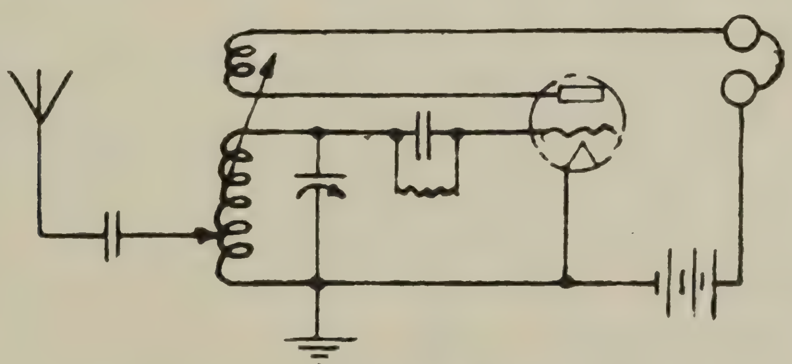


FIG. 355.

There is, unfortunately, some confusion in the use of the two terms "autodyne" and "regenerative". Strictly speaking, the term "autodyne" should be confined to the operation when the tube is

oscillating and the term "regenerative" to its operation when, by regulation of the feed-back, the tube is not oscillating but is supplying energy to replace some of that absorbed by the circuit losses. In the former case the set is adapted for receiving continuous waves (the grid input circuit being of necessity detuned slightly from the incoming signal for the production of audible beats); while in the latter it may be used for the reception of modulated waves when, if the feed-back is adjusted just below the point of oscillation, maximum response is obtained. For both cases, the feedback action of the tube supplies the losses of the detector circuit, a condition which sharpens its tuning to a great degree and is believed to more than make up for the loss in response claimed for the off resonance adjustment required in CW reception. Radiation of the local oscillating energy by the antenna, in the case of the simple circuits shown, will be somewhat stronger with this system (in the strictly autodyne sense) than with the heterodyne type; but since both systems are generally preceded by one or more stages of radio frequency amplification (and followed by audio amplifiers) means can usually be employed in the former circuits which will eliminate this evil for both types. Both systems may be employed universally throughout the full range of radio frequencies now in use. The Navy uses this autodyne regenerative principle in most of its receivers.

The super-regenerative receiver. This receiver was designed to provide a means of increasing the amplification obtainable with a simple regenerative receiver. It is therefore, meant to function primarily with modulated signals. Everyone who has used a regenerative receiver must have observed the very great increase in the detected signal which results just when the feed-back is increased to the point of self-oscillation. The signal of course becomes unreadable or "mushy" at this point. To take advantage of this large increase in the detected signal which occurs at this point and at the same time to avoid the distortion due to the self oscillations is the object aimed at in the super-regenerative idea. This is realized by interrupting the signal frequency oscillations circuit periodically, the period of the interruption being usually at or near the upper range of audibility. This interrupting frequency is most efficiently furnished from a second tube oscillator. Many circuit arrangements have been devised which will accomplish this purpose, one of which is shown in Figure 356.

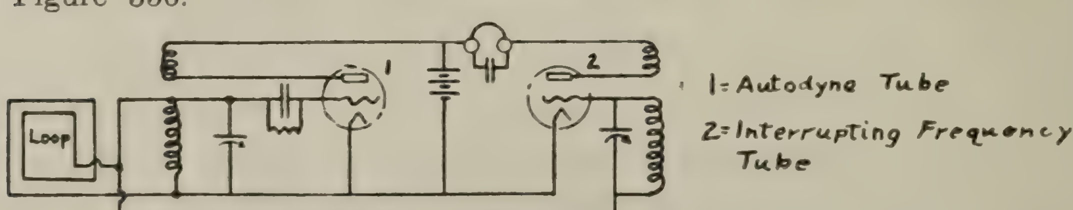


Fig. 356.

The main advantage of this type of receiver (which, it must be remembered, is only useful for modulated signals), is the very great amplification which can be secured. Taking into account the small number of tubes used, this type of receiver will probably give the greatest amplification which has yet been secured. The main disadvantages are: first, that the principle is not applicable to continuous wave reception, and second, that the receiver is very noisy. All of the causes of these noises have not been found as yet and no way of entirely eliminating them is known. Furthermore, the system is highly efficient only when the difference between the signal and interrupting frequencies is great. Its application is therefore limited to reception of the shorter wave-lengths, since at low signal frequencies the period of interruption falls well within the audible range and thus interferes with its own reception.

The super-heterodyne receiver. In the heterodyne or autodyne receivers everything that is picked up by the tuning coils is amplified together with the desired signal. In the super-heterodyne receiver this condition is eliminated or greatly improved by adjusting the frequency of the local oscillator so that its beat frequency with the signal is **above audibility**; this beat frequency passes thru a filter and is then amplified to the desired amount and detected after such amplification. After this second detection the usual stages of audio frequency amplification can be added when desirable. The intermediate beat or transfer frequency which is amplified is in practice anything between 30 and 400 kcs. depending on the range of frequencies to be received. Its high efficiency therefore, is limited to the reception of frequencies well above that of the transfer frequency employed. Figure 357 illustrates the simple principle:

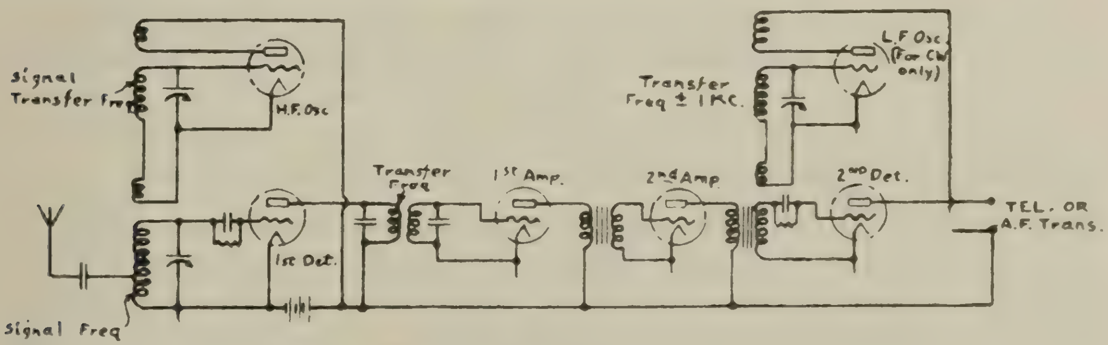


FIG. 357.

This system is probably one of the most satisfactory, at least for broadcast reception, which has yet been developed. For the reception of unmodulated signals, however, a second or heterodyne oscillator must be added and made to beat audibly with the transfer frequency at the second detector. Although extremely selective to ordinary signals, the necessity for these local oscillators with the unavoidable presence of their harmonics forms an unfortunate

limitation to the use of this type of set in such close proximity to transmitting sets as exists in the radio room on shipboard. Energy from the first or high frequency oscillator will also radiate from the antenna unless preceded by suitable balanced circuits, in which case the equipment becomes over-cumbersome and difficult to operate. In a certain commercial broadcast receiver the effect of this radiation has been reduced by operating the oscillator at one-half its required frequency, depending on its second harmonic for production of the desired intermediate frequency beat. Being so widely detuned from the fundamental of the collector circuit, the strength of the actual radiation is thus greatly reduced, and such as does get out is on a frequency outside the band covered by this class of service. This "second harmonic" principle does not constitute any improvement over the straight super-heterodyne system as concerns its utility in the Naval service, where the reception band is essentially universal and harmonic relationships constitute the principal obstruction to reception simultaneous with local transmission.

Tuned and untuned radio frequency amplification. From the fact that it takes an appreciable voltage change to cause any detecting device to operate it is apparent that with some means of bringing the too feeble voltages up to a point where detection can be accomplished is desirable for increasing the range in reception. This is accomplished with the aid of vacuum tube amplifiers, the type of which is named from the method of coupling. The coupling of two stages can be considered the same as the coupling of the two ordinary radio circuits. If two circuits of low loss are loosely coupled with a primary winding of a few turns and a secondary of many, we can expect a step-up in voltage by virtue of the turns ratio step-up characteristic of the transformers as well as from the amplification constant of the tube. With such a stage, amplification would be at a maximum at but one point or frequency and would drop off rapidly on either side, so in order to make a usable stage over a band of frequencies one of two things must be done. First, a variable tuning condenser could be supplied across the secondary or grid circuit to obtain resonance at such frequencies as are within its range. Second, the coupling and losses can be so regulated as to amplify over a band of frequencies with nearly uniform amplitude. In the first case such a stage is called a tuned radio frequency amplifier and is employed in many commercial broadcast receivers including the neutrodyne shown in Figure 358. This type of circuit gives relatively high amplification for the frequency to which it is tuned and assists in giving selectivity to the receiver employed by virtue of the low loss circuits. The utility of such an amplifier is however, limited in application for several reasons. If used for an extended band of frequencies, the inductances would have to be tapped. This has never been accom-

plished successfully due to losses from tapped arrangements and due to feed-back or oscillation difficulties which result from the switching mechanisms required. If employed for lower frequencies than those used in the broadcast band the coils become very large electrically and physically, making it very difficult to prevent reactions from couplings and feed-back. If employed for the higher frequencies little or no amplification will be obtained by virtue of the fact that, the inductance becomes so small that it is impossible to obtain a high L/R ratio. This type of circuit, in order to function perfectly, requires considerable space per stage and if more than two stages are employed, each must be absolutely shielded and perfectly balanced for each tube employed. Each stage requires a variable condenser and although it is possible to construct a mechanical means for tuning several stages simultaneously, such construction can be accomplished satisfactorily only in a plant where every operation is effected with the greatest of precision, or where a plant has a large enough production on one type of equipment to make every employee a specialist.

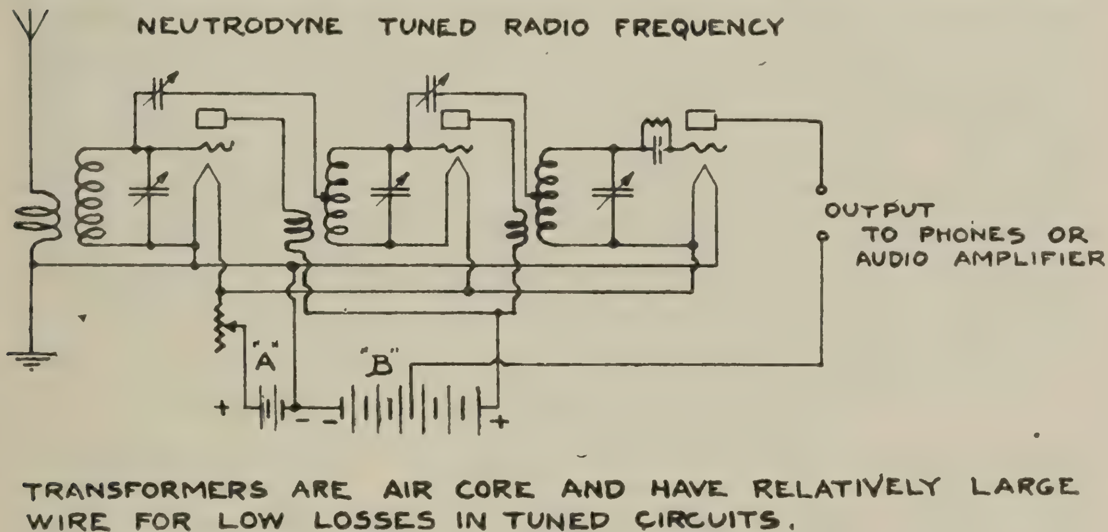


Fig. 358.

It has been further found that in spite of the relatively high selectivity of tuned radio frequency sets that they do not function under the influence of such strong interference as is experienced on shipboard where the transmitting and receiving antennas are in very close proximity. In order to make multiple reception thru this strong interference a possibility, a very loose coupling to the antenna circuit is provided. This calls for radio frequency amplification to make up in the receiver that which has been lost by this loose and inefficient coupling. Service requirements call for maximum amplification in minimum space together with a minimum number of controls and absolute stability. For these reasons the broad band or iron core type of transformer coupled amplifiers are generally employed in the Navy. The amplification per stage which can be expected from this type

is far less than would obtain with tuned radio frequency but in as much as several stages can be placed in a much smaller space than even two tuned stages giving even greater amplification can be maintained with stability and a minimum of controls. As an example the entire range of one of the Navy type receivers is included in one switch setting of the amplifier, whereas if tuned amplification were employed at least 3 coils or taps per stage would be required with tuning facilities for each setting.

Reflex Amplification. This is a scheme developed with a view to cutting down the actual number of vacuum tubes normally required for a receiver incorporating a plurality of stages of any type of radio frequency amplification. The process consists of proceeding thru the desired number of stages of radio frequency amplification; using a vacuum tube for each such stage, detecting in a separate tube, then returning thru the same R. F. tubes for the audio amplifier stages, the windings of the audio frequency transformers being placed in series with the low potential ends of the R. F. transformers, it being assumed that each tube may be caused to do this double duty by reason of the wide discrepancy in the frequencies involved (which automatically limits its adaptability to reception of the higher frequencies). Both windings of each audio transformer must therefore be by-passed for radio frequency by suitable fixed condensers, and these condensers serve to reduce their efficiency and produce audio frequency distortion. The system is shown schematically in Figure 360 for the straight reflex principle:

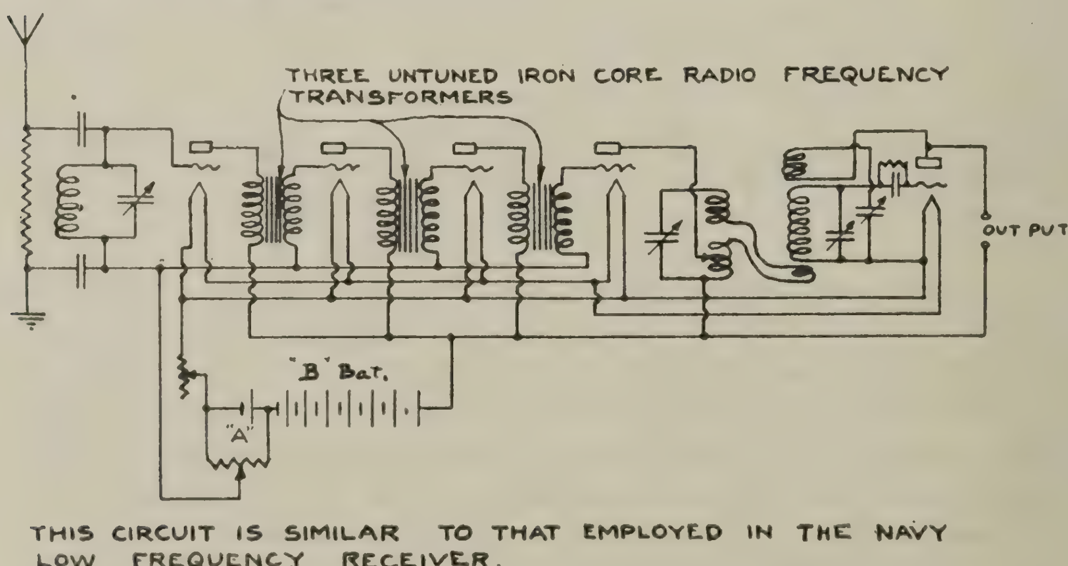


Fig. 359.

In another system, known as the "Inverse Reflex", the progression of the audio stages thru the radio amplifier tubes is in the reverse order to that shown; that is, the 2nd R. F. tube is used for

the 1st A. F. stage, and the 1st R. F. tube for the 2nd A. F. stage, thus tending to equalize the load on the various grids. Since but two stages

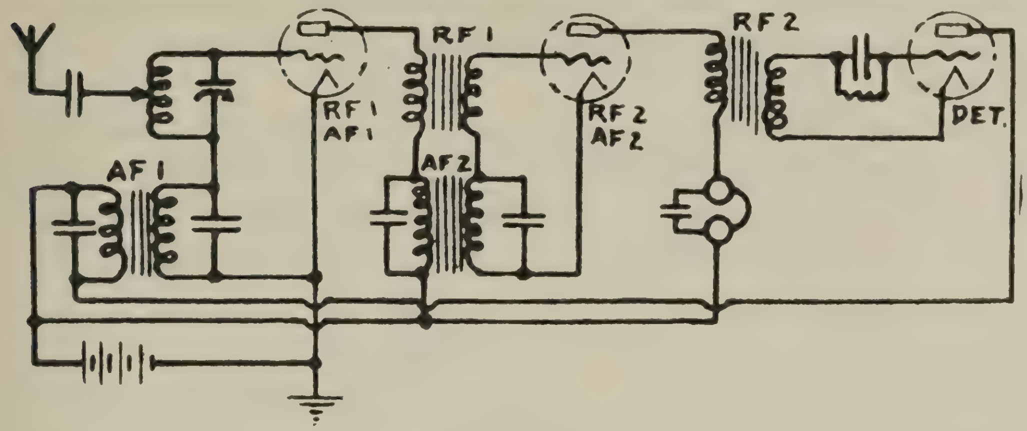


FIG. 360.

of audio frequency amplification constitute the maximum used in ordinary practice, this is the greatest number of tubes that may be

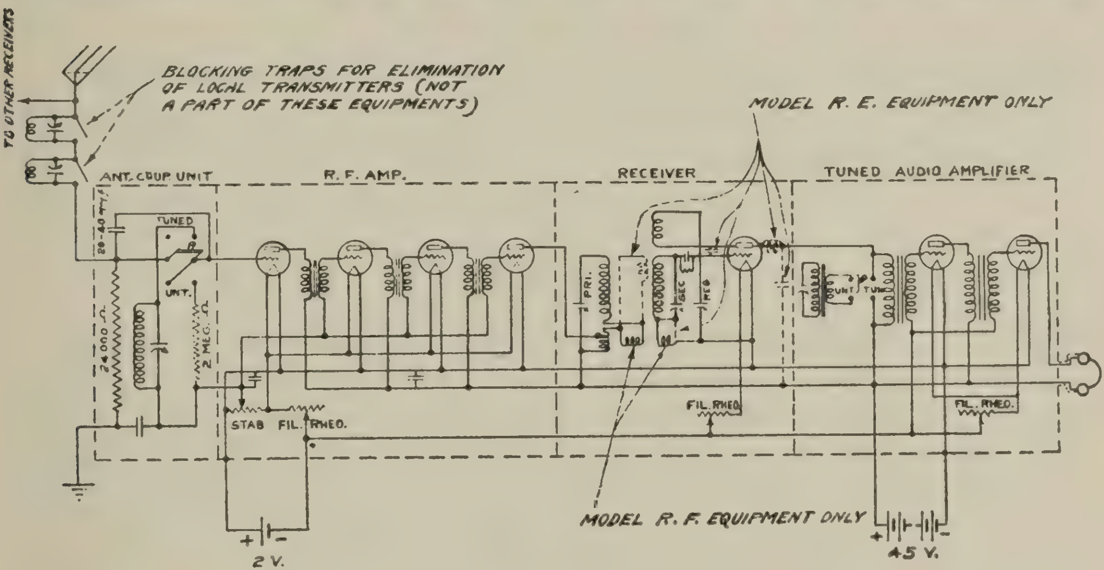


FIG. 361.

eliminated by the system regardless of the number of radio stages employed. Although both radio and audio amplification is obtained

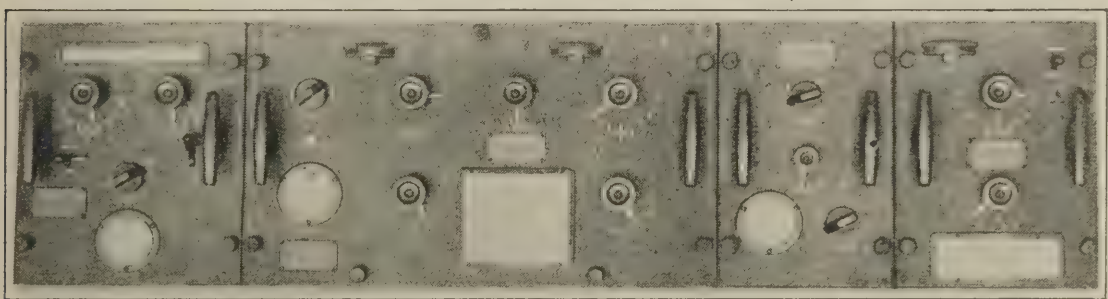


FIG. 362.

from the double duty tubes, the actual overall voltages step-up does not equal that which is obtained when a separate tube is supplied

for each function. Moreover, where three or more stages of radio amplification are employed, extreme difficulty is experienced in securing stability, even two stage commercial broadcast receivers built on this system usually resorting to a crystal detector to aid in this respect.

Navy type receiver. The complete elementary circuit of a Navy type receiver on board ship for multiple radio reception is shown in Figure 361. The appearance of the receiver is shown in Figure 362.

CHAPTER VII. RECEIVER BATTERIES.

General. Batteries in radio service are, in general, used for two purposes:

(*a*) In connection with vacuum-tube equipment,

(*b*) For emergency power supply for transmitters.

The dry battery and storage battery are both used in connection with vacuum-tube receiving equipment. The filament circuit is usually supplied by a storage battery, because a larger current than can be supplied by the dry battery is required. (Special generators are sometimes used for transmitting tube filaments.) For the plate circuit, where the voltage is relatively high and the current of the order of a few milliamperes, the dry battery, a special storage battery or generator is used.

The emergency power supply for transmitters is always obtained from storage batteries, because the current demand is greater than can be supplied by the dry battery. The storage battery is frequently used for the filament supply for vacuum-tube transmitters.

A. DRY BATTERIES.

A satisfactory **dry battery** has been developed especially for use as the plate voltage supply for receiving vacuum-tube equipment. It has a rated voltage, when new, of 22.5 volts and will deliver a current of approximately 4.3 milliamperes over a period of 34 days before the voltage is reduced to 17 volts, measurements being made by a voltmeter across the terminals of the battery while it is discharging at approximately the normal rate. The average discharge voltage is 19.2 volts. The internal resistance is 315 ohms, and the ampere-hour capacity 3.45.

The battery is hermetically sealed and water-proof. Three leads are brought out from the case, one from each end and one from the middle. The positive lead is red, and is further designated by a plus sign stamped in the compound near the lead. When one half the battery is used (either end lead and the center lead) the lead in the center becomes negative or positive, according as the positive or negative end lead is used for the other lead.

These batteries have a **low rate of deterioration** on the shelf. Batteries held in stock over a year have proved to be in excellent condition when put into service, but it is not wise to carry a quantity in stock for a longer period than is required to meet the demands of the Service. Frequent small shipments from the central supply depot are preferable to less frequent shipments of large quantities.

This battery will supply sufficient current for the plate circuit of any piece of receiving apparatus, including the multi-stage amplifier, and will function properly and efficiently on the latter for at least one month. When used with apparatus which employs only a few vacuum tubes, such as a vacuum-tube detector or a detector tube and one 2-stage amplifier, the current drawn from the battery is less and its life prolonged in proportion. Intermittent duty also prolongs the life, since only the actual ampere-hours used are to be deducted from the total ampere-hours available in the battery.

It is important that the batteries be used in the order of their receipt, that is, the oldest first, so that those held in stock will be reliable. **This battery should not be tested by short-circuiting,** because this will appreciably reduce its life on account of the small size of the cells. The proper method of testing is to put the battery in service and read the voltage by a voltmeter connected across the battery terminals while the battery is discharging. If up to standard, repeat after approximately 1,000 tube-hours' use, which is equal to continuous service on a 6-stage amplifier for one week. If the voltage is then less than 20 volts, the battery is poor and should be renewed as soon as unsatisfactory results are obtained from it. The voltage under the condition noted, namely, after 1,000 tube-hours use, should be approximately 20.5 volts. A 6-stage amplifier draws a current of approximately 4.3 milliamperes from the battery.

Defective, or **exhausted, batteries** are indicated when the filaments of the tubes must be burned above normal brilliancy, when the back coupling must be tightened more than the usual amount, or both, and when the amplifier or detector circuit becomes erratic in its behavior, and a frying or hissing tone or noise is heard in the telephones. The hissing or frying noises may also be caused by a poor contact or defective tube, but it is generally an indication that the battery is exhausted. As soon as it has been definitely determined that a battery is exhausted, it should be replaced by a fresh one. Exhausted batteries are of no further use.

The fact is to be emphasized that **the battery will discharge whenever the filament is lighted and the plate circuit is closed.** For this reason, among others, the filament should not be lighted except when necessary for receiving. Satisfactory service can not be obtained from the battery after the voltage has dropped below 17 volts on discharge.

Batteries in stock should be stored in a cool, dry place, with the side having the sealing compound up. Care should be taken also that no battery is accidentally short-circuited by the leads touching, and thereby exhausted.

B. EDISON (NICKEL-IRON TYPE) STORAGE BATTERY.

The Edison B Battery. A special storage battery has been developed by the Edison Co. for use in the plate circuit. The cells are made very small and 48 are connected in series to give a normal discharge voltage of 60 volts. The charging rate is one-half ampere for 7 hours when the battery is fully discharged.

The Edison B battery has the advantage over the dry battery in that it can be kept in good condition by proper care and charging

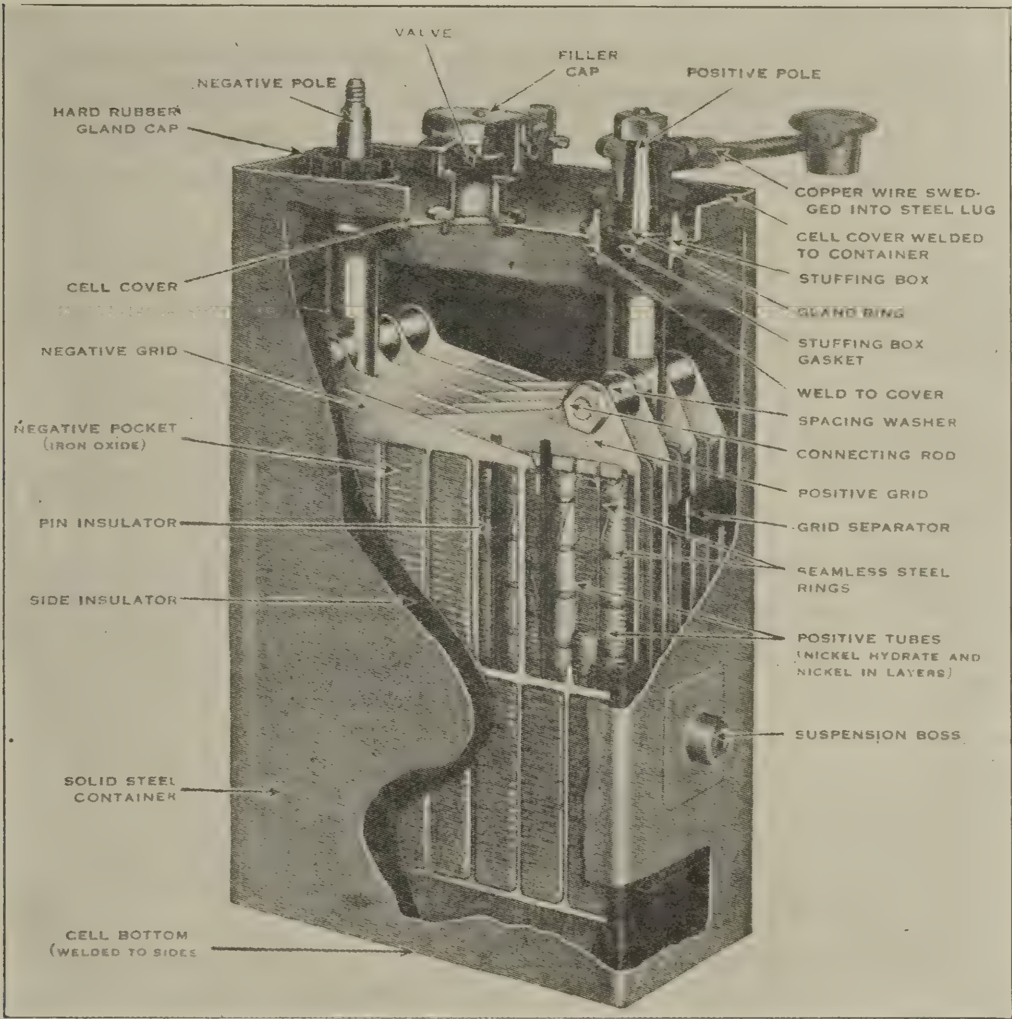


Fig. 363.—Cell Construction in United States Navy—Edison A. N. S. Battery, Type CU-119.

when needed. On the other hand, the dry battery occupies but little space, but has the disadvantage of a comparatively short life, and after exhaustion must be replaced by a new battery.

The Edison A Battery. Considerable current at a low voltage is required for lighting filaments. The 6-volt storage battery (either the Edison or the lead-acid type) is used for most receiving tubes. The Edison battery is rated at 75 ampere-hours, while the lead-acid type of battery generally used has a capacity of either 80 or 100 ampere-hours.

A fairly high capacity battery is preferable, especially where multi-stage amplifiers are in use, in order to maintain the required voltage for a reasonable length of time and to reduce the frequency of charging.

General. The active materials used in the Edison type storage battery are nickel hydrate in the positive plates, and iron oxide in the negative plates. The electrolyte is mainly a solution of potassium hydrate. All of the cell parts such as grids, containers, perforated tubes and pockets, etc., are made of steel, heavily nickel plated and thoroughly annealed. The insulating material which separates the plates and insulates the elements from the steel container, is specially treated hard rubber. The cell-to-cell connectors are made up of copper rods swedged into suitable drop forgings, drilled and reamed on a taper to fit over the cell poles. The trays which hold the cells are made of hard wood. Figure 363 shows a cut-away view of an Edison (nickel-iron) storage cell.

Electrolyte. The electrolyte shipped with an Edison battery consists primarily of a 21 per cent solution of potassium hydrate in water, to which is added a small quantity of additional chemicals.

Polarity. The **positive pole** of any Edison cell is designated by a **red** hard rubber bushing around the positive pole, and by a **plus** mark plainly stamped on the cell cover, near the positive pole. The **negative pole** is designated by a black bushing and has no marking on the cell cover.

Charging. When charging any of the Navy Edison radio type batteries, the cover of the steel battery box (not the cell caps, valves, or plugs) should be lifted and left open during the entire charging period.

The filler caps or plugs on the tops of the cells should not be removed while charging. There is no occasion to lift or remove them except when watering or renewing the electrolyte.

When a battery is to be charged, an inspection should be made to see if any water or solution has seeped into the steel battery boxes; if so, this should be emptied out before the charge is started.

Before placing the battery on charge, inspect the height of electrolyte in each cell. If the level is found to be low, **distilled water** should be added.

To charge a battery, the positive side of the line should be connected to the positive terminal of the battery and the negative side to the negative terminal.

The normal charging rate for different types of Edison cells, varies according to the size and type of plate used, and according to the number of plates in the cells.

The initial charge to be given a new battery should be longer than a normal charge. For large batteries, the initial charge should be for 12 hours at the normal rate. For small batteries, the initial charge should be for 8 hours at the normal rate.

The normal charging time for large batteries is 7 hours at the normal rate. For small batteries it is 5 hours at the normal rate.

A normal length charge should be given a battery if it is in a completely discharged condition. If it is only half discharged it should be, charged for one-half the normal length of time at normal rate. If only one-quarter discharged, it should be charged for one-quarter of normal time, at normal rate.

If the extent of previous discharge is unknown, charge at the normal rate until the voltmeter has remained constant for 30 minutes at 1.8 volts or slightly above per cell, with normal charging current flowing, and the cells are gassing freely.

It is possible to charge Edison batteries at high rates (booster charge) during brief periods of idleness. The following table gives figures that may be used under average conditions. Battery can be given—

5 minutes charge at 5 times normal rate, or
10 minutes charge at 4 times normal rate, or
20 minutes charge at 3 times normal rate, or
40 minutes charge at 2 times normal rate.

In case of emergency, these values may be exceeded somewhat. Frothing at the filter opening is an indication that the boosting rate is too high (provided the electrolyte level is not too high). If this is seen to occur, the rate should be cut down. High rate boosting will cause the temperature of the cells to rise. If conditions will permit, excessively high cell temperature should be guarded against. A battery will not take its charge as satisfactorily when the temperature is in excess of 115° F.

The average voltage of any Edison type cell, discharging at manufacturer's normal rate, is 1.2 volts. At rates lower than normal, the voltage will average slightly higher. High rate discharges may be employed without fear of injury to the cells. As the rate of discharge is increased, however, the average voltage is lowered.

Edison cells are not injured by short-circuit discharges. When given a dead short-circuit for any length of time, cells will heat excessively. An occasional discharge to complete exhaustion after a discharge at normal rate, will keep the negative plates in good active condition and is, therefore, desirable.

An Edison battery is in no way injured by being allowed to stand in either a partially or completely discharged condition, but should be given an overcharge after remaining thus for an extended period of time.

Capacity. Under proper treatment Edison storage batteries improve with use. A new cell will continue to increase in capacity for a period of at least 30 cycles of charge and discharge. If a new battery, or

one which has been standing idle for a long time, operates somewhat sluggishly, use it as much as possible, giving it occasional complete discharges, and it will soon return to normal capacity. If the capacity of a battery falls off, it is usually an indication that the electrolyte needs to be changed. Experience indicates that years of continuous service under normal conditions may be reasonably expected from Edison batteries. Hence, a battery should not be condemned as useless because it is perhaps a year or two old, as a thorough inspection will more than likely determine that all that is needed is a renewal of the electrolyte and a forming charge.

Watering. During charge, water of the electrolyte is driven off as gas, and must be replaced. Under normal conditions, it will probably be found necessary to add water after each three complete cycles of charge and discharge. The evaporation which takes place during initial or long overcharges is considerable. Upon completion of such a charge the cells should be watered.

Distilled water, if available, should be used for flushing (filling) a battery. If distilled water is not to be had, use any pure water which may be at hand. Clean rainwater is very good. Any water which is fit to drink may be used. The principle precaution necessary is to avoid water containing acids or sulphur. Water containing moderate amounts of iron or lime, although not approved by the manufacturer, may be used without fear of causing serious injury to the battery.

The level of the electrolyte must be kept above the tops of the plates. When the level gets down to the plate tops, the cells should be watered. The proper level of the solution is one-half inch above the plate tops.

Care should be exercised when filling a battery to avoid spilling water over and around the cells, and not to exceed the specified level above the plate tops.

The cells should always be filled before placing them on charge, and not after they are connected to the charging source. When a battery is placed on charge, the gas formed lifts the electrolyte to a false level; therefore, **level testing and watering should be done before connection to the charging source is made.**

To test the height of electrolyte in a cell, a glass tube should be used. Insert the tube into the cell through the filler opening until the tops of the plates are touched. Close the upper end of the tube with the fingers and withdraw. The height of liquid in the tube indicates the height of the electrolyte above the plate tops. The glass must not be less than three-sixteenths inch inside diameter, and its ends must be cut straight.

Electrolyte data and renewal. The Edison Co. supplies two solutions for use in filling batteries of their manufacture. They are known as

refill and **renewal solutions.** These terms have been adopted by the manufacturer, and should be adhered to to avoid confusion. The refill solution is that solution which is supplied for use in a new battery. The proper specific gravity of refill solution before being poured into a cell will vary from 1.210 to 1.220. The renewal solution is that which should be used when renewing the electrolyte in a cell, or battery of cells, which has been in service. The proper specific gravity of renewal solution before being poured into a cell is approximately 1.250.

Some confusion may result from the use of the two terms **solution** and **electrolyte.** The liquid before being poured into a cell is commonly known as solution. After being poured into a cell it is known as electrolyte.

The normal strength of electrolyte in a cell should be such that the specific gravity is about 1.200 as measured by the hydrometer, but at times, when newly introduced, it may be as high as 1.230.

After a period of use, the electrolyte will weaken and must be completely renewed. The interval between necessary electrolyte renewals will vary greatly, according to the nature of the service and the care the battery has received. It may be anywhere from 12 months to 4 or 5 years. The question of electrolyte renewal should be determined by specific gravity readings, taken at intervals. If the electrolyte is found to have a specific gravity of 1.160 or less, when tested after a full charge and with the level at the proper height, it should be renewed. Before measuring the specific gravity always make sure that the cell is filled to the proper height and, when freshly made up or has been watered, that the electrolyte is thoroughly mixed. If a gravity reading is taken directly after the addition of water, unless the cell is thoroughly shaken, a false reading will result.

With some types of batteries, to determine the specific gravity of the electrolyte, it will be necessary to empty the electrolyte from a number of cells. This is occasioned by reason of the relatively small amount of electrolyte contained in each cell. Cells emptied for this purpose must not be allowed to remain thus for more than a few minutes—just long enough to get a reading—as they are not in a discharged condition.

Potassium mixture for making renewal solution is usually shipped in dry powder form, put up in steel containers. Complete instructions for mixing it with water are pasted on the side of the container. The mixing of this powder with water will cause the solution to heat. Allow the solution to cool before pouring it into the cells.

The renewal of the electrolyte in a cell is very simple, and is as follows:

(a) Discharge the battery at normal rate to one volt per cell and short circuit to zero.

(b) Empty out the old electrolyte.

(c) Refill immediately with new solution to the proper height of one-half inch over the plate tops.

(d) Give the battery a long charge similar to an initial charge.

Do not pour out the old electrolyte until the new is received and is ready to be poured into the cells.

In putting the electrolyte into a cell, an earthenware pitcher and glass or black iron funnel should be used. **Do not use a tinned or enamelware funnel, because impurities will be introduced into the cell.**

Low temperature effects. If the specific gravity is normal, an Edison cell will not freeze until the electrolyte temperature gets below 20° below zero Fahrenheit. If electrolyte temperatures lower than this are encountered, the solution will not freeze solidly, but will congeal into a snowy consistency. This, of course, temporarily discontinues the action of the battery, but does not injure it in the least. The rate of cooling of the electrolyte is very slow, particularly when being discharged, even at a very low rate. This is due to the heat-insulating effect of the dead air around the cells in the battery box. It is, therefore, doubtful if conditions causing a battery to freeze will ever be encountered.

An edison battery will absorb its charge quite readily even though the electrolyte temperature is quite low. If batteries are discharged at continuous relatively high rates, with a low electrolyte temperature, a temporary loss of capacity will be noted. If rates of discharge are low, such as prevail in the filament circuit of vacuum tubes, or if the discharge is of an intermittent nature, the effect of low temperatures will be of no material consequence.

If conditions will permit, it is advantageous to keep the temperature of the charging room above 32° F.

Cleaning and coating cells. Edison cells do not require any internal cleaning, as there is practically no sediment precipitated to the bottom of the container. However, they should be kept clean externally. The outsides of the containers, trays, etc., should be kept as dry as practical.

A slight deposit of potash salts will collect, under normal conditions, on the tops of the cells. This is not injurious but, if allowed to collect, will short circuit from container to container of adjacent cells. It should be removed from time to time.

The outsides of the cell containers are coated with a vaseline compound containing a small amount of rosin, the purpose of which is to protect the steel containers from possible corrosion. If this coating is wiped off it should be replaced. Any ordinary commercial vaseline may be used for the purpose.

Dirt or potassium incrustations, which very often collect on the tops of the cells, can be very effectively and easily removed by means of a

jet of dry steam. If this method of cleaning off the tops of the cells is adopted, the tray containing the cells should be removed from the steel box before the cleaning is attempted.

DO NOT UNDER ANY CONDITIONS PUT ACID INTO AN EDISON CELL. IT WILL COMPLETELY RUIN THE BATTERY.

Edison type number	W–1T	W–2T	B–4	L–20
Number of cells	42	24	5	4
Manufacturer's ampere hour rating	1.25	2.5	75	12.5
Manufacturer's normal discharge time (hours)	5	5	5	3.33
Manufacturer's normal discharge rate (amperes)	0.25	0.5	15	3.75
Average discharge voltage per cell (5-hour rate)	2.4	2.4	1.2	1.23
Average discharge voltage per cell (3.33-hour rate)	....	....	....	1.2
Manufacturer's normal charging time (hours)	7	7	7	5
Manufacturer's normal charging rate (amperes)	0.25	0.5	15	3.75
Maximum charging voltage, constant current method (per battery)	155.4	88.8	9.25	7.4
Maximum charging voltage, tapering current method (per battery)	147	84	8.75	7.00
Amount of solution[1] per cell (pounds) liquid renewal	0.38	0.32	1.72	0.44
Amount of dry electrolyte[1] per cell (renewal)	0.114	0.096	0.56	0.132
Weight of battery complete with box (pounds)	50	23	55	15.8
Size of steel battery box (inches) not including brackets:				
Length	$15\frac{1}{2}$	$15\frac{5}{8}$	$18\frac{5}{16}$	$8\frac{7}{16}$
Width	$12\frac{5}{8}$	$6\frac{1}{8}$	7	$5\frac{1}{2}$
Height	$8\frac{1}{2}$	$8\frac{1}{2}$	$10\frac{3}{16}$	$11\frac{5}{16}$

[1] Amount of solution given for W–1T and W–2T is sufficient to refill a twin unit (2 cells).

C. LEAD-ACID BATTERIES.

Care and upkeep. Assign each cell a number, beginning with the positive terminal of the first cell, and number each one consecutively through the total number in the battery.

In each battery select, at random, one cell to be used as a **pilot cell,** and watch it carefully during the entire life of the battery. If the battery is divided into two or more groups, select a pilot cell in each group.

All batteries should be charged to full capacity once every two months, whether or not in use.

Do not charge battery at such a rate that **excessive gassing** takes place. Be careful not to allow **flames** or fire of any sort in the compartment with a battery.

Remember that the **lower the state of charge,** the more injurious will be the effects of local action on the battery.

Remember that the **higher the specific gravity** of the electrolyte, for a low charge, the more injurious will be the effects of local action on the battery.

Remember that the **higher the temperature** of the electrolyte, the greater will be the injurious effects on the battery.

A Navy cell should be considered charged when the density of 1.215 corrected to a temperature of 80° F. is reached (unless otherwise specified on battery nameplate). The cell should not be discharged to a density of less than 1.130 and should not be left idle after the gravity reaches 1.170.

Never allow the temperature of electrolyte to exceed 110° F., as plates are likely to buckle and will need to be replaced.

The **height of the electrolyte** should be such that very little of it will spill during charging.

The **maximum gravity and equalizing charge** should be applied whenever it becomes necessary to offset local action or correct the specific gravity of a cell.

This charge may be satisfactorily applied by starting a charging period at the high rate given on the battery name plate, continue this charge until the cells gas freely, after which time reduce the rate of charge, and charge until they again gas freely. Then reduce the charge to one-fourth the original charge and continue until five hydrometer readings, taken at 15 minute intervals, on each cell indicate that a constant gravity has been reached.

A test discharge may be performed to determine the ampere-hour capacity of a battery by first performing the maximum gravity and equalizing charge, then discharging the battery at a steady rate equal to the capacity of the battery in ampere hours divided by ten (this time is frequently given as 8) until the voltage of each cell reaches 1.75 volts measured during discharge. An accurate record of time should be kept during discharge. This time multiplied by the discharge current in amperes gives the ampere-hour capacity of the cell.

Immediately after a test discharge, the maximum gravity charge should again be applied.

Distilled water may be added at any time to replace evaporation, preferably just before the battery is placed on charge. This is especially true in freezing weather, as the water will remain on top of the electrolyte until mixed with it by charging.

Never add acid to a battery. When mixing acid and water always use a lead or earthenware vessel. Pour the acid slowly into the water (**never pour water into acid**) and stir with a glass rod or tube.

Never use a hydrometer in a lead-acid battery after it has been used in an Edison battery.

Never mix electrolyte from one cell with that from another.

PART 3.

THE RADIO COMPASS.

1. GENERAL.

The purpose of the radio compass is to permit the determination by radio of the absolute direction and geographical position of a radio transmitter. The principle involved in both ship and shore radio-compass installations is the same, while the operation and equipment of the two types of installation differ slightly on account of the particular requirements.

Shore radio-compass stations are located at advantageous points along hazardous coast lines or at important harbor entrances for the

FIG. 364.—Radio-Compass House.

purpose of providing vessels navigating in the vicinity with radio bearings from known positions. Where two or more stations are located adjacent to a particular harbor entrance, two or more simultaneous bearings of a vessel may be obtained, thus furnishing a fix which will enable the vessel to plot its position. This is accomplished by building stations similar to the one shown in figure 364 in groups of two or more, several miles apart, and interconnecting them by telegraph or radio. One of the stations in the group is designated as the master station, where the observed bearings are plotted and then forwarded to the vessels requesting them.

Ship radio-compass stations. The radio compass is installed on vessels for the purpose of enabling commanding officers to determine the true bearing from the vessel to another vessel or to a radio station, or radio beacon on shore. Such an installation determines along what line a given transmitter lies, but does not indicate the absolute direction. This is because the radio compass receives equally well from two diametrically opposite points. For this reason, there exists an ambiguity in direction of 180°. In order to determine direction—it is necessary that the observing vessel make two observations with a run between. When the course and bearings are plotted, the position of the transmitter is indicated by the intersection of the two lines of bearing.

The radio compass aboard ship determines the direction of the transmitter relatively to the heading of the ship; therefore, to obtain the true bearing, the heading of the ship must be observed simultaneously with the radio-compass bearing and applied to the latter.

The 180° ambiguity also exists in the shore station installations, but, because a bearing from a vessel would not come from inland (there are a few exceptions to this in the case of aircraft and of river navigation when the vessel is near the mouth of the river), this discrepancy would not occur. The possibility of the direction of the line of bearing being in error by approximately 180° is entirely eliminated in the case of group stations, because the intersections of the lines of bearing would indicate in which direction the transmitter is located.

It is more difficult to obtain an accuracy comparable to that obtained at shore stations on account of the metallic structures in close proximity to the coil system, as shown in figure 365, the dissymmetry of the vessels, the difficulty in conducting frequent calibrations, as well as of the lack of experience of the radio personnel. Despite these adverse conditions, when the above obstacles are overcome, it is possible to obtain radio bearings having almost the same degree of accuracy as that obtained by the usual means of navigation.

In order that the desired conditions just mentioned may be the rule rather than the exception, it will be necessary for officers in charge of the installation, calibration, and operation of the radio compass equipment on shipboard to make certain that the installations are made in conformity with the standard practice, that the calibrations are conducted with extreme care and accuracy, and that the equipment is operated at regular periods, in order that the personnel may become expert and reliable.

Comparison of ship and shore radio compass. While it is possible to use the radio compass aboard ship for coastwise navigation and piloting, there are certain problems that handicap this method:

(a) The difficulties encountered in recalibrating and checking the accuracy of the ship radio compass. Shore radio-compass stations are frequently checked and calibrated.

(b) The comparative infrequent use by the operating personnel does not permit them to become expert as is the case with operators on shore where the observers become expert by constant practice and are able to measure as close as one-half degree.

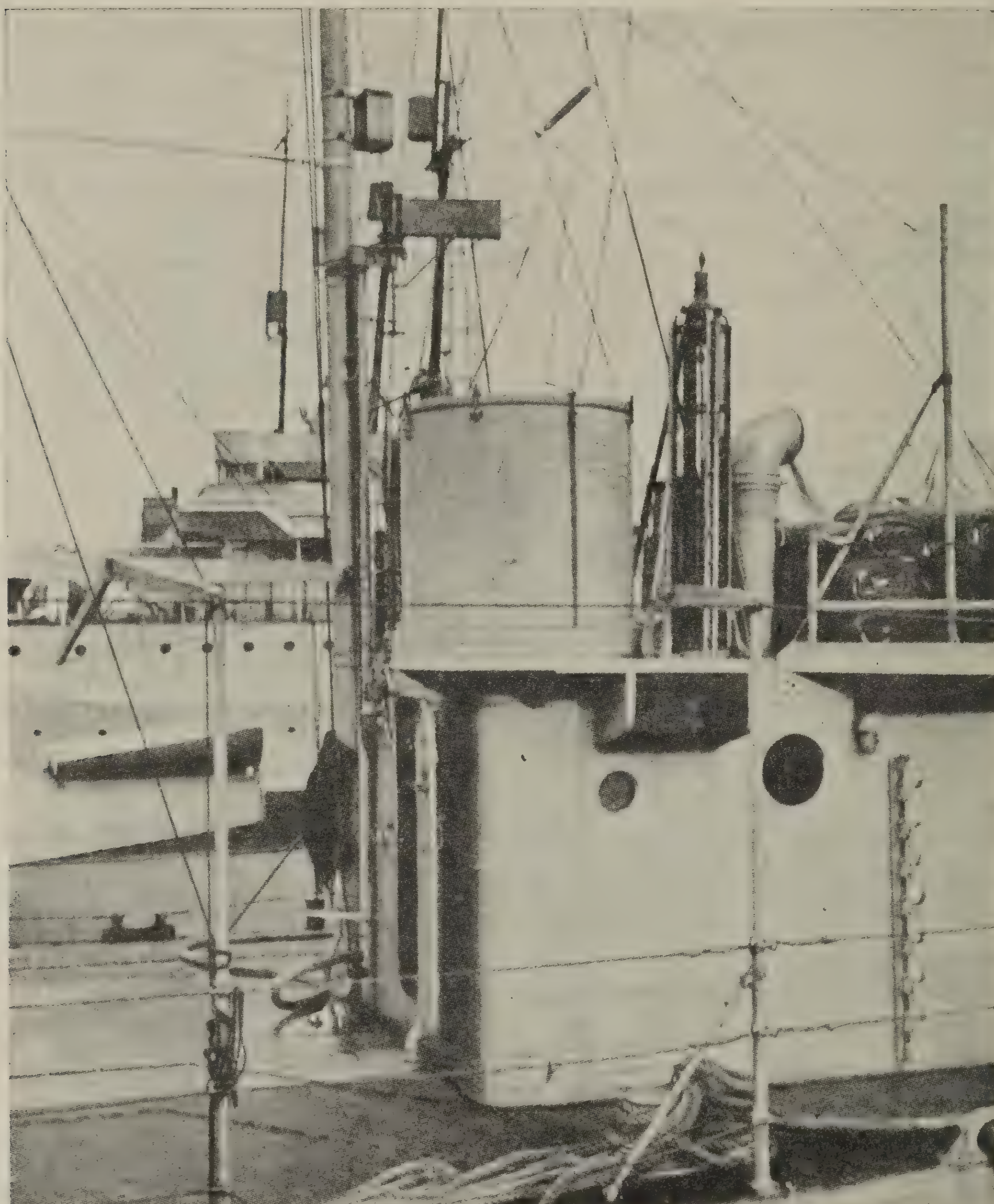


FIG. 365.—View of Radio-Compass House on Shipboard, Showing Antenna Lead-in.

(c) The bearings observed by the ship radio compass are relative to the ship's head; the true bearing of the transmitting station is ascertained by adding to the true course the radio compass bearing (corrected for radio deviation). Example: ship's head 350°, radio bearing 50° relative, true bearing: $400° - 360° = 40°$ true.

The observations made at radio-compass shore stations are read directly from true north, and are sent out as true bearings.

The aircraft radio compass is similar in principle to the radio compass used on ship and shore. Two methods of determining direction are employed, namely, the minimum and the maximum.

The first method is the same as that used on ships and at shore stations if the received signal is very loud with the coil set in the maximum position. The maximum method is employed when the signals are too weak to use the minimum method, that is, when the signals are weak on the maximum setting, or when a very broad zone of silence is obtained by the minimum method. In the minimum method, the plane of the coil is at right angles to the line of bearing of the transmitting station when zero signal is received, while in the maximum method the plane of the coil coincides with the line of bearing of the transmitting station when maximum signal is received.

2. THEORY.

Nondirectional antennas. The antenna of moderate horizontal dimensions is nondirectional. This can be proved by moving the radio transmitter so as to make a complete circle about the receiving antenna and plotting the emf induced in the antenna against the angular position of the transmitter. The resulting polar diagram will be a circle, showing that the emf induced in the antenna is the same for reception from all directions. Therefore, the signal will not vary in intensity, and the direction of the transmitter can not be determined.

Directional antennas. If a loop consisting of several turns of wire wound on a fairly large form is arranged for receiving, as shown in figure 43, so that it can be rotated on a vertical axis, the loop will make a varying angle with the direction of the source of electromagnetic waves produced by a fixed radio transmitter. If the emf induced in the loop is plotted in a polar diagram against the angular position of the loop, the resulting curve will be similar to that shown in figure 367. This is called the **figure 8,** or bilateral characteristic curve of the loop. Figure 367 shows a top view of the coil and the dial, which is securely attached to the vertical shaft of the loop. The pointer in this case is set on the north-south line. It will be seen from the position of the figure 8 curve that a maximum signal will be received when the transmitting station is either east or west of the loop, that is, when the plane of the loop coincides with the east-west line. If the transmitting station were in the north-south line, it would not be heard. Thus, it is clear that two maxima and minima are passed through as the loop is rotated through 360°.

The electromagnetic wave from the transmitter passing through the loop induces in the vertical sides of the loop equal emfs which are slightly out of phase. The resultant emf, which is extremely small, sets up a flow of current in the loop. This current either lags behind or leads the electrical component of the wave by 90°.

Maximum and minimum signal positions. The maximum signal will be received when the plane of the loop coincides with the line of bearing of the transmitting station. As the loop is rotated from this position, the signal will decrease in strength, finally becoming zero when the plane of the loop is at right angles to the direction of the source. The maximum signal method of determining direction is not usually used in radio-compass work because the bearing cannot be determined very accurately. This is made clear by the shape of the characteristic curve. It will

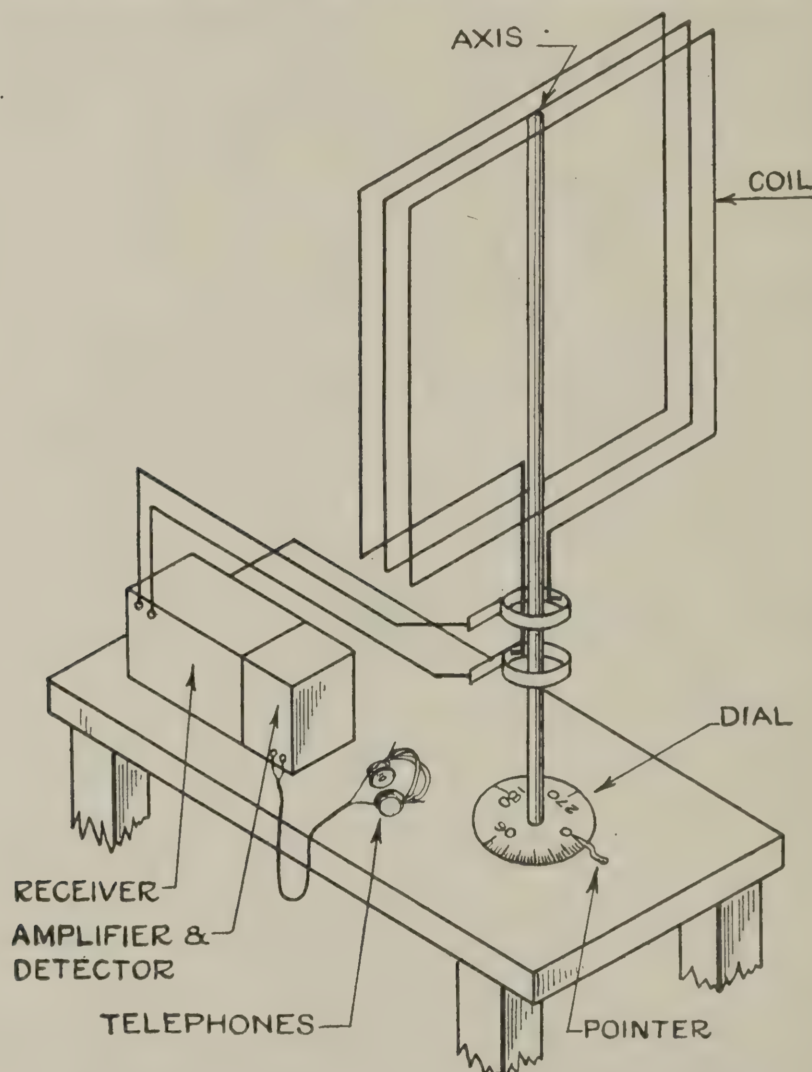


FIG. 366.—Elementary Diagram of a Radio-Compass Installation.

be seen, however, that the point of zero signal is very sharply defined, and for this reason the **null method** is used for determining direction.

The shape of the figure 8 curve, given in the figure is that of the resultant emf. The variation in signal intensity will be the same, provided that the detector used follows the first power law (autodyne or heterodyne reception). In the case of the plain detector, with which the signal response is proportional to the square of the current in the loop, the variation in the signal response will have a form obtained by squaring these values.

Practical application. The previous discussion relates to the theory of an ideal compass system acted upon by an undistorted electromagnetic wave. This condition is not realized in practice. The departures from the ideal compass are due to the defects or nonsymmetry of the apparatus itself, and to the distortion of the electromagnetic wave before it reaches the coil. The defects and nonsymmetry of the apparatus can be compensated for to a large extent. The distortion of the wave, due to reradiation, diffraction and reflection can be partly eliminated by a proper choice of the location of the station.

It was stated above that all bearings are obtained by observing the position of the coil in relation to true north when the minimum signal is being obtained. This holds for all ship and shore radio-compass

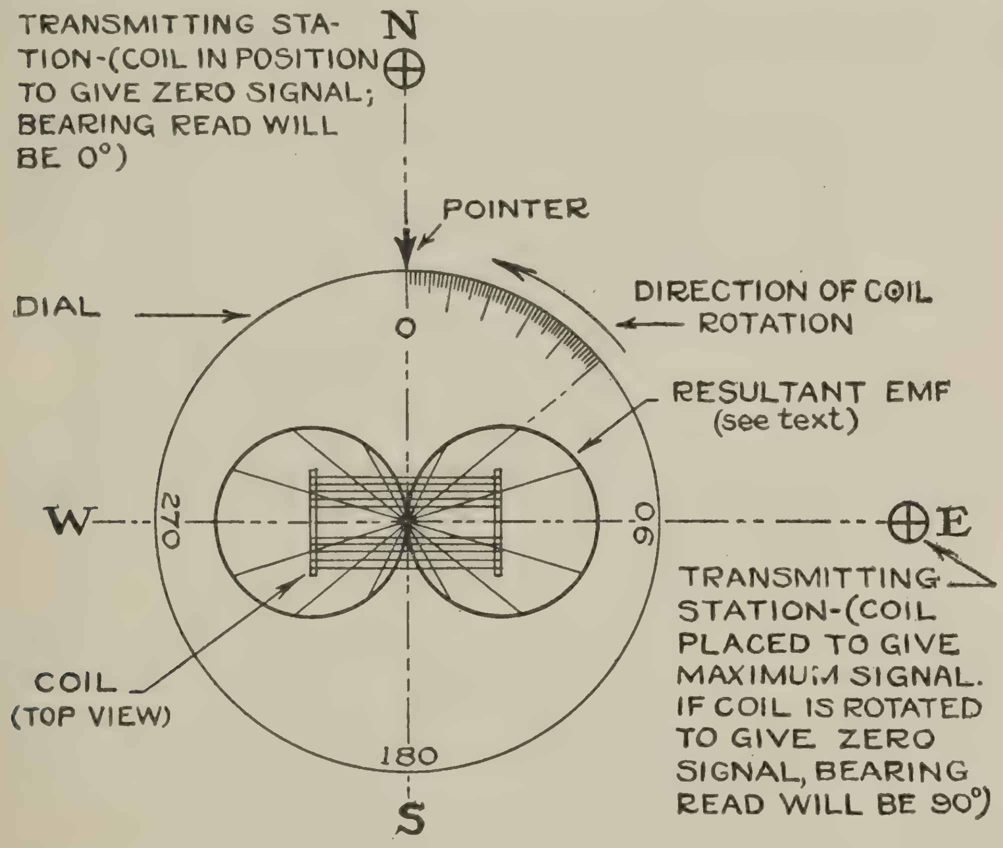


FIG. 367.—Relations of Coil, Dial, Pointer, Transmitter and Induced emf.

stations, but not for aircraft installations when the maximum method is used; therefore, in order to make an accurate observation, it is essential that the minimum be clearly defined and cover as small an arc (zone) as possible; or better still, that there be a **point** on the arc at which no signal is heard (**null point**) such that the slightest change from this will result in the signal being heard.

The above applies to a perfect compass. In practice, the null point is seldom found, but in its place there is either a xone of minimum signal, or a zone of silence, caused by the nonsymmetry of the apparatus, reradiation, and intensity of signal, etc. The breadth of the zone of silence is decreased as the signal strength is increased. The breadth of this

zone determines the possible accuracy of the equipment. Therefore, by the proper design of the circuit and with sufficient amplification this zone of silence can be reduced to approach the null point as a limit.

Antenna effect. The most common cause for the destruction of the null point in the practical radio compass is the antenna effect.

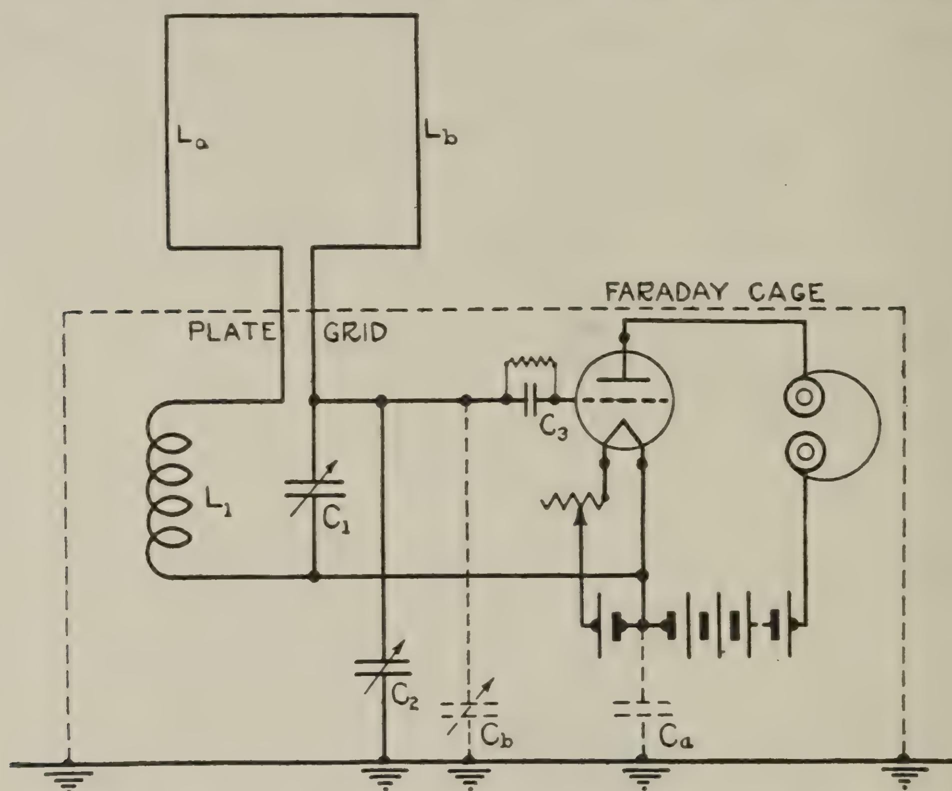


FIG. 368.—Electrical Dissymmetry of the Practical Radio Compass.

This effect is due to the electrical dissymmetry of the radio-compass circuit. Consider figure 368, where the radio-compass coil is installed above a symmetrical Faraday cage containing the receiving apparatus and the operator. The terminal from the vertical side of the coil marked L_a is connected to the filament of the vacuum tube of the receiving apparatus (detector or amplifier), while the side marked L_b is connected to the grid. An electrical dissymmetry exists on account of the unequal capacities to ground of the filament C_a and of the grid C_b caused mainly by the proximity of the filament battery to ground. This unbalancing of the circuit permits the coil to function additionally as an open oscillator (simple antenna). Therefore, when the plane of the coil is at right angles to the direction of the propagation of the electromagnetic wave and the current induced in the coil is zero, there is also a current induced in the circuit due to the coil system acting as an antenna. For this reason, the zone of silence is obscured by a residual signal having a practically constant intensity. This disturbing effect is eliminated, and the symmetry of the circuit restored by the addition of sufficient capacity from the grid to ground balance that of the filament to ground. This capacity C_2 is

called the **compensating condenser,** and consists of a small continuously variable air-dielectric condenser which is thoroughly shielded and has a very small capacity at the zero setting.

Compensation. The presence of the antenna effect and the use of an artificial capacity balance makes it necessary to use extreme care in tuning the circuit, because any adjustment of the compensating condenser C_2 will disturb the wave length of the circuit and require a readjustment of the condenser C_1. This change in wave length, or detuning from resonance with the incoming signal, causes a shift in the position of the minimum, thereby giving rise to an error in the determination of direction.

More freedom from interference and the possibility of decreasing the zone of silence is obtained with the use of a coil system having a very low radio-frequency resistance. A low resistance circuit is especially important when the nonoscillating detector and audio-frequency amplifier or the radio-audio-frequency amplifier is used. On the other hand, if the autodyne or heterodyne method of reception is employed, the resistance of the loop can be neglected.

3. RADIO-COMPASS SHORE STATIONS.

A. Description.

Location. The problem of deciding upon the location of a compass station merits considerable study. It entails a thorough investigation of the local shipping and the relative positions of existing

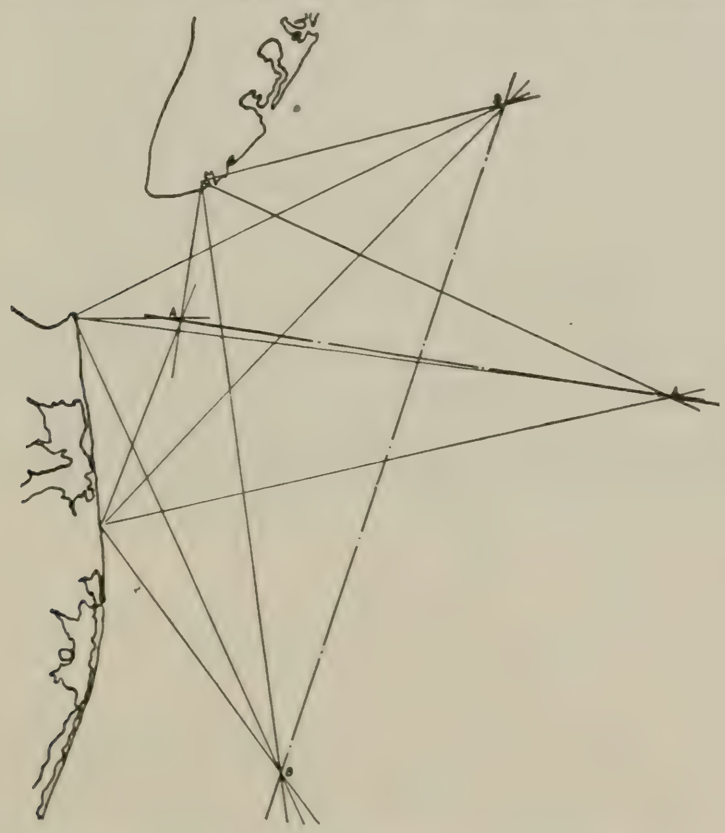


FIG. 369.—Arrangement of Radio-Compass Stations in the Vicinity of a Harbor.

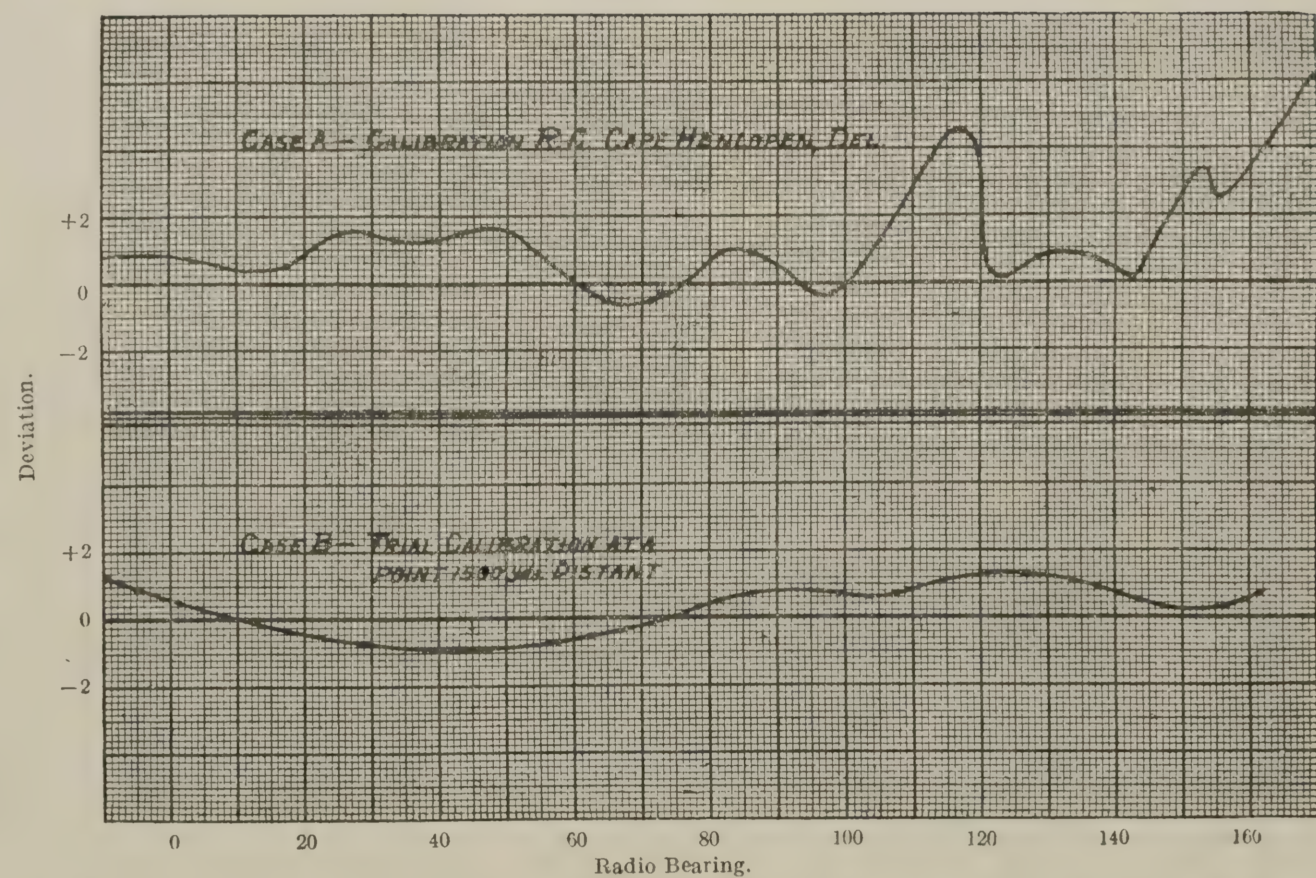


FIG. 370.—Comparison of two calibration curves. Curve A shows the calibration of the original location which in operation proved very unsatisfactory. Curve B shows the deviation curve obtained using portable equipment at a point 1,500 yards distant.

compass stations or groups of stations. Thus, the placing of each station bears an important relation to the entire compass system of that coast; and, since the expense involved is considerable, the location selected is not considered final until every factor has received attention.

Badly broken coast lines are to be avoided if smoother ones are available. Operation behind uniform, level coast lines is much more consistent even though this desired condition obtain only within the immediate locality of the station.

Sites in the proximity of swamps or where conditions vary greatly with the weather are to be avoided. A large gully in the vicinity which is normally dry but which retains water after a rainfall will generally cause inconsistent deviations. Hills in the vicinity may produce excessive deviation, as may deeply wooded sections or metallic veins in the earth. If any suspicion exists regarding such a location, resort should be made to a trial calibration.

The difference in the results that can be expected from a different location is clearly shown in figure 370.

Any structure, the conductivity of which is indeterminate, will produce marked effects upon the operation of a radio compass within its vicinity. In this category are communication lines, metallic towers or buildings, or even buildings containing electric wiring or piping. Two hundred feet, at the least, must separate the radio compass from any of the above, and in no case should the structure in question be between the compass and the sea within the working sector.

FIG. 371.—Layout of Radio-Compass Stations Protecting a Navigational Hazard.

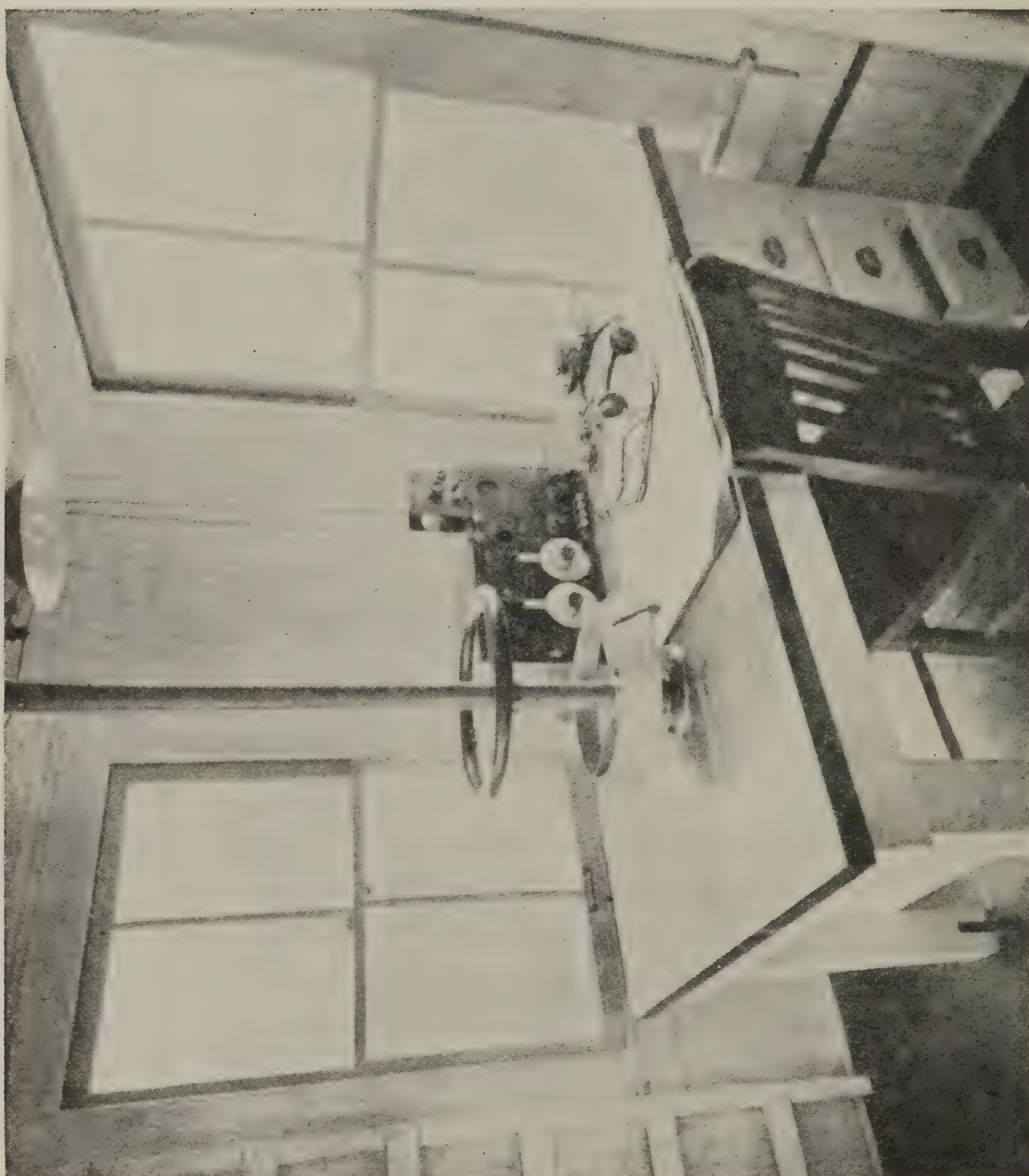


Fig. 372.—Interior View of Radio-Compass Receiving Room.

The Radio-compass shore station. The structures which comprise a radio-compass station include a compass house, quarters for the personnel, a power house, and a shed for fuel storage.

The distribution of the various structures will be influenced somewhat by the size and shape of the plot available. It is desirable, however. that the compass house be 200 feet to the seaward of any other structure on the reservation. The distance between the quarters, power house and fuel shed should be about 50 feet in order to reduce fire risk. If an antenna is to be erected it should be at right angles to the coast line and centered over the power house, as shown in figure 373.

Specific plans and specifications have been prepared for the erection of the radio-compass house. Every detail must be as planned, since the satisfactory operation of the equipment depends upon the proper surroundings. The foundations must be carefully placed, in order that there will be no settling nor vibration during storms. No metal sheet nor strip is to be used in the construction of the roof or observation platform. The arrangement of the instruments is carefully planned and they should be installed according to definite specifications, as shown in figure 372.

Arrangement of shore stations. The usual arrangement is a group of stations placed at the entrance of a harbor, as shown in figure 369, or in the vicinity of navigational hazards as shown in figure 371. The various stations in the group are interconnected by some method of communication to a master station having control of a radiotelegraph transmitter. The stations in the groups take observations simultaneously upon the vessel requesting radio-compass service. These observed bearings are transmitted to the vessel to be plotted by the navigator. The intersection of the bearings indicates the position of the vessel at the time the observations were made. If there are more than two stations involved, the accuracy of the work will be indicated by the precision with which any bearing crosses the intersection of the other two.

Equipment. The rapid development of the radio compass has resulted in frequent changes and improvements in the apparatus designed by the U. S. Navy. The principle features of design are given in figure 374. The apparatus shown consists of a rectangular coil frame $4' \times 4' \times 6'$ wound with 14 turns. This coil is mounted on a shaft that is free to rotate on the main supporting bearing (piece 1).

The position of the coil is determined by reading the dial, which is graduated in degrees ($0° - 360°$). The hair-line indicator is so set that when the reading is zero the coil is pointing north and south on shore stations, or to bow and stern when aboard ship.

The receiving equipment employed usually consists of the circuit shown in figure 374. The important feature of the receiving equipment is the amplifier which consists of three stages of radio-frequency

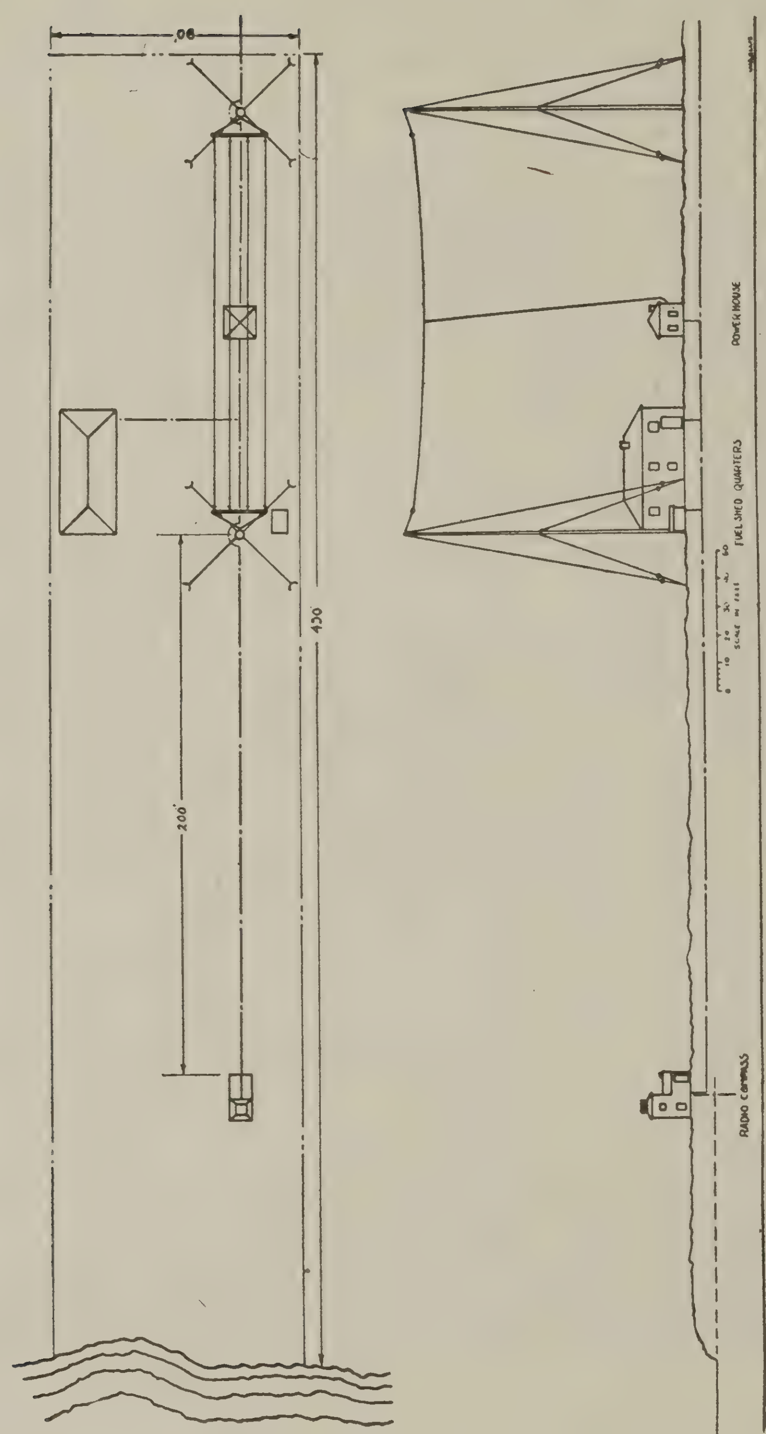
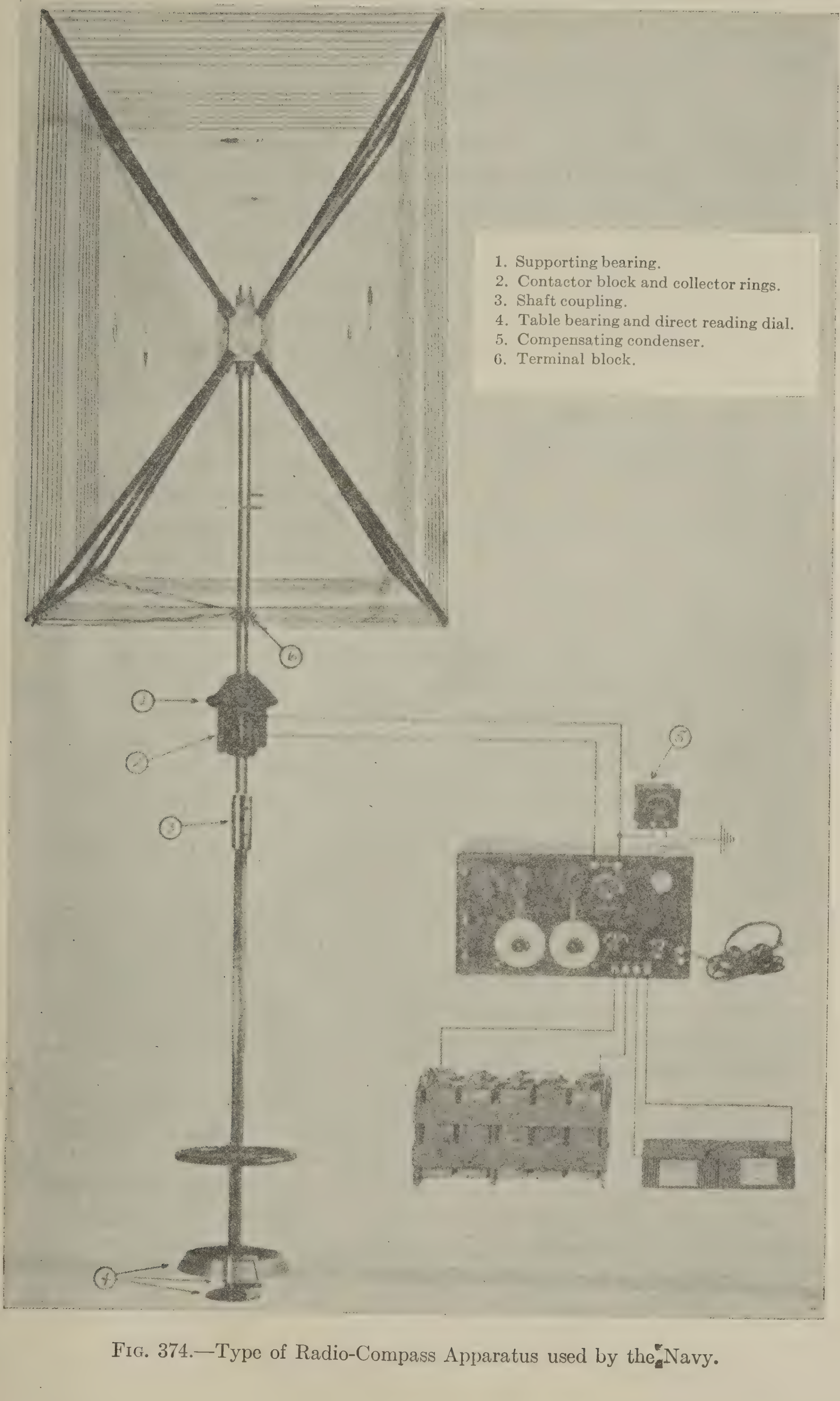


FIG. 373.—Arrangement of Radio-Compass Site.



1. Supporting bearing.
2. Contactor block and collector rings.
3. Shaft coupling.
4. Table bearing and direct reading dial.
5. Compensating condenser.
6. Terminal block.

FIG. 374.—Type of Radio-Compass Apparatus used by the Navy.

amplification, a detector and two stages of audio-frequency amplification.

B. Calibration of Radio-Compass Shore Stations.

The necessity for the calibration of the radio compass, whether installed on shipboard or on shore, arises from the fact that certain deviations are observed between the true bearing and the radio-compass bearing when rotating the loop through its azimuth. Repeated tests have shown that these deviations have, normally, a constant value when the incoming signal is within a certain band of wave lengths, and that the radio compass can be relied upon to give accurate bearings which are corrected by a table of deviation values to be applied to the radio-compass values. The procedure is to rotate a radio transmitter about the radio compass to be calibrated, taking simultaneous readings of both true and radio-compass bearings.

Calibration instructions. In the case of radio-compass stations on shore, all land lines must be completely installed and in operating condition previous to calibration. All radio antennas should be erected and proper precautions taken to insure that the antennas are disconnected at all times by means of an anchor gap, and that the transmitting apparatus is in working order.

The calibrating vessel should be maneuvered to a position approximately at right angles to the shore line. If the vessel is head-on to the radio compass house in this position there will be normally very little deviation of the wave front, and the true bearing as observed by the transit may be used as the setting for the radio-compass dial. In certain cases, a fixed radio transmitting station, preferably one from which the signals travel entirely over water, may also be conveniently used for this purpose, provided that the signal intensity is sufficient to give a good minimum.

Previous to starting the run, tests are made for tuning, wave length, clarity of note, and requisite power, followed by tests for the setting of the dial at the radio-compass station. The vessel should send continuous dashes of ten seconds duration with five-second intervals for five minutes, followed by a one-minute period of listening for communications from the radio compass station.

The transmitter on the calibrating vessel should be capable of withstanding abnormally heavy duty. The apparatus should preferably be a 500-cycle quenched-spark transmitter, power rating not less than 2 kilowatts, and be able to maintain a clear note under continuous operating conditions. The apparatus must be carefully tuned to the calibrating wave length. The radio-compass operating wave length for shore stations established for the United States is **800 meters** (**375 kc**).

The following features are important:

(a) The vessel must be visible from the compass station, or in case there is an obscured sector, from the triangulation points.

(b) The optimum radius of the circle is approximately 5 miles.

(c) As viewed from the compass station, the vessel should move with an angular advance of not greater than 2° in a minute of time.

(d) During the continuous calibration, at the end of the course, the vessel reverses her direction, repeating the run to balance out any errors due to time lag between compass-transit readings, as well as those caused by the center of radiation of the vessel's antenna not being at the lead-in.

Should a sudden change of deviation value be observed, the procedure in seeking the cause of this trouble is as follows:

(a) Recheck transit set-up on landmarks.

(b) Test **peep sight** alignment of coil system.

(c) Note any possibility of grounds or open circuits in the wiring of the apparatus.

(d) Investigate any possible change in communication lines, also in the antenna system at the station if there be one.

Preparation of the data. During the calibration, the divergence of the radio-compass bearing from the true bearing should be plotted in the form of a deviation curve, using cross-section paper. When the radio-compass bearing is greater than the true bearing, the deviation is considered **negative.** The value of this deviation is the ordinate, while the abscissa is the radio bearing for this particular observation. Distinctive markings should be used in plotting the two courses of the ship, as shown in figure 375.

A curve showing the variation of the compensating condenser in degrees over the calibrated sector should be plotted. This curve proves of value in a theoretical study of the deviation curve, and may also be used as a guide in the future operation of the station. This curve may conveniently be plotted with the deviation curve. The two curves should be distinctly marked.

Installation difficulties. Aboard the average ship the most desirable locations are either not available, or there are various metallic structures in the vicinity of the loop that produce deviations of varying magnitude unless certain precautions are taken. The stays to the mast and stack must be broken near their upper ends with strain insulators, and securely grounded to the hull of the vessel at their lower ends by means of flexible copper jumpers. The railings placed around the edge of deck houses must not extend around the compass coil. These railings should end at points several feet from the coil house, and it is advisable to insert nonmetallic rope horizontal members in place of metallic rail adjacent to the compass coil. If the space surrounding the loop is occupied by movable metallic lockers, or structures of a

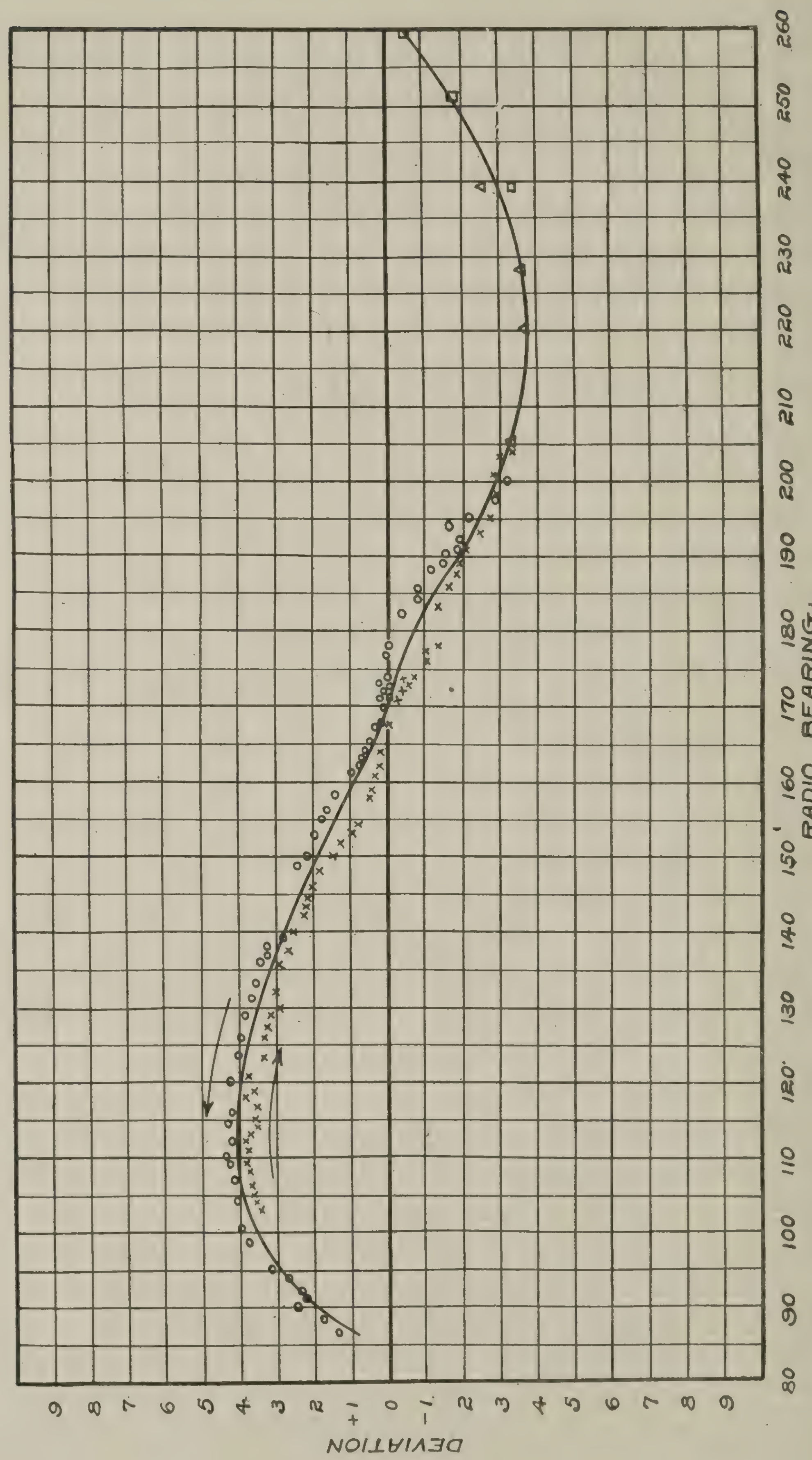


FIG. 375.—Method of Plotting Radio-Compass Deviation Curve.

similar nature, satisfactory performance cannot be expected. Closed metallic loops and metal structural details in close proximity to the compass coil cause excessive deviation.

All radio-compass bearings are taken as angles turned from the center line of the ship and the bow as zero degrees. In order that this may be accomplished, the dial must have the correct relative position with respect to the loop. The pointer should be placed so that the dial reading may be easily observed by the operator. The loop should then be turned so that the plane of the turns is at right angles to the center line of the vessel, and the dial made to register 0° or 180° and then clamped securely to the shaft.

Installation of equipment. The practice of installing the compass in the same compartment with the service radio installation, while being more compact, is not particularly desirable from the consideration of compass operation. When it is installed in the radio compartment it is difficult to place the antenna lead so that it will not seriously affect the compass.

The metallic hull of a vessel has an important influence on the direction of travel of the incoming radio waves. Instead of continuing in a straight line, the waves are bent and tend to follow the length of the vessel. It is this condition which produces the major portion of the deviations noted. Therefore, it is desirable to have the installation so located with respect to the hull as to distribute the deviations symmetrically.

The advantage gained by any particular height is small, provided that the height is 6 feet or over. This dimension refers to the loop itself and not to the operating room, and is subject to the further condition that there is no metallic superstructure within 50 feet of it. If it is impossible to obtain such a spacing at any point on the vessel, it is advantageous to mount the loop above all the superstructure except the funnels and stacks.

Should the location selected place the compass loop in the vicinity of a mast, the mast stays must be broken by strain insulators at a point near their upper end. The lower end of stays should be connected electrically to the metal structure of the vessel, in order that the electrical condition of these stays will remain constant.

Antenna leads will cause wide deviations which, furthermore, will vary with different conditions of the antenna circuit. It is, therefore, extremely desirable to locate the radio compass at some distance from all antenna leads. If such a location is impossible, the influence of the antenna lead may be reduced by carrying it in a grounded trunk to a point above the top of the compass loop. Precautions will then be necessary to keep the antenna in the condition existing during calibration at all times when the compass is being used.

If the compartment provided for the instruments is not of metal, it will be necessary to line it completely with a fine mesh copper screening. This lining should include the doors, with suitable grounding strips to complete the shield when the door is closed. The screening should be extended over all of the ports.

The loop system should be mounted on the deck over the operating compartment so that the control shaft will extend into it.

The equipment, as shown in figure 376, should be placed within convenient reach of the operator and, at the same time, be well away (approximately 4 inches) from the grounded screening or metal bulkheads. The batteries must be kept above metal decks and the leads run directly and as short as possible. No two leads should be fastened together, but are to be run separately and well supported.

The tests necessary to insure that the installation is operative have previously been described. The installer should lay particular stress upon the condition of surroundings in giving instructions to the operators. It is particularly desirable that an efficient system of communication with the bridge be available in order that the bearings obtained by the compass may be intelligible, since they are dependent for their true direction upon the heading of the ship itself.

Quality of minima. It is desirable that some investigation be made as to the quality of minima obtainable. In extreme cases, the capacity of the compensator may not be sufficient. This should be determined and corrected before the vessel leaves for calibration.

Installation defects. The most common cases of installation defects, their effects and remedies, follow:

The battery leads to the receiver are frequently run in lead-covered wire, which covering is generally grounded. This results in greatly increasing the capacity to ground of these circuits.

It is imperative that the entire radio-compass circuit, inside the receiving cabinet, as well as the connections between the various pieces of apparatus comprising the unit, including also the batteries, be kept as far from the metallic hull of the vessel as is practicable. For this reason, the connecting wires must **not** be encased in a metallic sheath. The **batteries must be raised from the deck and set out from the bulkhead,** and the receiver must be kept at least 3 inches from bulkheads. The leads from the compass coil should be carried on column insulators down to the receiver.

The most common defects in apparatus are:

(a) Defective receivers and amplifiers,

(b) Commutation (dc) noises,

(c) Dirty collector rings.

Test of apparatus. Very thorough tests should be made of the receiving apparatus previous to calibrating, noting the control of

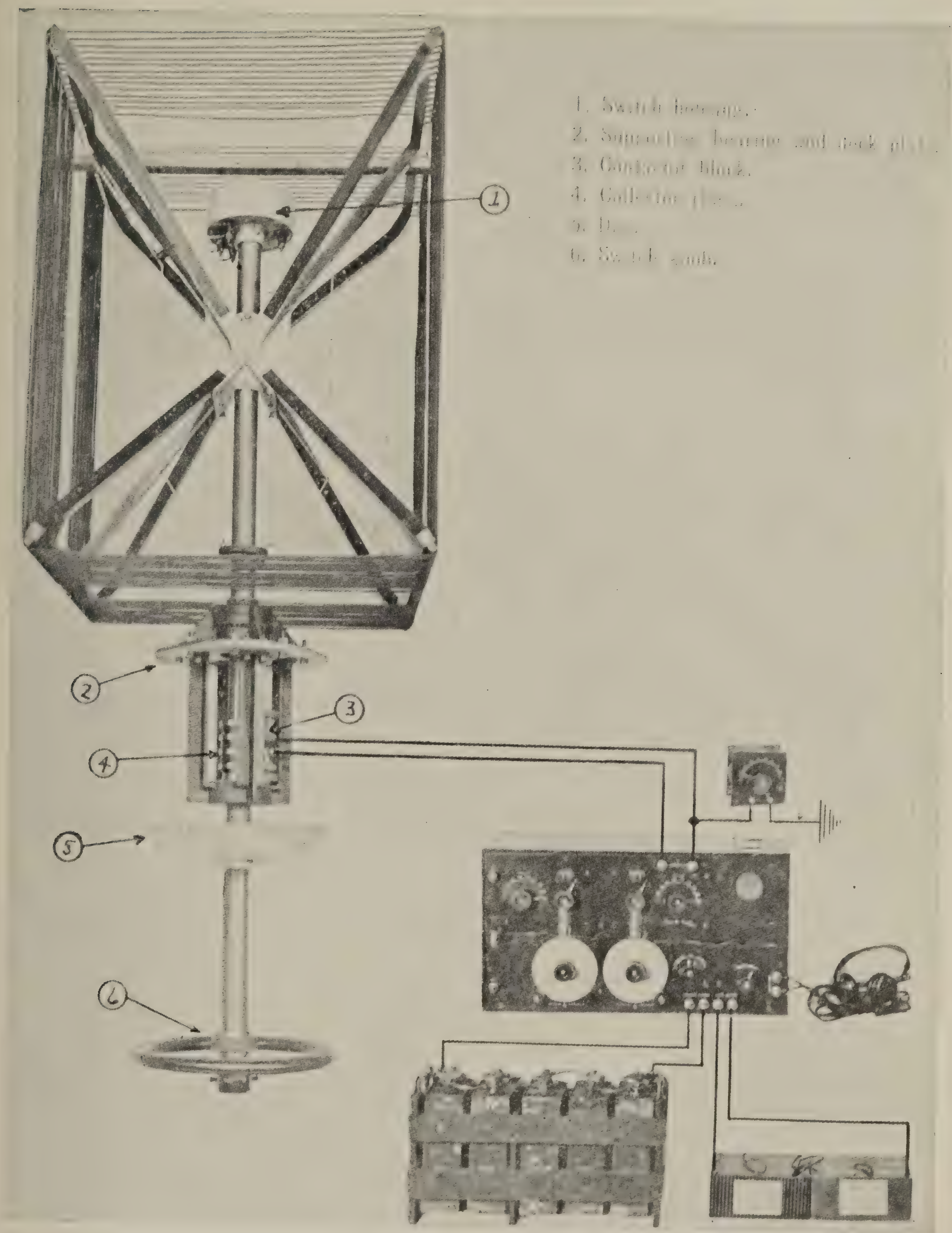


FIG. 376.—Shipboard Radio-Compass.

oscillations or the presence of excessive noises in the telephones, particularly with receiver oscillating.

A wave-length calibration of the receiver should be made by coupling a wavemeter to the compass coil. Curves should be plotted and the tuning condenser dial of the receiver suitably marked. This wave length calibration is a very good method of testing the operation of the receiving apparatus.

While testing the receiving apparatus, the coil system should be rotated to note the condition of the collector rings. Dirty collector rings will cause very loud, grating noises in the telephones, especially with receiver oscillating. The rings may be cleaned with either an oiled rag or the fingers or, if seriously corroded, the use of crocus cloth is recommended. If these rings are thoroughly cleaned, no noise will be heard while rotating the coil system.

Following the tests of the receiver, the quality of the minima and the operation of the compensating condenser must be tested. Any powerful near-by radio station may be used for this purpose. In taking a bearing, whether during calibration or in subsequent operation, **all antennas must be open—not grounded.**

Previous mention has been made that two minima are obtainable on any particular signal, and the observer will notice that one of these is much the more sharply defined. The coil system may be lined up manually. To do this, remove the top of the coil housing. The lead from the grid terminal of the receiver (the right-hand compass loop terminal to which the compensating condenser is attached) must be traced through to the coil winding. Turn the compass coil so that the grid end—i. e., the end to which this lead is attached—faces the starboard side of the vessel, with the plane of the winding fore and aft, estimating this alignment of the coil winding as closely as possible. The coil system is then held rigidly, the dial set screws loosened and the dial rotated until the 180-degree mark is **directly** in line with the pointer.

B. Calibration of the Radio Compass on Shipboard.

Purpose. The fact that the electromagnetic wave front is distorted in passing over a vessel's hull has been well established. The purpose of calibrating the radio compass on shipboard is to determine the magnitude of this distortion, technically known as **deviation**; also the direction in which the bending takes place, in order that these deviation values may be applied in subsequent operation of the radio compass. In practice, these deviation values are the only correction that need be applied to radio-compass observations.

Effective operation of the radio compass requires a certain amount of skill on the part of the observer, which cannot be expected of an untrained man. This skill can be developed only by constant application of the proper operating methods.

An idea of the utility of the radio compass should be borne in mind. The most important fact to be remembered is that the radio-compass bearings aboard ship are relative bearings and that, in order to obtain the true bearing of a radio transmitting station, the heading of the vessel must be known. The present day radio compass on shipboard has reached such a stage of development as to be comparable in accuracy to other instruments indispensable to the navigator. The potentiality of the radio-compass as a military asset and as a device for locating vessels in distress has likewise been well established.

As the effectiveness of future operation depends greatly upon the accuracy of the calibration, the necessity for painstaking care cannot be too greatly emphasized.

Method of calibrating. The general scheme of calibration consists in the orientation of a radio transmitting station about the radio compass while simultaneous observations are taken both visually and by radio compass. It is assumed that the ambiguity of the radio compass (that is, the fact that two bearings **approximately** 180° displaced are obtainable) is common knowledge. The fact that the two bearings are **only approximately** 180° displaced is the most important fact to be remembered. Therein lies the difficulty of utilizing the bilateral compass on shipboard, where the operating sector is 360°. This difficulty has been overcome by utilizing what is known as the **half-scale method of calibration.** To understand this method clearly, picture the calibrating vessel commencing her circuit of the vessel about to be calibrated at the bow and moving in a clockwise direction. From a position dead ahead to a position astern (0° to 180°), the radio-compass bearings differ from the visual bearings only by the amount of the deviation. When the calibrating vessel has reached a radio-compass bearing of 180°, the coil system is rotated to the region of 0°, and the bearing noted. This bearing, known as the **reciprocal bearing,** will not be exactly 0° when the direct bearing was 180°, because the two minima are not exactly opposite. The calibration is continued in this manner on the port side of the vessel, the visual bearings of from 180° to 360° corresponding to a second set of radio bearings of from 0° to 180°.

In the above description, the 180-degree sector to be used was selected arbitrarily; but, in practice, certain peculiarities of the station should be used as a guide in this selection. It will usually be found that one minimum is much sharper than the other, except at two opposite points, where the minima are practically alike. Between these points the sharper minima is always on the same side; i. e., with the grid end of the coil in the same general direction, regardless of the side from which the wave approaches. In general, the minima are poorest between these axes of symmetry and best in their general direction. On new type destroyers, the axis of symmetry usually bears about 30° and 210°, the

sharper minimum occurring when the grid end of the coil is toward the starboard side of the vessel. In this sharper sector, less capacity is required for compensation, and it is the logical sector to adopt for a half-scale calibration. The above mentioned method is depicted in figure 377.

In this way, only one of the two possible minima of the compass is utilized for the complete circuit of 360°, although there are two relative or true bearings corresponding to any radio bearing. In practice, it is impossible to determine the proper bearing from a single

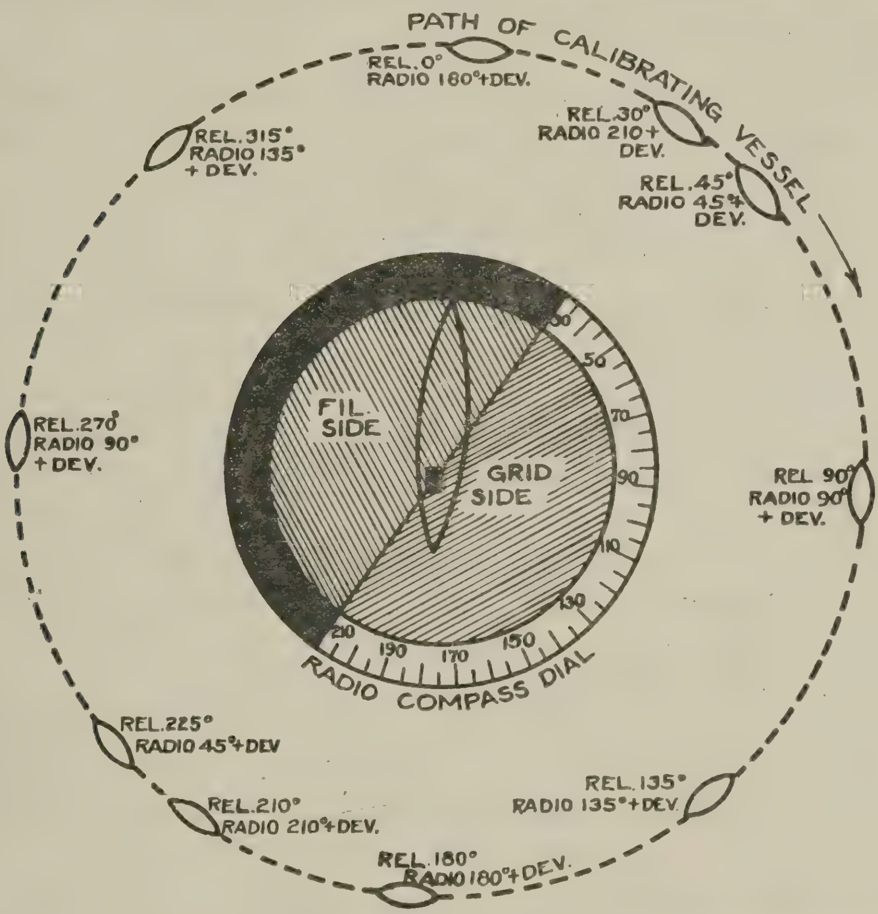


FIG. 377.—Graphical Illustration of Half-Scale Method of Calibration.

observation except in coastwise sailing where, knowing the vessels head, the proper bearing becomes evident. Where it is impossible to select the proper bearing, two observations must be made with a run between. The intersection of the two lines of bearing indicates the location of the transmitter.

The calibration should be carefully planned, particular attention being given to the choice of personnel for observers. The next consideration is the type of calibrating vessel, or, to express it differently, the source of radio signals.

The use of shore stations for calibration purposes is not urged, although they may be used if no other source is available. With a fixed transmitter it becomes necessary to swing the vessel. If sufficient care

is exercised in swinging the vessel within a very restricted circle, this method should prove satisfactory.

Having selected the calibrating vessel, the next step is to select a suitable stretch of navigable water in which to perform the calibration. It is well to state here that it is impossible to calibrate the radio compass with the vessel tied up at the dock. When using a vessel with a transmitter of 2 kw or over as the calibrating vessel, the radius of the vessel's encircling course should preferably be not less than 4 miles. If the power of the transmitter is insufficient to give a minimum of one-half degree or less, the radius may be reduced to 2 miles, although there is danger of considerable error at a short radius, because it is difficult to determine the exact center of radiation of the calibrating vessel's antenna. To express this differently, the center of radiation of an antenna is not necessarily the lead-in. In fact, on certain types of vessels (with the **F** antenna, a main radio room forward) experimental data indicate the true center of radiation to be slightly forward of the bow. Thus, if the calibrating vessel were brought within a radius of a mile or so from the compass and swung, using the lead-in of the main antenna as a pivot, the observed radio bearing would alternately increase and decrease, becoming correct only when head on or away from the compass. If swung with the bow as a pivot, no appreciable variation of the radio bearing would be observable. When the calibrating vessel possesses a **T** antenna, the lead-in may be assumed as the approximate center of radiation. Errors that may be appreciably great at a short radius become vanishingly small as the distance is increased. In the calibration of radio-compass shore stations, any possible error due to this condition is balanced out by making a reverse run, using the same visual target on the vessel, which may be a mast or a stack, for both runs. As time is usually not available in ship calibrations for such a degree of refinement, it becomes more essential to keep the radius large and to sight on a point as close to the true center of radiation as possible.

Smooth water should preferably be chosen when performing the calibration, since the yawing or sudden swinging of the vessel to be calibrated may seriously interfere with the accuracy of both the compass and visual observations.

The instrument most commonly used for determining the visual or relative bearing, is the pelorus. The general procedure is to use the two bridge peloruses, changing from one to the other to obtain an unobstructed view. It is most important that these peloruses be checked previous to calibration, as considerable errors in both peloruses have frequently been found. This method necessitates the use of a stadimeter or range finder, with subsequent computation of parallax arising from the displacement of the radio-compass installation and the bridge.

The laborious procedure of computing parallax, as well as the possible errors arising from it, may be obviated by mounting a pelorus and stand on top of the radio-compass coil housing. This method has been very successfully used—the obscured sectors due to the superstructure of the vessel and the main mast are not extensive, and the angular increment of the calibrating vessel may be readily plotted against a time abscissa.

Due to the numerous adjustments of the receiving apparatus and the possibility of interference from extraneous signals, compass observations cannot always be made at a prescribed instant. The most reliable method is for the compass observer to give a **mark** signal the instant he has obtained the bearing, followed a second or so later by the value of the bearing in degrees and estimated half or quarter degree. The mark signal should be relayed instantly through the voice tube to the bridge by the assistant, and the observed radio bearing recorded. On the bridge, the mark signal must be repeated instantly by the assistant to the pelorus observer, who is standing by. The pelorus observer, having obtained his bearing to a half or quarter degree (greater accuracy not being required), informs his assistant, who in turn relays the data through the voice tube to the recorder. This plan has been used satisfactorily during many calibrations. It requires quick action by all hands, and a short drill immediately preceding the actual calibration will insure perfect cooperation.

If the pelorus is mounted on top of the compass coil housing, the assistant may conveniently stand on the deck to relay the mark signal from the compass observer to the pelorus observer and take down the data. With this method, as the calibrating vessel is approaching or emerging from either obscured sector, the pelorus observer should inform the data recorder, so that the time in minutes and seconds at which the observations were made may be recorded. The time at which the radio bearings are observed, within the obscured sector, should, of course, be noted and plotted.

The angular velocity of the calibrating vessel and the rate of taking bearings are important considerations. It has been found that the most satisfactory results are obtained when the calibrating vessel moves at a rate of approximately 2 degrees in a minute of time. At this rate, the entire circuit may be made in three hours. A greater rate increases the possibility of inaccurate observations, and a lesser rate requires more time than is actually necessary. A radio-compass observer should bear in mind the fact that a few good bearings are vastly superior to a multitude of scattered ones. Under normal conditions he should be able to average one to two observations per minute.

As an example of the calibrating procedure, using the half-scale method, consider the calibrating vessel starting her circuit at a relative bearing of 0°. The zero section of the radio-compass dial is painted black.

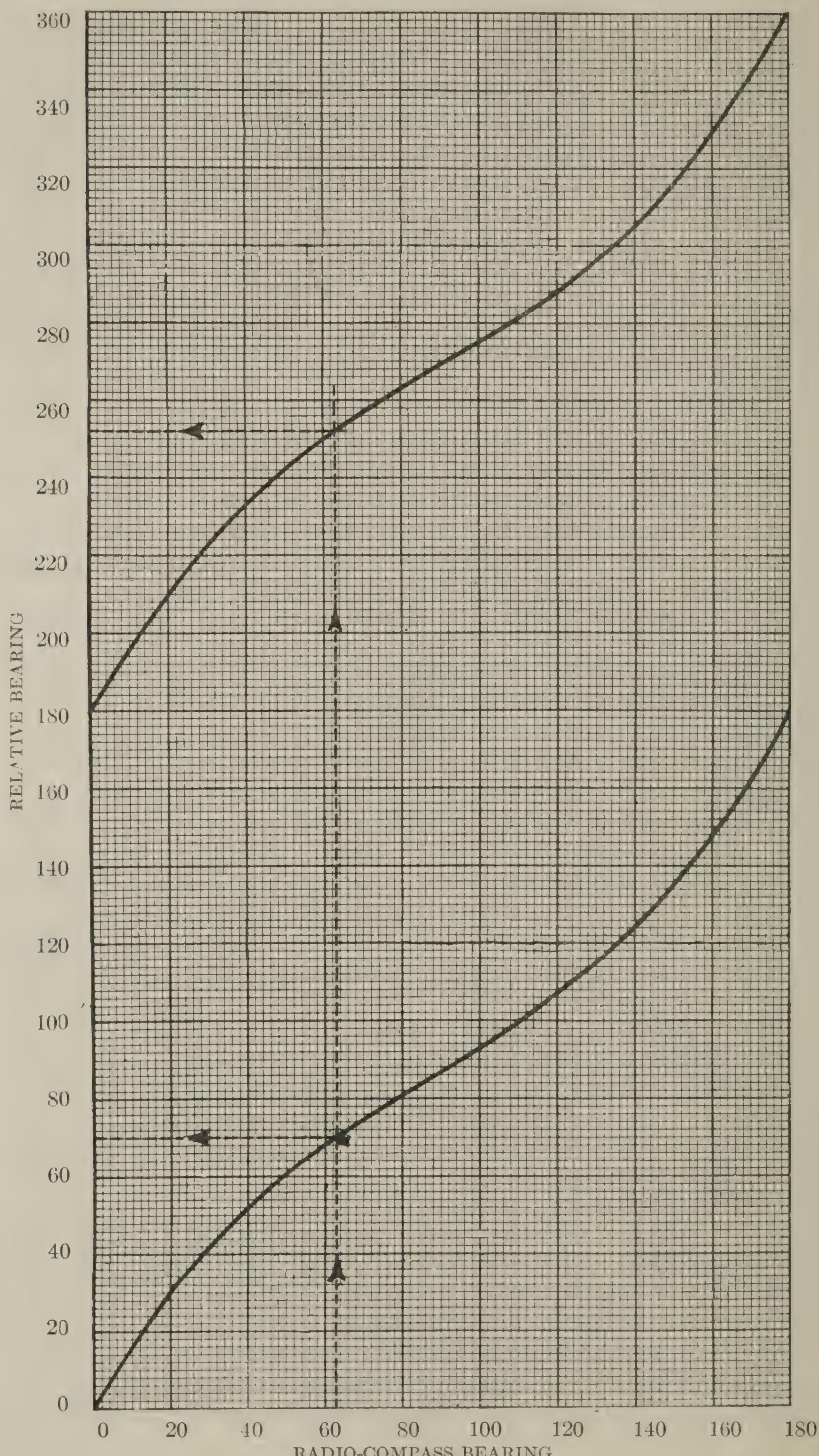


FIG. 378.—Method of Plotting the Calibration Curve.

However, a compass bearing will be obtainable in the region of 180° on the dial. At the instant of obtaining the radio bearing, the observer calls "mark"—the assistant relays the "mark" to the pelorus observer—the radio-compass observer calls off the bearing in degrees and estimated fraction (half or quarter degrees). Meanwhile, the pelorus observer takes his observation (to the nearest half or quarter degree) and relays the bearing (through his assistants) to the data recorder, who writes down the pelorus observation in the proper column adjacent to the corresponding radio bearing. With a little practice, this system may be worked easily and without confusion. Ten seconds should be ample time to obtain and record a single bearing. Should any confusion arise, the questionable bearing must be eliminated.

This procedure is then carried on until the calibrating vessel has reached a radio bearing of 210° (or over). When the observed bearing falls on the blackened section of the dial, the coil must always be reversed, and the reciprocal bearing used. If the last observed radio bearing was 210°, the reciprocal bearing will be found in the vicinity of 30°.

The calibration is then continued, using dial reading from 30° on, until the observed radio bearing once more falls on the blackened scale at 210°, corresponding to the calibrating vessel's relative position of approximately 210°. As the vessel continues to advance beyond this point the coil system must once more be reversed and, as the vessel continues from 210° to 360°, relative bearing, the corresponding radio compass readings will advance from 30° to 180°, approximately.

The variation of the compensating condenser should be noted and recorded. These observations (in degrees) may be recorded at about every 10° of azimuth.

The calibration curve. The data may best be plotted in the form of a curve, as shown in figure 378. The large size cross-section paper (10 lines to the inch) is suitable, 18 spaces being required on the axis of abscissas (radio compass bearing) and 37 spaces on the axis of ordinates (relative bearing). When the paper has been properly drawn up and the degrees marked, the observed points may be placed in position. Begin at the first observed radio bearing on data sheet and note its value and the corresponding pelorus reading (corrected pelorus reading if the bridge pelorus has been used). Next, place a point on the cross-section paper at the exact intersection of the lines corresponding to these values of radio-compass and relative (pelorus) bearings. Continue placing these points until all the observations have been recorded. A curve may then be drawn (with the aid of a french curve) through the average of these observed points. The closeness with which these points adhere to this line is an indication of the accuracy with which both sets of observations have been made. This curve represents the deviation of the radio compass. Curves showing variation of the compensating condenser should

be plotted, using the observed bearings as the abscissas, as in plotting the deviation curve.

Correction Table. The next step is to make up a typewritten form of deviation values. The corresponding relative bearings (two) for every radio compass bearing are to be obtained by noting where the vertical line corresponding to the particular radio-compass bearing intersects the curve which has been drawn through the average of the plotted points. At the **exact intersections,** the corresponding relative bearings (two) whose horizontal lines intersect the curves must be noted and recorded. This is to be done for each degree (compass bearing).

5. OPERATION.

The operation of the radio compass in taking a bearing is the same whether it is a ship or shore installation. The same procedure is followed when taking a bearing with a calibrated radio compass as when calibrating. The deviation of the radio compass was determined during the calibration, and the curve drawn. In that case—the bearing of the transmitter was taken visually. After calibration, entire dependence is placed in the calibration in order to find the bearing of the transmitter without resort, of course, to any visual means. It is therefore necessary that the deviation, as found during the calibration, be applied to the bearings read on the radio-compass dial in conjunction with the bearing of the ship's head. The result will be the true bearing of the transmitter from zero degrees.

Bearings should invariably be taken in accordance with the following procedure:

(*a*) Take bearing preferably with clear tone signal, or else full oscillating condition. With receiver oscillating, the minima are much more sharply defined. However, interference will obscure an otherwise good minimum, because tonal selectivity is lost.

(*b*) Tune signal carefully, with coil system at or near the maximum.

(*c*) Rotate coil system through the null point of the signal, oscillating it to and fro, finally bringing it to rest at estimated center of the **minimum.**

(*d*) Adjust compensator **carefully,** in the same manner as the coil system is rotated, seeking the exact center of the minimum.

(*e*) Retune carefully. (Always tune with compass coil **off** the minimum.)

(*f*) Repeat (*c*).

(*g*) Repeat (*d*). This operation is now to be repeated in conjunction with the rotation of the coil system. The final adjustment of compensator and coil should give a point of silence on some particular bearing. It is well to note here that there are some compass stations at which an absolutely silent minima is unobtainable, although these minima are

exceedingly sharp and the station operation is excellent. The width of the zone of silence depends on the power of the transmitter, the distance of the transmitter, and the sensitivity of the receiver.

The following **precautions** should be observed:

(a) Never attempt to obtain bearing with receiver regenerating or barely oscillating, as in this condition the circuit is very sensitive to any changes in its decrement which will result from the adjustment of either tuning condenser or compensating condenser. Due to the variation in signal intensity resulting from these changes, proper operation will be difficult. If signal intensity is low, regeneration should be used after all adjustments of tuning and compensation have been made.

(b) Adjust tuning condenser for maximum signal intensity. **All tuning must be done with the coil off the minimum. Incorrectly tuned circuits will result in erroneous bearings.**

(c) Do not swing coil through a large arc. Ten degrees either side of null point is sufficient at start; after retuning and recompensating, the swinging of the coil should be further restricted to about 1 to 3 degrees on either side of null point, depending upon the sharpness of the minimum.

(d) This should be restricted as in (c).

(e) At stations where compensator values change greatly throughout the azimuth the detuning may be considerable. Therefore, the necessity for retuning is important.

(f) See (c).

(g) When coil system and compensator have been successively adjusted to give a silent point, the entire attention may be devoted to the coil system. The coil system should be slowly rotated within restricted arc, careful attention being given to estimating the true center of the minimum.

Salvage and distress emergencies. Methods for solving 180° ambiguity. When the occasion arises requiring a vessel fitted with radio-compass equipment to proceed toward another vessel in distress whose position is unknown, it is necessary to solve the 180° ambiguity with all possible dispatch. Probably the best procedure under such circumstances would be to steam at right angles to the observed radio bearing for several miles and, in the meantime, obtain several bearings over this base line. By a plot of the resulting bearings from the plotted course of the vessel it will be observed that on one side of the vessel the bearings converge while on the other they diverge. The position of the vessel in distress is obviously at the intersection of the various plotted bearings. This actual fix, however, will in all probability be only approximate as far as the actual distance to the objective is concerned, due to the short base line available. By setting a course along the last radio bearing obtained and in the direction indicated by the point of convergence, the navigator may rest assured

that he is proceeding in the correct direction. As the rescuing vessel draws near the objective, it may be possible to make further radio observations so that no time will be lost in proceeding directly to the vessel in distress.

6. FAULTS AND REMEDIES.

New stations. Should the compass station be a new one and difficulties arise during the calibration, the remedies outlined in the following should remove the difficulty. On the other hand, if the previous calibration of the particular station has proved satisfactory and good minima have been obtained throughout the operating sector, it is but reasonable to expect the station to continue to operate in the same manner. If, however, a considerable change in the quality of the minima throughout the operating sector, or in any particular part of the sector should be detected, the following remedies are to be studied and applied.

1. Broad with residual signal.

Possible causes.—(a) No compensation (open circuit).

(b) Nearby grounded antenna.

(c) Undrained communication lines.

(d) Grounding of radio-compass apparatus, wiring or batteries.

(e) Moisture on coil windings, or terminal rings.

(f) Weak signals.

(g) Faulty screening of operating room.

Remedies.—(a) Overhaul wiring, particularly compensating condenser connections.

(b) Note condition of anchor gap in antenna lead. This should be open. Test across this gap with battery and buzzer.

(c) All communication lines should be adequately drained of radio-frequency currents.

(d), (e) Test insulation of radio-compass circuits for grounds.

(f) Broad minima are caused by weak signals.

(g) The screening of the operating room must be kept intact. This is very important at certain compass stations. On ships, the room is usually built of metal.

2. Broad without residual signals.

Possible causes.—(a) Insufficient power at transmitter.

(b) Transmitter beyond normal working range.

(c) No oscillations or no amplification.

Remedies.—(a) The normal operating range of radio compass is from 100 to 150 miles with 2-kilowatt transmitter. When signals from 2 kw transmitter are silent over a dial reading of about 2°, the transmitter is beyond the normal working range of station.

3. Very sharply defined.

Possible causes.—(a) Transmitter with excessive power.

(b) Transmitter very near at hand.

(c) Excessive amplification.

Remedies.—(a), (b) This condition is not considered as unfavorable. In fact, it contains all that is desirable in radio compass operation. During calibration of a shore station the best radius is 5 miles. This results in a minimum about one-half of a degree in width. If, however, the calibrating vessel approaches the compass station to within a radius of a mile or so, the minimum may be so sharply defined as to be scarcely obtainable; in fact, merely touching the compass wheel may result in a tremendous change in signal intensity.

(c) Amplification may be reduced.

4. Varying degrees of sharpness through azimuth.

Possible causes.—(a) Grounded antenna nearby.

(b) Undrained communication lines.

(c) Location of station.

(d) Closed metallic loops nearby.

Remedies.—(a), (b) To a certain degree, slight variations of the quality of the minima are found at all compass stations. If this defect should become very pronounced, it may be concluded that the trouble is purely local. Test antenna. Test communication lines.

(c) Move station.

(d) All mast stays near compass coil should be insulated, preferably at the top.

5. Erroneous bearings.

Possible causes.—(a) Nearby grounded antenna.

(b) Undergrounded communication lines.

(c) Closed metallic loops nearby.

Remedies.—(a), (b) With nearby grounded antenna, especially if tuned to certain wave lengths, the compass will point to antenna, due to its reradiation. This is prevalent on shipboard. Undrained communication lines have been the source of stationary minima at shore stations.

(c) See paragraph 4 (c).

6. Apparent shift of minimum (erroneous bearings).

Possible causes.—(a) Mechanical slip in coil system.

(b) Undrained communication lines.

(c) Grounded antenna nearby.

(d) Grounding of radio compass circuit wiring or batteries.

(e) Diurnal variations of fixed transmitting station.

(f) Diurnal variations at compass stations.

(g) Faulty screening.

(h) Improper operation.

Remedies.—(a) Test coil system alignment. Any slip will give a constant error for all bearings.

(b), (c) See paragraph 4 (a), (b).

(d) Test.

(e) This is a research problem at present being studied. Certain variations in the bearings of fixed transmitting stations have been observed, although it is difficult to differentiate between the variations in the transmitter and those prevailing at the compass station.

(f) Experiments indicate that bearings on fixed transmitting stations within the operating range and situated at points where no land intervenes remain constant to within one degree. Bearings on fixed transmitting station over land and particularly when tangent to the coast line are subject to some diurnal variations. (See par. 8 (c), (d).)

(g) See paragraphs 1 (g), 8 (e).

(h) Study operating instructions carefully.

7. Not opposite (180° displacement).

Possible causes.—(a) Extraneous in-phase current in compass coil.

Remedies.—(a) The ideal condition of having the two minima exactly opposite is seldom found. A displacement of 3 degrees is not considered abnormal; however, if displacement exceeds this value, the operation of the station becomes questionable, as this indicates that an excessive amount of extraneous in-phase current is being induced into the coil system, which effect is subject to some variation. The amount of this displacement will vary throughout the azimuth. Test antenna. Test communication lines.

8. Variation of compensator values for a given azimuth.

Possible causes.—(a) Grounded antenna nearby.

(b) Undrained communication line.

(c) Diurnal variations at station.

(d) Tuning of transmitter.

(e) Condition of screening.

Remedies.—(a), (b) Test antenna. Test communication lines.

(c) The compensator values for particular azimuths do not remain absolutely constant, but vary within restricted limits, due to changing conditions in the surrounding bodies, chiefly caused by the degree of moisture in the air or on the surface of the ground. Considerable change in the deviation curve and compensator values between the two extremes of very dry and very wet spells of weather has been observed at some compass stations. The importance of having a permanent ground connection at the compass station must not be overlooked. Then, too, at compass stations where the compensator values are sharply defined, which are associated with sharp minima, the capacity of the operator's body becomes very apparent, and changes, such as lifting or lowering the feet, may affect the compensation to a noticeable degree.

(d) The type of transmitter on which the bearing is being obtained may noticeably affect the compensation values. There are, in fact, certain types of transmitters on which it is impossible to obtain bearings;

these are likewise unresponsive to adjustments of the compensator condenser. (See par. 4 (e), (f).)

(e) Any radical change in the condition of the operating room screening may affect the compensation and also the deviation. This is more apparent at certain compass stations. The screening should be kept intact and frequently inspected.

Compass coil defects, in general, may be considered as mostly mechanical. The construction of the various parts comprising the coil system should be thoroughly studied. The electrical defects that can possibly develop in the coil system may be readily located.

Radio-compass receiver and amplifier troubles may be readily located by anyone who has had some general experience with radio receiving apparatus. The compass coil system may be treated simply as a large inductance inserted in series with the secondary inductance contained within the receiver. Any troubles that may develop, such as lack of signals, howling, etc., may be treated as in an ordinary receiver.

Testing of circuits and apparatus. Testing of the receiving qualities of the radio-compass equipment may best be performed with a wavemeter. Place the wavemeter close to the compass coil, allowing clearance for rotation of coil system to obtain proper coupling; adjust wavemeter to proper wave length (800 meters) and start buzzer. In this manner the signal intensity, tuning of the coil system, control of oscillations, and control of amplification may be tested. Should there be no response, note if all switches are thrown to proper positions, tubes lighted, all wiring connected, etc. Test telephones with low-voltage battery for characteristic click. To test continuity of all circuits, the telephones and battery, or buzzer are very useful.

The compensating condenser. This is a small variable condenser (0.0003 μf) and is connected between the compass coil lead extending to the grid side of the detector tube from the ground. Some compass stations require a much greater value of capacity for compensation than others. This condenser was designed for and has sufficient capacity for the normal compass station. Should a particular compass station require a greater value of capacity for adequate compensation, it is an indication that the station is not well situated or that the condition of the antenna or communication lines is at fault. To test its operation (for open-circuits) select some sharply tuned signal (wavemeter may be used), place this condenser on either zero or 180°, tune the compass coil to the signal very carefully with the tuning condenser of the receiver, then reverse the position of the compensating condenser. A large change in the compensating condenser capacity will alter the tuning of the compass coil and, hence, the signal intensity.

Test for short-circuit with battery and telephones, or buzzer.

The **dc resistance** of any compass coil, wound with No. 16 Navy standard bell wire, may be checked by measuring the length of the conductor and allowing 260 feet for one ohm.

Communication lines and other lines entering the radio-compass house, particularly when they are carried near the receiving circuits, are frequently a great source of trouble, causing excessive amplitude in deviation and resulting in faulty operation of the radio compass. The trouble results from the presence of radio-frequency currents (signals) picked up by these wires and induced into the receiving circuits. In practice, all communication and power lines entering a radio-compass station are treated by inserting radio-frequency chokes in each line at several points, with the necessary drain condensers added.

Installation defects usually consist of excessive capacity to ground, or direct grounding of radio-compass circuits. In ship installations the most flagrant cases are:

(a) Battery leads in lead-covered wire.

(b) Batteries placed too close to metal bulkhead or deck.

(c) Coil leads or connecting leads too close to metal bulkhead.

(d) Receiver too close to metal bulkhead.

(e) Mast stays in vicinity of compass coil form closed loop.

(f) Antenna lead-in.

In shore installations, the usual defects are the presence of lead and armored cable or any grounded conductors in close proximity to:

(a) The receiver.

(b) The batteries.

(c) The telephone cords.

(d) The remedies are obvious. In general, a minimum distance of 5 inches should be allowed between any of the circuits or apparatus and ground, the ground leads to the compensating condenser and to the ground terminal on the receiver (ship compass installation) being the exceptions; these leads, where they run parallel to the receiver, should be properly spaced. The importance of breaking up any closed loops of large dimensions near the compass coil must not be overlooked, as such loops have been known to cause deviations as high as 45°. The best method is to insert a strain insulator at the top of each of the stays, grounding these stays at the deck. Any change of this kind should be done before the calibration.

The antenna lead-in (on shipboard) should be brought into a grounded trunk where it passes the compass coil housing, and the rat-tail should be securely held in a definite location.

Excessive deviation and compensation. At shore radio-compass stations, a certain amplitude of deviation is normal to the station, depending on the characteristics of the site. Thus, it has been found that a compass station situated very close to the surf on a flat, sandy

beach will exhibit a deviation amplitude (maximum) of about three-fourths of a degree, increasing to about 2° for a station located 300 yards inland and 25 feet above sea level. At compass stations such as these, the compensation values throughout the azimuth will vary from 100 to 200 $\mu\mu$f. Should a compass station in a similar location exhibit a deviation curve whose amplitude greatly exceeds these values, the condition of all antennas and nearby metallic circuits, should be investigated for the purpose of reducing this deviation to normal.

In cases where compass stations are situated on or near high rocky ground, it is impossible to estimate the deviation that will be normal. Such locations should be avoided as much as possible and, if circumstances necessitate a compass station in such a locality, the site should first be tested by a trial calibration. In general, it has been found that the operation of compass stations exhibiting a deviation curve of low amplitude is more consistently accurate.

In the case of radio-compass installations on board ships, the same general conditions apply, although not so closely, as the factors which determine the amplitude of deviation are more constant.

Neglecting the effect of local metal bodies on shipboard, it has been found that deviation is nearly proportional to the length and height of vessel. When extraneous effects are removed, the average for destroyers should not exceed 6°. For battleships and larger vessels it will be from 20° to 25°. It has been very evident from work on destroyer compasses, that any metal body in the vicinity of the coil house which forms a closed electrical loop will cause excessive deviation. Brass rails or wire rope, awning supports and, particularly, mainmast stays forming loops will cause deviations as high as 45° on destroyers. When these influences are removed, the calibration curve passes through zero deviation very close to 0°, 90°, 270°, and 360°, which is normal.

While battleships generally show a maximum of 20° to 25° and are approximately 600 feet in length, one vessel 700 feet in length (the **Mount Vernon**) had a maximum of 15°. The later installation was located much higher than is ordinarily the case.

SECTION III.

RADIO MEASUREMENTS AND PRECISION INSTRUMENTS.

CHAPTER I. WAVEMETERS.

The **wavemeter** is an instrument which measures the frequency and, hence, the wave length of radio-frequency currents, and determines the natural frequency and the natural wave length of radio circuits. The wavemeter is not only the fundamental radio instrument, but is also the most generally useful measuring device that the radio engineer has at his disposal. By its use, inductances and capacities can be compared, the decrement and resistance of circuits can be determined, resonance curves can be obtained, and circuits tuned to be resonant to a predetermined wave length; in fact, all radio measurements are directly, or indirectly, dependent upon the wavemeter.

Essential wavemeter circuit. The essential wavemeter circuit consists of a capacity and an inductance in series, with some device to indicate either (a) the current flowing in the series circuit, or (b) the voltage across a part or the whole of the capacity or the inductance. The wavemeter is thus seen to be a simple radio circuit, the natural frequency of which is determined solely by the capacity and the inductance included in the circuit. Hence, the wave length can be obtained from the formula

$$\lambda = 1885\sqrt{LC}$$

where λ = wave length in meters,
C = capacity in microfarads,
L = inductance to microhenries.

Since a circuit containing a given capacity and inductance in series can be resonant only to the wave length which corresponds to its oscillation constant LC, it is usual to provide a variable capacity, a variable inductance or both, so that the wavemeter can be made resonant to any wave length within a given range. In its usual form, the wavemeter consists of an air-dielectric, variable condenser and a set of inductance coils. The coils are so chosen that the range of wave lengths obtainable with one coil and the condenser overlaps considerably the ranges obtained with the next smaller and the next larger coil. In this manner, the natural wave length of the wavemeter can be varied, in a continuous manner, over the entire range of wave lengths for which the wavemeter is designed.

Calibration of wavemeters. A wavemeter must be calibrated before it can be used to measure wave lengths. It is usual to have a standard wavemeter and to calibrate other wavemeters by comparing them with the standard. A method of calibrating a standard wavemeter in absolute value is given in Measurement No. 1. Other wavemeters are then calibrated in terms of wave length by comparing them with the standard wavemeter, according to the methods given in Measurement No. 2. A later practice in calibrating wavemeters was in the use of a quartz crystal oscillator as a driver, and utilizing the harmonics of this crystal to ascertain points on the calibration curve.

The calibration of wavemeters was formerly made in terms of wave length, although frequency calibrations are now used, especially for the very long wave lengths. After calibration, the wave lengths corresponding to the settings of the condenser and coil can be plotted on cross-section sheets with divisions the condenser as abscissas and the corresponding wave lengths, or frequencies, as ordinates; or, the calibration can be engraved on the condenser scale at every 10 or 20 divisions. The first method is compulsory in the case of standard wavemeters, because it is frequently necessary to read the wave length corresponding to a condenser setting which, in turn, must be read to a decimal part of a division. The second method is in general use in commercial wavemeters where such precision is not necessary.

Classes of wavemeters. Wavemeters are divided into two general classes, **standard** and **commercial.** The standard wavemeter is essentially a precision instrument for use in a radio laboratory. It is especially designed with a view toward obtaining constancy, low rf resistance, and, hence, sharpness of resonance indication due to the resulting small decrement. The condenser is ordinarily equipped with a vernier as an aid to precision. Since the condenser is the weakest part of the wavemeter, and constancy of calibration is of prime importance, the standard wave meter is not portable. In addition, the standard wavemeter is generally constructed in skeleton form, that is, it is not enclosed in a box. Compactness and portability are sacrificed in order to gain the characteristics given above.

The **portable commercial wavemeter** is built in a light and compact form for ease in transportation and manipulation about circuits under measurement. The condenser must be of rugged construction to withstand the jarring, etc., incident to normal careful use in the field. The inductance coils are generally of one form and physical size, so that they can be substituted one for another. Such a wavemeter is usually enclosed in a shielded box, with only the coil on the outside. All of these points of construction affect the electrical characteristics and result in a wavemeter that is less sensitive, having a larger decrement and, therefore, a less sharply defined resonance indication than the standard.

Devices for indicating resonance visually. When a wavemeter is tuned to resonance with the impressed emf, the current induced in the wavemeter is a maximum. Consequently, a suitable radio-frequency ammeter connected in series with the wavemeter circuit can be used to indicate resonance. Resonance between the wavemeter and the driving circuit is obtained when the ammeter shows a maximum deflection.

This method of indicating resonance is shown in figure 379. The indicating instrument is usually a thermogalvanometer with a scale

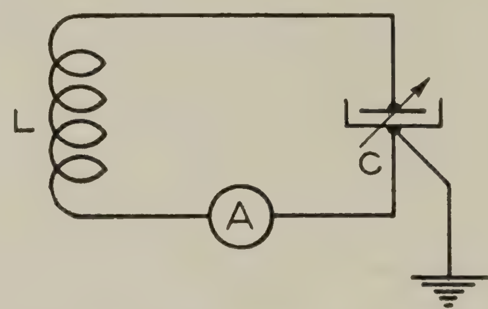


FIG. 379.—Wavemeter with Unshunted Thermo-Galvanometer.

marked in arbitrary divisions, the deflection of the pointer being proportional to the square of the rf current flowing in the circuit.

The rf resistance of such an instrument is rather high, being approximately 4 ohms. More sensitive instruments generally have even higher resistances than this. The addition of even 4 ohms to the total rf resistance of the wavemeter results in broad resonance indications. The thermogalvanometer is, therefore, frequently shunted with a noninductive resistance or other type of shunt, thereby reducing the decrement of the wavemeter, but at the expense of current sensitivity.

Alternate methods of using the thermogalvanometer are given in figure 380. In figure 380 (*a*), the instrument is inductively coupled to the wavemeter circuit by means of a single-turn loop. Less resistance is inserted into the wavemeter by this method. It is essential, however, that the loop be **fixed** in position relatively to the wavemeter coil, since any change in the position of the loop circuit will affect the calibration of the wavemeter. One method of accomplishing this is to imbed the single turn in each coil form.

In figure 380 (*b*), the thermogalvanometer is connected across a few turns of the inductance coil, which act as a shunt. This method necessitates three terminals on each coil and some mechanical device to ensure that the same turns on each coil are used as the shunt. The current sensitivity of the wavemeter is, of course, somewhat reduced, but the decrement is also lowered.

When a current measuring instrument is used as the resonance indicating device, measurements of decrement can be made and reso-

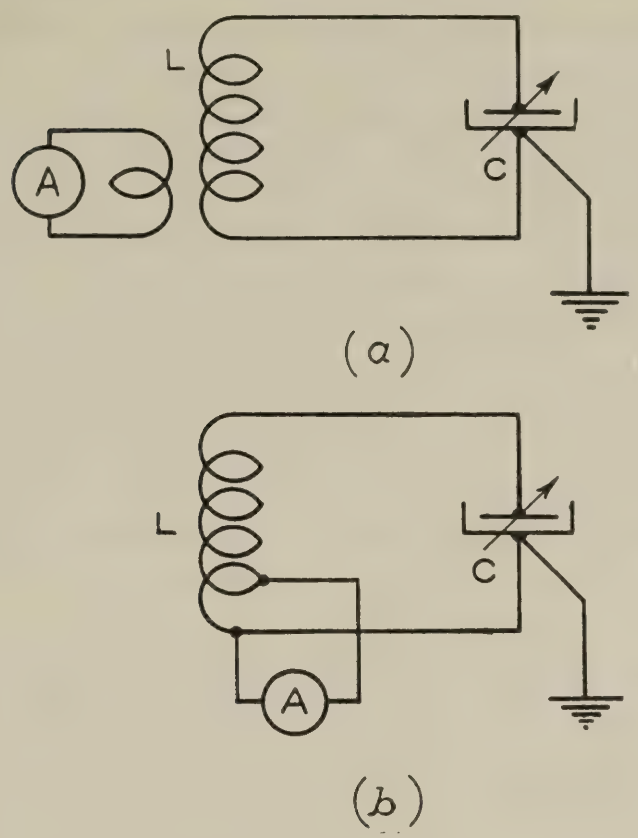


FIG. 380.—Wavemeter Circuits with Thermo-Galvanometer (a) Coupled Inductively and (b) Shunted by a Few Turns of the Inductance Coil.

nance curves obtained. The rf resistance of a circuit can also be obtained indirectly from the decrement.

Another device for indicating resonance visually is shown in figure 381. An incandescent lamp, such as a 2 volt, 1-2 candle power lamp, is inserted in series with the wavemeter circuit. The lamp filament is brought to a dull red glow by means of a two or three cell

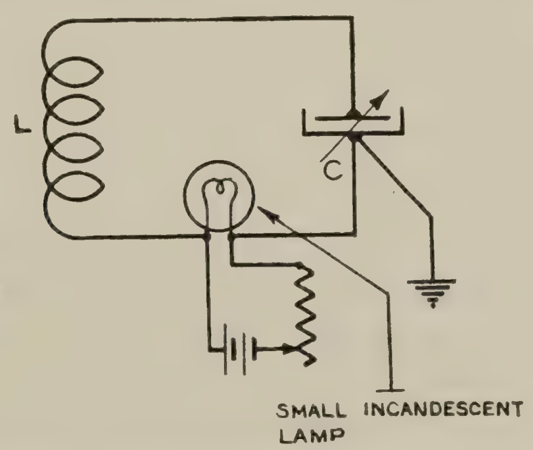


FIG. 381.—Wavemeter with Incandescent Lamp for Visual Indication of Resonance.

dry battery in series with a rheostat. Resonance is indicated by an increase in the brilliancy of the filament. The device is quite sensitive, because only a small extra current is required to raise the filament from

dull red to white. This type of indicating device is suited only to the determination of resonance.

The **Geissler tube** furnishes still another method of showing when resonance is obtained. The Geissler tube is simply a glass tube with an electrode sealed into each end and filled with an inert gas, such as neon or argon. The tube is connected across the condenser terminals. Resonance is indicated by a vivid glow in the tube.

The indicating devices described above can be used only when the driving circuit has considerable power, as the power consumed in the wavemeter is considerable. Wavemeters so equipped can be used in conjunction with any type of transmitter—spark, arc, vacuum-tube telegraph or telephone. It will be noted that the above devices are usually connected, or coupled, at the low potential (ground) point of the wavemeter circuit. It is good practice to do this to reduce the capacity effect of the operator's body.

Wavemeters without indicating device. A wavemeter that is suitable as a standard can be constructed without a resonance indicating device. The wavemeter circuit simply consists of a capacity and an inductance in series. This type of wavemeter is used in conjunction with a low-power, continuous-wave driver which has a sensitive and quick acting dc milliammeter in the grid circuit. The driver is used to excite the wavemeter being calibrated. When the resonance between these two has been obtained, this wavemeter is removed and the standard substituted. Resonance between the driver and either wavemeter is indicated by a pronounced dip in the deflection of the milliammeter. This method of calibrating wavemeters, and the special driver, are described in Measurement No. 2.

Points of design. The increasing use of continuous-wave transmission has decreased the need for a decrement calibration of wavemeters, but on the other hand, has made the use of the visual method of resonance indication imperative. The fact that the rf resistance of a wavemeter should be kept as low as possible should, however, not be ignored. The advent of the vacuum-tube driver, and the refinements in radio measurements made possible thereby, make it relatively easy to compare wavemeters as to their suitability. The use of the Multivibrator as a fundamental standard simplifies, and makes exact, the wave length calibration of wavemeters. The various points of design are treated in the following paragraphs.

The **condenser** used should fulfill the requirements mentioned in Chapter II of Part 8, Section I, both mechanically and electrically. The condenser should be shielded, with the movable section connected to the shield. The air-dielectric, variable condenser is the preferred type, although oil dielectric is sometimes used. Constancy and low losses are highly desirable.

The **inductance coils** should be sufficient in number so that each can be used with a considerable amount of capacity at the lower end of the wave-length range for that coil and yet overlap the next smaller coil. By this means, each coil will be employed at wave lengths where its rf resistance is low. The design of the coils themselves, including kind of wire, method of winding and coil form, should be such as to give the lowest rf resistance, highest inductance and smallest distributed capacity compatible with the design of the wavemeter, whether it is of the standard or portable type. Separate coils should be used rather than one or two tapped coils. These points were discussed in Chapter I, Part 8, Section I.

The **resonance indicating device** should have a low resistance and be of the required sensitivity for the use to which the wavemeter will be put.

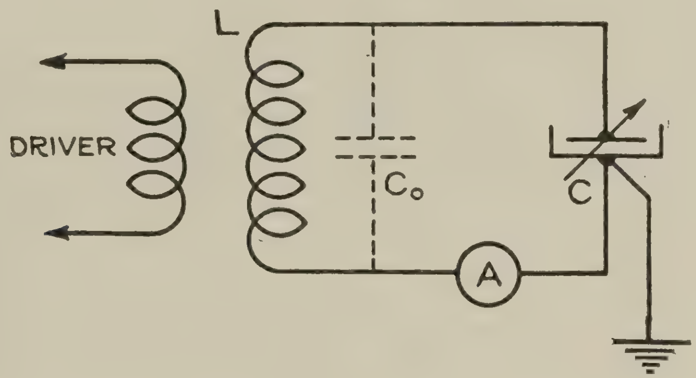


FIG. 382.—The Usual and Preferred Method of Coupling a Wavemeter with the Circuit Under Measurement.

The **leads** between the various parts should be made as short and straight as possible and kept away from the shielded case if one is used. Coil contacts should be positive and of negligible resistance. In the case that the coils are connected to the wavemeter by flexible leads, the lead wires should be maintained at a constant separation so that the constants of the wavemeter will not be affected. The coil should be mounted so that the minimum amount of metal will be included within its field.

The **coupling** between the wavemeter and the driving circuit should preferably be made through the wavemeter coil, as shown in figure 382. Frequently, a special coupling coil L_1, provided with a connecting strap, is connected in series with the main wavemeter inductance L and is used as shown in figure 383. This coil is generally called a **search coil** and aids in obtaining a coupling when the driving circuit is so situated that the wavemeter box cannot be placed near enough to obtain readings.

The wavemeter used in the Navy for service or field use is combined with a vacuum tube heterodyne driver so that it can be used for

calibrating both transmitter and receiver. The circuit used and the appearance of the meter is shown in figures 382 and 383, respectively.

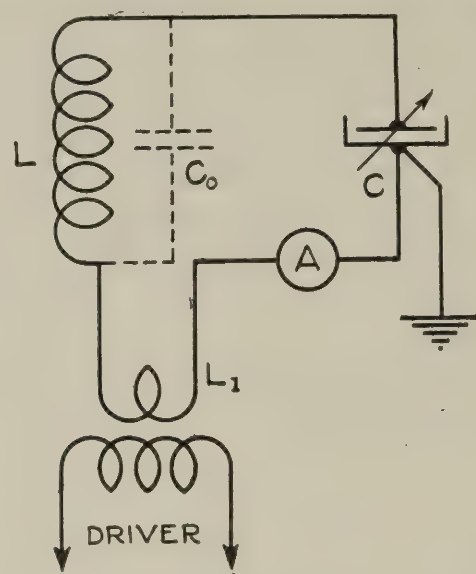


FIG. 383.—An Alternate Method of Coupling a Wavemeter by a Search Coil with the Circuit being Measured.

MEASUREMENT NO. 1.

STANDARDIZATION OF STANDARD WAVEMETER IN ABSOLUTE VALUE.

The Multivibrator. A very exact method of calibrating wavemeters in absolute value has been developed by two French scientists, M. Abraham and E. Bloch, who have given the name **Multivibrator** to the instrument used in their method. This method is comparatively easy to use, does not require a previous calibration of the capacity and inductance of the wavemeter, and can be operated over practically the entire wave-length range used in radio.

The multivibrator is a source of wave lengths which includes the fundamental wave length and an unbroken series of harmonics of this fundamental. The fundamental and harmonic frequencies are related to each other as the numbers 1, 2, 3, 4, 5, etc.; that is, each harmonic frequency is an exact integral multiple of the fundamental frequency. The fundamental is called the first harmonic; the frequency of, say, the tenth harmonic will be 10 times that of the fundamental. It will readily be seen that if the fundamental frequency can be accurately determined then all the frequencies and hence the wave lengths corresponding to the harmonics of the fundamental will also be determined. In practice, the fundamental is adjusted to have an audible frequency that can be compared directly with a standard tuning fork by the **beat** method. The frequency of the standard tuning fork is 1,000 per second, the corresponding wave length being 300,000 meters.

The standard tuning fork is adjusted to a frequency of 1,000 per second. The primary standard is time, accurately measured by a standard clock which is carefully checked by means of astronomical observations. Thus, the fundamental frequency can be accurately determined.

Therefore, if a source of electrical oscillations very rich in harmonics is available, and the fundamental frequency can be determined in absolute value, then all the harmonics will be accurately known and the corresponding wave lengths will be absolute in value.

A system including two vacuum tubes coupled together by capacities and resistances can be arranged to give alternate discharges spontaneously, and the long series of harmonics needed for the calibration of a wavemeter is thus obtained. The type A multivibrator consists of such a circuit and is the best type suited for practical purposes.

Detailed description of multivibrator. A diagrammatic sketch of the type A multivibrator is given in figure 384. L_1 and L_2 are the vacuum tubes. The circuits of these two tubes are identical and are also symmetrically arranged. The filaments, operating normally with 4

volts drop, are supplied from the common filament battery *A*. No regulating resistance is employed. Current for the plate circuits is supplied at 80 to 100 volts from the common battery *B* through the 70,000 ohm resistances r_1 and r_2. The grids are connected to the positive terminal of the filament battery through resistances R_1 and R_2, each approximately 50,000 ohms, and in addition, each grid is connected to

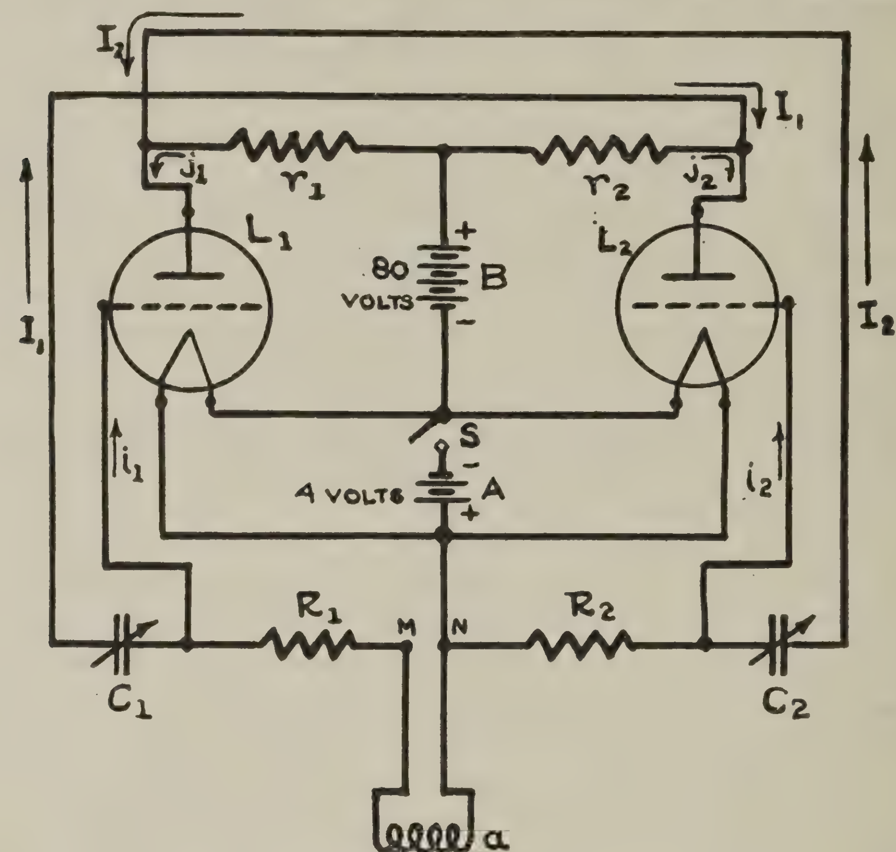


FIG. 384.—Circuit diagram of the Multivibrator, Type A.

the plate of the other tube through a capacity C_1 or C_2. The coupling coil *a*, consisting of a few turns of wire, is inserted between terminals *M* and *N*. The only adjustment to be made is that of the two capacities.

Harmonic systems and their interrelation. The fundamental frequency of the type A multivibrator can be varied between 400 and 30,000. In practice, the fundamental frequency is adjusted to 1,000, 15,000 and 30,000 cycles, and the harmonics of each are employed up to about the fiftieth. Beyond this point the difference in wave length between successive harmonics is too small to be readily found on the scale of the wavemeter condenser. The main purpose in increasing the fundamental is to place the harmonics farther apart on the condenser scale. The table of harmonics on page 597 has been made in order to make clear the interrelation of the harmonics of each of the fundamental frequencies employed.

Referring to the table, it will be seen that the sixtieth harmonic of the 1,000-cycle fundamental, 5,000 meters, appears as the fourth

harmonic of the 15,000-cycle fundamental and as the second of the 30,000-cycle fundamental. It will also be noted that the sixtieth harmonic of 1,000 frequency is an octave higher than the thirtieth harmonic, which, in turn, is an octave above the fifteenth of the same series. In each case, the frequency has been doubled and the wave length halved. The use of this law will be explained more fully later.

Arrangement of the circuit used for calibration. The circuit used for the calibration of wavemeters is shown in figure 385. The coupling coils, *a*, *b*, and *c* consist of about 10 turns of wire wound on 4-inch micarta tubing and are connected as shown in the diagram. Coupling coil *d* is usually given different values, depending upon the range of wave lengths being used. Coil *a* is connected to the terminals *M* and *N* of the multivibrator, figure 384. Coils *b* and *c* are in series and are connected to the input terminals of the amplifier, while coil *d* is in series with the oscillatory circuit of the heterodyne.

TABLE OF HARMONICS.

Wave length, in meters.	Frequency in kilocycles.	Number of harmonic.		
		$f_0 = 1,000$.	$f_0 = 15,000$.	$f_0 = 30,000$.
20,000	15	15	1	
10,000	30	30	2	1
6,666	45	45	3	
5,000	60	60	4	2
4,000	75	75	5	
3,333	90	90	6	3
2,858	105	105	7	
2,500	120	120	8	4
2,222	135	135	9	
2,000	150	150	10	5
1,848	165	165	11	
1,666	180	180	12	6
1,538	195	195	13	
1,429	210	210	14	7
1,333	225	225	15	
1,250	240	240	16	8
1,176	255	255	17	
1,111	270	270	18	9
1,053	285	285	19	
1,000	300	300	20	10

The calibrating circuit consists of four parts:

(a) Multivibrator with its coupling coil,

(b) Wavemeter being calibrated,

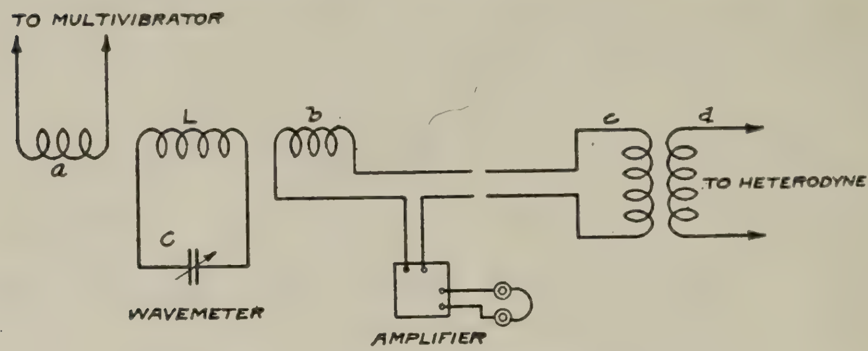


FIG. 385.—Arrangement of Calibrating Circuit.

(c) Amplifier with its two coupling coils (listening-in circuit),

(d) Heterodyne with its coupling coil.

The multivibrator has been described.

The wavemeter to be calibrated is placed between coils *a* and *b* and loosely coupled to them.

The amplifier is shown diagrammatically in figure 386. It has four tubes, three of which are used for radio-frequency amplification and one as detector. It is of the resistance coupled, nonoscillating type.

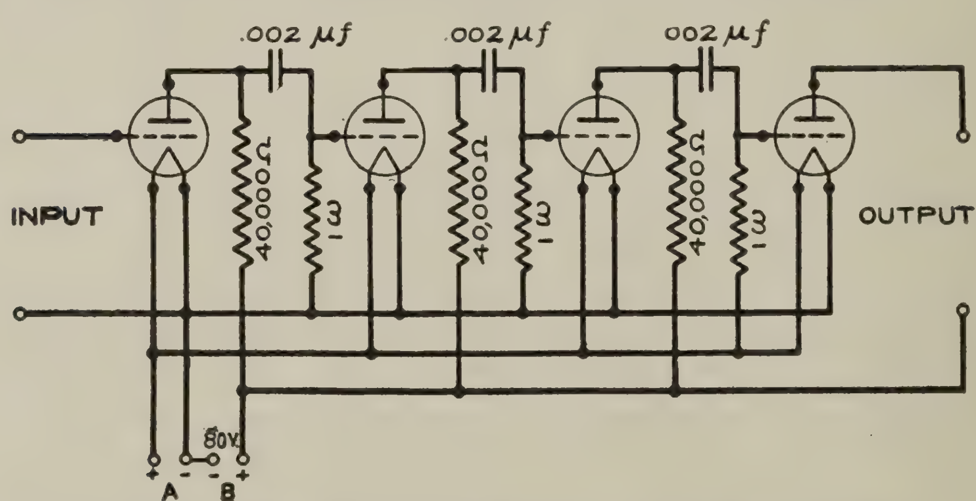


FIG. 386.—Resistance Coupled Type of Radio-Frequency Amplifier.

The purpose of the amplifier is to allow loose couplings to be used in the calibrating circuit and, although the amplifier is not very powerful, it will be found that more amplification can not be employed unless there is marked freedom from induction. Another feature of the amplifier circuit is that it is untuned and will amplify and detect over the entire range of wave lengths employed in calibrating wavemeters.

The heterodyne is the usual type of radio-frequency driver, and is shown in figure 387.

The purpose of the heterodyne in the calibrating circuit is to render audible the radio-frequencies of the harmonics given out by the multivibrator and transmitted to the amplifier circuit through the wavemeter. In addition, the usual increase in sensitivity due to the heterodyne principle is obtained.

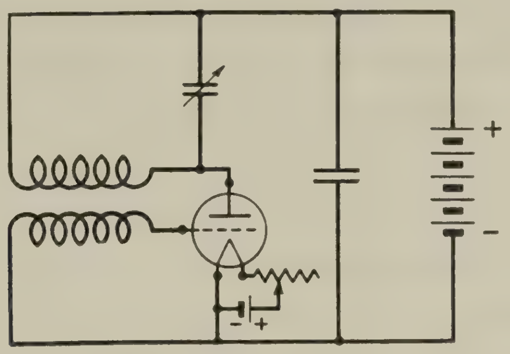


FIG. 387.—Circuit Diagram of Heterodyne.

The apparatus used in the set-up should be arranged in the sequence shown in figure 385. The actual distance between multivibrator and heterodyne should be about 15 feet; the leads connecting the coupling coils *b* and *c* together and to the input terminals of the amplifier are made long enough to accomplish this separation. The telephones should be equipped with extra long chords, so that the operator can adjust any part of the circuit without the necessity of removing the telephones.

Theory of the circuit used for calibration. The multivibrator is the source of power. The current supplied by the multivibrator has a distorted wave form and is, therefore, rich in harmonics. The discharge current of the condenser *C* flows through resistance *R* and through the coupling coil *a*, the fundamental frequency and all the harmonics, both even and odd, being present in coil *a*, and is thence transferred to the amplifier circuit by inductive coupling through the wavemeter.

The power residing in any harmonic and available in coil *a* can be transferred to the amplifier circuit, where it is amplified and detected, **only** when the wavemeter is in resonance with that harmonic. In the condition of resonance, the wavemeter serves to tighten the coupling between coils *a* and *b*, because the emf impressed in the wavemeter circuit then produces maximum current in that circuit, which current induces maximum emf in the amplifier circuit. The heterodyne renders the harmonic audible by the well-known **beat** principle. Thus the action of the wavemeter is to select the fundamental, or any one of the harmonics, to the exclusion of all others.

Calibration of a wavemeter. (a) Procedure. Start the multivibrator by closing the filament switch and allow the tubes to warm up and become steady. A few minutes will suffice. The filament and plate batteries should be of the lead-acid type and well charged for this duty. Couple the amplifier circuit closely to the multivibrator by

coil *b*. Neither the heterodyne nor the wavemeter under calibration is yet required. Close all the capacity switches on the multivibrator and listen in on the amplifier. A clear note of about 400 cycles should be heard. If the note is roughened, it indicates that the amplifier is oscillating. Check the relation of the coupling between coils *a* and *b*. If a louder signal is obtained when one of these coils is turned through 180°, leave it in that position. Now increase the frequency of the multivibrator by reducing the capacity equally on each side. This can be done by the fixed steps and the continuously variable condensers. Compare the note of the multivibrator with that of the standard fork. The tone of the fork can be heard very distinctly through that of the multivibrator, the telephones being on the head and the fork held fairly close to the operator. Beats will be heard very distinctly when the multivibrator and fork are **nearly** in resonance. The multivibrator has a frequency of 1,000 when no beats are heard; in other words, when it is in resonance with the tuning fork. If one beat per second is heard, the two are out of resonance by 0.1 per cent, and the error in wave length will be the same. The steadiness of the multivibrator should be checked from time to time by comparison with the tuning fork. If the frequency is unsteady, or shifts, it indicates that the batteries are not well charged, or that there is a loose contact.

The wavemeter should be inserted between coils *a* and *b* in such a way that the wavemeter inductance is coupled closely to each of the coils. It will be found that the note of the multivibrator is no longer heard, due to the looseness of the coupling between *a* and *b*. Set the wavemeter in the vicinity of some harmonic (it is preferable to start the calibration at the longest wave length possible), and also start the heterodyne and set it (by inspection) at about the same wave length as that of the harmonic sought. Next, vary the wavemeter slowly back and forth over the setting desired. The beat note of the heterodyne on the frequency of the harmonic should be heard. As soon as this is heard, adjust the heterodyne to give any pleasing note, high or low, loosen the couplings throughout the circuit, and proceed with the calibration. **Loose couplings must always be employed,** and can be obtained if the fundamental frequency of the heterodyne is approximately that of the harmonic sought.

Both even and odd harmonics are present and should be found. It will be noted that as soon as a few successive harmonics have been found, it will be possible to predict the location of the next with considerable accuracy, thereby minimizing the possibility of overlooking it. The heterodyne frequency should be changed from time to time to follow the harmonics. The fundamental of 1,000 should be used to about the one-hundredth harmonic, frequency 100,000, wave length 3,000 meters, for all coils that cover the range of wave lengths above 3,000 meters. The fundamental frequency of the multivibrator should

then be changed to 10,000, 15,000 or 30,000, as required, and the calibration proceeded with.

The new fundamental frequency is inaudible, and is found by trial and error. The multivibrator is first adjusted to resonance with the fork; and the location of the fifteenth or thirtieth harmonic on the wavemeter condenser, with the necessary wavemeter inductance, is then very accurately determined by the method previously described for locating any harmonic. The wavemeter and heterodyne are now left untouched, and the capacity in the multivibrator reduced to approximately one-fifteenth, if the fifteenth harmonic is desired. A beat note should again be heard. The wavemeter is then varied in the usual manner to determine resonance, and if resonance does not fall on the point required, the multivibrator is readjusted until the wavemeter indicates correctly. The new fundamental has then been determined and the calibration can be continued. An inspection of the table of harmonics given above will show the relation of the harmonics of the different fundamentals. The separation of the harmonics of the new fundamental is considerable, but the second harmonic of fundamental 15,000 must fall exactly on the point previously determined to be the thirtieth harmonic of the fundamental 1,000; if otherwise, it indicates an error in the new fundamental.

Determination of the order of the harmonic. The condenser settings where harmonics are heard should be carefully noted and tabulated. The next step is the determination of the number of the harmonic, in order that the corresponding wave length may be found. There are three general methods used in determining the number of the harmonic:

(1) By inspection,

(2) By the octave method,

(3) By the sound method.

(1) A previous calibration of the wavemeter, which need be only approximate, will serve as a guide and is the simplest method. The wave length of the wavemeter for any setting can be roughly determined from the LC value and this used as a guide for numbering the harmonic. In either case, the harmonics should fall well apart on the condenser scale in order to preclude any possibility of selecting the wrong wave length for the harmonic and thus giving the harmonic the wrong number.

(2) The octave method requires a fairly accurate calibration of the condenser and makes use of the fact that if the capacity in a wavemeter is quartered, the inductance remaining constant, the wave length is halved. The method is best explained by the following example. Assume that at 160°, capacity 4,000 $\mu\mu$f, a harmonic is detected, and that between this point on the condenser and the point of one-quarter capacity 1,000 $\mu\mu$f, 40°, 15 harmonics are detected, none being overlooked, and the number 15 includes the 40° but not the 160° setting. Then, the number of the harmonic that fell on 160° is 15 and that for 40° is 30. The

corresponding wave lengths are 20,000 and 10,000 meters, respectively, for the fundamental 1,000 cycles. Care should be taken to give proper consideration to the distributed capacity of the wavemeter inductance. This capacity is usually negligible when the capacity of the condenser is 4,000 $\mu\mu$f, but at the quarter-capacity value it becomes important, and due allowance should be made for it, so that the number of harmonics counted will be correct, because the quarter-capacity point on the condenser will be higher than that read for one-half the wave length by the amount of the distributed capacity of the inductance.

If, now, the fundamental of the multivibrator is to be changed to 15,000 cycles, it should be adjusted to fall exactly on 160°. The second harmonic would not be detected until the condenser reads 40°, and should fall exactly on this point. **No harmonics should be heard between 160° and 40°.**

The wave lengths determined by the harmonics of the multivibrator are correct only when the fundamental has been accurately adjusted. An error of one-tenth of 1 per cent in the fundamental will cause an error of the same amount in all the harmonics.

(3) The sound method is similar to the octave method just described and requires no previous knowledge of the wavemeter under calibration. This method is based on the principal of beats and is very useful. The heterodyne is adjusted so that its fundamental is very nearly equal to the first harmonic found on the wavemeter condenser, say the fifteenth, frequency 15,000 cycles and wave length 20,000 meters. The beat note should be from 25 to 100 cycles. If, now, the heterodyne is left set at this frequency, the half-wave length point will be determined by the doubling of the beat frequency; the beat note heard will be an octave higher than the original note. For example, let the multivibrator fundamental be 1,000 cycles and the harmonic chosen for the first be the fifteenth, frequency 15,000 cycles. The heterodyne is set to give 15,100 cycles. The beat note is 100 cycles. The one-half wave length has double the frequency of the fifteenth harmonic, or 30,000 cycles, while the second harmonic of the heterodyne is 30,200; the beat note is therefore doubled, or 200 cycles.

With the use of the multivibrator, a wavemeter can be calibrated to a degree of accuracy such that the average error in wave length is not greater than about 0.1 of 1 per cent.

The multivibrator simplifies many measurements that were heretofore considered difficult on account of the inaccuracy of wavemeters. These measurements pertain mainly to the determination of inductance and capacity. If the capacity of the condenser used is accurately known, the apparent and true inductance and the distributed capacity of a coil can be determined with great accuracy, or if the true inductance and distributed capacity of a coil are known, the condenser can be accurately calibrated.

Calibration of standard wavemeter at short wave lengths. The multivibrator cannot readily be used for standardizing a wavemeter below about 400 meters, because at the shorter wave lengths the harmonics of the highest frequency to which it can be adjusted, fall so closely together on the wavemeter condenser scale that the order of the harmonics cannot be determined with certainty. The following method, however, is accurate and has been employed satisfactorily in the calibration of wavemeters down to 50 meters.

The apparatus necessary for calibration from 50 to 400 meters consists of a powerful continuous-wave driver with a range from 100 to 800 meters, a receiver with oscillating vacuum-tube detector, range 50 to 400 meters, with a two stage audio-frequency amplifier.

This method requires that the wavemeter be tuned through the medium of the receiver to the second harmonic of the wave length to which the driver is adjusted.

The driver is first tuned to approximately 800 meters by the standard wavemeter, and the wave length checked by adjusting the wavemeter to resonance with the driver. Then the receiver is tuned to the second harmonic or 400 meters, by means of the zero beat note method, employing loose coupling between receiver and driver, which is permitted by the use of the amplifier. This harmonic will be found at one-fourth the capacity at which the fundamental is located. Next, the receiver is left untouched, the driver plate circuit opened, and the wavemeter is tuned to the receiver by the **click method,** using just enough coupling to get an audible click. The wavemeter is now adjusted to 400 meters, and this setting must check with that obtained for the same wave length by the multivibrator method.

The driver is now tuned to 760 meters and the above operation repeated. When adjusted to the second harmonic as before, the wavemeter will be set at 380 meters. The process is continued through the range of wave lengths desired. Below about 400 meters, which is the approximate lower limit of the multivibrator calibration, the wavemeter is used at the settings just obtained to tune the driver to the shorter wave lengths.

In lowering the range of the receiver to 50 meters, coils of small diameter and few turns with variable coupling between them should be used, together with a condenser of low minimum capacity.

The use of a vacuum-tube detector having a low capacity between the elements and a high vacuum, with shortest possible leads throughout the receiver, will also assist in reaching this short wave length.

MEASUREMENT NO. 2.

CALIBRATION OF A WAVEMETER BY COMPARISON WITH A STANDARD WAVEMETER.

The calibration of a wavemeter can be effected by adjusting it to resonance at a number of wave lengths with a standard wavemeter. When so adjusted, the wave length of the wavemeter under calibration is the same as that of the standard. Wavemeters are compared for the purpose of calibration by tuning them in turn to resonance with a source of oscillations, preferably a continuous-wave driver. Three methods of detecting resonance between a wavemeter and the driver will be described, with a brief statement of the type of driver to be used in each case. These methods are: (1) the maximum deflection method, (2) the click method and (3) the grid meter method.

(1) **Maximum deflection method.** This method, which is generally used, will first be given, together with a detailed description of the procedure to be followed in calibrating a wavemeter. A high-power vacuum-tube driver may be employed, and resonance indicated by maximum deflection on the current-squared meter in the wavemeter.

In beginning the calibration of a wavemeter, the detector contact is opened and the buzzer switch is thrown to the **off** position. Now insert the coil of lowest inductance and set the condenser at about 20°. Tune the driver roughly to the wavemeter and then, leaving the driver unchanged, adjust the wavemeter accurately to resonance with it, and record the condenser setting. Detune the wavemeter and tune the standard to the driver, noting the setting. The wave length corresponding to this setting is tabulated as that of the wavemeter under measurement at the setting recorded.

The observation should be repeated at about 30°, and thereafter at intervals of approximately 20° on the wavemeter condenser scale, with readings at both 160° and 170°. At every setting above about 130°, determine if a reading can be obtained on the next larger coil. This assures an accurate calibration of the overlap between consecutive coils.

In case either wavemeter has fixed condenser units, it will be necessary to adjust the driver accurately to resonance with it, and then to tune the wavemeter with the variable condenser to the driver.

The data thus obtained for each coil should be plotted on cross-section paper with the condenser settings in degrees as abscissas and the corresponding wave lengths in meters as ordinates. A smooth curve is drawn through the points. If a point lies outside the curve, that observation should be checked to correct the error. A typical wave-length calibration curve is shown in figure 388. From such a wave-

length calibration curve the wave length corresponding to any condenser setting with a given coil can be read.

(2) **Click method.** A circuit in which oscillations are being maintained by a receiving vacuum tube, with telephones in the plate circuit, may be used. When a wavemeter is coupled to such a circuit and its condenser is rotated through resonance from a lower setting, a click is

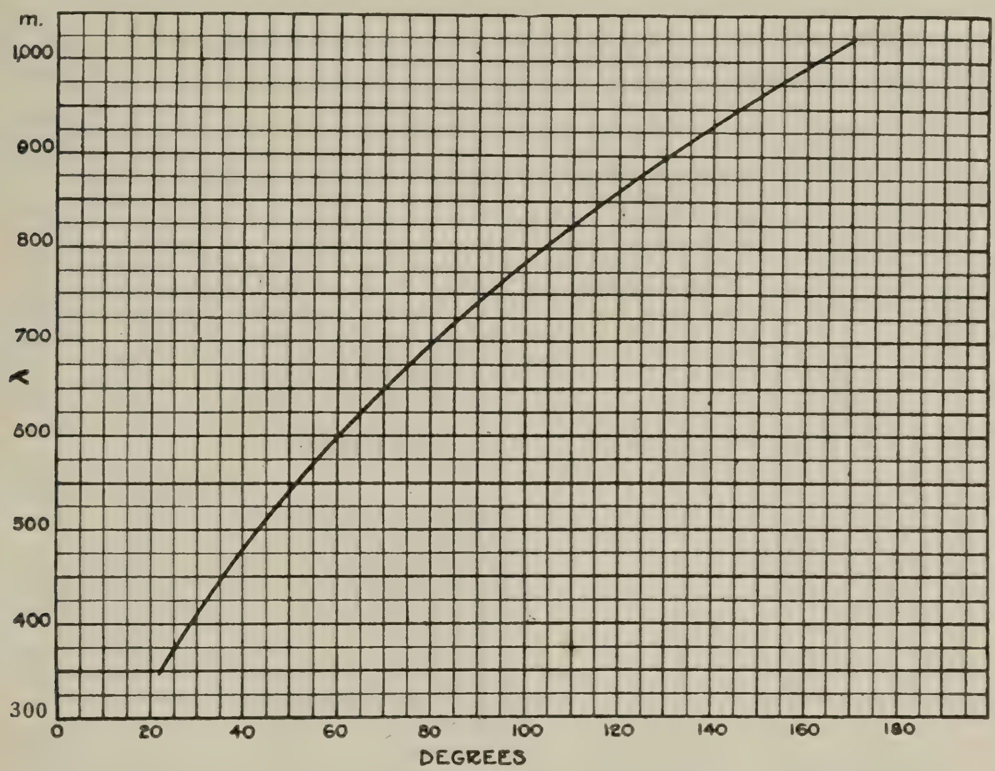


FIG. 388.—Typical Wave-Length Calibration Curve of a Wavemeter.

heard in the telephones and another click is heard when the condenser is turned through resonance from a higher setting. The clicks occur one above resonance and one below it where the frequency of the oscillating vacuum tube circuit is changed by the action of the resonant wavemeter circuit. If the coupling is very close, or if the resistance of the wavemeter is high, the wave length of the driver will be radically affected, tending to follow any change in the period of the coupled circuit over a range of wave lengths in the vicinity of resonance. This will be indicated by a wide separation of the settings at which the clicks occur, and under this condition accurate wave-length measurements can not be made.

However, with very loose coupling and by the use of a wavemeter of low radio-frequency resistance, the clicks can be made practically to coincide. Then the click method becomes a rapid and accurate means of indicating resonance. It is particularly valuable in determining the wave length of oscillating vacuum-tube circuits where the output is too small to give a deflection on the current-squared meter, and may be used in the calibration of wavemeters when the clicks can be made to fall within 0.5° at about the middle of the condenser scale for all coils.

The mean of the settings at which the clicks occur is the point of resonance between the circuits.

To insure accurate results, two precautions must be observed: (a) Use the loosest possible coupling at which the distinct clicks can be heard; (b) Use wavemeter with low radio-frequency resistance.

(3) **Grid meter method.** This method of detecting resonance makes use of the fact that the value of grid current in an oscillating vacuum tube circuit drops off sharply at resonance with a coupled circuit. This decrease in grid current is indicated by a pronounced dip, or negative deflection, on a sensitive grid ammeter. The point of minimum deflection denotes resonance between the coupled circuits.

A receiving vacuum tube is used as the source of oscillations, thus dispensing with a power driver in the calibration of wavemeters. A suitable circuit is shown in figure 389. This is of the tuned-grid type

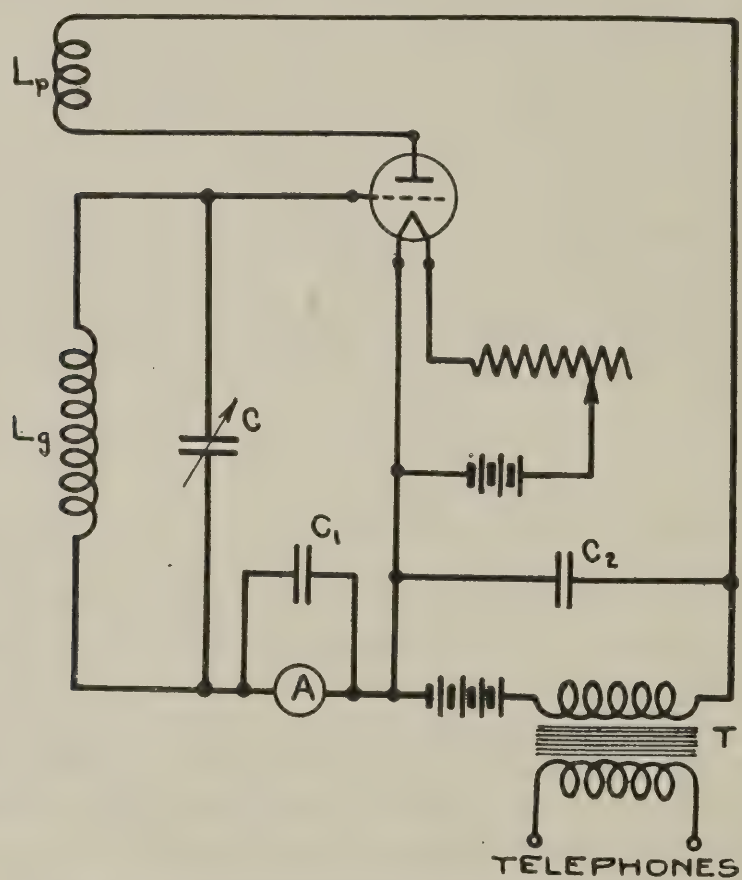


Fig. 389.—Tuned Grid Circuit Vacuum-Tube Driver.

with a back coupling coil in the plate circuit. A dc microammeter is inserted in the grid circuit between the filament and the tuned circuit and is shunted by a capacity C_1, of about 0.1 μf. This ammeter registers the average direct current flowing in the grid circuit. Variable coupling between the plate and grid coils is used to control the strength of oscillations. Telephones are connected in the plate circuit by means of a telephone transformer T, or they may be inserted directly in the circuit.

By variation of the back coupling and filament temperature, the circuit is adjusted so as just to permit the generation of oscillations, since in this condition the dimunition in grid meter reading at resonance with a coupled circuit is most pronounced. The presence of oscillations may be determined by tapping with the finger the grid side of the tuning condenser or a grid meter contact. If, with each tapping, the grid current drops suddenly and then returns to its former value, the circuit is oscillating.

When a wavemeter is tuned to this driver, the click in the telephones is a rapid means of determining the position of the dip. The coupling is then made loose and the wavemeter is adjusted to resonance with the driver as indicated by minimum deflection on the grid meter.

When the capacity of the driver is increased or decreased, as in tuning it to a wavemeter using fixed condenser units, the grid current will rise or fall, usually in a steady manner. At resonance with a coupled circuit a sharp dip in deflection is superimposed upon this steady change. This dip is more noticeable when the grid current is increasing as resonance is approached.

As the capacity of the driver is varied over a considerable range, the back coupling and filament temperature must be adjusted to keep the circuit oscillating at a point just above that at which oscillations are sustained.

To protect the grid meter from injury, all increases in back coupling or in filament current should be made slowly, particularly at the point at which the circuit begins to oscillate, and the meter reading observed. The same care should be taken in varying the capacity when the grid current is rising rapidly with rotation of the condenser.

The grid meter method does not require a resonance indicating device in the wavemeter. It permits much looser coupling than is possible with the click method, and is satisfactory even in wavemeters of high radio-frequency resistance. The tuning also is very sharp. This method is second to none in the degree of accuracy that can be attained by its use.

An exact means of detecting slight inaccuracies in the observations taken in the calibration of a wavemeter may be employed provided an accurate calibration of the wavemeter condenser is available. The capacity of the condenser at every setting at which a wave length reading has been taken is plotted against the square of the corresponding wave length. This method is explained in Measurement No. 13, and illustrated by a graph. All the points should lie on a straight line. Any error will thus be readily observed, and should be corrected by repeating the observations.

Crystal calibrator. In the Navy the latest methods of calibrating the service wavemeters is by means of a crystal calibrator which has

been most accurately checked with the absolute value. This calibrator consists merely of a vacuum tube driver controlled in wave lengths by an extremely accurate ground quartz crystal on a frequency of 25 or 50 kilocycles. The harmonics of this crystal are used to plot the curve of the wavemeter being calibrated.

MEASUREMENT NO. 3.

MEASUREMENT OF THE WAVE LENGTH OF RECEIVED SIGNALS.

1. **Continuous waves.** To measure the wave length of continuous-wave signals, a receiver with oscillating vacuum tube detector is used. The transmitting station, whose wave length is to be determined, is tuned in, in the usual manner. The secondary coupling is then loosened until the signal is just audible, and the antenna circuit is retuned for maximum signal intensity. With coupling and antenna circuit as adjusted, the secondary condenser is set to give zero beat note with the incoming wave. If no signal is heard over several degrees of the condenser scale, the mean of this zone of silence is taken as the point of resonance. In case the zone of silence is too broad, the coupling may be increased slightly.

The condenser is now set at this mean position and the antenna circuit opened. Then a wavemeter is coupled to the secondary circuit and tuned to resonance with it by means of the click method, which is described in Measurement No. 2.

If the wavemeter cannot be coupled to the secondary circuit, due to the use of a shielded receiver, a slightly different procedure is made necessary. The circuits are tuned as above, and a continuous wave driver is coupled to the antenna lead and adjusted to resonance with the incoming wave by the zero beat method. A wavemeter is then tuned to the driver by the click method. The wave length thus determined is that of the transmitter.

2. **Damped waves.** A turn of wire is inserted in the ground lead of the antenna circuit, and the receiver is tuned to the incoming signals as usual. A wavemeter excited by a buzzer is loosely coupled to this coil and tuned to resonance with the receiver as indicated by maximum sound in the telephones. If the wavemeter has been calibrated as a buzzer driver, it will register the wave length of the transmitter. But if not, it will indicate a wave length lower than that of the received signals, due to the added capacity of the buzzer circuit. At low settings of the wavemeter condenser the error is considerable, but it is slight at the higher settings when the capacity is larger in comparison with that of the buzzer circuit. Therefore, when the wave length to be measured falls within the range of two wavemeter coils, the one requiring the larger capacity should be used.

MEASUREMENT NO. 4.

CALIBRATION OF CONDENSERS.

The capacity of a condenser is measured usually by comparison with a variable standard condenser. In the calibration of condensers, two cases may arise: one in which the maximum capacity of the condenser under measurement lies within the range of the standard, and the other in which it exceeds the capacity of the standard. A method to be used in each of these cases is here described.

1. **Calibration of condenser within the range of the standard.** The method of substitution is employed with a circuit such as that shown in figure 390. C_1 is the standard, and C_2 the condenser to be calibrated.

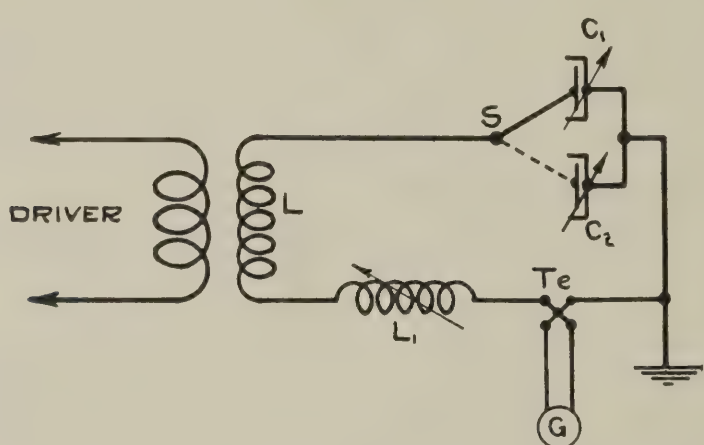


FIG. 390.—Circuit used in Calibrating Condensers by the Substitution Method.

The shields of C_1 and C_2 are connected together and grounded. An *SPDT* switch *S* is used to connect either of the condensers into the circuit. The leads from the switch to the condensers are made as short as possible by mounting the switch on a level with the condenser terminals, and should be of equal length so as to have equal capacity. A variable inductance L_1 may be employed to compensate for changes in capacity in the circuit instead of returning the driver with every change in condenser setting. The inductance L should be of fair size so as to avoid the use of very short wave lengths. A quick-acting current-squared meter, or thermoelement and galvanometer, may be used to indicate resonance.

The switch is first thrown to C_2, which is set at any desired position, and the driver is loosely coupled to L and adjusted to resonance with the circuit. The setting of C_2 is noted and C_1 is substituted in the circuit for C_2 by throwing the switch to the other position. C_1 is now varied to bring the circuit back into resonance with the driver. The capacity

of C_1 at this setting equals that of C_2 at the setting noted. By repeating this process at a number of settings, C_2 may be calibrated throughout its range, provided C_2 is not greater than C_1, and a capacity calibration curve plotted with condenser settings in degrees as abscissas and the capacity in $\mu\mu$f as ordinates.

To determine if any error is being introduced into the measurements due to the position of the condensers with respect to the rest of the circuit, one condenser should be tuned to the driver first in its own position and then in that of the other condenser. The settings at resonance should, of course, be the same.

Direct substitution may be employed, by eliminating the switch and connecting one condenser in the circuit. After a reading is obtained, it is removed and the other is inserted in its place. But with due care the more rapid method above described should be found as accurate as the latter.

2. Calibration of a variable condenser beyond the range of the standard. If the capacity of the condenser C_2 is greater than that of the standard C_1, it should first be calibrated by the substitution method up to the maximum capacity of C_1. For example, assume that a 0.005 μf variable condenser is to be calibrated and that the maximum capacity of the standard condenser is only 0.001 μf. By the substitution method readings are taken up to 0.001 μf, which may be at about 35° on C_2. The same circuit is used as in figure 390 with one change: a jumper is placed from the middle point of the switch to the end point to which C_2 is connected, as indicated by the dotted line in the figure. This connects C_2 permanently into the circuit. The condensers are now placed in parallel by closing the switch to C_1.

C_2 is set at the highest point for which the capacity has been determined by the substitution method, say 0.001 μf. C_1 is next set at a capacity that is equal to a change of about 10° or 20° on C_2, for example, 0.0005 μf. The driver is brought into resonance with the circuit, then the switch is opened, disconnecting C_1 from the circuit, and C_2 is varied to retune the circuit to the driver. The setting is noted and the corresponding value of capacity equals that previously determined by substitution plus the amount used in C_1. Thus for this point on C_2 the capacity equals 0.001 μf+0.0005 μf or 0.0015 μf.

C_2 is left at this new setting, the switch is closed, and the desired amount of capacity again added at C_1, and the procedure repeated. In this manner C_2 may be calibrated to its maximum capacity.

Since an error made in any observation by this capacity variation method will be passed on to all successive readings, it is important to exercise the greatest care in making all resonance adjustments and in all scale readings. When C_2 has been adjusted to tune the circuit as described above, the setting should not be changed until C_1 has been again inserted in the circuit and resonance with the driver again ob-

tained. An attempt to reset C_2 other than by the resonant method will introduce an avoidable error in the succeeding readings.

If readings on C_2 are to be taken at differences in capacity that are small compared to the total capacity of C_1, the following procedure will be found more rapid than that just outlined.

Assume that observations are to be made at intervals of 0.0002 μf, when the maximum capacity of C_1 is 0.001 μf or larger. The switch is closed and left closed. C_2 is set at the highest known value as before, but C_1 is now set at maximum, and resonance obtained with the driver. The capacity of C_1 is now reduced by about 0.0002 μf, and that of C_2 is increased to compensate. The decrease in C_1 is equal to the increase in C_2. The driver is left unchanged and C_1 is again reduced, and the circuit tuned to resonance by increasing C_2. When C_1 can no longer be decreased, the two condensers are again set at their maximum known capacities and the driver tuned to the circuit. C_1 may now be decreased or C_2 increased by the desired step, and the circuit returned by the proper adjustment of the other condenser.

When the capacity of the circuit is increased, resonance with the driver is brought about either by decreasing the inductance of the circuit at L, or by increasing the wave length of the driver. In either event, the fine adjustment is effected by tuning the measuring circuit carefully to the driver. In the capacity variation method when the condensers are in parallel, either C_1 or L_1 is varied, but never C_2.

No change in coupling should be made from the time the circuit is tuned to the driver until the measurement is completed.

MEASUREMENT NO. 5.

CALIBRATION OF A THERMOCOUPLE AND GALVANOMETER.

The combination of a thermocouple and galvanometer is an ammeter, and is generally used to measure small radio-frequency currents.

The three types of thermocouples used in radio work are:

1. Crosswire type, in vacuum or in air;
2. Thermal ammeter type;
3. Tellurium-platinum type; and may be distinguished by the following symbols, figure 391.

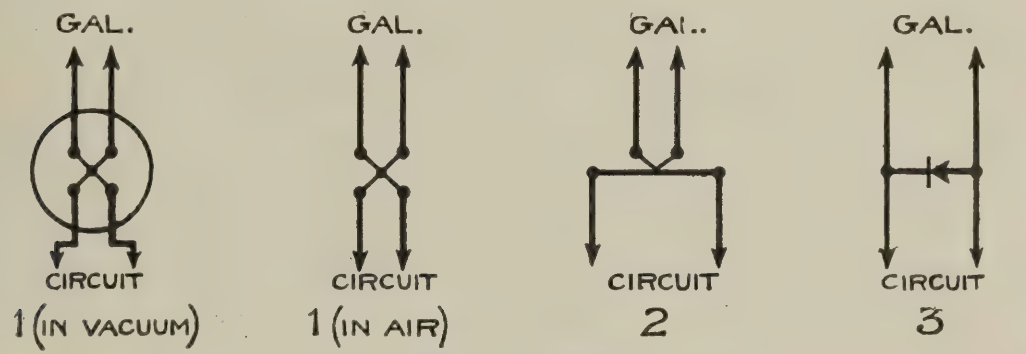


FIG. 391.—Types of Thermocouples.

Thermocouples, having couple resistances of not more than 50 ohms, should be used in conjunction with galvanometers having coil resistances from 10 to 200 ohms. Couples having higher resistances than 50 ohms should be used with galvanometers having coil resistances exceeding 200 ohms.

The deflections of a galvanometer used in conjunction with a thermocouple are generally assumed to be **proportional** to the **square** of the current passing through the thermocouple heater, but the combination should be calibrated both to ascertain whether or not this proportionality actually exists and to determine the current sensitiveness. The calibration can be made by using:

1. Alternating current,
2. Direct current.

Of the two methods, the first is the more reliable and should be used whenever possible.

1. **Alternating-current method.** The circuit for calibrating thermocouples and galvanometers is shown in figure 392.

Transformer *T* should step down 110 volts, 60 cycles, to approximately 10 or 20 volts, which is measured on voltmeter *V*. *R* is a non-inductive resistance having a maximum resistance of approximately 100,000 ohms and variable in steps down to tenths of an ohm.

With all of the resistance R in circuit, close switch S. If no deflection is registered on galvanometer, decrease resistance at R until a readable deflection is obtained. Take simultaneous readings of the voltmeter V and the deflection of galvanometer G. Record these readings. Take readings for a number of points on the galvanometer scale and note deflections and corresponding voltage in separate columns.

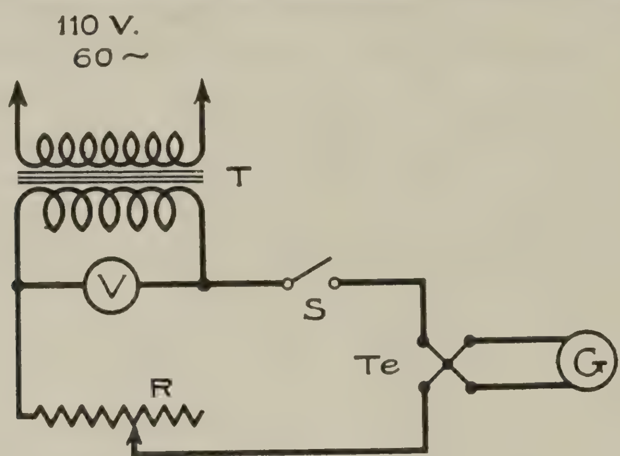


Fig. 392.—Circuit for Calibrating Thermocouples and Galvanometers, Ac Method.

If the voltmeter has a key, it should be kept closed while the galvanometer is being read. The current corresponding to each galvanometer deflection may be calculated from the formula:

$$I = \frac{E}{R+R_0} \cdot 10^6$$

where I = current in microamperes,
E = voltage read on V in volts,
$R+R_0$ = total resistance in ohms between voltmeter terminals, **including the resistance R_0 of the thermocouple heater.**

Care should be taken not to exceed the current-carrying capacity of the thermocouple heater. It is always best to start with the maximum resistance R in the circuit to prevent burning out the thermocouple heater.

Having calculated the current corresponding to the deflections taken, a curve may be drawn that can be used to find the value of current for any deflection. Such a curve is shown in figure 394, curve A. The values of current are plotted as the abscissas, while the deflections are plotted as the ordinates.

2. **Direct-current method.** Thermocouples and galvanometers, with the exception of the tellurium-platinum type, can be calibrated with direct current. Figure 393 shows the circuit used for this purpose. The thermocouple and galvanometer to be calibrated are connected to one side of the *DPDT* reversing switch S. R should be variable and have a maximum resistance of approximately 10,000 ohms. A is a milliam-

meter or microammeter, depending upon the current sensitiveness of the thermocouple and galvanometer. The calibration is then made in the following manner.

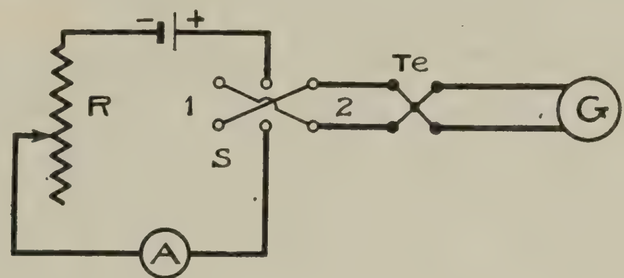


FIG. 393.—Circuit for Calibrating Thermocouples and Galvanometers. Direct Current Method.

Set R to have maximum resistance; and having made certain that the galvanometer reads zero when no current is flowing, throw switch S to position 1 and adjust R until the desired deflection on the galvanometer is obtained. Read meter A for the current corresponding to this deflection. Then throw switch S to position 2, thereby reversing the direction of the current through the heater of the thermocouple. Read the deflection and the current. The current should be the same as before, but the deflection may be different. In case the difference in the deflections is small, the mean of the two deflections may be taken. If, however, the deflections differ considerably, the alternating-current method of calibration should be used.

If the deflections are equal, or the mean taken when they are unequal, the calibration may be continued, larger deflections being

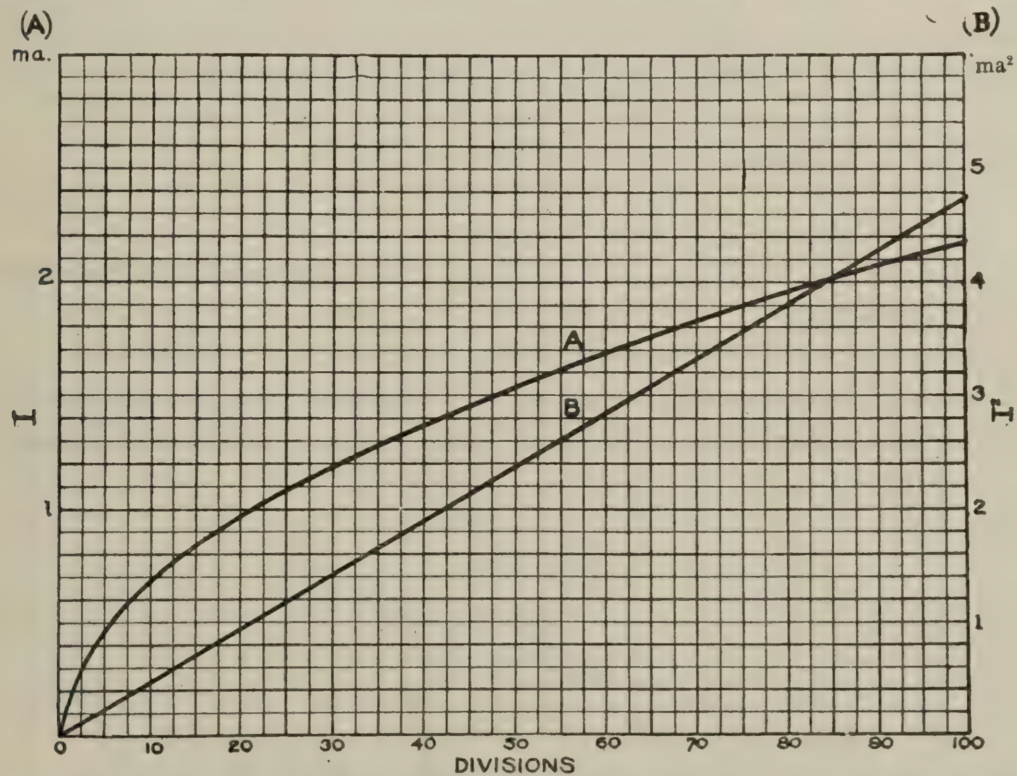


FIG. 394.—Curve A. Calibration Curve of Thermocouple and Galvanometer. Curve B is obtained by Squaring the Values of Observed Current.

obtained by decreasing the resistance of R. The switch should be thrown to both positions for every value of current used. The galvanometer should frequently be checked for zero reading with no current flowing, and kept corrected.

The readings may be plotted in curve form as described in the alternating-current method. Figure 394, Curve A, gives the current in milliamperes plotted against divisions of the galvanometer. Curve B is plotted from the squares of the observed current. It will be seen that the thermocouple and galvanometer gave deflections which were proportional to the square of the current throughout the range, because a straight line can be drawn through all points. The data from which the curves were drawn are given in the first two columns of the following table.

I_θ(ma)	θ	$\sqrt{\theta}$	$I_1=\frac{I_\theta}{\sqrt{\theta}}$ (ma)
0.487	5	2.23	0.218
0.689	10	3.16	0.218
0.975	20	4.47	0.218
1.38	40	6.33	0.218
1.69	60	7.75	0.218
1.95	80	8.95	0.218
2.18	100	10.00	0.218

where I_θ = current in milliamperes for corresponding deflection,
θ = deflection of galvanometer in divisions,
I_1 = current in milliamperes for 1 division.

Under these conditions,

$$I_\theta = I_1\sqrt{\theta}$$

and in this case $I_1 = 0.218$ ma (for 1 division over the entire range).

Example:

Find the value of the radio-frequency current flowing in a circuit when the galvanometer deflection is 50 divisions and the current for 1 division is 0.218 ma, the deflections being proportional to the square of the current.

Solution:

Formula $I_\theta = I_1\sqrt{\theta}$

substituting $= 0.218\sqrt{50} = 0.218 \times 7.07 = 1.54$

whence $I_\theta = 1.54$ ma.

MEASUREMENT NO. 6.

CALIBRATION OF A SHUNTED DETECTOR AND GALVANOMETER.

The shunted detector and galvanometer circuit constitutes a radio-frequency current meter which is capable of accurately measuring radio-frequency currents of the order of a few microamperes. The circuit consists of a high-sensitivity, high-resistance galvanometer, shunted by a condenser having a capacity of about 0.5 μf, connected in series with a silicon-antimony detector and the whole shunted by a non-inductive resistance variable in small steps with a maximum value of 100 ohms. The circuit is shown in figure 395.

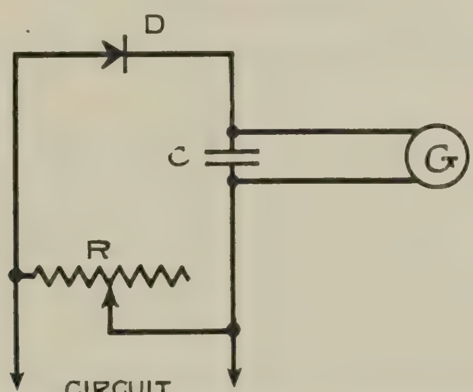


Fig. 395.—Shunted Detector and Galvanometer Circuit.

The circuit is calibrated at radio frequencies, using a calibrated thermocouple and galvanometer as the standard. It is always necessary to calibrate the circuit both before and after it is used for measuring, because the sensitivity of the detector varies, due to vibration and changes in temperature. It is the usual practice to place the detector on a heavy felt pad and cover it with a wooden box. The connections between the galvanometer, detector and shunt resistance should be made very short to prevent their acting as a loop collector in picking up induction or other local interference.

The calibrating circuit is shown in figure 396. The thermocouple and galvanometer used in calibrating the shunted detector and galvanometer circuit are connected in series with an inductance and capacity. The combination composing the measuring circuit is shown in the figure. This measuring circuit is then coupled to a continuous-wave driver and the amount of current flowing in the circuit varied as desired, either by varying the coupling to the driver or by detuning, or both, until a moderate deflection is obtained on G_2. The shunt R is then set at a low value and the detector adjusted to a sensitive and stable contact, and the deflection of G_1 noted.

The current sensitivity of the shunted detector and galvanometer circuit is then calculated by the formula:

$$I_1=\frac{I}{\sqrt{\theta}}$$

where I_1 = current for 1 division on scale of G_1,
I = current read on G_2,
θ = deflection of G_1 in divisions, corresponding to I.

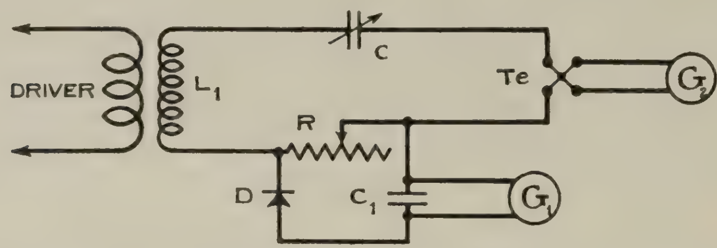


FIG. 396.—Circuit for Calibrating a Shunted Detector and Galvanometer.

If the current sensitivity of the shunted detector and galvanometer circuit is sufficiently high for the purpose, the current in the circuit should be increased until the deflection of G_1 is as high as will be used in the measurement, and the current sensitivity again calculated. The value obtained should be the same as that for the lower deflection; that is, the deflection should be proportional to the square of the current.

If the current sensitivity is not high enough, the resistance of R should be increased, and the current sensitivity determined, until the desired sensitivity is obtained.

Thus, having the current sensitivity for one division for galvanometer G_1, and, since the combination is a radio-frequency current-squared meter, the current for any number of divisions on the scale of galvanometer G_1 may be computed from the formula:

$$I=I_1\sqrt{\theta}$$

where I_1 = current sensitivity of G_1 for 1 division,
θ = deflection in divisions of G_1.

Since the fraction of the total current passing through the detector is, for shunt values up to 100 ohms, practically proportional to the shunt resistance, it is possible to calibrate the apparatus approximately with a 1 ohm shunt which, with a galvanometer of a sensitivity of $5 \cdot 10^{-9}$ ampere, and with an average silicon-antimony detector, gives a deflection of 1 mm. for about $2 \cdot 10^{-3}$ ampere rf current. Thus, 300 mms. on the galvanometer scale represent about $3.4 \cdot 10^{-4}$ ampere. This can be read conveniently on the small hot-wire instruments found in most laboratories, which give full scale deflection for from 80 to 100 ma. The sensitivity for other shunts can then be found by dividing the sensitivity by the shunt ratio.

The following table gives the approximate rf current required for a deflection of 1 mm. on a galvanometer having a direct-current sensitivity of $5 \cdot 10^{-9}$ ampere. If the sensitivity of the galvanometer is $5 \cdot 10^{-10}$, a deflection of 1 mm. with a 100-ohm shunt will represent approximately $6 \cdot 10^{-6}$ ampere, rf current. The third column gives the maximum rf current which can be safely sent through the system without danger of changing the detector resistance. The values are based on a detector resistance of 3,000 ohms and a maximum safe detector current of $12 \cdot 10^{-6}$ ampere.

SILICON-ANTIMONY DETECTOR

Shunt (ohms)	Rf sensitivity in μa	Maximum rf current in μa
1	2,000	36,000
5	400	7,200
10	200	3,600
25	80	1,400
50	41	710
100	21	360

MEASUREMENT NO. 7.

MEASUREMENT OF THE RADIO-FREQUENCY RESISTANCE OF A CIRCUIT.

The resistance of a circuit at radio frequencies is important in all radio work, and the measurement of this quantity is a fundamental operation underlying a large number of measurements. For example, the losses in insulators, condensers and antennas are found by measuring the resistance of the circuit of which they are a part. The method described below, is known as the **Resistance Variation Method** and is preferred because of its speed, accuracy, and ease of operation.

The apparatus used in determining the radio-frequency resistance of a circuit is arranged as shown in the circuit diagram, figure 397.

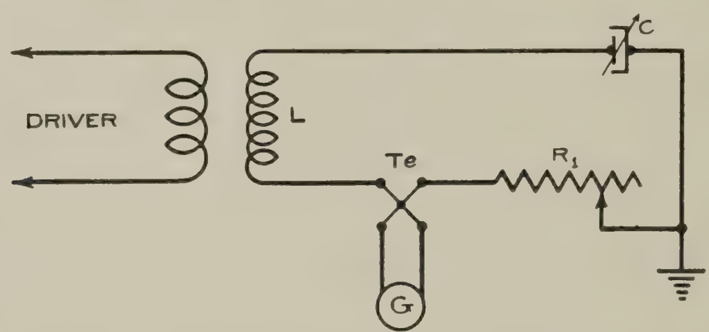


Fig. 397.—Method of Arranging a Circuit when Measuring its Rf resistance.

The driver is a source of radio-frequency current; usually an oscillating vacuum tube. It is essential for accurate measurements that the output of this driver remain constant; hence, lead-acid storage batteries are used whenever possible, because variations in filament temperature or plate voltage will cause variations in the output. Condenser C should be shielded and the shield grounded, thus eliminating the capacitive effect due to the observer's body. A resistance box R_1 of the non-inductive type is generally used, and should be connected next to the ground as shown in the figure.

The straight-wire type of resistance should, however, be used when making radio-frequency resistance measurements at very short wave lengths. At wave lengths below 200 meters, the resistance box is very likely to cause detuning of the measuring circuit and thus lead to inaccurate measurements. The straight-wire type of resistance, having an extremely low inductance, will not cause detuning, especially when it is substituted for a copper jumper of the same length.

The thermoelement *Te,* and galvanometer G must, of course, be calibrated so that the current flowing through the circuit may be known for any deflection of the galvanometer. The resistance of the thermoelement should not be high compared with the resistance of the

circuit, in order to obtain accurate results. Thus, if measurements are being made on a circuit having a resistance of three or four ohms, the thermoelement used should not have a resistance of more than one ohm; otherwise, it would be difficult to detect and measure small changes in the resistance of the circuit.

The procedure is as follows: Tune the circuit to the same wave length as that of the driver. Resonance is indicated by maximum deflection of the galvanometer. **The tuning must be very exact.** Note this deflection. Insert a resistance R_1 such that the deflection is reduced to about one-half or one-quarter of that with no resistance inserted. Note this deflection.

In order to verify the work, other values of resistance may be inserted and readings taken, and when the results are calculated they should check. In this manner an average value, which will probably be more accurate, will be obtained.

Since the circuits are tuned to resonance,

$$I=\frac{E}{R}$$

where I is the current as measured by the thermoelement and galvanometer, with no inserted resistance, E is the induced electromotive force and R the circuit resistance. When the resistance R_1 is inserted the current changes to a new value I_1. Since the voltage remains constant,

$$I_1=\frac{E}{R+R_1}$$

from which the circuit resistance is given by

$$R=\frac{R_1}{\frac{I}{I_1}-1}$$

It was assumed that the induced voltage remained constant. It is therefore necessary that the apparatus be protected from electric couplings that may change during the measurement. This is accomplished by grounding the shielded side of the condenser. The protection thus obtained should be such that the observer can place his hand on the outside of the condenser without a change occurring in the deflection of the galvanometer.

Another precaution to be observed is to make sure that the **coupling** between the two circuits **is not too tight.** In this case, too much power would be taken from the driver, thus changing its output and the voltage induced in the measuring circuit. This would lead to poor results. One method of testing the coupling is to repeat a measurement with reduced coupling, and, if the first coupling was not too tight, the

results will check. A second method is to open and close the measuring circuit, at the same time watching the ammeter in the output circuit of the driver. If a change in current is noted, it is certain that the coupling is too tight.

As has been stated before, it is very important that the two circuits be in resonance. Slight detuning will give resistance values that are too high. Detuning is indicated when there is a progressive change in the several observed values of the circuit resistance; that is, the smaller the inserted resistance, the higher the value obtained for the resistance of the circuit.

Single measurements of the circuit resistance by this method should be accurate to about two per cent, while the average value obtained when several values of inserted resistance are used should be accurate to one per cent, or better.

A typical measurement is given below.

R_1	Defl.	$I \cdot 10^{-3}$	$\frac{I}{I_1}-1$	R	$R_{\text{average.}}$
0	85.0	110.9			
2	32.5	68.6	0.618	3.23	
3	22.2	57.5	0.932	3.22	3.23
4	15.8	49.7	1.233	3.24	
0	85.3	111.2			

The first column gives the values of the inserted resistances, and the second column the corresponding deflections. The third column gives the corresponding currents in milliamperes, which are read from the calibration curve of the thermocouple and galvanometer. The numbers in the fourth column are obtained by subtracting 1 from the ratio of the average current, with zero inserted resistance, to the current with the corresponding inserted resistance; thus, the first number 0.618 is equal to $\frac{111.0}{68.6}-1$. The figures in the fifth column are the result of dividing the value of the inserted resistance by the corresponding number in the fourth column and, hence, give

$$R=\frac{R_1}{\frac{I}{I_1}-1}$$

The result, 3.23 ohms, is the resistance of the total circuit, which includes the resistances of the coil, condenser, thermocouple and leads.

If the resistance of the coil alone is desired, an air-dielectric condenser with negligible losses should be used. The resistance of the coil

is then found by subtracting the resistance of the leads and thermocouple. The latter can usually be determined with sufficient accuracy with direct current, using a Wheatstone bridge. Care should be taken to keep the coil away from the condenser case or other masses of metal, otherwise its resistance will be increased by reason of the eddy currents induced in the metal.

MEASUREMENT NO. 8.

MEASUREMENT OF THE PHASE DIFFERENCE OF CONDENSERS.

The measurement of the resistance, or the phase difference, of a condenser due to dielectric losses, depends upon the fact that when a condenser having losses is substituted in a circuit for a standard condenser having negligible losses, there is a change in the resistance of the circuit, which can be measured.

A convenient circuit for this measurement is given in figure 398. C is the standard condenser having losses so low as to be negligible.

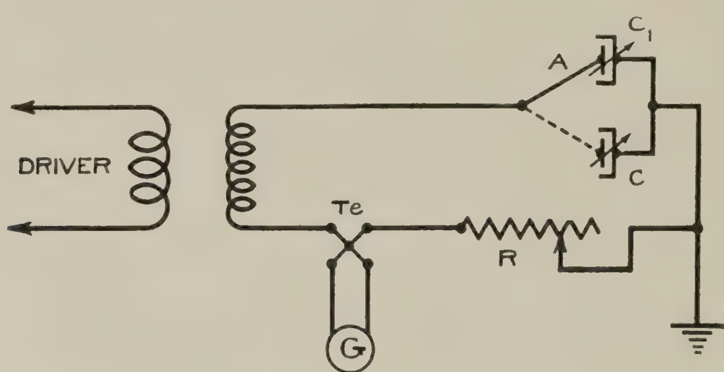


FIG. 398.—Circuit Used in Measuring the Phase Difference of Condensers.

C_1 is the condenser to be compared with the standard. C and C_1 should be so placed that the lead A can be connected to either by a minimum change in position of the lead. The resistance and thermocouple are the same as those used in the circuit resistance measurement described in Measurement No. 7.

The procedure is as follows: Connect the lead A to C and, by the resistance variation method, measure the circuit resistance. Change the lead A to C_1. Tune the circuit by means of C_1 and measure the circuit resistance again. The difference in the two values of the circuit resistance gives the resistance ρ of the condenser C_1.

The phase difference ψ of condenser C_1 is given by,

$$\tan \psi = \omega C \rho \cdot 10^{-6}$$

where $\omega = 2\pi f$,

C = the capacity in μf,

ρ = the change in circuit resistance, i. e., the equivalent series resistance.

For small angles,

$$\tan \psi = \psi \text{ in radians,}$$

hence $\psi = \omega C \rho \cdot 10^{-6}$ (radians)

whence $\psi = 3.4 \cdot 10^{-3} \omega C \rho$ (minutes)

Example:

At a wave length of 14,250m. a certain mica condenser, when substituted for a standard air condenser, increased the resistance of the measuring circuit 0.30Ω. The capacity was 0.0103 μf. Calculate the phase difference of the mica condenser.

Solution:

Formula $\psi = 3.4 \cdot 10^{-3} \omega C \rho$

substituting $= 3.4 \cdot 10^{-3} \times 6.28 \times 2.11 \cdot 10^{-4} \times 1.03 \cdot 10^{-2} \times 3.0 \cdot 10^{-1} = 1.39$

whence $\psi = 1.39$ minutes.

The resistance, ρ, of a fixed condenser is proportional to the wave length. In the case of an **air-dielectric variable condenser** in which the part of the capacity that has dielectric losses does not vary with the condenser setting, as is the usual case, the resistance ρ varies inversely as the square of the capacity.

The value of ρ for an **air-dielectric variable condenser** can be calculated for any capacity and wave length when one value is known.

Example:

A certain air-dielectric variable condenser has a resistance of 0.50Ω for a capacity of 1,000 $\mu\mu$f and at a wave length of 1,000m. What will be its resistance for a capacity of 2,000 $\mu\mu$f and at 3,000m.?

Solution:

According to the law given above,

$$\rho = \frac{\lambda}{C^2}$$

In this case the wave length is to be tripled and the capacity doubled. Hence, the resistance will be

$$\rho = \frac{3}{(2)^2} \cdot 0.50$$

$$= \frac{3}{4} \cdot 0.50 \text{ or } 0.375$$

whence $\rho = 0.375\,\Omega$

It is assumed that no additional, poor dielectric is introduced when the capacity is increased from 1,000 $\mu\mu$f to 2,000 $\mu\mu$f. Should this assumption be violated, the result obtained will be incorrect.

It should be pointed out that the above measurement determines the resistance of the condenser C_1 **due to any and all power losses.** Thus, if in addition to dielectric loss, the resistance of the leads or plates inside of C_1 is appreciable, this will be included. Thus, an otherwise perfect condenser will sometimes show a high resistance due to bad contacts between the plates and the washers used to space the plates.

Such a resistance would not show a variation dependent upon frequency, as is the case with dielectric loss.

Further use can be made of the circuit shown in figure for the calibration of condensers.

Tune the circuit by means of C. Connect A to C_1 and retune. The capacity of C then equals the capacity of C_1. Thus, C_1 can be calibrated by comparison with C.

MEASUREMENT NO. 9.

MEASUREMENT OF ANTENNA RESISTANCE.

In the measurement of antenna resistance, the resistance variation method described in Measurement No. 7 is used.

The circuit connections are made as shown in figure 399. The source of radio-frequency oscillations is a vacuum-tube driver. A 5-watt vacuum tube can be used, but somewhat higher power is desirable. The driver is coupled to the loading coil *L*, which should have a low

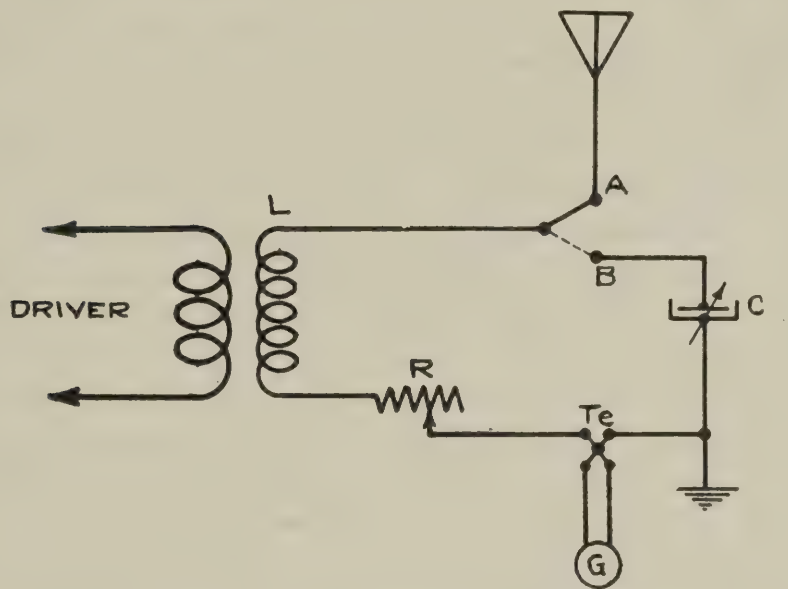


FIG. 399.—Circuit Used in Measuring Antenna Resistance.

radio-frequency resistance. A different coil is substituted for each wave length at which a measurement is desired. The thermocouple used for this work should have a low resistance. One having a resistance of about one ohm is suitable. For convenience, the galvanometer *G* should be of a portable type. The thermocouple and galvanometer must be calibrated as in the previous case of resistance measurement. The resistances are inserted at *R*. In general, satisfactory results are obtained using a non-inductive type of resistance box. The straight-wire type of resistances are used at very short wave lengths. The condenser *C* must have negligible dielectric losses and be shielded. The shield should be connected to ground. A condenser equipped with a vernier adjustment is very desirable because the tuning must be accurate. Since the condenser is to be substituted for the antenna, it should have sufficient capacity for this purpose.

The method of measurement is as follows: With the connection made to *A*, figure 399, the driver is tuned to resonance with the loaded antenna and the resistance of the circuit determined by the resistance

variation method, previously described. The connection is then shifted to B and the condenser C used to tune the circuit to resonance. This change is **not** accomplished by a switch, but by shifting the lead to the inductance L from A to B and, at the same time, changing its position, relative to the rest of the circuit as little as possible. The coupling should also be decreased somewhat, because the condenser will probably have a lower resistance than the antenna.

The total resistance of the circuit, with the condenser substituted for the antenna, is determined as in the previous case. Then, the resistance of the antenna, exclusive of the loading coil, is equal to the difference between the resistance of the circuit with the antenna, and that of the circuit with the substitution condenser. Different coils are used for each wave length. A previous knowledge of the resistance of the inductances used is not necessary, as they are common to both the antenna and the substitution circuit.

Any method of measurement that depends upon an equality of current when the connection is shifted from A to B is questionable, in as much as the coupling with the driver will not be the same for the two cases. **The only safe way is to measure the circuit resistance in each case.** It is then assumed that the coupling is constant only during each measurement. Between measurements, coil L must not be moved, relative to metal masses, such as the condenser case. **Tuning must be accurate,** and body effects—that is, electric coupling or detuning by means of the observer's body, must be eliminated. The galvanometer and resistance box should be at a sufficient distance from the antenna lead-in and other high-potential wires so that the movements of the observer will not affect the tuning. The driver coupling coil should be so located that the observer will not affect the electric coupling between the driver and coil L, or the antenna lead-in.

A shielded vernier condenser should be used in tuning the driver, and the shield grounded to the filament battery so that the hand will not affect the tuning.

Loose coupling with the driver is essential. The 5-watt vacuum-tube driver is not perfectly safe in this respect, and somewhat more power is desirable. Too much power is also objectionable because of electric coupling effects. With a low-power driver, less than full scale deflection should be used rather than tight coupling.

MEASUREMENT NO. 10.

MEASUREMENT OF THE PHASE DIFFERENCE AND LOSSES IN INSULATORS.

The determination of the losses in insulators is similar to the measurement of the resistance of condensers, Measurement No. 8. Measurements of the circuit resistance with and without the insulator are made, the difference between the observed values obtained by the two measurements furnishing the basis for the calculation of the phase difference.

A convenient circuit with which to make this measurement is given in figure 400. This consists of the same apparatus that was used in the measurement of the radio-frequency resistance of a circuit. The insulator under measurement is connected across the condenser, the grounded side remaining connected at all times.

With the lead to the high-potential side of the condenser open at A, the circuit resistance is measured by the resistance variation method described in Measurement No. 7. The setting of the condenser is noted. Connection is made at A and a second resistance measurement made. An increase in resistance will be noted. The reason for this is that the material of which the insulator is made is a poor dielectric, compared with air. Hence, the capacity through the insulator is an absorbing capacity and, when connected in parallel with the low-loss condenser, an increase in the total circuit resistance is the result. When the insulator is connected, the setting of the condenser must be lowered. The difference between the two capacities, corresponding to the two settings of the condenser, is equal to the capacity through the insulator.

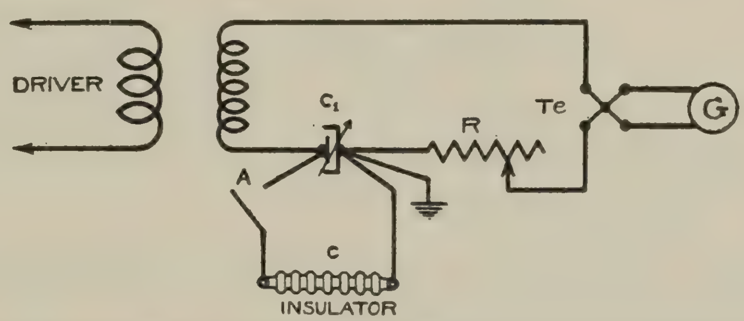


Fig. 400.—Circuit Used in Determining Phase Angle and Losses in Insulators.

From Measurement No. 8 the phase difference ψ in minutes is given by

$$\psi = 3.4 \cdot 10^{-3}\omega C\rho$$

where $\omega = 2\pi f$,

C = the capacity in $\mu\mu$f (through the insulator),

ρ = the equivalent series resistance which, in this case, is given by

$$\rho = r\left(\frac{C_1}{C}\right)^2$$

where r = the change in circuit resistance,
C_1 = the capacity of the condenser,
C = the capacity through the insulator.

Example:

The following data was obtained on an entering insulator; $\lambda = 2{,}000$ m.; $C_1 = 17.6 \cdot 10^{-5}\,\mu$f; $C = 2.4 \cdot 10^{-5}\,\mu$f; $r = 1.62\Omega$.
Calculate ψ.

Formulas (A) $\psi = 3.4 \cdot 10^{-3}\omega C\rho$

(B) $\rho = r\left(\frac{C_1}{C}\right)^2$

Substituting in (B) $= 1.62\left(\frac{17.6 \cdot 10^{-5}}{2.4 \cdot 10^{-5}}\right)^2$

$= 1.62(53.7) = 87$

whence $\rho = 87$

Substituting in (A) $\psi = 3.4 \cdot 10^{-3} \times 6.28 \times 1.5 \cdot 10^5 \times 2.4 \cdot 10^{-5} \times 87 = 6.68$

whence $\psi = 7$ minutes

A precaution to be observed, in addition to those relative to the measurement of the circuit resistance, is that the capacity of condenser C_1 be small. Since the capacity through the insulator is small, a large additional capacity would mask the losses in the insulator and thus decrease the accuracy of the measurement. The position of the lead A must not be changed when the connection is broken.

The power loss in watts in an insulator may be calculated from

$$P = IV \sin \psi$$

or

$$P = 2.94 \cdot 10^{-10}\, V^2 \omega C \psi$$

where I = the current in amperes,
V = the voltage in volts,
$\omega = 2\pi f$,
C = the capacity through the insulator in μf,
ψ = the phase difference in minutes.

For small angles, $\psi = \sin \psi$; hence, ψ is used in the formula. The constant $2.94 \cdot 10^{-10}$ is introduced in order to change ψ from radians to minutes and C from farads to microfarads.

Example:

Calculate the power loss in the entering insulator used above when $\lambda = 4{,}000$ meters and $V = 40{,}000$ volts.

Solution:

Formula $P = 2.94 \cdot 10^{-10}\, V^2\omega C \psi$

substituting $= 2.94 \cdot 10^{-10}\,(4.0 \cdot 10^4)^2 6.28 \times 7.5 \cdot 10^4 \times 2.4 \cdot 10^{-5} \times 7 = 37$

whence $P = 37$ watts.

The insulator used in the above measurements and example shows a low loss, and is a well-designed porcelain insulator.

MEASUREMENT NO. 11.

MEASUREMENT OF THE FUNDAMENTAL AND HARMONIC FREQUENCIES OF OSCILLATION OF A COIL

The apparatus required to make this measurement consists of an oscillating vacuum-tube driver, and an aperiodic resonance indicating circuit. The latter consists of a coupling coil of a few turns, crystal detector and galvanometer connected in series, figure 401. The galvanometer should be sensitive to a microampere or less.

The procedure is as follows: Place the coil to be measured in such a position that the driver may be coupled to it. Place the coupling coil of the resonance indicating circuit in such a position that it will be coupled to the coil but **not** to the driver. Vary the wave length of the driver until resonance is indicated by maximum deflection of the galvanometer. Measure the wave length of the driver. This should give the fundamental wave length of the coil under measurement.

Two precautions are to be observed in making this measurement. First, make certain that the wave length represents an oscillation in the coil, and not a natural period of the indicating circuit. This can be tested by removing or shorting the coil under measurement, which should cause the deflection of the galvanometer to drop. Second, it is necessary to ensure that the resonant wave length corresponds to the fundamental, rather than to a harmonic of the coil. If resonance is not indicated at a longer wave length, then the measured wave length represents the fundamental of the coil. By decreasing the wave length of the driver, harmonic oscillations can be found.

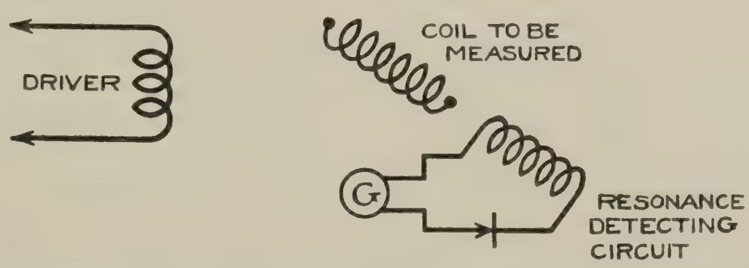


Fig. 401.—Arrangement of Circuit for Measuring the Fundamental of a Coil.

It is also possible to demonstrate the nodes and loops of voltage across a coil while it is oscillating at its fundamental or a harmonic frequency. This is done by passing a finger across the coil. A drop in the deflection of the galvanometer will be observed when the finger rests upon a potential loop (antinode) of the coil. When the finger passes over a node the deflection of the galvanometer is a maximum.

It is possible to make this measurement with or without leads, also with or without one end of the coil under measurement grounded,

so that the conditions obtaining in practice can be simulated. When one end of the coil is grounded, the coil and ground lead can act as an antenna system, and oscillate as such. This condition can usually be recognized because of the considerably greater deflection obtained in the indicating circuit. Also, when this condition exists a sensitive radio-frequency ammeter in the ground lead will indicate a flow of current to ground.

MEASUREMENT NO. 12.

MEASUREMENT OF THE REACTANCE, INDUCTANCE AND DISTRIBUTED CAPACITY OF RADIO-FREQUENCY CHOKE COILS.

1. **Reactance.** The reactance of radio-frequency choke coils may be measured by the use of the circuit shown in figure 402.

Condenser C_1 is for approximate tuning and C is a calibrated condenser. Both are shielded and grounded. L is the coupling coil and L_1 is the coil under measurement.

One side of L_1 is connected to the ground side of C and the circuit is tuned to resonance with the driver. Resonance is indicated by the galvanometer G. The setting of C is noted. A is connected and C retuned, again noting the setting.

The reactance, X_L, of L_1 is given in ohms by

$$X_L = \frac{1}{\omega \Delta C \cdot 10^{-6}}$$

where $\omega = 2\pi f$,

$\Delta C =$ change in capacity in μf.

By varying the wave length, a curve of reactance against frequency can be obtained.

The position of L_1 is important. The coil should be so placed that the condenser will not be in its field. The connection at A should be made and broken by moving only the end of the lead. **The tuning must be exact.**

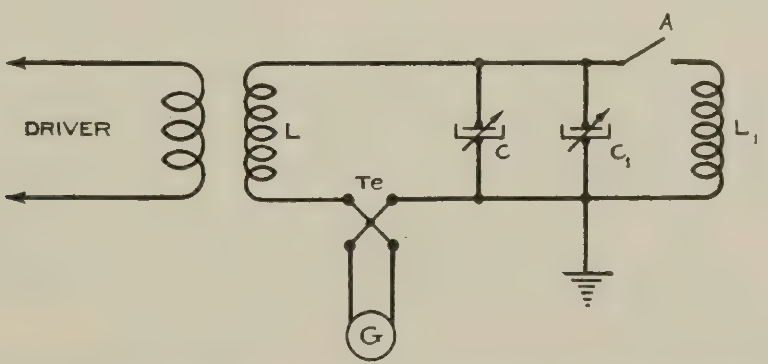


FIG. 402.—Circuit Used in Measuring the Reactance, Inductance and Distributed Capacity of Coils.

2. **Inductance and distributed capacity.** From the data obtained during the measurement just described, the inductance and distributed capacity of L_1 may be calculated.

When the wave length of the driver is longer than the fundamental of L_1, the coil offers an **inductive reactance**; and, when L_1 is connected,

C must be increased to obtain resonance. The apparent inductance of L_1 in μh is then given by

$$L_a = \frac{1}{\omega^2 \Delta C \cdot 10^{-12}}$$

where ΔC is the difference between the capacity of C in μf, when L_1 is connected and disconnected.

For a wave length shorter than the fundamental, L_1 offers a **capacitive reactance** and C must be decreased when retuning. The apparent distributed capacity, C_a, of L_1 is given directly by ΔC, which is the change in capacity of C when the circuit is retuned after connecting L_1.

If the measurement of L_a is made at a wave length that is 5 times the fundamental of the coil, the value obtained will be 4% higher than L_0. If the wave length is 10 times the fundamental, the value will be 1% higher.

The capacity found at 1/5 the fundamental wave length will be 4% lower than C_0. At a wave length 1/10 of the fundamental, the measured capacity will be 1% low.

3. **Correction terms for L_a and C_a.** By using the values of L_a and C_a obtained, and the formulas given below, it is possible to determine the difference between the measured values and the true values of the inductance and distributed capacity.

The apparent inductance in μh at any wave length is given in terms of the true inductance L_0 by

$$L_a = \frac{L_0}{(1 - \omega^2 L_0 C_0 \cdot 10^{-12})}$$

where L_0 and C_0 are given in μh and μf, respectively. From this equation, it is obvious that

$$L_a = L_0$$

when $\omega^2 L_0 C_0 \cdot 10^{-12}$ is small compared to 1. Hence, the value of L_a found by the method described above will be more nearly equal to the true inductance the longer the wave length at which the measurement is made. The term $\omega^2 L_0 C_0 \cdot 10^{-12}$ determines the percentage difference between the measured apparent inductance and the true inductance.

The apparent distributed capacity in μf is given in terms of the distributed capacity by

$$C_a = C_0\left(1 - \frac{1}{\omega^2 L_0 C_0 \cdot 10^{-12}}\right)$$

where L_0 and C_0 are in μh and μf, respectively.
Approximately,

$$C_a = C_0$$

when $\omega^2 L_0 C_0 \cdot 10^{-12}$ is very large. Hence, a short wave length should be used.

The uses of the equations given above are illustrated in the examples that follow. The measured values of C_a and L_a will be substituted for C_0 and L_0 in the last two equations. This is permissible because any error in the correction term will be a very small percentage of the total value. The process may be repeated, using corrected values of C_a and L_a if greater accuracy is required. The relations between L_a and L_0, C_a and C_0, and the formulas for reactance hold for all coils, while the method of measurement is given primarily for choke coils.

Example:

At $\lambda = 20{,}000$m. it was found necessary to increase C when L_1 was connected into circuit, the increase being $7.35 \cdot 10^{-5}$ μf. Calculate the reactance and apparent inductance at this wave length.

Solution:

Formula $X_L = \dfrac{1}{\omega \Delta C \cdot 10^{-6}}$

substituting $= \dfrac{1}{6.28 \times 1.5 \cdot 10^4 \times 7.35 \cdot 10^{-11}} = \dfrac{1}{6.92 \cdot 10^6} = 1.445 \cdot 10^5$

whence $X_L = 144{,}500\Omega$ (Inductive reactance at 20,000 m.)

Formula $L_a = \dfrac{1}{\omega^2 \Delta C \cdot 10^{-12}}$

substituting $= \dfrac{1}{8.87 \cdot 10^9 \times 7.35 \cdot 10^{-17}} = \dfrac{1}{6.52 \cdot 10^{-7}} = 1.53 \cdot 10^6$

whence $L_a = 1.53 \cdot 10^6$ μh or 1.53 h. (Inductance at 20,000 m.)

Example:

At $\lambda = 430$m. it was necessary to decrease C when L_1 was connected by $4.16 \cdot 10^{-6}$ μf. Calculate the reactance and apparent distributed capacity at this wave length.

Solution:

Formula $X_C = \dfrac{1}{\omega \Delta C \cdot 10^{-6}}$

substituting $= \dfrac{1}{6.28 \times 6.98 \cdot 10^6 \times 4.16 \cdot 10^{-12}}$

$= \dfrac{1}{1.83 \cdot 10^{-5}} = 5.46 \cdot 10^4$

whence $X_C = 54{,}600\Omega$ (Capacitive reactance at 430 m.)

$C_a = \Delta C$

since the distributed capacity of the coil at 430m. is $4.16 \cdot 10^{-6}$ μf, or 4.16 $\mu\mu$f.

Example:

Find the per cent correction to be applied to the value of L_a, at the wave length given in the first example, to obtain the true inductance.

Solution:

The term $$\omega^2 L_0 C_0 \cdot 10^{-12}$$

in the formula $$L_a = \frac{L_0}{(1 - \omega^2 L_0 C_0 \cdot 10^{-12}}$$

determines the per cent correction.

Substituting the values of L_a in μh and C_a in μf therein

$$\% \text{ correction} = [(6.28 \times 1.5 \cdot 10^4)^2\ 1.53 \cdot 10^6 \times 4.16 \cdot 10^{-18}]$$
$$= 8.98 \cdot 10^9 \times 6.36 \cdot 10^{-12} = 5.65 \cdot 10^{-2} = 0.06$$

whence correction $= 6\%$

and the value of L_a previously found is high by 6%.

Example:

Find the per cent error in the distributed capacity as found in the second example.

Solution:

The correction term $$\frac{1}{\omega^2 L_0 C_0 \cdot 10^{-12}}$$

in the formula $$C_a = C_0\left(1 - \frac{1}{\omega^2 L_0 C_0 \cdot 10^{-12}}\right)$$

is used.

substituting L_a in μh and C_a in μf for L_0 and C_0

$$\% \text{ error } = \frac{1}{[(6.28 \times 6.98 \cdot 10^5)^2 1.53 \cdot 10^6 \times 4.16 \cdot 10^{-18}]}$$
$$= \frac{1}{1.92 \cdot 10^{13} \times 6.36 \cdot 10^{-12}} = \frac{1}{122} = 0.008$$

whence error $= 0.8$ of 1%

and the value of C_a is 0.8 of 1% lower than C_0.

MEASUREMENT NO. 13.

MEASUREMENT OF THE RESISTANCE, INDUCTANCE AND DISTRIBUTED CAPACITY OF COILS.

1. **Measurement of resistance.** A method of measuring the radio-frequency resistance of a circuit is given in Measurement No. 7. Now, if the capacity used in the circuit shown in figure 397 is a **low-loss** condenser and may, therefore, be neglected, the resistance of the coil will be the difference between the circuit resistance and the direct current resistance of the thermocouple and leads. Figure 403 shows a curve where the radio-frequency resistance of a coil similar to a radio compass loop is plotted as ordinates against wave lengths as abscissas. This curve was obtained by measuring the circuit resistance, and since the losses in the condenser used were negligible, the correction to be

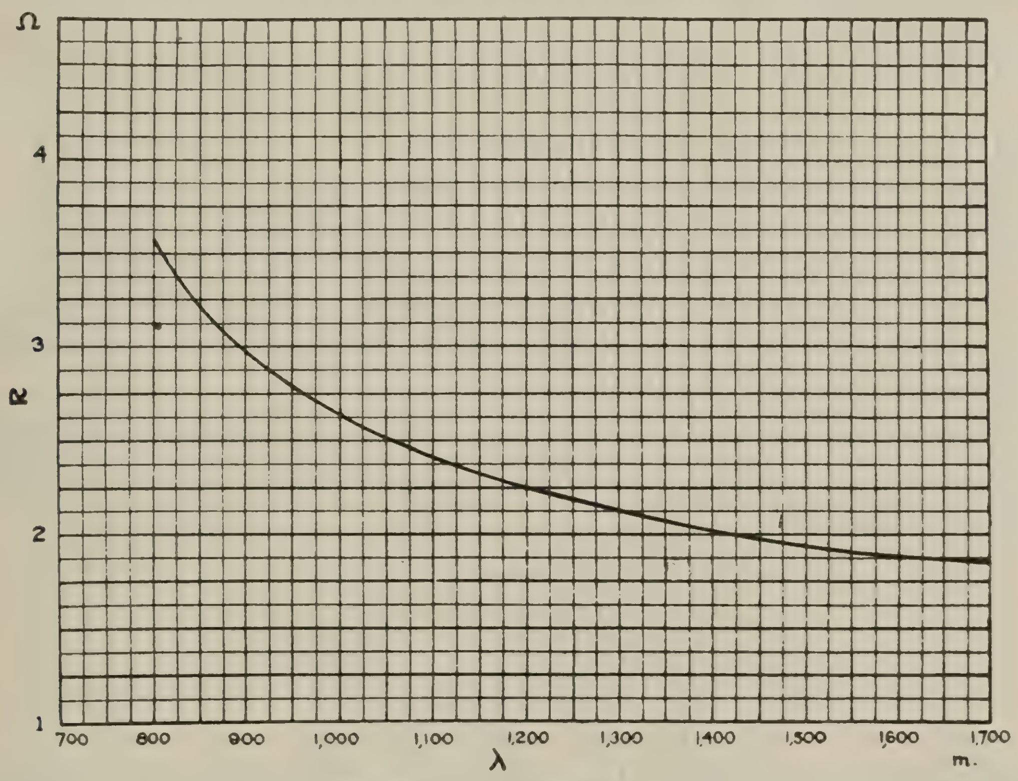


Fig. 403.—Curve Showing Decrease in Rf Resistance of a Coil with Increasing Wave Length.

deducted from the measured values was the direct-current resistance of the thermocouple and leads. The curve clearly shows how the radio-frequency resistance of a coil decreases as the capacity, and hence the wave length, is increased.

2. **Measurement of apparent inductance.** The values of the apparent inductance of the coil under measurement for radio-frequency resistance can be found if the condenser used in the measuring circuit has been calibrated.

The *LC* method is used for this purpose and is illustrated in the following.

Example:

At $\lambda = 1{,}000$m. the capacity of condenser used in series with the loop was $C = 0.0007111\ \mu$f. Calculate the apparent inductance.

Solution:

From *LC* Tables, Table No. 13 Section III, for $\lambda = 1{,}000$m.

$$LC = 0.281$$

but
$$L = \frac{LC}{C}$$

substituting
$$L = \frac{0.281}{0.0007111} = \frac{2.81 \cdot 10^{-1}}{7.111 \cdot 10^{-4}} = 395.$$

whence
$$L_a = 395\ \mu\text{h}.$$

Thus, the apparent inductance of a coil at any wave length can be found if the wave length and capacity are known. The formula is:

$$L_a = \frac{\lambda^2}{3.553 \cdot 10^6 C}$$

where λ = wave length in meters,
L_a = apparent inductance in μh,
C = capacity in μf,

and may be used in the event that no *LC* tables are at hand.

3. **Measurement of distributed capacity.** The distributed capacity of a coil may be measured by means of the harmonics of an oscillating tube.

Figure 404 shows a circuit containing the inductance under measurement, a capacity and an indicating instrument in series coupled to a vacuum-tube driver.

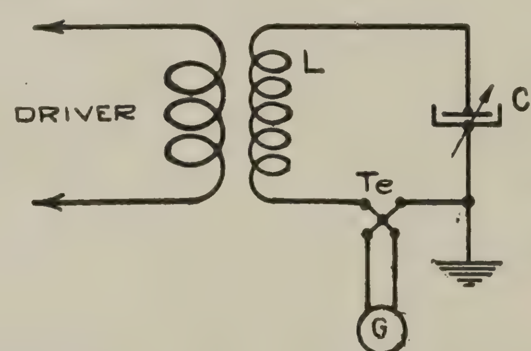


FIG. 404.—Circuit Used to Measure the Distributed Capacity of a Coil.

To find the distributed capacity of L, tune the circuit by means of C, to resonance with the driver. Note the setting of C. Now reduce the capacity of C until the second harmonic is found. Let C_f and C_{2f} represent the capacities of C at the fundamental and second harmonic, respectively. If the coil had no distributed capacity, C_{2f} would be

one-fourth of C_f, but it is actually less than one fourth. The distributed capacity, C_0, of L is then given by

$$C_0 = \frac{C_f - 4C_{2f}}{3}$$

where all the capacities are expressed in the same units, usually $\mu\mu$f. The measurement should be repeated using different wave lengths as the fundamental, and an average value obtained.

Example:

With the same coil as above, the capacity used at 1,000 meters was found to be 711.1 $\mu\mu$f. At the second harmonic, that is, 500 meters, the capacity used was 111.7 $\mu\mu$f. Calculate C_0.

Solution:

Formula $$C_0 = \frac{C_f - 4C_{2f}}{3}$$

substituting $$= \frac{711.1 - (4 \times 111.7)}{3} = \frac{711.1 - 446.8}{3} = 88.1$$

whence $C_0 = 88.1\ \mu\mu$f.

C_0 may be found by another method. Having found the apparent inductance of a coil at two or more wave lengths, the distributed capacity C_0 is given by the following formula:

$$C_0 = \left(\frac{L_1}{L_2} - 1\right)\frac{C_1C_2}{C_2 - C_1}$$

where L_1 and L_2 are the values of the apparent inductance determined at different wave lengths and C_1 and C_2 the values of capacity, used in the corresponding cases. It is the usual practice to express C_0 in $\mu\mu$f; hence, C_1 and C_2 should be expressed in $\mu\mu$f. In addition, L_1 and L_2 should always be expressed in units of equal size, such as μh or mh.

Substituting in the above formula, the distributed capacity of the coil used in the first example was found to be 88.4 $\mu\mu$f.

Example:

Given $L_1 = 427.1\ \mu$h; $L_2 = 379.6\ \mu$h; $C_1 = 421.7\ \mu\mu$f; $C_2 = 1{,}066.3\ \mu\mu$f. Calculate value of C_0.

Solution:

Formula $$C_0 = \left(\frac{L_1}{L_2} - 1\right)\frac{C_1C_2}{C_2 - C_1}$$

substituting $$= \left(\frac{427.1}{379.6} - 1\right)\frac{4.217 \cdot 10^2 \times 1.0663 \cdot 10^3}{10.66 \cdot 10^2 - 4.217 \cdot 10^2}$$

$$= \left(1.125 - 1\right)\frac{4.56 \cdot 10^5}{6.446 \cdot 10^2}$$

$$=1.25\cdot 10^{-1}\times 7.07\cdot 10^{2}=88.4$$

whence $$C_0=88.4\ \mu\mu f.$$

4. **Calculation of true inductance.** Having found the distributed capacity of the coil, it is now possible to find the true inductance L_0 by use of the following formula:

$$L_0=\frac{L_1}{\left(1+\frac{C_0}{C_1}\right)}$$

Example:

At $\lambda=1{,}000$ m. the apparent inductance of the coil was found to be 396.8 μh, the distributed capacity 88.4 $\mu\mu$f, and the tuning capacity was 711.1 $\mu\mu$f. Calculate L_0.

Solution:

Formula $$L=\frac{L_1}{\left(1+\frac{C_0}{C_1}\right)}$$

substituting $$=\frac{396.8}{\left(1+\frac{88.4}{711.1}\right)}=\frac{396.8}{1+0.124}=\frac{396.8}{1.124}=353$$

whence $$L_0=353\ \mu\text{h}.$$

A graphical method of determining the true inductance and distributed capacity of a coil, illustrated in figure 405, makes use of a plot of wave length squared against capacity. Since the true inductance L_0 is constant, the relation is linear and the plot will be a straight line. The slope of this line, that is, the tangent of the angle it makes with the axis of abscissas, determines the inductance. Thus

$$\text{Slope}=\frac{\lambda^2}{C+C_0}=3.553\cdot 10^6\, L_0$$

hence $$L_0=\frac{\lambda^2}{3.553\cdot 10^6(C+C_0)}$$

That is, L_0 in μh is found by dividing any value of wave-length squared by the product of the capacity $(C+C_0)$ in μf corresponding to the same point on the line, and the constant $3.553\cdot 10^6$.

Figure 405 shows the wave-length squared—capacity line. The line is extended until it cuts the axis of abscissas. The distance from the origin to this intercept gives the distributed capacity of the coil.

When using the graphical method to determine the true inductance, it must be remembered that the capacity must be counted from the

intercept on the axis of abscissas and not from the origin. In other words, the wave-length squared divided by $(C+C_0)$ gives the slope of the line.

Example:

From the λ^2-C line, figure 78, $\lambda^2=8 \cdot 10^5$ m., $C=550\ \mu\mu f+87\ \mu\mu f =6.37 \cdot 10^{-4}\ \mu f$. Calculate L_0.

Solution:

Formula $$L_0=\frac{\lambda^2}{3.553 \cdot 10^6(C+C_0)}$$

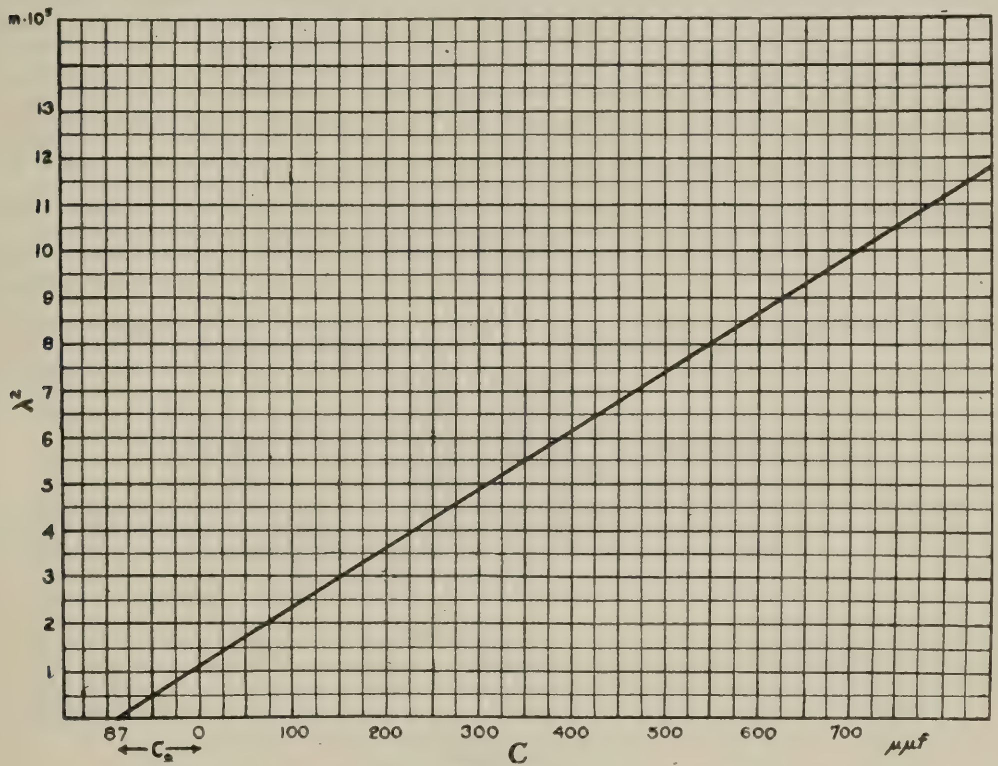


FIG. 405.—Wave Length Squared—Capacity Graph.

substituting $$=\frac{8 \cdot 10^5}{3.553 \cdot 10^6 \times 6.37 \cdot 10^{-4}}=\frac{8.10^5}{2.265 \cdot 10^3}=353$$

whence $L_0=353\ \mu h$.

The method described in Measurement No. 12 is useful when the coil is of such dimensions that it is impossible to couple a driver to it, or when its resistance is so high that the above methods are impractical.

MEASUREMENT NO. 14.

MEASUREMENT OF THE CONSTANTS OF AN ANTENNA.

In order to determine the inductance L and capacity C of an antenna two loading coils are successively inserted and the resulting wave length of the antenna circuit in each case is determined.

1. **Measurement of the inductance and capacity.** Figure 406 shows the driver, the antenna with one of the loading coils L_1 and indicating instrument in series, and a wavemeter, *Wm*.

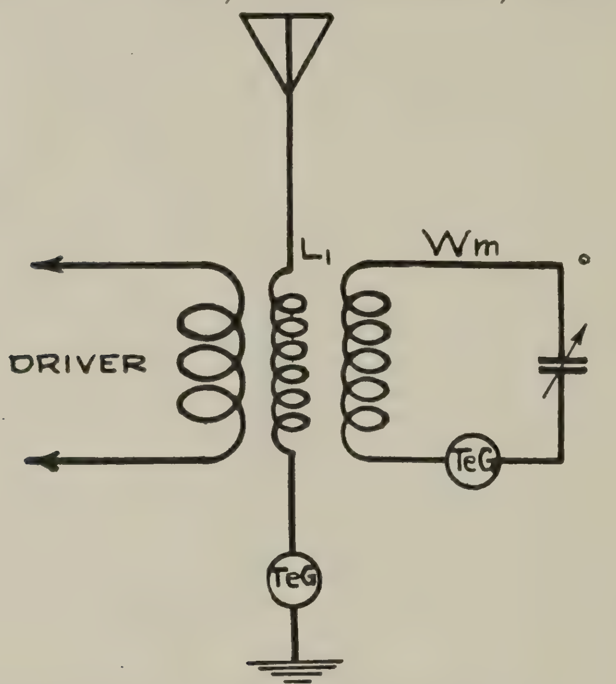


FIG. 406.—Circuit Used for Determining the Constants of an Antenna.

The wave length of the driver is varied until the resonant wave length of the antenna is found, as indicated by the meter in the antenna circuit. The antenna is then detuned and the wave length of the driver determined by the wavemeter. Two coils L_1 and L_2 are inserted, and the corresponding wave lengths λ_1 and λ_2 are determined. Using the equation for lumped capacity and inductance,

$$\lambda_1 = 1885\sqrt{(L_1 - L)C}$$

$$\lambda_2 = 1885\sqrt{(L_2 - L)C}$$

where L_1 and L_2 are the apparent inductances of the inserted coils at the measured wave lengths and L is the inductance and C the capacity of the antenna in μh and μf, respectively. Eliminating C between these two equations and solving for L,

$$L = \frac{L_2\lambda_1^2 - L_1\lambda_2^2}{\lambda_2^2 - \lambda_1^2}$$

The value of L is obtained from the known values of L_1, L_2, λ_1, and λ_2. Substituting for L in one of the original equations (preferably the one corresponding to the larger loading coil), the value of C is found.

For any antenna, L_1 should have a value of about four or five times the antenna inductance and L_2 should have about ten times the inductance of L_1.

The values of L and C determined in the above manner give the low-frequency values, which are the values to be used in calculations. The inductance values will be accurate to a few per cent, and the capacity will be accurate to about one per cent. These values are sufficiently accurate for most antenna measurements.

Example:

Two loading coils having true inductance values of 188.8 μh and 5,680 μh were inserted in a certain antenna, and the corresponding wave lengths were found by measurement to be 1,088m. and 5,350 m., respectively. Calculate the inductance and capacity of the antenna.

Solution:

The apparent inductances of the inserted coils were calculated using the formula

$$L_a = \frac{L}{1-\omega^2 L C_0}$$

given in measurement No. 12 and found to be

$$L_1 = 189.5\ \mu\text{h}$$

$$L_2 = 5{,}742\ \mu\text{h}$$

Formula
$$L = \frac{L_2\lambda_1^2 - L_1\lambda_2^2}{\lambda_2^2 - \lambda_1^2}$$

substituting
$$= \frac{5.742 \cdot 10^3(1.088 \cdot 10^3)^2 - 1.895 \cdot 10^2(5.35 \cdot 10^3)^2}{(5.35 \cdot 10^3)^2 - (1.088 \cdot 10^3)^2}$$

$$= \frac{6.805 \cdot 10^9 - 5.42 \cdot 10^9}{2.74 \cdot 10^7} = 50.5$$

whence
$$L = 50.5\ \mu\text{h}.$$

Solving for capacity

Formula
$$\lambda = 1885\sqrt{(L_2+L)C}$$

whence
$$C = \frac{\lambda^2}{3.553 \cdot 10^6(L_2+L)}$$

substituting
$$= \frac{(5.35 \cdot 10^3)^2}{3.553 \cdot 10^6(5.742+50.5)}$$

$$= \frac{2.86 \cdot 10^7}{2.058 \cdot 10^{10}} = 1.390 \cdot 10^{-3}$$

whence
$$C = 0.001390\ \mu\text{f}.$$

2. Measurement of the capacity by the substitution method. The capacity may be measured by a somewhat easier method, known as the substitution method, in which a calibrated condenser is substituted for the antenna in the same manner as in the measurement of antenna resistance, Measurement No. 9. The important requirement of this method is that a large inductance be inserted in the antenna circuit. Sufficient inductance should be used to tune the antenna to ten or twenty times its fundamental wave length.

With a large inductance inserted, a vacuum-tube driver is tuned to resonance with the antenna. Connection is now shifted from the antenna to the condenser, and the latter is tuned to resonance with the driver. The capacity of the condenser and antenna are equal. The capacity found by substitution was

$$C = 0.001388 \ \mu\text{f}.$$

3. Measurement of the fundamental wave length. The fundamental wave length of an antenna is found approximately by inserting a small single turn and indicating instrument in the ground lead and measuring the resonant wave length. If a driver, damped or undamped, be tuned to resonance with the antenna circuit, its wave length will be approximately the fundamental of the antenna. The measured wave length will be slightly high, due to the inductance of the single turn inserted.

The fundamental wave length of the antenna used in the above measurements was found to be

$$\lambda_f = 520 \text{ m}.$$

MEASUREMENT NO. 15.

MEASUREMENT OF THE AMPLIFICATION CONSTANT AND INTERNAL PLATE CIRCUIT RESISTANCE OF A VACUUM TUBE.

The amplification constant, μ, of a vacuum tube is the ratio of the change in plate voltage to the change in grid voltage that will produce the same change in plate current. It is dependent upon the geometrical characteristics of the tube.

Figure 407 shows a circuit, given by H. J. van der Bijl, for the determination of μ by a direct-current method. The resistance r_2 should have a value of 10 ohms. For convenience in measurement, r_1 consists of

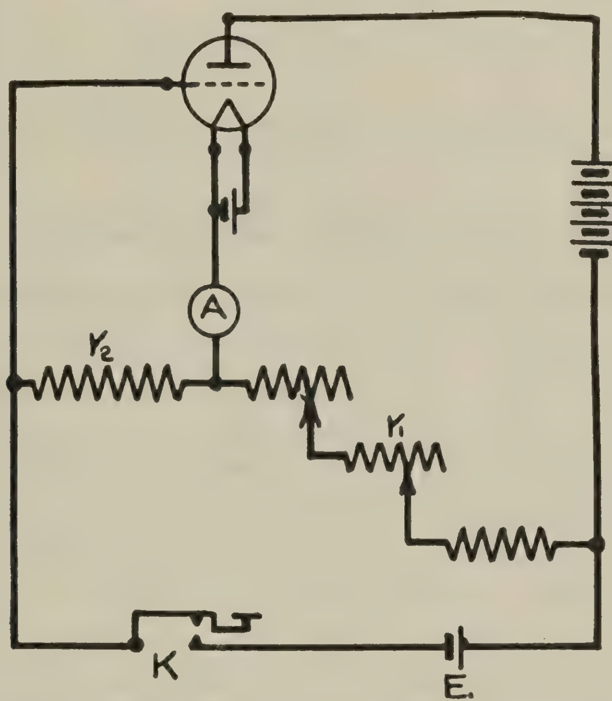


Fig. 407.—Circuit Using Direct Current in the Measurement of the Amplification Constant.

three dial rheostats of 10, 100 and 1,000 ohms arranged in steps of 1, 10 and 100 ohms, respectively. If the rheostats are marked in tenths of the actual resistances, the setting of the dials will give μ directly. E is a battery having a voltage of 10 or 20 volts.

By closing the key K, opposite potentials are applied to the grid and plate. The values of these potentials are proportional to the resistances r_1 and r_2. Since a potential applied to the grid produces μ-times the effect of a potential applied to the plate, no change will be produced in the reading of the milliammeter A when

$$r_1 = \mu r_2.$$

Hence
$$\mu = \frac{r_1}{r_2}$$

A similar method, but which uses an alternating-current source instead of the battery E, has the advantage that it also allows a simple determination of the internal plate circuit resistance.

Figure 408 shows the circuit in which E is replaced by an alternating-current source, and the milliammeter A by the telephones. The

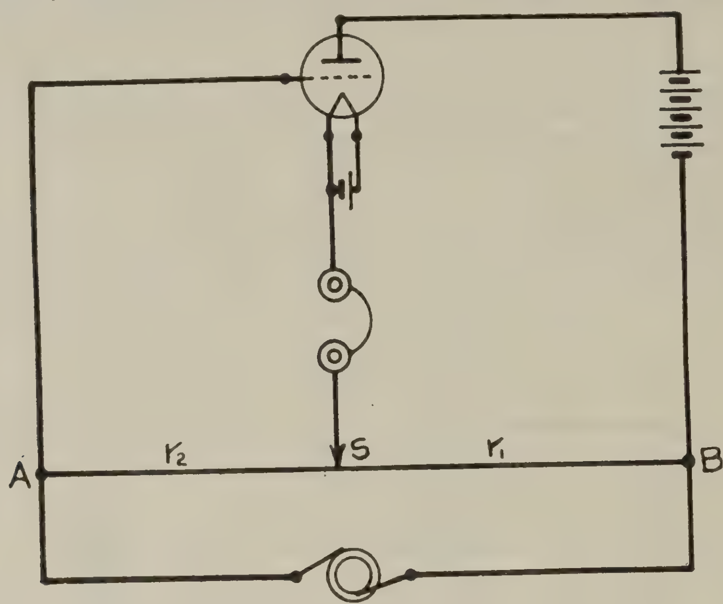


FIG. 408.—Circuit Using Alternating Current in the Measurement of the Amplification Constant.

resistance units r_1 and r_2, figure 407 have been replaced by a slide wire AB, but either the units or slide wire may be used. The slide wire may be of the straight-wire type or it may be wound on a cylinder.

A low voltage, frequency 1,000 cycles, is impressed across the slide wire AB. Voltages are thus applied to the grid and plate, their values depending, as in the previous case, upon the values of r_1 and r_2. As in the case with direct current, the voltage applied to the grid has μ-times the effect of that applied to the plate on the plate current. If the position of the slider S be varied until

$$r_1 = \mu r_2$$

there will be no change of current in the plate circuit, that is, there will be silence in the telephones. As in the previous case

$$\mu = \frac{r_1}{r_2}$$

The internal plate circuit resistance is easily obtained by a slight variation in the circuit shown in figure 408. A resistance r is connected as shown in figure 409 and may have values up to 10,000 ohms, in some cases more, depending upon the vacuum tube under measurement.

The slider S is again varied until silence is obtained in the telephones. The internal or plate resistance R_p is now given by

$$R_p = r\left(\mu \frac{r_2}{r_1} - 1\right)$$

If r_2 is made equal to r_1, and r is varied until silence is obtained, a simpler equation is obtained. In this case R_p is given by

$$R_p = r\ (\mu - 1)$$

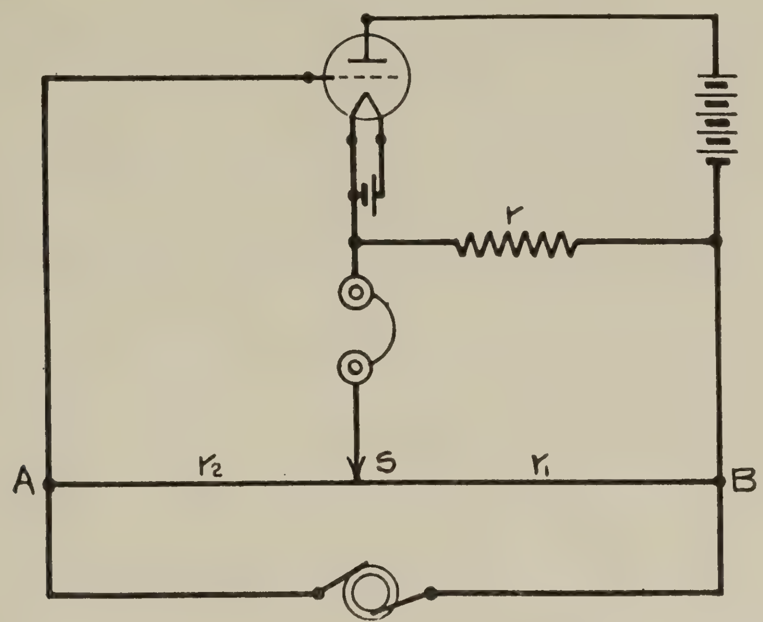


FIG. 409.—Circuit for Determining the Internal Plate Resistance.

A simple method for making both measurements may be used by changing the circuit just given as indicated in figure 410. Let $r_2 = r_3$. Close S_2 and open S_1. Adjust r_1 until silence is obtained. Then

$$\mu = \frac{r_1}{r_2}$$

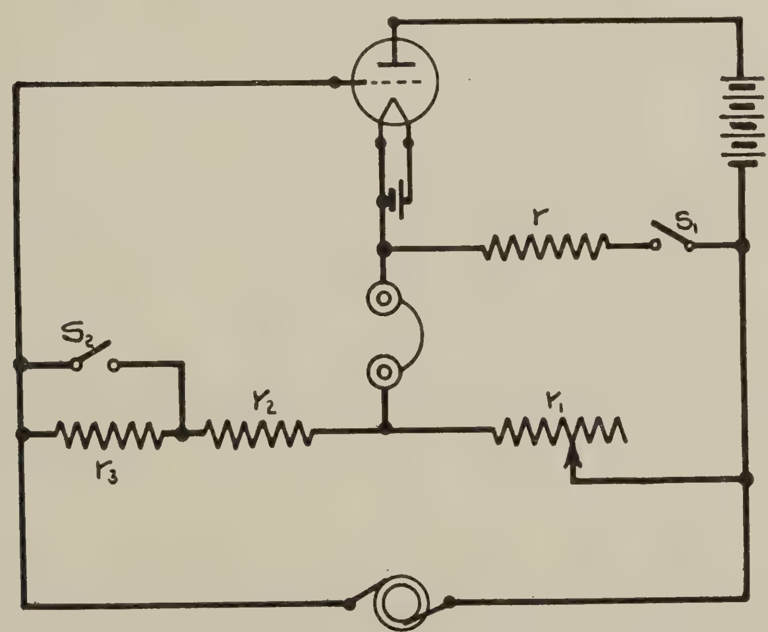


FIG. 410.—Circuit Used in Measuring the Constants of a Vacuum Tube.

close S_1. It may be seen from the equation above that it would be impossible to obtain a balance under these conditions. If, however,

r_2 be doubled, which is accomplished by opening S_2, and r be now varied, a balance can be obtained. The internal resistance is given directly by r. Thus

$$R_p = r\left(\frac{r_1}{r_2} \times \frac{2r_2}{r_1} - 1\right)$$

and hence

$$R_p = r.$$

MEASUREMENT NO. 16.

MEASUREMENT OF THE AMPLIFICATION OF AUDIO-FREQUENCY AMPLIFIERS.

This measurement is accomplished by means of the following circuit.

In figure 411, the resistances R_1 and R_2 should be kept about equal, and may have values up to 10,000 ohms. R is a lower resistance, its value being determined by the voltage given by the audio-frequency

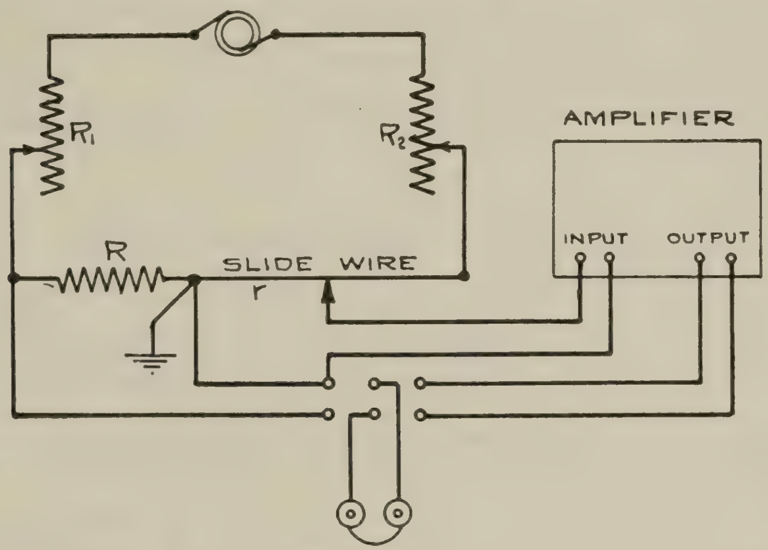


FIG. 411.—Circuit Used in Measuring the Amplification of Af Amplifiers.

alternator, and by the intensity of the signal desired. r is a slidewire resistance of either the straight or circular type. As the diagram shows, when the *DPDT* switch is thrown to the left, the telephones are connected across the resistance R, and when thrown to the right the telephones are connected to the output of the amplifier.

The method of measurement is one of comparison; that is, the signal in the telephones, when they are connected across R, is compared with the signal produced by the amplifier, connected across r.

The generator is adjusted to the voltage and frequency desired. The voltage is usually adjusted to about 5 volts. The frequency may be varied over the desired range, and a curve plotted of amplification against frequency. For single measurements, it is customary to use a frequency of 1,000 cycles.

After the signal intensity has been adjusted by means of the resistances R_1 and R_2, the switch is thrown back and forth and, at the same time, r is varied until the intensity of the signal in the telephones is the same for both positions of the switch. The resistances are in series; hence, the current is the same through both. The voltage is proportional to the resistance; hence, the ratio of the voltage across the telephones to that across the input of the amplifier is equal to the

ratio of R to r. Therefore, when the signal is the same for both positions of the switch, then

$$\text{Voltage amplification} = \frac{R}{r}$$

It is advisable to ground one end of the slide wire and with it one side of the input of the amplifier, in order to eliminate any residual signal which might be introduced by capacitive couplings.

Example:

In a certain measurement, silence was obtained with the following resistances:

$$R_1 = R_2 = 5{,}000\Omega$$
$$R = 20\Omega$$
$$r = 0.95\Omega$$

Solution:

Formula $\qquad \text{Voltage amplification} = \dfrac{R}{r}$

substituting $\qquad = \dfrac{20}{0.95} = 21.0$

whence $\qquad \text{Voltage amplification} = 21.0$

MEASUREMENT NO. 17.

MEASUREMENT OF THE AMPLIFICATION OF A VACUUM TUBE AND TRANSFORMER.

The measurement of the audio-frequency amplification obtained with a vacuum tube and transformer is made by the use of the circuit shown in figure 412.

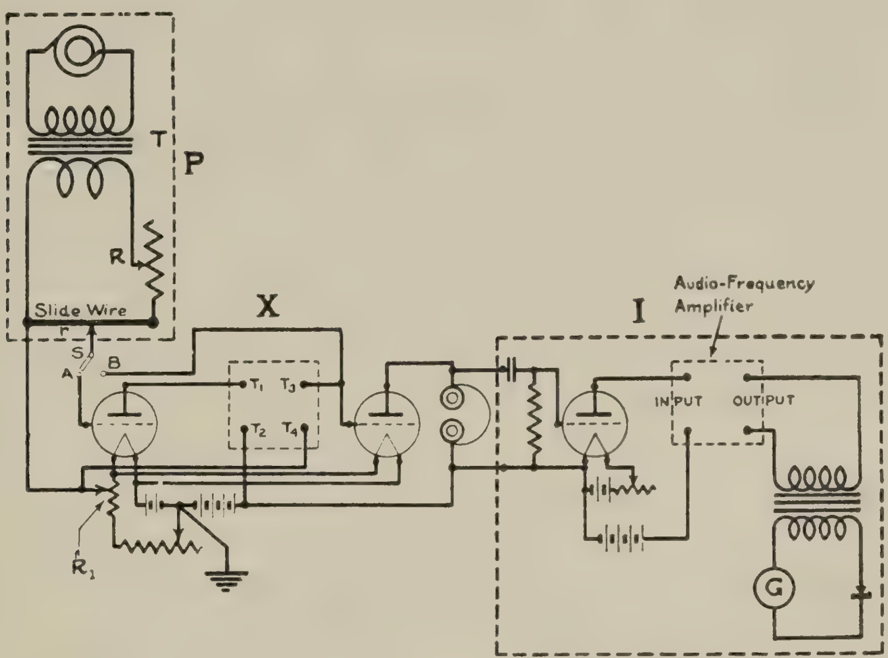


FIG. 412.—Circuit to be Used in Measuring the Amplification of a Vacuum Tube and Transformer.

The apparatus may be divided into three parts: the power supply or source P, the circuit under measurement X, and the indicating circuit I. Power is supplied by an alternator, the frequency of which may be varied over the desired audible range. T is a step-down transformer. Resistance R is used to adjust the current flowing through the slide wire.

The circuit under measurement X, consists of two tubes, connected by the audio-frequency transformer under test, the connections for this transformer being shown at T_1, T_2, T_3 and T_4. A *SPDT* switch is used to connect the slider of the slide wire to the input of either the first or second tube. R_1 is a low resistance used to obtain a negative bias for the grids, its value depending upon the vacuum tube used.

The indicating circuit I consists of a tube which connects the circuit X to the input of an audio-frequency amplifier. The output of the amplifier is connected by means of a one-to-one ratio transformer to the indicating device, which is the crystal detector and galvanometer. This galvanometer should read microamperes. The crystal detector is used to convert the audio-frequency current into direct current, which gives an

indication on G. This detector should be capable of stable adjustment. As may be seen from the figure, the voltage drop across the telephones is used as the input voltage for circuit I.

The method of measurement is as follows: Connect the transformer to T_1, T_2, T_3 and T_4. Throw the switch S to B, and set the slider of the slide wire at or near the maximum position. Adjust the filament current and grid bias to the proper values. With these adjustments made, R should be varied until the signal produced by the telephones is about normal, not too loud. R will need no further attention and the current flowing through it will have a steady value.

Circuit I should next be put into operation by lighting the tubes and adjusting the detector. A deflection of G that can be easily read should be obtained. The readings to be noted are the deflections of G and the value of r. Next reduce r to a low value, throw S to A and re-adjust r until the deflection of G is the same as in the previous case. Call this value r_a. Since the current through the slide wire was constant, the voltage applied to the vacuum tubes was proportional to the resistance; and since the output at the telephones was the same in each case, the amplification obtained by the use of the vacuum tube and transformer is given by the ratio of r to r_a. Thus,

$$\text{Amplification} = \frac{r}{r_a} \cdot$$

Changes, such as frequency, plate voltage, grid bias or type of transformer, may be made and the measurement repeated.

The indicating circuit I may be applied to the circuit shown in figure 411, under Measurement No. 16, and the measurement made by equalizing the deflections of G, instead of the signal in the telephones. The method described herein permits the measurements to be made with a higher degree of accuracy than with the audibility method.

MEASUREMENT NO. 18.

COMPARISON OF VACUUM TUBES AS DETECTORS.

A practical method of comparing vacuum tubes as detectors is simply to place the tubes to be compared in a receiver and take audibility measurements on the same signal. It is necessary, of course, to adjust the filament current and plate voltage to the correct value should the ratings for the several tubes call for different values. Retuning will probably be necessary.

There are other methods whereby values representing the merit of tubes as detectors may be obtained, but the simple comparison described herein furnishes a practical method which is all that is required in most instances.

MEASUREMENT NO. 19.

MEASUREMENT OF EXTREMELY SMALL CAPACITIES—HETERODYNE METHOD.

The heterodyne method of measuring extremely small capacities, such as those between the elements of a vacuum tube, is described in the following.

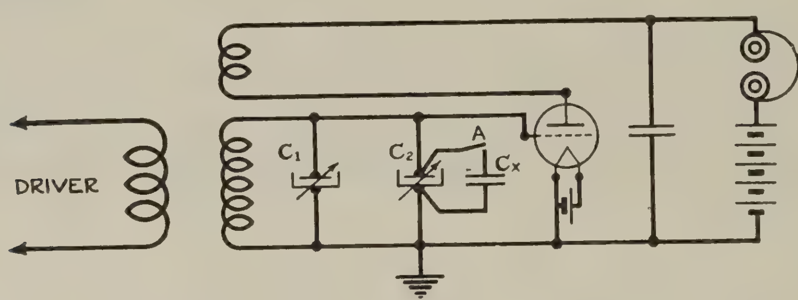


FIG. 413.—Circuit Used in Measuring Very Small Capacities.

The circuit used in this measurement is shown in figure 413. It consists of a short-wave vacuum-tube driver and an oscillating vacuum-tube receiving circuit. C_1 is a condenser used to tune the circuit to approximate resonance with the driver. C_2 is a calibrated condenser having a small maximum capacity, and preferably equipped with a vernier adjustment. Both of these condensers should be shielded and their shields grounded as shown in the figure. C_x is the piece of apparatus the capacity of which is to be measured. One side of C_x is connected to the grounded side of C_2, while the other side is to be connected by a jumper already placed in the required position and fixed in relation to the other connection so that the capacity between these two leads will have a constant value throughout the measurement, and therefore will not need to be considered.

Set the driver to a wave length of approximately 600 m. and tune the receiving circuit by C_2 until a beat note is heard. With C_x in position near C_2 and connected as stated above, tune the receiving circuit by C_2 until zero beat note is obtained. Note the setting of C_2 for this condition. Next, complete the connection to C_x as shown in the figure by lead A. A beat note should now be heard. Reduce the capacity in C_2 until zero beat note is again obtained. Note this new setting of C_2. The capacity C_x of the device being measured is then equal to the difference in the capacity at the two settings of C_2.

This method of measurement is sensitive enough to measure the change in capacity produced by placing a piece of bakelite one inch square on top of one of the terminals of C_2, and is also very accurate and entirely satisfactory for use in the measurement of any small capacity.

MEASUREMENT NO. 20.

VACUUM TEST FOR VACUUM TUBES.

A circuit for making qualitative determinations of the residual gas in a vacuum tube is shown in figure 414. The method depends upon the following facts. If the plate voltage of a tube is made quite high, so as to cause a fairly large plate current to flow through the tube, there will be an increase in the number of collisions between the electrons which constitute the plate current, and the molecules of gas which are left in the tube. The number of collisions which occur will also depend upon the amount of residual gas left in the tube, or the degree of evacuation. When an electron collides with a gas molecule, it will in general ionize it; that is, it knocks off an electron and leaves a positive ion. If, now, the grid of the tube is the most negative electrode, the positive ions which are thus formed will flow to the grid as fast as they are formed, causing a flow of current in the grid circuit. The direction of the current is opposite to that which flows when negative electrons move to the grid, and is called a **reversed current.** The direction of flow is from grid to filament outside the tube. A sensitive direct-current meter in the grid circuit will serve to measure this current and will, in effect, count the rate at which ions are being formed. Since the rate of formation of positive ions depends upon the amount of residual gas, the reversed current is a measure of the degree of evacuation of the bulb.

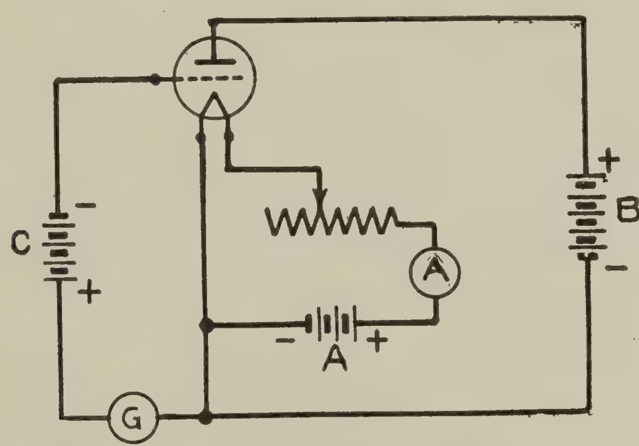


FIG. 414.—Circuit Used in Making Qualitative Measurements of Residual Gas.

The plate battery *B*, figure 414, should have a voltage of 100 to 200 volts, the grid battery *C*, 5 to 10 volts, both depending upon the tube under test. *G* is a direct-current instrument, sensitive to a microampere or less.

Under these conditions, the reversed current in receiving tubes should not exceed a few tenths of a microampere, if the pressure of the residual gas is low.

In a lot of 8 receiving tubes that were tested under the above conditions, 6 showed no reversed grid current, one showed 0.1 and one 4 microamperes. The last tube, showing 4 microamperes, was unusual. These tubes were considered good high vacuum tubes.

To test transmitting tubes, the voltages of batteries *B* and *C* are increased proportionally.

MEASUREMENT NO. 21.

MEASUREMENT OF THE CURRENT SENSITIVITY OF TELEPHONES.

The current sensitivity of telephones is measured by the value of the alternating current flowing through the windings of the magnets which will produce a signal of unit audibility.

The circuit for measuring the current sensitivity of telephones at a certain frequency is shown in figure 415.

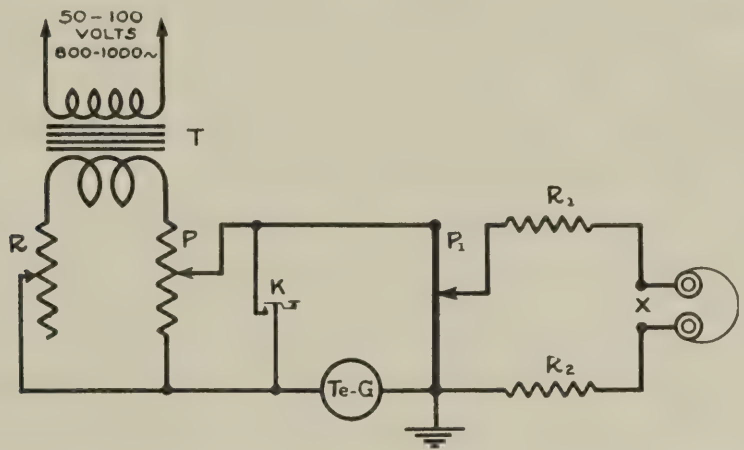


FIG. 415.—Circuit Used in the Determination of the Current Sensitivity of Telephones.

An alternating current having a frequency between 800 and 1,000 cycles which can be supplied by a low-voltage generator (50–100 volts) is required. The audio-frequency current is stepped down through the transformer T, and fed through the variable resistance R into the potentiometer P. TeG is a calibrated thermo-galvanometer having a range from 0–100 milliamperes. P_1 is a slide-wire potentiometer which may be made by stretching a wire having a resistance of one ohm per meter on a meter stick and providing it with a sliding contact; then, for any position of the slider, the exact resistance of the wire may be read directly from the meter stick. R_1 and R_2 are resistances of approximately 0.5 megohm. A ground should be connected as shown in the figure. The generator used should not be grounded. K is a back contact key. The circuit is operated in the following manner.

With all resistance in at R and P, start the generator. Connect the telephones to be measured at X. With the key K make signals. These should be easily distinguishable in the telephones. Next, decrease resistance at P until the signals are very weak, then vary the slide wire P_1 until separate letters sent as signals can just be read. At this point the signal has an audibility of one, and the voltage drop across P_1 can be calculated from the formula:

$$E = IR$$

where I = current read at TeG,
R = resistance in slide wire P_1.

Thus, having found the voltage drop, and since the resistances R_1 and R_2 are known and the impedance (ac resistance) of the telephones is approximately known, the current flowing through the telephones can be calculated by using Ohm's law:

$$I = \frac{E}{R}$$

where E = voltage drop across P_1,
R = sum of resistances R_1, R_2 and that of the telephones.

The average value of the impedance of the telephones used in the Navy is 25,000 ohms, one type having an impedance of 22,000 ohms, and another type approximately 30,000 ohms.

In case a generator suitable for this measurement is not available, a tuning-fork oscillator, such as the one described in Chapter VII, Part 7, may be used in itsplace. Either the 800-cycle or 1,000-cycle oscillator may be employed, but the 800-cycle is preferred because most of the modern telephones have their sensitivity peak in the neighborhood of 800 cycles. However, since the sensitivity of telephones varies with the frequency, a determination of the point of maximum sensitivity will require the use of an audio-frequency source, the frequency of which can be varied at will.

Two observers should make this test. One should operate the key while the other reads aloud the letter signals, being checked by the other. This will reduce the possibility of error. The room should be quiet and the apparatus free from induction. Care should be taken to have the ground connected as shown in the diagram. The ground connection eliminates residual currents which may cause considerable error.

MEASUREMENT NO. 22.

CALIBRATION OF THE SENSITIVITY OF A RECEIVER.

A receiver can be calibrated by two methods: (1) the watt sensitivity (local transmitter) and (2), using signals from a nearby transmitting station of known antenna effective height and antenna current.

1. The **watt sensitivity** of a receiver is defined as the power (in watts) in the antenna that is required to produce a signal of unit audibility in the telephones and is, therefore, a measure of the efficiency of the entire receiving system.

The method of determining the watt sensitivity of a receiver by the use of a local continuous-wave driver and artificial antenna is difficult, and the results obtained thereby are of somewhat doubtful value unless the measurement is made under ideal conditions. It is preferable to use a nearby transmitter and measure the known electric field intensity produced by it using the telephone current comparator method. Its value lies in the fact that it serves as a means for determining the sensitivity of a receiver when a nearby transmitter is not available.

The measurement of the watt sensitivity of a receiver requires the use of the sensitive shunted detector with galvanometer described in Measurement No. 6. The circuit for measuring the watt sensitivity of any receiver is shown in figure 416.

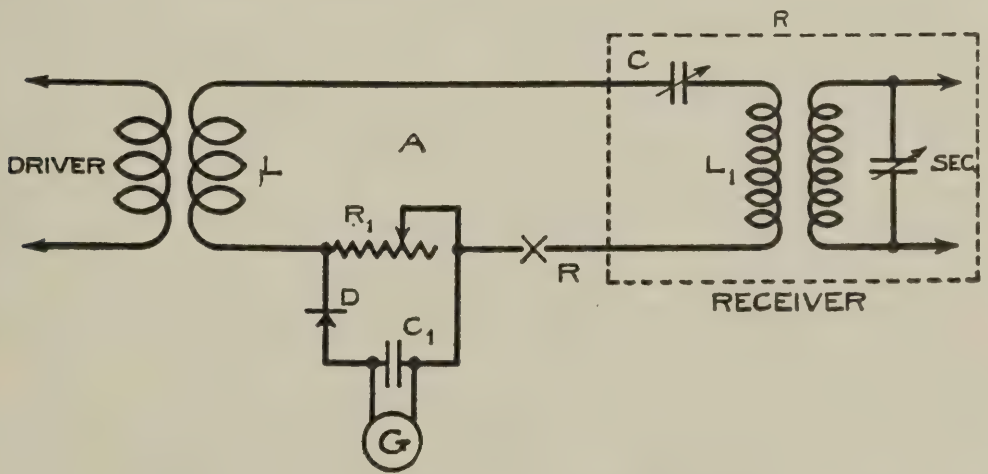


FIG. 416.—Circuit Used for Measuring the Watt Sensitivity of a Receiver.

The driver is of the continuous-wave type. Circuit *A* is the artificial antenna in which is connected the shunted detector and galvanometer. It is not necessary that the artificial antenna have the same constants as those of the antenna ordinarily used with the receiver, nor that the same wave length be employed, as is evident from the formula given later. *R* is the receiver proper, including the antenna circuit and the secondary circuit with the oscillating vacuum-tube detector, telephones and audibility meter, or other device for measuring audibilities.

The current sensitivity of the shunted detector and galvanometer is then found as explained in Measurement No. 6. The shunt resistance R_1 should not exceed 100 ohms. Having found the current sensitivity for a deflection of 1 **division** on galvanometer G, circuit A is then tuned to resonance with the driver and the coupling adjusted to give a deflection of one or two divisions on G. This deflection should be read accurately, and the corresponding current calculated. Couple the secondary of the receiver to L_1 and tune in the signal from the driver in the usual manner, using **optimum coupling.** Measure and record the audibility of the signal, using either (a) the audibility meter method or (b) the telephone current comparator method.

(a) The **audibility meter** is a variable inductive resistance shunted across the telephones, figure 417, and so designed that the impedance inserted by it in the detector circuit is not changed when the resistance across the telephones is varied. The method of connecting it in the circuit is shown in the figure.

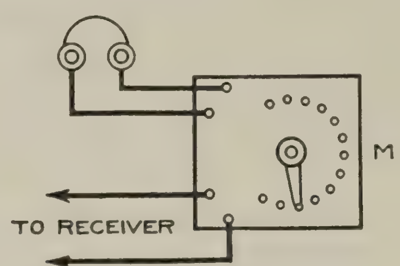


FIG. 417.—Method of Connecting an Audibility Meter into the Circuit.

The receiver is tuned to the frequency of the signal to be received, using **optimum coupling,** as described in Part 2 Chapter VI of Section II. The audibility of the signal is measured in the usual manner, that is, by rotating the switch lever on the audibility meter from the contact point 1 to a point one lower than that at which the signal vanishes. The number engraved on the face at this point represents the audibility of the signal; for example, if the number is 500, then the signal is said to have an intensity of 500 times audibility, or an audibility of 500.

(b) **Telephone current comparator method.** The **telephone current comparator** is an instrument that generates a current of audible frequency that can be varied in strength over a wide range, for comparison with the currents produced in the telephones of the receiver by signals from a transmitting station.

The telephone current comparator circuit is shown in figure 418. A is an audio oscillator containing a 1,000-cycle tuning fork which is electrically driven through a microphone button and operates on 6 volts. Output leads are taken from a step-up transformer. The milliammeter *ma*, has a range of approximately 0 to 50 milliamperes. C is a 2 μf, paper-dielectric condenser. The potentiometer P has a total resistance of 100 ohms and can be varied in 10-ohm and 1-ohm steps. The resistances R_1 and R_2 are each 100,000 ohms (non-inductive).

The current from the battery flowing through the field coil, and thence to the microphone button, causes the tuning fork to vibrate. The output from the audio-frequency generator is connected through the milliammeter *ma*, and the potentiometer *P*. The current flowing in this circuit

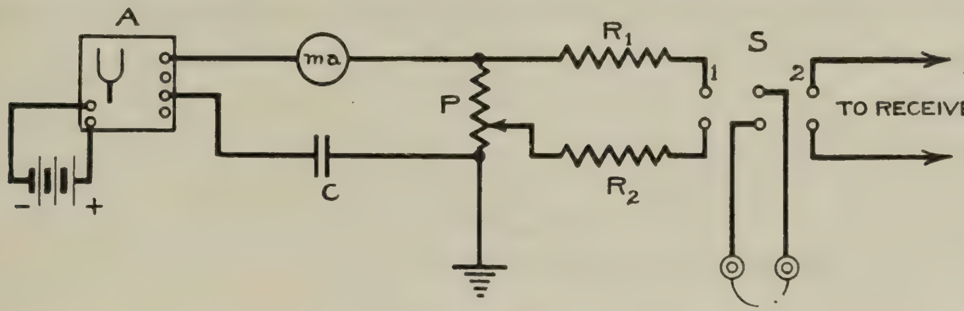


FIG. 418.—Telephone Comparator Circuit.

will cause a current to flow in the telephones when switch S is thrown to 1, and its strength can be varied by varying the part utilized of the total voltage drop across the potentiometer. The frequency of the signal heard in the telephones from the audio oscillator remains constant. An omnigraph, or hand operated key, placed across the telephone leads of circuit PR_1R_2, may be used to make dot-dash signals from the comparator. This is of some advantage when the radio signal is also broken up into dots and dashes.

To compare the strength of incoming CW signals with signals from the comparator, a switch S is provided to throw the telephones from the comparator to the receiver. With the switch thrown to position 2, tune the desired radio signal in the usual manner, using **optimum coupling.** Change the beat frequency of the signal to equal the frequency of the tuning fork. This operation can be facilitated by connecting the telephones alternately to the comparator and to the receiver by means of the switch. When the frequencies of the two signals have been adjusted to be the same, the intensity of the signal from the comparator is varied until it equals the intensity of the radio signal. When this equality has been obtained, it is evident that the current produced in the telephones by the radio signal is equal in value to the current flowing in the circuit PR_1R_2 from the comparator.

Since the resistance of the potentiometer and the current flowing through it are known, the voltage drop across that part of the potentiometer included in the circuit PR_1R_2, when equality of signals has been obtained, can be calculated from the formula:

$$E = IR$$

where E = voltage drop in volts,
I = current flowing through potentiometer in amperes,
R = resistance of any setting of potentiometer in ohms.

Having found the voltage drop, the current flowing in circuit PR_1R_2 and in the telephones can be determined by Ohm's law, the impedance of the telephones being given an average value of 25,000 ohms.

See measurement No. 21.

Example:

Equality of signals is obtained when 4 ohms are in the potentiometer through which 10 milliamperes are flowing. $R_1 = R_2 = 100{,}000\ \Omega$ and the impedance of the telephones used is 30,000 Ω. Calculate the current in the telephones, I_t.

Solution:

Formula $E = IR$

substituting $= 10 \cdot 10^{-3} \times 4 = 40 \cdot 10^{-3}$

whence $E = 40 \cdot 10^{-3}$ volts or 40 millivolts,

which is the voltage drop across utilized part of P.

Formula $I_t = \dfrac{E}{R}$

In this example $E = 40 \cdot 10^{-3}$ volts.

$R = 100{,}000 + 100{,}000 + 30{,}000 = 2.3 \cdot 10^5 \Omega.$

substituting $I_t = \dfrac{40 \cdot 10^{-3}}{2.3 \cdot 10^{-5}} = 1.74 \cdot 10^{-7}$

whence $I_t = 1.74 \cdot 10^{-7}$ ampere or 0 174 microampere.

The telephone current that will give unit audibility, using the average modern telephone, is approximately

$$I_a = 1 \cdot 10^{-9} \text{ ampere.}$$

Therefore, knowing the value of the current required to give unit audibility, and also the telephone current produced by the incoming signal, the audibility of the signal can be calculated. The formula is:

$$A = \frac{I_t}{I_a}$$

where A = audibility,

I_t = telephone current as measured by comparator,

I_a = current sensitivity of telephones $= 1 \cdot 10^{-9}$ ampere.

Example:

The telephone current produced by an incoming signal is found to be 0.174 microampere, and the current sensitivity of the telephones is $1 \cdot 10^{-9}$ ampere. Calculate the audibility.

Solution:

Formula $A = \dfrac{I_t}{I_a}$

substituting $= \dfrac{1.74 \cdot 10^{-7}}{1 \cdot 10^{-9}} = 1.74 \cdot 10^2 = 174$

whence $A = 174$

Having measured the audibility by either of the above methods, preferably the latter, uncouple the secondary circuit and open the filament circuit of the receiving vacuum tube. Tighten the coupling of the circuit A to the driver until a deflection of about one-half scale is obtained on G, and check the circuit for resonance with the driver. Measure the resistance of circuit A, by inserting noninductive resistance at R, until the deflection of G is reduced to one-fourth its maximum value (one-half current). The resistance of the total artificial antenna circuit is then equal to the inserted resistance, see Measurements Nos. 7 and 9. Record the resistance of the antenna circuit.

The watt sensitivity of the receiver can now be calculated using the formula:

$$P = \frac{I^2R}{A^2}$$

where P = power in watts in antenna for unit audibility,
I = current measured at G in amperes,
R = resistance of antenna circuit in ohms,
A = audibility of received signal.

Example:

A signal of 3,000 audibility was obtained with a current of 13.6 microamperes in an artificial antenna having a resistance of 150 ohms. Calculate the watt sensitivity of the receiver.

Solution:

Formula $P = \dfrac{I^2R}{A^2}$

substituting $= \dfrac{(13.6 \cdot 10^{-6})^2 \times 1.5 \cdot 10^2}{(3.10^3)^2} = \dfrac{2.78 \cdot 10^{-8}}{9.10^6} = 3.1 \cdot 10^{-15}$

whence $P = 3.1 \cdot 10^{-15}$ watt for signal of **unit audibility.**

This measurement is, in reality, a determination of the power in the antenna required to give unit audibility in a given receiver secondary; that is, it is a calibration of the secondary circuit. If the total antenna resistance is known, then the current in the antenna can be calculated; and if the effective height of the antenna is also known, the electric field intensity in microvolts per meter can also be calculated.

The value of the watt sensitivity of the receiver depends to a great extent upon the detector tube and the telephones used in the receiving circuit. Therefore, when either the tube or the telephones, or both, are replaced, the watt sensitivity value no longer holds good and the measurement should be repeated. An average value of $3 \cdot 10^{-15}$ watt may be used, bearing in mind the factors mentioned above.

It is usually best to have the driver as far away as possible from the receiving circuit, in order to eliminate electrostatic coupling effects which are likely to result in a residual signal in the receiver.

The leads in circuit A may be made as long as desired, but should be well insulated from any ground that might cause dielectric losses.

2. Sensitivity of receiver. From nearby transmitting station. The sensitivity of the receiving set is best determined from the electric field intensity (microvolts per meter) produced at the receiving antenna by a nearby station having an antenna of known effective height (See Measurement No. 25), and a known but small antenna current.

The calibrating transmitting station should be a little more than one wave length away from the receiving station to be calibrated; that is, if the transmitting station is 6.0 kilometers distant from the receiving station, the calibration should be conducted on a wave length of not more than 6,000 meters.

An ordinary vacuum-tube transmitter (CW) having a coupled antenna circuit may be used for transmission. The antenna current can then be regulated to give any desired strength of signal at the receiving station.

Thus, having a known current in the transmitting antenna, its intensity is measured at the receiving station with the telephone current comparator, care being taken to use optimum coupling in the receiver.

The **calculated value** of the electric field intensity can be determined from the formula:

$$\mathcal{E}_{calc} = 3.77 \cdot 10^8 \frac{I_s h_s}{\lambda d}$$

where $\mathcal{E}_{calc}$ = calculated electric field intensity in microvolts per meter,
I_s = current in transmitting antenna in amperes,
h_s = effective height of transmitting antenna in meters,
λ = wave length in meters,
d = distance between stations in meters.

Example:

Given: $I_s = 50$ milliamperes; $h_s = 80$m.; $\lambda = 6{,}000$m.; $d = 6{,}000$m. Calculate $\mathcal{E}_{calc}$.

Solution:

Formula $$\mathcal{E}_{calc} = 3.77 \cdot 10^8 \frac{I_s h_s}{\lambda d}$$

substituting $$= 3.77 \cdot 10^8 \frac{5 \cdot 10^{-2} \times 8 \cdot 10^1}{6 \cdot 10^3 \times 6 \cdot 10^3} = 4.19 \cdot 10^1 = 41.9$$

whence $\mathcal{E}_{calc} = 41.9$ microvolts per meter.

The electric field intensity **observed** at the receiving station is found by the formula:

$$\mathcal{E}_{obs} = \frac{B I_t \sqrt{R}}{h_r}$$

where I_t = the telephone current measured with the telephone current comparator by method 1 (b),

R = radio-frequency resistance of receiving antenna in ohms,

h_r = effective height of receiving antenna in meters,

B = the sensitivity constant; equals ratio of $\sqrt{P}$ to I_a of telephones (see Measurement No. 21).

Then $\mathcal{E}_{obs}$ is the observed value of electric field intensity derived from the above quantities.

Example:

Given $B=100$; $I_t=1.74 \cdot 10^{-7}$ amp; $R=49\Omega$;
$h_r=16$m. Calculate $\mathcal{E}_{obs}$.

Solution:

Formula
$$\mathcal{E}_{obs}=\frac{BI_t\sqrt{R}}{h_r}$$

substituting
$$=\frac{1 \cdot 10^2 \times 1.74 \cdot 10^{-7} \times \sqrt{49}}{1.6 \cdot 10^1}$$

$$=\frac{12.18 \cdot 10^{-5}}{1.6 \cdot 10^1}=7.61 \cdot 10^{-6} \text{ volts}$$

whence $\mathcal{E}_{obs}=7.61$ microvolts per meter.

The sensitivity constant

$$B=\frac{\sqrt{P}}{I_a}$$

where P = sensitivity in watts of receiver,

I_a = unit current sensitivity in amperes of telephones.

It can be seen that this ratio should be constant in the functioning of the receiver, while all of the other factors in the $\mathcal{E}_{obs}$ formula may be variable. Therefore, the current sensitiveness of the receiving system, as a whole, depends upon B.

The value of B can be checked from current comparator observations taken on signals from any nearby transmitting station, provided that the effective height and the radio-frequency resistance of the receiving antenna are known.

Then

$$B=\frac{\mathcal{E}_{calc}\, h_r}{I_t\sqrt{R}}$$

where all the quantities are the same as have previously been given.

MEASUREMENT NO. 23.

MEASUREMENT OF STRONG AND MEDIUM RECEIVED CURRENTS.

Measurement of received current. **Received current** is defined as the current produced in an antenna, or loop circuit of a receiver, by signals from a transmitting station.

Received current can be measured **directly** by two methods: (a) with thermocouple and galvanometer, and (b) with shunted detector and galvanometer.

(a) **Thermocouple-galvanometer method.** If the transmitted signal is strong, the received current can be measured by using a calibrated thermocouple and galvanometer in series with the antenna or loop circuit, as shown in figure 419 (a) and (b).

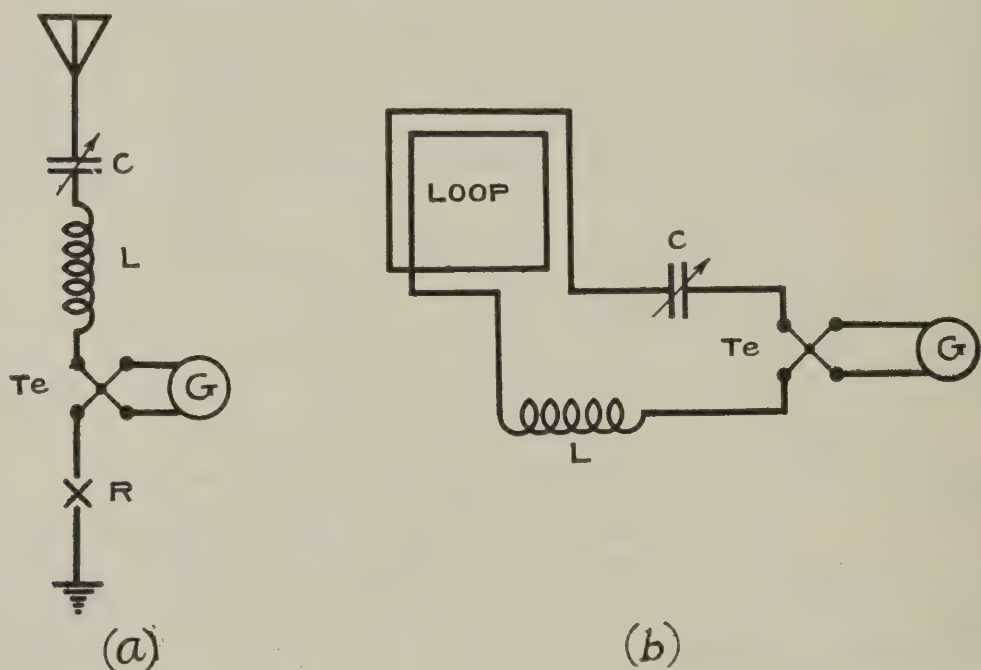


FIG. 419.—Thermocouple-Galvanometer Connected in Series with (a) Antenna Circuit, (b) Loop Circuit.

The antenna or loop circuit should be tuned to resonance with the signal to be measured, in the usual manner. In case the loop is used, it should be pointed in the direction of the transmitter; that is, after having obtained resonance and noted the deflection, the loop should be swung into the position where maximum deflection is indicated. The current flowing in the circuit may then be calculated from the deflection noted. This is the received current.

The radio-frequency resistance of the antenna or loop circuit should have as low a value as possible. No tuned circuits should be in the vicinity of the measuring circuit. Condenser C should be equipped with a vernier adjustment. The success of this measurement depends

greatly upon the degree of freedom from interfering signals, atmospheric disturbances, and stray currents. It is best to listen in both before and after the measurement, in order to determine whether or not there is any interference.

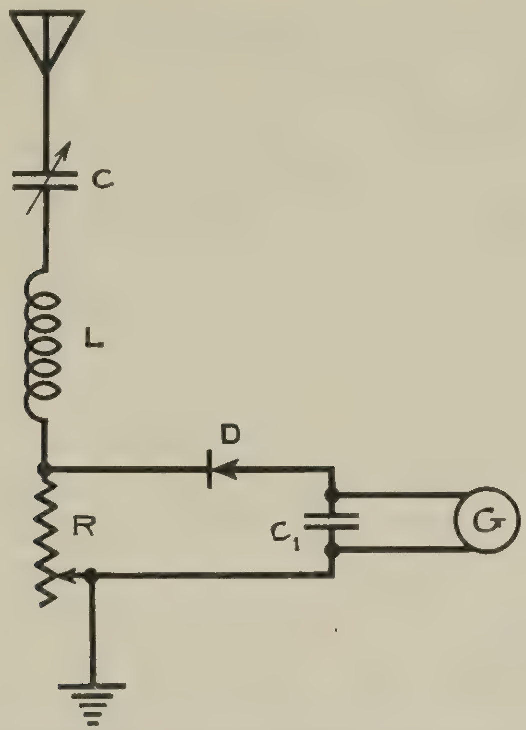


FIG. 420.—Shunted Detector and Galvanometer Connected into Antenna Circuit.

(b) **Shunted detector and galvanometer method.** The received current produced by medium signals can be measured by using a calibrated shunted detector and galvanometer. The connections are shown in figure 420. The method of measurement and the precautions to be observed are the same as those given for the thermocouple-galvanometer method. Greater care will need to be exercised in the use of this circuit because of its inherent instability and high sensitivity.

MEASUREMENT NO. 24.

MEASUREMENT OF ELECTRIC FIELD INTENSITY (ELECTRIC GRADIENT) FROM VERY DISTANT TRANSMITTING STATIONS.

Measurement of electric field intensity. The electric field intensity (electric gradient) $\mathcal{E}$ produced at the receiving point by a very distant transmitter may be determined by the measurement of the audibility of the signal using (a) an audibility meter with an error which is, in the absence of static and noise, generally not greater than $\pm 20\%$ or (b) the telephone current with the telephone current comparator.

(a) **The audibility meter method.** The audibility meter measures the telephone current in terms of audibility. This current is proportional to the received current, and to the square root of received power, when the oscillating vacuum-tube detector or the heterodyne is used, and is proportional to the square of the received current, and to the received power, when the non-oscillating vacuum tube or crystal detector is used. The coupling between antenna and secondary circuit should be at the **optimum value** in both cases, and **regeneration should not be employed in the latter case.** The method of tuning using optimum coupling is described in Part 2, Chapter VI of Section II.

The signal is tuned in, using optimum coupling, and the lever of the audibility meter is rotated from zero to the point just below where the signal vanishes. The reading at this point is the audibility of the received signal. The tuning of the secondary of the receiver may be varied to give best signal during this operation, but **the antenna circuit and coupling should not be changed.**

The electric field intensity can now be calculated from the audibility measurement, provided that the sensitivity of the receiver and the effective height of the antenna are known.

The electric field intensity is calculated from the formula:

$$\mathcal{E}=\frac{A\sqrt{PR}}{h_r}\cdot 10^6$$

where $\mathcal{E}$ = electric field intensity in microvolts per meter,
A = audibility,
P = watt sensitivity of receiver in watts,
R = radio-frequency of receiving antenna circuit in ohms,
h_r effective height of receiving antenna in meters.

When the non-oscillating vacuum tube or crystal detector is used (without heterodyne), the resistance of the antenna should be increased 70% due to optimum coupling of resonant secondary circuit.

Example:

The following values were found for a given antenna and signal: $P=3.09 \cdot 10^{-15}$ watts; $A=1{,}000$; $R=50\Omega$; $h=16$m. Calculate $\mathcal{E}$.

Solution:

Formula $$\mathcal{E}=\frac{A\sqrt{PR}}{h}\cdot 10^6$$

substituting $$=\frac{1\cdot 10^3\sqrt{3.09\cdot 10^{-15}\times 50}}{16}\cdot 10^6$$

$$=\frac{1\cdot 10^3\times 3.93\cdot 10^{-7}}{1.6\cdot 10}\cdot 10^6=24.6$$

whence $\mathcal{E}=24.6$ microvolts per meter.

When a loop antenna is used instead of an antenna the electric field intensity is calculated from the following formula:

$$\mathcal{E}=A\sqrt{PR}\cdot\frac{\lambda}{2\pi lhn}\cdot 10^6$$

where the terms $\mathcal{E}$, A, P, and R are expressed in the same units as in the preceding formula, and

λ = wave length in meters,
l = length of loop in meters,
h = height of loop in meters,
n = number of turns.

(b) **The telephone comparator method.** The electric field intensity can also be calculated from telephone current measurements described in 1 (b), Measurement No. 22, from the formula

$$\mathcal{E}_{obs}=\frac{BI_t\sqrt{R}}{h_r}$$

which is also explained in Measurement No. 22.

The radio-frequency comparator is being used to some extent, but is not being used at present in the Navy.

MEASUREMENT NO. 25.

MEASUREMENT OF THE EFFECTIVE HEIGHT OF ANTENNAS.

The **effective height** of an antenna, theoretically, is equal to one-half the length of the equivalent hertzian oscillator. In practice, the effective height of a given flattop antenna is usually estimated to be from fifty to seventy per cent of its mean height. It is always best, however, to measure the effective height, and a method of making this measurement follows.

The effective height of an antenna may be determined by making received current measurements, using the antenna under measurement as the receiving antenna, and a loop antenna as the transmitter. Figure 421 shows the antenna circuit arranged for making the received current measurement. The antenna circuit is tuned to the signal to be received, and its radio-frequency resistance measured at that wave length by the method given in Measurement No. 9 or No. 22. The resistance of the antenna circuit including the thermocouple, the inductance and series capacity as used for the measurement should be found. This is usually done immediately after the measurement of the received current.

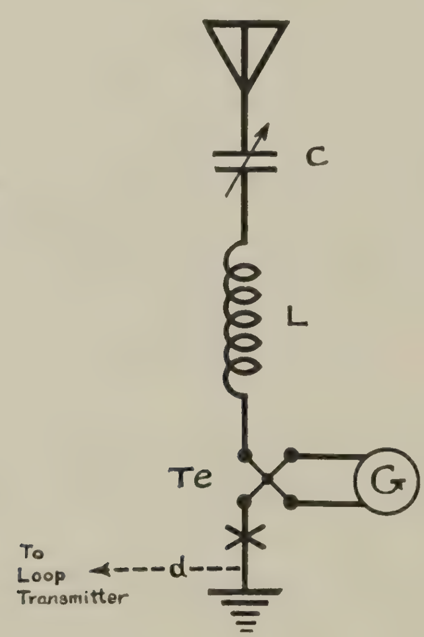


Fig. 421.—Circuit used in Measuring the Effective Height of an Antenna.

The loop antenna used as the transmitter is located at a distance of approximately one wave length from the receiving antenna, with the plane of the loop lying in the direction of the receiving antenna. The effective height of the transmitting loop must be known, and is calculated from the formula:

$$h_s = \frac{2\pi lhn}{\lambda}$$

where h_s = effective height of transmitting loop in meters,
l = length of transmitting loop in meters,
h = height of transmitting loop in meters,
n = number of turns,
λ = wave length at which measurement is to be made in meters.

A large rectangular loop about 40 feet long and 20 feet high, having 5 turns spaced 2.5 feet apart, is suitable for this measurement. A continuous-wave driver capable of supplying enough power to produce a moderate deflection of the galvanometer in the receiving antenna should be used. Measurements should be made at several wave lengths. Long waves are preferable, because they suffer least from refraction and reflection. The path between the two stations should also be carefully chosen.

The received current is calculated from the deflection of the calibrated thermocouple and galvanometer. The shunted detector and galvanometer circuit may be used if found necessary. The measurement of the resistance of the antenna circuit should then be made, using a local continuous-wave driver. The expression for the effective height of an antenna is:

$$h_r = \frac{I_r d R \lambda}{120\pi I_s h_s}$$

where h_r = effective height of receiving antenna in meters,
I_r = received current in amperes,
d = distance in meters,
R = radio-frequency resistance of antenna in ohms,
λ = wave length in meters,
I_s = current in transmitter (loop or antenna) in amperes,
h_s = effective height of transmitter (loop or antenna) in meters.

The calculated effective height of the transmitting loop is substituted for h_s in the above formula, and the effective height of the antenna under measurement can then be found.

Thus having determined the effective height of one antenna, the effective height of any other antenna can readily be determined from it. This measurement is identical to that just explained. The loop transmitter is replaced by the transmitting antenna to be measured. Then the effective height of the transmitting antenna is given by the formula:

$$h_s = \frac{I_r d R \lambda}{120\pi I_s h_r}$$

where h_s = effective height of the transmitting antenna in meters,
I_r = received current in amperes,
d = distance in meters,
R = radio-frequency resistance of receiving antenna circuit in meters,
λ = wave length in meters,
I_s = current in transmitter antenna in amperes,
h_r = effective height of receiving antenna in meters.

Example:

Given: $I_r = 2 \cdot 10^{-3}$ ampere; $\lambda = 6{,}000$m.; $d = 7{,}800$m.; $R = 100\Omega$; $I_s = 25$ amperes; $h_r = 16$m. Calculate the effective height of the transmitting antenna h_s.

Solution:

Formula $$h_s = \frac{I_r d R \lambda}{120\pi I_s h_s}$$

substituting $$= \frac{2 \cdot 10^{-3} \times 7.8 \cdot 10^3 \times 1 \cdot 10^2 \times 6.0 \cdot 10^3}{3.77 \cdot 10^2 \times 2.5 \cdot 10^1 \times 1.6 \cdot 10^1}$$

$$= \frac{9.36 \cdot 10^6}{1.508 \cdot 10^5} = 6.2 \cdot 10^1 = 62.0$$

whence $h_s = 62$m.

The usual precautions should be taken in the use of the thermocouple-galvanometer or shunted detector and galvanometer. No tuned circuits should be in the vicinity of the measuring circuit. Other antennas or any objects acting as wave collectors near the transmitting or receiving antennas absorb a portion of the current. This will lower the effective height of the antenna.

MEASUREMENT NO. 26.

MEASUREMENT OF LOGARITHMIC DECREMENT.

1. **Calculation of the decrement of an isolated circuit.** The logarithmic decrement of a circuit containing inductance, capacity and resistance and which is unaffected by coupling to other circuits can be calculated. The factors which must be known are the frequency, the radio-frequency resistance at the wave length employed and either the inductance or the capacity. The formulas to be used are:

$$\delta=\frac{R}{2fL} \text{ and } \delta=\pi R\omega C$$

where δ = logarithmic decrement,
R = radio-frequency resistance in emu,
f = frequency,
$\omega=2\pi f$,
L = inductance in emu,
C = capacity in emu.

The resistance of the circuit can be found by the method given in Measurement No. 7, the capacity by Measurement No. 4 and the inductance by Measurement No. 13.

Example:

A given circuit contains an inductance of 188.8 μh and a capacity of 838 $\mu\mu$f ($C_0=6\mu\mu$f included) in series and has a resistance of 7.9 ohms at $\lambda=750$m. Calculate δ by both formulas.

Solution:

From Table 14, $R=7.9\Omega=7.9\cdot 10^9$ emu; $L=188.8\ \mu\text{h}=188.8\cdot 10^3$ emu;
$C=838\ \mu\mu\text{f}=838\cdot 10^{-21}$ emu;

From Table 13, $f=4\cdot 10^5$; $\omega=2.513\cdot 10^6$.

Formulas $\delta=\frac{R}{2fL}$, $\delta=\pi R\omega C$.

substituting $=\frac{7.9\cdot 10^9}{2\times 4\cdot 10^5\times 1.888\cdot 10^5}=3.1416\times 7.9\cdot 10^9\times 2.513\cdot 10^6\times 8.38\cdot 10^{-19}$

$=\frac{7.9}{151.0}=5.23\cdot 10^{-2}$ $=5.23\cdot 10^{-2}=0.0523$

whence $\delta=0.0523$ $\delta=0.0523$.

The above formulas are useful in determining the decrement of a single circuit, such as a wavemeter. The usual case, however, is where it is impossible to know the resistance with sufficient accuracy, for example,

the antenna circuit of transmitter. In such a case, the decrement is measured with the aid of a wavemeter, the decrement of which has been previously determined.

2. **Measurement of the decrement of a wavemeter.** Instead of calculating the decrement of a wavemeter as described above, it is much easier to measure it by using the reactance-variation method, and a continuous-wave driver to which the wavemeter is loosely coupled. The driver should be fairly powerful on account of the tendency of the driver to follow the variations made in the wave length of the wavemeter during the measurement, this tendency being more pronounced when the coupling is tightened in order to obtain sufficiently large deflections on the meter in the wavemeter.

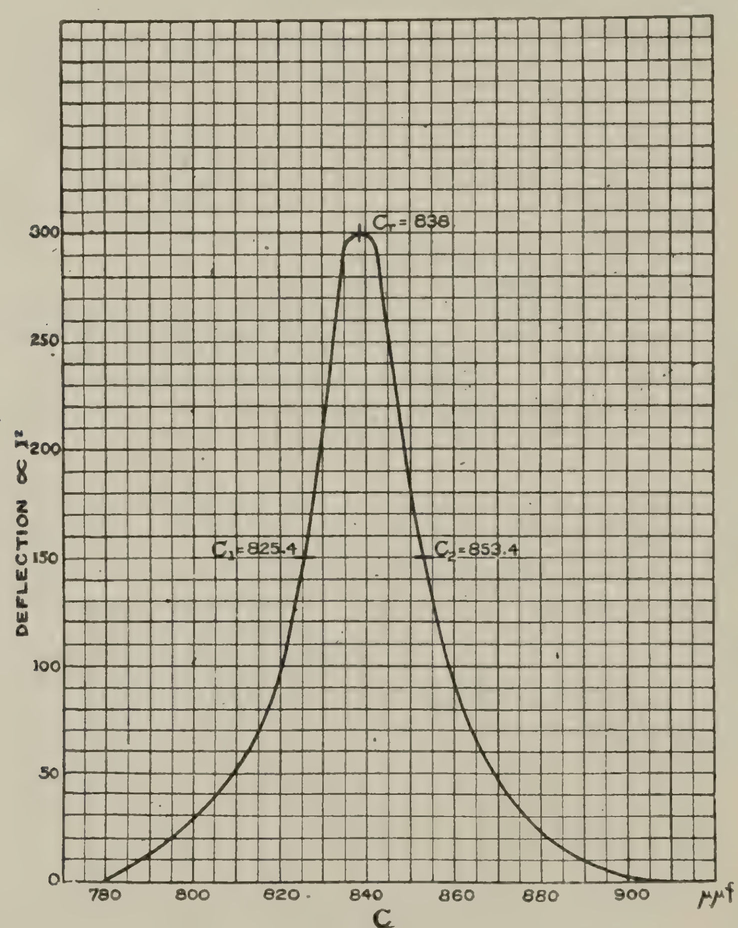


FIG. 422.—Resonance Curve. Deflections Plotted Against Capacity.

The reactance-variation method of determining the decrement of a circuit consists in varying the reactance of the circuit by varying the capacity, usually on both sides of resonance, until the induced current flowing in the circuit is reduced to one-half its resonant value. The readings of the condenser at these two points are recorded. The decrement may then be calculated, using either the corresponding wave lengths or the corresponding values of capacity. When there is reason to suppose that **humps** are present, it is the usual practice to take

sufficient simultaneous readings of both the condenser and indicating instrument, from which a resonance curve may be plotted.

Practically all wavemeters are equipped with current-squared meters, that is, they have current measuring instruments which are graduated in terms of current squared. Hence, the half-deflection or half-reading method can be used, where $I_1^2 = I_2^2 = \frac{I_r^2}{2}$

where I_r = current at resonance,
I_1 = current below resonance,
I_2 = current above resonance.

When the decrement is determined by the above method, the sum of the decrement of the driver plus that of the wavemeter is found. However, since the decrement of the driver is zero, only that of the wavemeter remains, and is, therefore, found directly.

The formulas for determining the decrement of coupled circuits, when the half-deflection method is used, are:

$$\delta_1 + \delta_2 = \pi \frac{C_2 - C_1}{C_2 + C_1} \quad \text{and} \quad \delta_1 + \delta_2 = 2\pi \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1}.$$

where δ_1 = decrement being measured,
δ_2 = decrement of wavemeter,
C_2 = capacity in $\mu\mu$f at half-deflection $\left(\frac{I_r^2}{2}\right)$ point above resonance,
C_1 = capacity in $\mu\mu$f at half-deflection $\left(\frac{I_r^2}{2}\right)$ point below resonance,
λ_2 = wave length at half-deflection $\left(\frac{I_r^2}{2}\right)$ point above resonance,
λ_1 = wave length at half-deflection $\left(\frac{I_r^2}{2}\right)$ point below resonance.

Example:

From the resonance curves, figures 422 and 423, that were plotted from readings obtained on the same circuit as was used in the first example, find the decrement of the wavemeter by the use of the above formulas.

Solution:

Values from curves: $C_2 = 853.4\ \mu\mu$f; $\lambda_2 = 756.5$m.
$C_1 = 825.4\mu\mu$f; $\lambda_1 = 744.0$m.

In this case $\delta_1 = 0$, and $\delta_1 + \delta_2 = \delta_2$

Capacity method:

Formula $\delta_1 + \delta_2 = \pi \frac{C_2 - C_1}{C_2 + C_1}$

substituting $$=3.1416\,\frac{853.4-825.4}{853.4+825.4}$$

$$=3.1416\,\frac{28}{1678.8}=5.24\cdot 10^{-2}=0.0524$$

whence $\delta_2=0.0524.$

Wave length method:

Formula $$\delta_1+\delta_2=2\pi\frac{\lambda_2-\lambda_1}{\lambda_2+\lambda_1}$$

substituting $$=6.2832\,\frac{756.5-744}{756.5+744}$$

$$=6.2832\,\frac{12.5}{1500.5}=5.24\cdot 10^{-2}=0.0524$$

whence $\delta_2=0.0524$

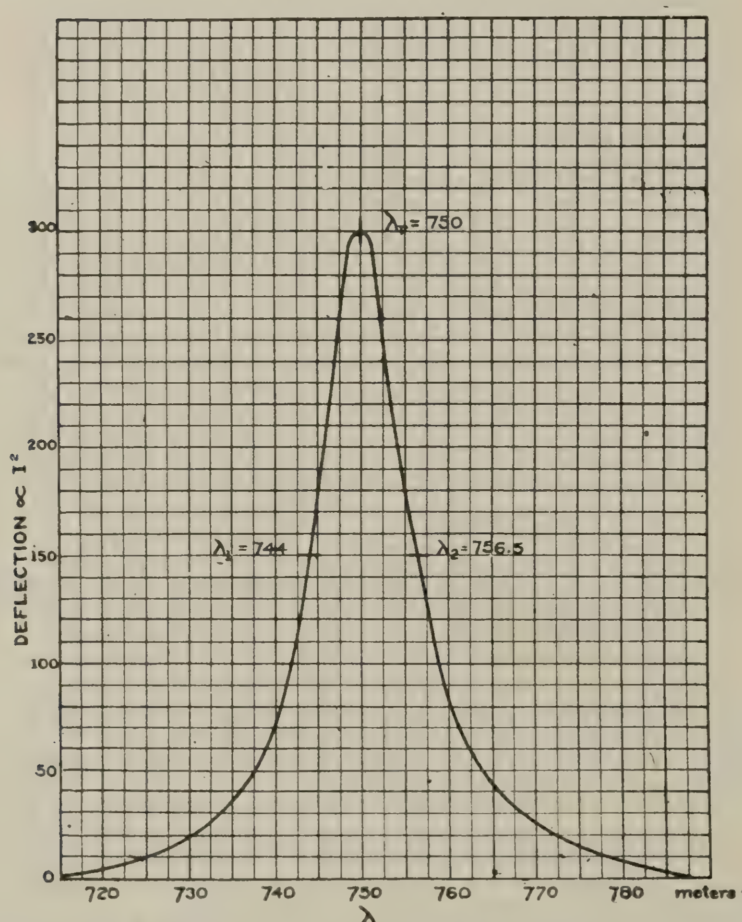


FIG. 423.—Resonance Curve. Deflections Plotted Against Wave Length.

In measuring the decrement of a wavemeter, at least three sets of readings should be taken, and the decrement then calculated from each set. The mean of these should be taken for the final result. It should be remembered that the deflection of the current-squared meter

and the condenser setting at resonance need to be recorded, because from the former the half deflection value is determined, and from the latter the resonant wave length.

The decrement of the wavemeter, determined in the manner given above, **holds only for the resonant wave length,** or $\lambda = 750$m. The decrement should be measured at a sufficient number of wave lengths within the working range of each coil of the wavemeter, and a curve plotted for each coil showing the change in decrement with change in capacity or wave length. These curves should be used when the wavemeter is employed to measure decrement because, with their aid, the decrement δ_2 of the wavemeter corresponding to any wave length can be readily found.

3. **Measurement of the decrement of a transmitter.** The decrement of a transmitter is measured by the same method as was used for determining that of the wavemeter. In this case, the decrement of the wavemeter must be known, because the value obtained is the sum of the decrements of the transmitter and wavemeter. Furthermore, most transmitters are not steady enough to permit the taking of a complete resonance curve, as was done in the case of the wavemeter, so that only the full and half-deflection points are taken. The formulas given above are used. The method in which the wave lengths are employed is much more difficult than the capacity method, and requires a very accurate determination of the wave length. Here again it will be necessary to measure the decrement at each different wave length.

The method assumes that the decrement of a transmitter is logarithmic and hence, **is not applicable to circuits in which there is a spark gap** and, therefore, have a **linear decrement.**

Care must be exercised when measuring the decrement of a transmitter that the wavemeter is not influenced by any circuit other than the antenna circuit. The transmitter should, of course, have been tuned to resonance before the decrement is measured. The decrement should also be measured under normal operating conditions, that is, the gap should be neither overheated nor cooler than is the normal condition. Several sets of readings should be taken, and the average value of the decrements calculated from the sets taken for the final decrement. Shifts in wave length should be followed, in order to determine the deflection at resonance and, hence, the half-deflection values. It is also important to check the current-squared meter for its zero mark, before and after each set of readings.

4. **Decremeter method.** The **Kolster decremeter** is a wavemeter especially designed for the measurement of decrement. It is provided with a logarithmic plate, air-dielectric, variable condenser having a vernier gear which is equipped with a knob, locking device and movable dial engraved with the decrement values. A current-squared meter is also a part of the instrument. The operation for measuring the logarithmic decrement of a transmitter by the decremeter is as follows:

The condenser is first set at the position of resonance as indicated by the maximum deflection of the sensitive hot-wire instrument, the scale readings of which are proportional to the current squared. This

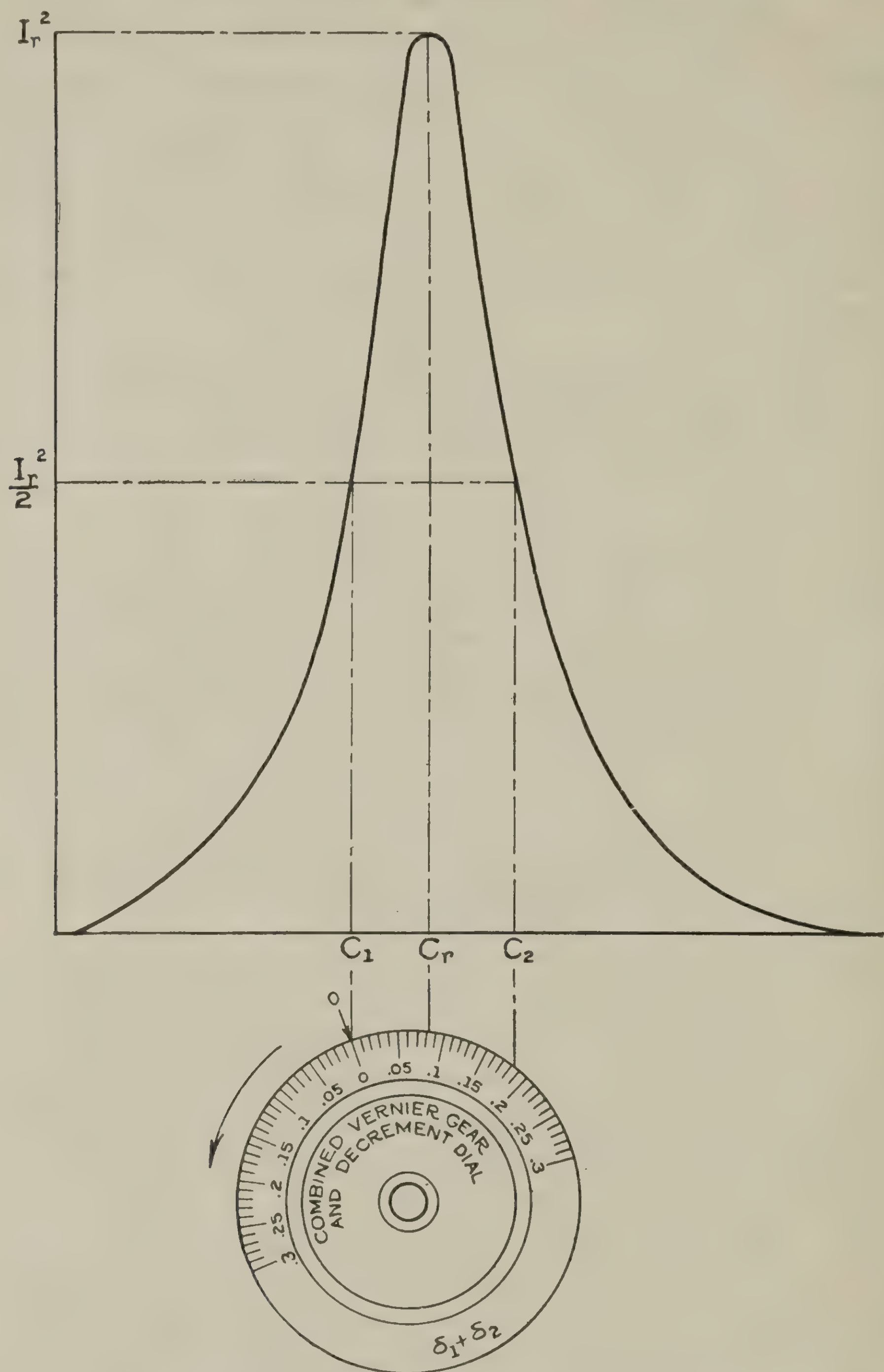


FIG. 424.—Diagram Showing Relation Between Decrement Scale and Resonance Curve.

maximum deflection is now reduced to one-half its value by decreasing or increasing the capacity of the condenser. The decrement scale, which may be rotated independently, is now set at zero, then clamped so that when the condenser is again varied it will rotate with it.

Starting at the zero setting of the decrement scale with the hot-wire instrument reading one-half the maximum deflection, the condenser is varied continuously in one direction until the needle of the hot-wire instrument makes a complete excursion from one-half deflection to maximum deflection, and back again to one-half deflection. The scale reading now opposite the index mark 0 is the value of $\delta_1+\delta_2$, δ_1 being the decrement of the circuit under test, and δ_2 the known decrement of the instrument.

It will be noted, by referring to figure 424, that it is desirable to make the zero setting of the decrement scale at the point of half deflection and also to take the final reading at the point of half deflection, because at these points the resonance curve is steep, and consequently the settings are sharply defined and easily made. In this connection, it will be noted that the formula

$$\delta_1+\delta_2=\pi\frac{C_2-C_1}{C_2+C_1}$$

does not involve the resonant value of capacity, C_r, but only those at the points of half deflection where the slope of the resonance curve is steep. This formula is, therefore, the most desirable one to use, and the decremeter is consequently operated in accordance with it.

The same precautions should be observed in measuring the decrement of a transmitter by the decremeter as given just above.

SECTION IV.

USEFUL INFORMATION.

PART 1.

Tables Adapted for Use in Radio Engineering.

TABLE 1.

THE TRIBO-ELECTRIC SERIES.

(Smithsonian Tables.)

No.	Material	No.	Material	No.	Material
1	Asbestos (sheet)	13	Silk	26	Shellac; resin; sealing wax
2	Rabbit's fur; hair	14	Hand	27	Ebonite
6	Mica	17	Cotton	30	Guttapercha
7	Wool	20	Woods	31	Sulphur
8	Glass (polished)	23	Cork; ebony	33	Celluloid
11	Cat's fur	24	Amber	34	India rubber

Explanation:

This table is so arranged that any material in the list becomes **positively** electrified when rubbed with one having a higher number in the list. The phenomenon depends upon surface conditions, and circumstances may alter the relative positions in the list.

Many of the materials have been omitted from the list.

Example:

When cat's fur (11) and sealing wax (26) are rubbed together, the cat's fur becomes positively electrified and the sealing wax negatively electrified.

TABLE 2.

THERMO-ELECTRIC POWER OF METALS.

(Smithsonian Tables.)

Substance	Microvolts	Substance	Microvolts
Aluminum	− 0.68		
Bismuth		Platinum (hardened)	+ 2.42
(comm'l pressed wire)	−97.0	Platinum (malleable)	− 0.818
Cadmium	+ 3.48	Selenium	+807.
Constantan	−22.0	Silver (pure hard)	+ 3.00
Copper (comm'l)	+ 0.10	Steel	+ 10.62
Gold	+ 3.0	Tantalum	− 2.6
Iron (piano wire)	+17.5	Tellurium β	+500.
Lead	0.00	Tellurium α	+160.
Molybdenum	+ 5.9	Tungsten	− 2.0
Nickel	−22.8	Zinc	+ 2.79

Explanation:

The **thermoelectric power** of a circuit is the emf produced by 1° C. difference in temperature between the junctions. Such a circuit would consist of two dissimilar metals forming two junctions, one at each end where they are fastened together. The two junctions would normally have a mean temperature, such as 20° C. The one to which heat is applied is called the hot junction and the other the cold junction.

The above table gives the thermoelectric power in μv of the more commonly used metals at room temperature, 20° C., for a rise of 1° C. in the temperature of the hot junction. The values of thermoelectric power in the table are **with respect to lead** as the other metal of the thermoelectric circuit.

If one metal is (1) and the other is (2), then the thermoelectric power of a couple composed of two metals (1) and (2) is given by subtracting (algebraically) the value for (2) from that for (1). When this difference is positive, the current flows from the hot junction to the cold in (1).

Example:

What is the thermoelectric power in μv of a thermocouple made of copper and constantan? Which terminal will be positive?

Solution:

Let the copper be metal (1) and the constantan be metal (2).

From table,	(1) copper	+ 0.10
	(2) constantan	−22.0
and algebraic difference		+22.1 μv

Hence, the current flows from the junction through the copper to the external circuit and the terminal of the copper is + and that of the constantan −.

TABLE 3.

ELECTRO-CHEMICAL EQUIVALENTS OF ELEMENTS.

Element	Symbol	Grams per Amphr.	Amphrs. per gram
Aluminum	Al	0.3369	2.969
Copper	Cu	1.186	0.843
Gold	Au	3.677	0.2720
Lead	Pb	3.858	0.2592
Nickel	Ni	1.094	0.914
Silver	Ag	4.025	0.2485
Zinc	Zn	1.219	0.820

Explanation:

The **electrochemical equivalent** of a substance is the definite quantity of that substance which is consumed per ampere-hour, or which is deposited electrolytically per ampere-hour.

Example:

A battery delivers 5 amperes to an external circuit for 10 hours. How much zinc is consumed?

Solution:

$$1 \text{ oz.} = 28.4 \text{ grams}$$
$$5 \text{ amps.} \times 10 \text{hrs.} = 50 \text{ amphrs.}$$

From Table 3, 1.219 grams of zinc are consumed per amphr. Hence,

$$50 \times 1.219 = 60.95 \text{ grams, or } 2.15 \text{ oz.}$$

TABLE 4.

ELECTRO-CHEMICAL SERIES OF ELEMENTS.

+		
Aluminum	Lead	Mercury
Manganese	Cadmium	Silver
Zinc	Tin	Gold
Iron	Bismuth	Platinum
Nickel	Copper	Carbon

Explanation:

If any two metals are placed in contact, one of them becomes positively charged with respect to the other and is said to be **electropositive,** and the other **electronegative.** Also, when two such metals are immersed in an electrolyte (to form a primary cell) and their exposed ends (poles) are connected by a wire, the electropositive metal tends to form **anions, cations** being deposited on the electronegative metal.

No element is of itself electropositive or electronegative, but is only so in relation to other elements. The above table gives the elements in electrochemical series. The order is such that any element is electropositive to all those that follow, and electronegative to all those preceding it.

The terminal or pole of an electropositive element of a primary battery is **negative,** and that of the electronegative element is **positive.**

TABLE 5.

WIRE TABLE, STANDARD COPPER.

American Wire Gauge (B & S).

(Cir. No. 31 of Bu. of St'ds.)

Gauge No.	Diameter in mils.	Diameter in mms.	Cross-section in Cir. Mils.	Cross-section in Sq. inches.	Cross-section in mm².	Resistance Ohms per 1,000 ft.	Resistance Feet per ohm.	Weight Pounds per 1,000 ft.	Weight Feet per pound
0000	460.0	11.68	211,600.	0.1662	107.2	0.04901	20,400.	640.5	1.561
000	409.6	10.40	167,800.	.1318	85.03	.06180	16,180.	507.9	1.968
00	364.8	9.266	133,100.	.1045	67.43	.07793	12,830.	402.8	2.482
0	324.9	8.252	105,500.	.08289	53.48	.09827	10,180.	319.5	3.130
1	289.3	7.348	83,690.	.06573	42.41	.1239	8,070.	253.3	3.947
2	257.6	6.544	66,370.	.05213	33.63	.1563	6,400.	200.9	4.977
3	229.4	5.827	52,640.	.04134	26.67	.1970	5,075.	159.3	6.276
4	204.3	5.189	41,740.	.03278	21.15	.2485	4,025.	126.4	7.914
5	181.9	4.621	33,100.	.02600	16.77	.3133	3,192.	100.2	9.980
6	162.0	4.115	26,250.	.02062	13.30	.3951	2,531.	79.46	12.58
7	144.3	3.665	20,820.	.01635	10.55	.4982	2,007.	63.02	15.87
8	128.5	3.264	16,510.	.01297	8.366	.6282	1,592.	49.98	20.01
9	114.4	2.906	13,090.	.01028	6.634	.7921	1,262.	39.63	25.23
10	101.9	2.588	10,380.	.008155	5.261	.9989	1,001.	31.43	31.82
11	90.74	2.305	8,234.	.006467	4.172	1.260	794.0	24.92	40.12
12	80.81	2.053	6,530.	.005129	3.309	1.588	629.6	19.77	50.59
13	71.96	1.828	5,178.	.004067	2.624	2.003	499.3	15.68	63.80
14	64.08	1.628	4,107.	.003225	2.081	2.525	396.0	12.43	80.44
15	57.07	1.450	3,257.	.002558	1.650	3.184	314.0	9.858	101.4
16	50.82	1.291	2,583.	.002028	1.309	4.016	249.0	7.818	127.9
17	45.26	1.150	2,048.	.001609	1.038	5.064	197.5	6.200	161.3
18	40.30	1.024	1,624.	.001276	0.8231	6.385	156.6	4.917	203.4
19	35.89	0.9116	1,288.	.001012	.6527	8.051	124.2	3.899	256.5
20	31.96	.8118	1,022.	.0008023	.5176	10.15	98.50	3.092	323.4
21	28.46	.7230	810.1	.0006363	.4105	12.80	78.11	2.452	407.8
22	25.35	.6438	642.4	.0005046	.3255	16.14	61.95	1.945	514.2
23	22.57	.5733	509.5	.0004002	.2582	20.36	49.13	1.542	648.4
24	20.10	.5106	404.0	.0003173	.2047	25.67	38.96	1.223	817.7
25	17.90	.4547	320.4	.0002517	.1624	32.37	30.90	0.9699	1,031.
26	15.94	.4049	254.1	.0001996	.1288	40.81	24.50	.7692	1,300.
27	14.20	.3606	201.5	.0001583	.1021	51.47	19.43	.6100	1,639.
28	12.64	.3211	159.8	.0001255	.08098	64.90	15.41	.4837	2,067.
29	11.26	.2859	126.7	.00009953	.06422	81.83	12.22	.3836	2,607.
30	10.03	.2546	100.5	.00007894	.05093	103.2	9.691	.3042	3,287.
31	8.928	.2268	79.70	.00006260	.04039	130.1	7.685	.2413	4,145.
32	7.950	.2019	63.21	.00004964	.03203	164.1	6.095	.1913	5,227.
33	7.080	.1798	50.13	.00003937	.02540	206.9	4.833	.1517	6,591.
34	6.305	.1601	39.75	.00003122	.02014	260.9	3.833	.1203	8,310.
35	5.615	.1426	31.52	.00002476	.01597	329.0	3.040	.09542	10,480.
36	5.000	.1270	25.00	.00001964	.01267	414.8	2.411	.07568	13,210.
37	4.453	.1131	19.83	.00001557	.01005	523.1	1.912	.06001	16,660.
38	3.965	.1007	15.72	.00001235	.007967	659.6	1.516	.04759	21,010.
39	3.531	.08969	12.47	.000009793	.006318	831.8	1.203	.03774	26,500.
40	3.145	.07987	9.888	.000007766	.005010	1,049.	0.9534	.02993	33,410.
41*	2.800	.07112	7.841	.000006160	.003973	1,323.	0.7562	.02374	42,130.
42	2.494	.06334	6.220	.000004885	.003152	1,667.	0.5998	.01882	53,100.
43	2.221	.05641	4.933	.000003873	.002500	2,103.	0.4756	.01495	66,970.
44	1.978	.05022	3.910	.000003073	.001982	2,652.	0.771	.01184	84,460.

* The values for wire sizes smaller than number 40 have been added on account of their increasing use in radio apparatus.

Explanation:

The values given in the table are for bare wire made from average commercial high-conductivity copper and at a temperature of 20° C

(68° F). Such copper is 99.9% pure, with silver and oxygen composing the larger part of the 0.1% of impurities. The conductivity is, therefore, taken as 100%.

Hard-drawn copper wire has the characteristic that its resistivity varies with wire size. An approximate average value of its per cent conductivity may be taken as 97.3. To find values of **Ohms per 1,000 feet** and **Feet per ohm** for this wire by means of the above table, increase the table values by 2.7% for the former and decrease the values by 2.7% for the latter. **Pounds per 1,000 feet** and **Feet per pound** may be considered to be given correctly by the table for either annealed or hard-drawn copper.

The insulation of wire varies greatly both in thickness and kind. For radio receiving apparatus, the following are used:

Kind	Abbreviation	Increase in diameter in mils.
Single cotton covered	SCC.	4
Double cotton covered	DCC.	8
Single silk covered	SSC.	2
Double silk covered	DSC.	4
Enameled		1 (average)

Enameled wire is also procurable with single or double silk covering, in which case the increase in diameter due to the extra insulation can be readily calculated from the foregoing.

TABLE 5A.

RELATIVE VOLUME RESISTIVITY AND THE TEMPERATURE COEFFICIENT OF VARIOUS METALS AND ALLOYS.

Metal or Alloy	Remarks	Relative Volume Resistivity	Temperature Coefficient	
			t_s	a_s
Advance	See constantan	......	....	....
Aluminum	Commercial, hard drawn	1.64	18°	+0.0039
Brass	......	4.06	20°	+0.002
Climax	......	50.46	20°	+0.0007
Constantan	60% Cu., 40% Ni.	28.43	12°	+0.000008
Copper	**Annealed**	**1.00**	**20°**	**+0.00393**
	Hard drawn	1.027	20°	+0.00382
Eureka	See constantan	......	...	....
German silver	18% Ni.	19.14	20°	+0.0004
Gold	pure, drawn	1.415	500° ann'ld	+0.0035
Iron	99.98% pure	5.80	20°	+0.0050
Lead	......	12.76	20°	+0.0039
Manganin	84 Cu, 12 Mn, 4 Ni.	25.52	12°	+0.000006
Mercury	......	55.55	20°	+0.00089
Molybdenum	drawn	3.31	25°	+0.0033
Nichrome	......	58.0	20°	+0.0004
Nickel	......	4.52	20°	+0.006
Platinum	......	5.80	20°	+0.003
Silver	99.98% pure	0.952	20°	+0.0038
Steel	manganese	40.6	20°	+0.001
	piano wire	7.28	0°	+0.0032
Tantalum	......	8.99	20°	+0.0031
Tin	......	6.67	20°	+0.0042
Tungsten	......	3.20	18°	+0.0045

Explanation:

The volume resistivity of standard commercial, annealed, high conductivity copper is 1.724 microhm-cms. at 20° C. The volume resistivity of all the substances given in the above table, which is adapted from the Smithsonian Tables, is given relatively to this grade of copper as the standard. The values of volume resistivity are those for a temperature of 20° C. The relative volume resistivity is given as a multiplier to simplify calculations. This table can be used in conjunction with Table 5 in order to find the resistance of wire made of any one of the materials listed in this table.

Example:

Find the resistance of 1,000 feet of #20 B & S gauge, hard-drawn aluminum wire at 20° C.

Solution:

From Table 5, the resistance of 1,000 feet of #20 B & S gauge standard copper wire at 20° C is 10.15Ω.

From Table 5A, the relative volume resistivity of aluminum at 20° C is 1.64.

Hence $$10.15 \times 1.64 = 16.65$$

whence $$R = 16.7\Omega$$

Use of the temperature coefficient. The resistance of a given material at temperatures below or above 20° C can be computed by using the values appearing in the last two columns for the given material. The formula to be used is

$$R_t = R_s\{1 + a_s(t - t_s)\}$$

where R_t = resistance at temperature desired (°C),
R_s = resistance at standard temperature (20° C),
a_s = temperature coefficient in last column of table,
t = temperature (°C) desired,
t_s = temperature (°C) corresponding to a_s.

Example:

The resistance of a length of copper wire is 2.5Ω at 20° C. What will be its resistance at 40° C? At 0° C?

Solution:

From Table 5A, $t_s = 20°$; $a_s = +0.00393$.

Formula $$R_t = R_s\{1 + a_s(t - t_s)\}$$

substituting $$= 2.5\{1 + 0.00393\ (40\text{–}20)\}$$
$$= 2.5 \times 1.0786 = 2.697$$

whence $$R_t = 2.7\Omega \text{ at } 40°\text{C}.$$

The resistance at 0°C is found in a similar manner by substituting in the formula.

Thus $$R_t = 2.5\{1 + 0.00393\,(0\text{–}20)\}$$
$$= 2.5 \times 0.9214 = 2.304$$

whence $$R_t = 2.3\Omega \text{ at } 0°\text{C}.$$

The **rise in temperature** of a wire can also be computed if the resistance at standard temperature and the second value of resistance are known. The formula is

$$t - t_s = \frac{R_t - R_s}{a_s R_s}$$

Example:

The cold (20°C) resistance of a solenoid wound with copper wire was 2.2 ohms. During continuous operation, its resistance rose to 2.7 ohms. What was the temperature rise? What was the operating temperature?

Solution:

$$R_t = 2.7\Omega;\ R_s = 2.2\Omega;\ a_s = +0.00393;\ t_s = 20°\text{C}.$$

Formula $$t-t_s=\frac{R_t-R_s}{a_sR_s}$$

substituting $$=\frac{2.7-2.2}{0.00393\times 2.2}=\frac{5\cdot 10^{-1}}{8.646\cdot 10^{-3}}=59.1$$

whence $$t-t_s=59.1°\text{C} \quad \text{(temperature rise).}$$

$$\textbf{operating temperature}=t+t_s$$

substituting $$=59.1+20=79.1$$

whence $$\text{operating temperature}=79.1° \text{ C.}$$

TABLE 6.

MAXIMUM DIAMETER OF WIRES FOR A RADIO-FREQUENCY RESISTANCE 1% GREATER THAN THE DC RESISTANCE.

(Cir. 74 of Bu. of St'ds.)

$f \cdot 10^{-6}$	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	3.0
λ in meters	3000	1500	750	500	375	300	250	214.3	187.5	166.7	150	100
Material	Diameter of wire in mms.											
Copper	0.356	0.251	0.177	0.145	0.125	0.112	0.102	0.095	0.089	0.084	0.079	0.065
Silver	0.345	0.244	0.172	0.141	0.122	0.109	0.099	0.092	0.086	0.082	0.077	0.063
Gold	0.420	0.297	0.210	0.172	0.149	0.133	0.121	0.112	0.105	0.099	0.094	0.077
Platinum	1.120	0.793	0.560	0.457	0.396	0.354	0.323	0.300	0.280	0.264	0.250	0.205
Mercury	2.640	1.87	1.32	1.080	0.936	0.836	0.763	0.706	0.661	0.623	0.591	0.483
Manganin	1.784	1.261	0.892	0.729	0.631	0.564	0.515	0.477	0.446	0.420	0.399	0.325
Constantan	1.892	1.337	0.946	0.772	0.664	0.598	0.546	0.506	0.473	0.446	0.423	0.345
German Silver	1.942	1.372	0.970	0.792	0.692	0.614	0.560	0.518	0.485	0.458	0.434	0.354
Graphite	7.65	5.41	3.83	3.12	2.71	2.42	2.21	2.04	1.91	1.80	1.71	1.40
Carbon	16.0	11.3	8.01	6.54	5.66	5.06	4.62	4.28	4.00	3.77	3.58	2.92
Iron $\mu = 1000$	0.0263	0.0186	0.0131	0.0108	0.0094	0.0083	0.0076	0.0070	0.0066	0.0062	0.0059	0.0048
$\mu = 500$	0.0373	0.0264	0.0187	0.0152	0.0132	0.0118	0.0108	0.0100	0.0093	0.0088	0.0084	0.0068
$\mu = 100$	0.0838	0.0590	0.0418	0.0340	0.0295	0.0264	0.0241	0.0223	0.0209	0.0197	0.0186	0.0152

Explanation:

This table shows the maximum diameter and frequency for which a wire of a given material has a radio-frequency resistance 1% greater than its dc resistance. The values hold only for straight single conductors, in which the current distribution is symmetrical with respect to the axis of the wire.

When several single conductors are stranded to form **litzendraht,** or when a single conductor is wound in coil form, the current distribution is no longer symmetrical, and the radio-frequency resistance per strand is increased.

TABLE 7.

LOGARITHMIC DECREMENT δ OF WAVE TRAIN AND THE APPROXIMATE NUMBER OF WAVES n IN THE WAVE TRAIN BEFORE THE AMPLITUDE FALLS TO 1/10 OF THE MAXIMUM AMPLITUDE.

δ	n	δ	n
0.20	12.5	0.05	47.0
0.15	16.0	0.04	58.0
0.10	24.0	0.03	78.0
0.09	27.0	0.02	116.0
0.08	30.0	0.01	231.0
0.07	34.0	0.005	462.0

Good tuning is not possible with less than 15 waves in each wave train.

The number of waves in a wave train is found from the formula:

$$n=\frac{2.3026+\delta}{\delta}$$

where 2.3026 = Napierian logarithm of 10,
δ = decrement of transmitter (antenna circuit).

Example: Find number of waves in each wave train emitted by a transmitter having a decrement of 0.05.

Solution:

substituting $$n=\frac{2.3026+0.05}{0.05}=\frac{2.3526}{0.05}=47.0$$

whence $n=47.0$

TABLE 8.

COMPARISON OF LOOP ANTENNAS AND ANTENNAS.

λ in meters	Effective height of equivalent antenna.				
	10 cms.	20 cms.	30 cms.	40 cms.	50 cms.
	Area-turns (lhn).				Area-turns (lhn).
200	3.19	6.37	9.56	12.32	15.92
300	4.77	9.55	14.28	19.10	23.88
400	6.38	12.74	19.10	25.48	31.84
500	7.96	15.92	23.88	31.84	39.80
600	9.55	19.10	28.56	38.20	47.76
700	11.14	22.28	33.45	44.58	55.76
800	12.74	25.48	38.20	50.96	63.70
900	14.28	28.65	42.84	57.30	71.64
1,000	15.92	31.84	47.76	63.70	79.60
2,000	31.85	63.70	95.52	127.4	159.2
3,000	47.76	95.52	142.8	191.0	238.8
4,000	63.80	127.4	191.2	254.8	318.4
5,000	79.6	159.2	238.8	318.4	398.0
7,000	111.4	222.8	334.5	445.8	557.6
10,000	159.2	318.4	477.6	637.0	796.0
12,000	191.1	382.2	573.3	764.4	955.2
15,000	238.8	477.6	716.4	955.2	1,194.0
20,000	318.4	637.8	955.2	1,274.0	1,592.0
25,000	398.0	746.0	1,194.0	1,592.0	1,990.0
30,000	477.6	955.2	1,428.0	1,910.0	2,388.0

Explanation:

The values in this table have been calculated from the formula:

$$h_{eff} = 2\pi \frac{lhn}{\lambda} = 6.28 \frac{lhn}{\lambda}$$

where h_{eff} = effective height of equivalent antenna in meters,
l = horizontal length of loop in meters,
h = vertical height of loop in meters,
n = number of turns,
λ = wave length in meters.

The area-turns in the table are for l in meters and h in meters, while the effective height is given in cms. The table may be used for both transmitting and receiving loops, although the values have been calculated primarily for receiving loops, which usually have an effective height of less than 1 meter. It is assumed, also, that the loop is not used very near its natural wave length, as in this case the current

distribution is no longer uniform on account of the effect of the distributed capacity.

Example:

Find the effective height of the antenna equivalent to a receiving loop antenna 8 feet square having 40 turns at a wave length of 10,000 m.

Solution:

$$8 \text{ ft.} = 2.44 \text{ m.}$$

Formula $$h_{eff} = 6.28 \frac{lhn}{\lambda}$$

substituting $$= 6.28 \frac{2.44 \times 2.44 \times 40}{1 \cdot 10^4}$$

$$= \frac{1.496 \cdot 10^3}{1 \cdot 10^4} = 0.1496$$

whence $h_{eff} = 0.15$m or 15 cms.

TABLE 9.

MEAN AND EFFECTIVE HEIGHTS OF VARIOUS ANTENNAS.

Station	Type of Antenna	h_m in meters.	h_{eff} in meters.	$\frac{h_{eff}}{h_m}$ in percent.
Annapolis	Quadrilateral platform	167	109	65
Arlington	Triangular platform	145	80	55
Marion	No data	No data	68	—
New Brunswick	Long Γ	115	66	57
Naval Radio Research Lab.	T	27.8	15.5	56
Sayville	Irregular umbrella	121 (approx.)	110	91
Tuckerton	Umbrella	No data	121	—
Washington Navy Yard	T	86	82	95

Explanation:

The values of effective height given in the table have been calculated from observations made at the U. S. Naval Radio Research Laboratory, Bureau of Standards, and dated Jan. 1922.

TABLE 10.

RADIATION RESISTANCE OF ANTENNAS.

λ \ h	40 ft.	60 ft.	80 ft.	100 ft.	120 ft.	160 ft.	200 ft.	300 ft.	450 ft.	600 ft.
	12.2m	18.3m	24.4m	30.5m	36.6m	48.8m	61m	91.5m	137.3m	183m
m	ohm	ohm	ohm	ohm	ohm	ohm	ohm	ohm	ohm	ohm
200	5.88									
300	2.61	5.88								
400	1.47	3.31	5.88							
600	0.65	1.47	2.63	4.08	5.88					
800	0.37	0.83	1.47	2.29	3.30	5.88				
1,000	0.24	0.53	0.99	1.47	2.11	3.76	5.88			
1,200	0.16	0.37	0.65	1.02	1.47	2.61	4.07			
1,500		0.24	0.42	0.65	0.94	1.67	2.61	5.88		
2,000		0.13	0.24	0.37	0.53	0.94	1.47	3.30		
2,500			0.15	0.24	0.34	0.60	0.94	2.11	4.76	
3,000			0.11	0.16	0.24	0.42	0.65	1.47	3.31	5.88
4,000			0.059	0.092	0.13	0.24	0.37	0.83	1.86	3.30
5,000				0.059	0.085	0.15	0.24	0.53	1.19	2.12
6,000					0.059	0.11	0.16	0.37	0.83	1.47
7,000							0.12	0.27	0.61	1.08
8,000							0.092	0.21	0.47	0.83
9,000							0.073	0.16	0.37	0.65
10,000							0.059	0.13	0.30	0.53
12,000							0.041	0.092	0.21	0.37
15,000							0.026	0.059	0.13	0.24
20,000							0.015	0.033	0.074	0.13
25,000							0.0094	0.021	0.048	0.085
30,000							0.0065	0.015	0.033	0.059

Explanation:

In the foregoing table, h is the effective height of the antenna in meters. This value is generally about 0.6 of the geometric mean height for any type of antenna. λ is the wave length is meters, and is considerably greater than the fundamental of the antenna. The radiation resistance is expressed in ohms.

The values of radiation resistance have been calculated from the formula

$$R_r = 160\pi^2\left(\frac{h}{\lambda}\right)^2 \qquad \text{(ohms)},$$

or

$$R_r = 1.58 \cdot 10^3\left(\frac{h}{\lambda}\right)^2 \qquad \text{(ohms)}.$$

This formula only takes account of the radiation from the vertical part of the antenna. In the case of flattop and similar antennas there is also, probably, a radiation from the flattop which is not included in this expression and is probably of no value in transmission.

TABLE 11.

ELECTRIC INTENSITY $\mathcal{E}$ IN MICROVOLTS/METER AT ANY RECEIVING STATION, PRODUCED BY A CURRENT OF 30 AMPERES IN A TRANSMITTING ANTENNA, HAVING AN EFFECTIVE HEIGHT OF 17.5 METERS.

Distance			Wave Length							
Nautical Miles	$\cdot 10^3$ mters		300m	600m	1000m	1500m	2500m	4000m	6000m	
20	37.	$\mathcal{E}$ in microvolts/meter	16,300.	8,350.	5,100.	3,430.	2,080.	1288.	880.	$\mathcal{E}$ in microvolts/meter
50	92.5		5,600.	3,020.	1,880.	1,282.	790.	498.	340.	
100	185.		2,170.	1,270.	817.	573.	361.	233.	161.	
200	370.		650.	433.	310.	228.	152.	101.	71.8	
300	556.		263.	205.	156.	122.	85.	58.6	42.7	
400	740.		116.	107.	89.	73.	53.6	38.3	28.6	
500	925.		55.4	60.2	53.5	46.3	35.9	26.7	20.5	
600	1110.		27.6	34.8	34.2	31.1	25.3	19.4	15.2	
800	1480.		7.72	12.8	14.7	14.8	13.1	11.0	9.06	
1000	1850.		2.27	4.97	6.7	7.5	7.42	6.66	5.80	
1200	2220.		0.66	2.0	3.24	3.95	4.36	4.3	3.89	
1500	2780.		0.12	0.54	1.10	1.58	2.04	2.2	2.18	
2000	3700.		0.0081	0.069	0.21	0.39	0.64	0.83	0.93	

Explanation:

The data given in the above table have been calculated from the formula:

$$\mathcal{E}=377\frac{I_sh_s}{d\lambda}\epsilon^{-\frac{4.75\cdot 10^{-5}d}{\sqrt{\lambda}}}$$

where $\mathcal{E}$ = electric intensity in volts/meter;
I_s = current in amperes in transmitting antenna;
h_s = effective height in meters of transmitting antenna;
d = distance in meters;
λ = wave length in meters;
ϵ = 2.7183.

The above formula has been changed to the following form so that the result obtained will be expressed in microvolts/meter.

$$\mathcal{E}=3.77\cdot 10^8\frac{I_sh_s}{d\lambda}\epsilon^{-\frac{4.75\cdot 10^{-5}d}{\sqrt{\lambda}}}$$

Example:

Find the electric intensity at a distance of 500 nautical miles produced by a current of 30 amperes in a transmitting antenna having an effective height of 17.5 meters for a wave length of 1000 meters.

Solution:
I_s = 30 amperes;
h = 17.5 meters;
d = $9.25\cdot 10^{-5}$ meters;
λ = $1\cdot 10^3$ meters;

$$\mathcal{E}=3.77\cdot 10^8\left(\frac{30\times 17.5}{9.25\cdot 10^{-5}\times 1\cdot 10^3}\right)\epsilon^{-\frac{4.75\cdot 10^{-5}\times 9.25\cdot 10^5}{\sqrt{1\cdot 10^3}}}$$

$$=3.77 \cdot 10^8 \left(\frac{5.25 \cdot 10^2}{9.25 \cdot 10^8}\right)\epsilon^{-1.39} = \left(\frac{19.79 \cdot 10^2}{9.25}\right)0.249$$

$$=214. \times 0.249 = 53.3$$

whence $\mathcal{E} = 53.3$ microvolts/meter, which corresponds with the value given in the table.

The table may also be used in the calculation of $\mathcal{E}$ for any other values of current and height. To do this: Multiply the value of $\mathcal{E}$, given in the table, by the ratio of the new Ih to that used in the table (30×17.5) or 525.

Example:

In the above example, assume $I = 60$ amperes and $h = 35$ meters.

$$Ih = 60 \times 35 = 2100$$

$$\text{Ratio} = \frac{2100}{525} = 4$$

$$\mathcal{E} = 53.3 \times 4 = 213.0 \text{ microvolts/meter.}$$

For values of ϵ^{-x} see Table 12.

TABLE 12.

VALUES OF ϵ^{-x}

(Smithsonian Tables.)

x	0	1	2	3	4	5	6	7	8	9
0.0	1.000	0.990	0.980	0.970	0.961	0.951	0.942	0.932	0.923	0.914
0.1	0.905	0.896	0.887	0.878	0.869	0.861	0.852	0.844	0.835	0.827
0.2	0.819	0.811	0.803	0.795	0.787	0.779	0.771	0.763	0.756	0.748
0.3	0.741	0.733	0.726	0.719	0.712	0.705	0.698	0.691	0.684	0.677
0.4	0.670	0.664	0.657	0.651	0.644	0.638	0.631	0.625	0.619	0.613
0.5	0.607	0.600	0.595	0.589	0.583	0.577	0.571	0.566	0.560	0.554
0.6	0.549	0.543	0.538	0.533	0.527	0.522	0.517	0.512	0.507	0.502
0.7	0.497	0.492	0.487	0.482	0.477	0.472	0.468	0.463	0.458	0.454
0.8	0.449	0.445	0.440	0.436	0.432	0.427	0.423	0.419	0.415	0.411
0.9	0.407	0.403	0.399	0.395	0.391	0.387	0.383	0.379	0.375	0.372
1.0	0.368	0.364	0.361	0.357	0.353	0.350	0.346	0.343	0.340	0.336
1.1	0.333	0.330	0.326	0.323	0.320	0.317	0.313	0.310	0.307	0.304
1.2	0.301	0.298	0.295	0.292	0.289	0.287	0.284	0.281	0.278	0.275
1.3	0.273	0.270	0.267	0.264	0.262	0.259	0.257	0.254	0.252	0.249
1.4	0.247	0.244	0.242	0.239	0.237	0.235	0.232	0.230	0.228	0.225
1.5	0.223	0.221	0.219	0.217	0.214	0.212	0.210	0.208	0.206	0.204
1.6	0.202	0.200	0.198	0.196	0.194	0.192	0.190	0.188	0.186	0.185
1.7	0.183	0.181	0.179	0.177	0.176	0.174	0.172	0.170	0.169	0.167
1.8	0.165	0.164	0.162	0.160	0.159	0.157	0.156	0.154	0.153	0.151
1.9	0.150	0.148	0.147	0.145	0.144	0.142	0.141	0.139	0.138	0.137
2.0	0.135	0.134	0.133	0.131	0.130	0.129	0.127	0.126	0.125	0.124
2.1	0.122	0.121	0.120	0.119	0.118	0.116	0.115	0.114	0.113	0.112
2.2	0.111	0.110	0.109	0.108	0.106	0.105	0.104	0.103	0.102	0.101
2.3	0.1000	0.0993	0.0983	0.0973	0.0963	0.0954	0.0944	0.0935	0.0926	0.0916
2.4	0.0907	0.0898	0.0889	0.0880	0.0872	0.0863	0.0854	0.0864	0.0837	0.0829
2.5	0.0821	0.0813	0.0805	0.0797	0.0789	0.0781	0.0773	0.0765	00.0758	0.0750
2.6	0.0743	0.0735	0.0728	0.0720	0.0714	0.0707	0.0609	0.0693	0.0686	0.0679
2.7	0.0672	0.0665	0.0659	0.0652	0.0646	0.0639	0.0633	0.0627	0.0620	0.0614
2.8	0.0608	0.0602	0.0596	0.0590	0.0584	0.0578	0.0573	0.0567	0.0561	0.0556
2.9	0.0550	0.0545	0.0539	0.0534	0.0529	0.0523	0.0518	0.0513	0.0508	0.0503
3.0	0.0498	0.0493	0.0488	0.0483	0.0478	0.0474	0.0469	0.0464	0.0460	0.0455
3.1	0.0450	0.0446	0.0442	0.0437	0.0433	0.0429	0.0424	0.0420	0.0416	0.0412
3.2	0.0408	0.0404	0.0400	0.0396	0.0392	0.0388	0.0384	0.0380	0.0376	0.0373
3.3	0.0369	0.0365	0.0362	0.0358	0.0354	0.0351	0.0347	0.0344	0.0340	0.0337
3.4	0.0334	0.0330	0.0327	0.0324	0.0321	0.0317	0.0314	0.0311	0.0308	0.0305
3.5	0.0302	0.0299	0.0296	0.0293	0.0290	0.0287	0.0284	0.0282	0.0279	0.0276
3.6	0.0273	0.0271	0.0268	0.0265	0.0263	0.0260	0.0257	0.0255	0.0252	0.0250
3.7	0.0247	0.0245	0.0242	0.0240	0.0238	0.0235	0.0233	0.0231	0.0228	0.0226
3.8	0.0224	0.0221	0.0219	0.0217	0.0215	0.0213	0.0211	0.0209	0.0207	0.0204
3.9	0.0202	0.0200	0.0198	0.0196	0.0194	0.0193	0.0191	0.0189	0.0187	0.0185
4.0	0.0183	0.0181	0.0180	0.0178	0.0176	0.0174	0.0172	0.0171	0.0169	0.0167
4.1	0.0166	0.0164	0.0162	0.0161	0.0159	0.0158	0.0156	0.0155	0.0153	0.0151
4.2	0.0150	0.0148	0.0147	0.0146	0.0144	0.0143	0.0141	0.0140	0.0138	0.0137
4.3	0.0136	0.0134	0.0133	0.0132	0.0130	0.0129	0.0128	0.0127	0.0125	0.0124
4.4	0.0123	0.0122	0.0120	0.0119	0.0118	0.0117	0.0116	0.0114	0.0113	0.0112
4.5	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102
4.6	0.0101	0.00995	0.00985	0.00975	0.00966	0.00956	0.00947	0.00937	0.00928	0.00919
4.7	0.00910	0.00900	0.00892	0.00883	0.00874	0.00865	0.00857	0.00848	0.00840	0.00831
4.8	0.00823	0.00815	0.00807	0.00799	0.00791	0.00783	0.00775	0.00767	0.00760	0.00752
4.9	0.00745	0.00737	0.00730	0.00723	0.00715	0.00708	0.00701	0.00694	0.00687	0.00681
5.0	0.00674	0.00667	0.00660	0.00654	0.00647	0.00641	0.00635	0.00628	0.00622	0.00616
5.1	0.00610	0.00604	0.00598	0.00592	0.00586	0.00580	0.00574	0.00568	0.00563	0.00557
5.2	0.00552	0.00546	0.00541	0.00535	0.00530	0.00525	0.00520	0.00514	0.00509	0.00504
5.3	0.00499	0.00494	0.00489	0.00484	0.00480	0.00475	0.00470	0.00465	0.00461	0.00456
5.4	0.00452	0.00447	0.00443	0.00438	0.00434	0.00430	0.00425	0.00421	0.00417	0.00413
5.5	0.00409	0.00405	0.00401	0.00397	0.00393	0.00389	0.00385	0.00381	0.00377	0.00374
5.6	0.00370	0.00366	0.00362	0.00359	0.00355	0.00352	0.00348	0.00345	0.00341	0.00338
5.7	0.00335	0.00331	0.00328	0.00325	0.00321	0.00318	0.00315	0.00312	0.00309	0.00306
5.8	0.00303	0.00300	0.00297	0.00294	0.00291	0.00288	0.00285	0.00282	0.00279	0.00277
5.9	0.00274	0.00271	0.00269	0.00266	0.00263	0.00261	0.00258	0.00255	0.00253	0.00250
x	0	1	2	3	4	5	6	7	8	9

Explanation:

This table gives the values of ϵ^{-x} where $\epsilon = 2.7183$, the base of Napierian logarithms, and $-x$ is any negative exponent of ϵ.

The term ϵ^{-x} appears in the transmission formula as the attenuation factor

$$\epsilon^{-\frac{4.75 \cdot 10^{-5} d}{\sqrt{\lambda}}}$$

To use the table, it will be necessary first to solve for the exponent x. The corresponding numerical exponent is then found under x in the table, and the value of ϵ^{-x} is read in the corresponding column. Example:

Find the value of $\epsilon^{-\frac{4.75 \cdot 10^{-5} \times 9.25 \cdot 10^{5}}{\sqrt{1 \cdot 10^{3}}}}$

Solution:

$$-x = -\frac{4.75 \cdot 10^{-5} \times 9.25 \cdot 10^{5}}{\sqrt{1 \cdot 10^{3}}} = -\frac{43.94}{31.62} = -1.39$$

Then, from the table, $\epsilon^{-x} = \epsilon^{-1.39} = 0.249$

TABLE 13.

TABLE OF WAVE LENGTHS AND THE CORRESPONDING FREQUENCY, OSCILLATION CONSTANT AND ANGULAR VELOCITY.

λ	f in kilo-cycles	LC	ω	λ	f in kilo-cycles	LC	ω
1,000	300.0	0.281	1,885,000	1,400	214.3	0.552	1,346,000
1,010	297.0	.287	1,866,000	1,410	212.8	.560	1,337,000
1,020	294.1	.293	1,848,000	1,420	211.3	.568	1,327,000
1,030	291.3	.299	1,830,000	1,430	209.8	.576	1,318,000
1,040	288.5	.304	1,812,000	1,440	208.3	.584	1,309,000
1,050	285.7	.310	1,795,000	1,450	206.9	.592	1,300,000
1,060	283.0	.316	1,778,000	1,460	205.5	.600	1,291,000
1,070	280.4	.322	1,762,000	1,470	204.1	.608	1,282,000
1,080	277.8	.328	1,745,000	1,480	202.7	.617	1,274,000
1,090	275.2	.334	1,729,000	1,490	201.3	.625	1,265,000
1,100	272.7	0.341	1,714,000	1,500	200.0	0.633	1,257,000
1,110	270.3	.347	1,698,000	1,510	198.7	.642	1,249,000
1,120	267.9	.353	1,683,000	1,520	197.4	.650	1,240,000
1,130	265.5	.359	1,668,000	1,530	196.1	.659	1,232,000
1,140	263.2	.366	1,653,000	1,540	194.8	.668	1,224,000
1,150	260.9	.372	1,639,000	1,550	193.6	.676	1,216,000
1,160	258.6	.379	1,625,000	1,560	192.3	.685	1,208,000
1,170	256.4	.385	1,611,000	1,570	191.1	.694	1,201,000
1,180	254.2	.392	1,597,000	1,580	189.9	.703	1,193,000
1,190	252.1	.399	1,584,000	1,590	188.7	.712	1,186,000
1,200	250.0	0.405	1,571,000	1,600	187.5	0.721	1,178,000
1,210	247.9	.412	1,558,000	1,610	186.3	.730	1,171,000
1,220	245.9	.419	1,545,000	1,620	185.2	.739	1,164,000
1,230	243.9	.426	1,532,000	1,630	184.1	.748	1,156,000
1,240	241.9	.433	1,520,000	1,640	182.9	.757	1,149,000
1,250	240.0	.440	1,508,000	1,650	181.8	.766	1,142,000
1,260	238.1	.447	1,496,000	1,660	180.7	.776	1,136,000
1,270	236.2	.454	1,484,000	1,670	179.6	.785	1,129,000
1,280	234.4	.461	1,473,000	1,680	178.6	.794	1,122,000
1,290	232.6	.468	1,461,000	1,690	177.5	.804	1,115,000
1,300	230.8	0.476	1,450,000	1,700	176.5	0.813	1,109,000
1,310	229.0	.483	1,439,000	1,710	175.4	.823	1,102,000
1,320	227.3	.490	1,428,000	1,720	174.4	.833	1,096,000
1,330	225.6	.498	1,417,000	1,730	173.4	.842	1,090,000
1,340	223.9	.505	1,407,000	1,740	172.4	.852	1,083,000
1,350	222.2	.513	1,396,000	1,750	171.4	.862	1,077,000
1,360	220.6	.521	1,386,000	1,760	170.5	.872	1,071,000
1,370	219.0	.528	1,378,000	1,770	169.5	.882	1,065,000
1,380	217.4	.536	1,366,000	1,780	168.5	.892	1,059,000
1,390	215.8	.544	1,356,000	1,790	167.6	.902	1,053,000

λ	f in kilo-cycles	LC	ω	λ	f in kilo-cycles	LC	ω
1,800	166.7	0.912	1,047,000	2,400	125.0	1.621	785,000
1,810	165.8	.922	1,041,000	2,420	124.0	.648	779,000
1,820	164.8	.932	1,036,000	2,440	123.0	.676	773,000
1,830	163.9	.943	1,030,000	2,460	122.0	.703	766,000
1,840	163.0	.953	1,024,000	2,480	121.0	.731	760,000
1,850	162.2	.963	1,019,000	2,500	120.0	.759	754,000
1,860	161.3	.974	1,013,000	2,520	119.1	.787	748,000
1,870	160.4	.984	1,008,000	2,540	118.1	.816	742,000
1,880	159.6	.995	1,003,000	2,560	117.1	.845	736,000
1,890	158.7	1.005	997,000	2,580	116.3	.873	731,000
1,900	157.9	1.016	992,000	2,600	115.4	1.903	725,000
1,910	157.1	.027	987,000	2,620	114.5	.932	719,000
1,920	156.3	.038	982,000	2,640	113.6	.962	714,000
1,930	155.4	.048	977,000	2,660	112.8	.991	709,000
1,940	154.6	.059	972,000	2,680	111.9	2.022	703,000
1,950	153.9	.070	967,000	2,700	111.1	.052	698,000
1,960	153.1	.081	962,000	2,720	110.3	.082	693,000
1,970	152.3	.092	957,000	2,740	109.5	.113	688,000
1,980	151.5	.103	952,000	2,760	108.7	.144	683,000
1,990	150.8	.115	947,000	2,780	107.9	.175	678,000
2,000	150.0	1.126	942,000	2,800	107.1	2.207	673,000
2,020	148.5	.148	933,000	2,820	106.4	.238	668,000
2,040	147.1	.171	924,000	2,840	105.6	.270	664,000
2,060	145.6	.194	915,000	2,860	104.9	.302	659,000
2,080	144.2	.218	967,000	2,880	104.2	.334	655,000
2,100	142.9	.241	898,000	2,900	103.5	.367	650,000
2,120	141.5	.265	889,000	2,920	102.7	.400	646,000
2,140	140.2	.289	881,000	2,940	102.0	.433	641,000
2,160	138.9	.313	873,000	2,960	101.4	.466	637,000
2,180	137.6	.338	865,000	2,980	100.7	.499	633,000
2,200	136.4	1.362	857,000	3,000	100.0	2.53	628,000
2,220	135.1	.387	849,000	3,020	99.3	.57	624,000
2,240	133.9	.412	842,000	3,040	98.7	.60	620,000
2,260	132.7	.438	834,000	3,060	98.0	.64	616,000
2,280	131.6	.463	827,000	3,080	97.4	.67	612,000
2,300	130.4	.489	820,000	3,100	96.8	.70	608,000
2,320	129.3	.515	812,000	3,120	96.2	.74	604,000
2,340	128.2	.541	806,000	3,140	95.5	.77	600,000
2,360	127.1	.568	799,000	3,160	94.9	.81	597,000
2,380	126.1	.594	792,000	3,180	94.3	.85	593,000

λ	f in kilo-cycles	LC	ω	λ	f in kilo-cycles	LC	ω
3,200	93.8	2.88	589,000	4,000	75.0	4.50	471,200
3,220	93.2	.92	585,000	4,020	74.6	.55	468,900
3,240	92.6	.95	582,000	4,040	74.3	.60	466,600
3,260	92.0	.99	578,000	4,060	73.9	.64	464,300
3,280	91.5	3.03	575,000	4,080	73.5	.69	462,000
3,300	90.9	.07	571,000	4,100	73.2	.73	459,700
3,320	90.4	.10	568,000	4,120	72.8	.78	457,500
3,340	89.8	.14	564,000	4,140	72.5	.82	455,300
3,360	89.3	.18	561,000	4,160	72.1	.87	453,100
3,380	88.8	.22	558,000	4,180	71.8	.92	450,900
3,400	88.2	3.25	554,000	4,200	71.4	4.97	448,800
3,420	87.7	.29	551,000	4,220	71.1	5.01	446,700
3,440	87.2	.33	548,000	4,240	70.8	.06	444,600
3,460	86.7	.37	545,000	4,260	70.4	.11	442,500
3,480	86.2	.41	542,000	4,280	70.1	.16	440,400
3,500	85.7	.45	539,000	4,300	69.8	.20	438,400
3,520	85.2	.49	535,000	4,320	69.4	.25	436,300
3,540	84.7	.53	532,000	4,340	69.1	.30	434,300
3,560	84.3	.57	529,000	4,360	68.8	.35	432,300
3,580	83.8	.61	527,000	4,380	68.5	.40	430,400
3,600	83.3	3.65	524,000	4,400	68.2	5.45	428,300
3,620	82.9	.69	521,000	4,420	67.9	.50	426,500
3,640	82.4	.73	518,000	4,440	67.6	.55	424,500
3,660	82.0	.76	515,000	4,460	67.3	.60	422,600
3,680	81.5	.81	512,000	4,480	67.0	.65	420,700
3,700	81.1	.85	509,000	4,500	66.7	.70	418,900
3,720	80.6	.89	607,000	4,520	66.4	.75	417,000
3,740	80.2	.94	504,000	4,540	66.1	.80	415,200
3,760	79.8	.98	501,000	4,560	65.8	.85	413,400
3,780	79.4	4.02	498,700	4,580	65.5	.90	411,600
3,800	78.9	4.06	496,000	4,600	65.2	5.96	409,800
3,820	78.5	.11	493,400	4,620	64.9	6.01	408,000
3,840	78.1	.15	490,900	4,640	64.7	.06	406,200
3,860	77.7	.19	488,300	4,660	64.4	.11	404,500
3,880	77.3	.24	485,800	4,680	64.1	.16	402,800
3,900	76.9	.28	483,300	4,700	63.8	.22	401,100
3,920	76.5	.32	480,900	4,720	63.6	.27	399,400
3,940	76.1	.37	478,400	4,740	63.3	.32	397,700
3,960	75.8	.41	476,000	4,760	63.0	.38	396,000
3,980	75.4	.46	473,600	4,780	62.8	.43	394,300

λ	f in kilo-cycles	LC	ω	λ	f in kilo-cycles	LC	ω
4,800	62.5	6.49	392,700	6,500	46.15	11.89	290,000
4,820	62.2	.54	391,100	6,550	45.80	12.08	287,800
4,840	62.0	.59	389,500	6,600	45.46	.26	285,600
4,860	61.7	.65	387,800	6,650	45.11	.45	283,500
4,880	61.5	.70	386,300	6,700	44.78	.63	281,300
4,900	61.2	.76	384,700	6,750	44.44	.82	279,300
4,920	61.0	.81	383,100	6,800	44.12	13.01	277,200
4,940	60.7	.87	381,600	6,850	43.80	.21	275,200
4,960	60.5	.92	380,000	6,900	43.48	.40	273,200
4,980	60.2	.98	378,500	6,950	43.17	.60	271,200
5,000	60.0	7.04	377,000	7,000	42.86	13.79	269,300
5,050	59.4	.18	373,300	7,050	42.55	.99	267,400
5,100	58.8	.32	369,600	7,100	42.25	14.19	265,500
5,150	58.3	.47	366,000	7,150	41.96	.39	263,600
5,200	57.7	.61	362,500	7,200	41.67	.59	261,800
5,250	57.1	.76	359,000	7,250	41.38	.79	260,000
5,300	56.6	.91	355,700	7,300	41.10	15.00	258,200
5,350	56.1	8.06	352,300	7,350	40.82	.20	256,500
5,400	55.6	.21	349,100	7,400	40.54	.41	254,700
5,450	55.0	.36	345,900	7,450	40.27	.62	253,000
5,500	54.5	8.51	342,700	7,500	40.00	15.83	251,300
5,550	54.1	.67	339,600	7,550	39.74	16.04	249,700
5,600	53.6	.83	336,600	7,600	39,47	.26	248,000
5,650	53.1	.98	333,600	7,650	39.22	.47	246,400
5,700	52.6	9.12	330,700	7,700	38.96	.69	244,800
5,750	52.2	.31	327,800	7,750	38.71	.90	243,200
5,800	51.7	.47	325,000	7,800	38.46	17.12	241,700
5,850	51.3	.63	322,200	7,850	38.22	.34	240,100
5,900	50.8	.80	319,500	7,900	37.98	.57	238,600
5,950	50.4	.96	316,800	7,950	37.74	.79	237,100
6,000	50.00	10.13	314,200	8,000	37.50	18.01	235,600
6,050	49.59	.30	311,600	8,050	37.27	.24	234,200
6,100	49.18	.47	309,000	8,100	37.04	.47	232,700
6,150	48.78	.65	306,500	8,150	36.81	.69	231,300
6,200	48.39	.82	304,000	8,200	36.59	.92	229,900
6,250	48.00	.99	301,600	8,250	36.36	19.16	228,500
6,300	47.62	11.17	299,200	8,300	36.15	.39	227,100
6,350	47.24	.35	296,800	8,350	35.93	.62	225,700
6,400	46.88	.53	294,500	8,400	35.71	.86	224.400
6,450	46.51	.71	292,200	8,450	35.50	20.10	223,100

λ	*f* in kilo-cycles	*LC*	ω
8,500	35.29	20.34	221,800
8,550	35.09	.57	220,500
8,600	34.88	.82	219,200
8,650	34.68	21.06	217,900
8,700	34.48	.30	216,700
8,750	34.29	.55	215,400
8,800	34.09	.80	214,200
8,850	33.90	22.04	213,000
8,900	33.71	.29	211,800
8,950	33.52	.55	210,600
9,000	33.33	22.80	209,400
9,050	33.15	23.05	208,300
9,100	32.97	.31	207,100
9,150	32.79	.56	206,000
9,200	32.61	.82	204,900
9,250	32.43	24.08	203,800
9,300	32.26	.34	202,700
9,350	32.09	.61	201,600
9,400	31.92	.87	200,500
9,450	31.75	25.1	199,500
9,500	31.58	25.4	198,400
9,550	31.41	.7	197,400
9,600	31.25	.9	196,400
9,650	31.09	26.2	195,300
9,700	39.93	.5	194,300
9,750	30.77	.8	193,300
9,800	30.61	27.0	192,300
9,850	30.46	.3	191,400
9,900	30.30	.6	190,400
9,950	30.15	.9	189,400

Explanation:

This table has been calculated using $c = 3 \cdot 10^8$ meters per second. λ is expressed in meters. The frequency *f* is in kilocycles per second. The oscillation constant *LC* is the product of the inductance *L* in microhenries and the capacity *C* in microfarads. The fourth column gives the values of $\omega = 2\pi f$.

The values are given for wave lengths between 1000 m. and 9950m., as these are most frequently employed. Interpolation will be required to find values not given in the table, but included within its limits.

Example:

Find *f*, *LC* and ω for λ = 1085m.

Solution:

1080 and 1090 are given. Difference between 1080 and 1090 = 10. Difference between 1080 and 1085 = 5. Hence 5/10, or 0.5, of difference in values for 1080 and 1085 should be added to or subtracted from, respectively, value for 1080 depending upon whether values are increasing or decreasing.

	λ	f in kilo-cycles	LC	ω	
	1,090 1,080	275.2 277.8	0.334 0.328	1,729,000 1,745,000	
Difference....... Multiplying.....	+ 10 0.5	− 2.6 0.5	0.006 0.5	− 16,000 0.5	
	+ 5. 1,080	− 1.3 277.8	0.003 0.328	− 8,000 1,745,000	to be added to values for 1080m.
	1,085	276.5	0.331	1,737,000	Values for 1085m.

Example:

Find wave length corresponding to an LC value of 10.19.

Solution:

An inspection of the table shows that 10.19 lies between 10.13 and 10.30, the respective wave lengths being 6000m. and 6050m.

	6050	10.30	10.19
	6000	10.13	10.13
Subtracting	50	0.17	0.06

Hence, the wave length corresponding to $LC = 10.19$ is

$$\frac{.06}{.17} \times 50 \text{ greater than } 6000.$$

$$\frac{.06}{.17} = 0.353$$

$$0.353 \times 50 = 17.65\text{m. to be added to } 6000\text{m.}$$

Hence $\lambda = 6018$m.

Values for wave lengths between the following pairs of limits can readily be found by moving the decimal point of the values for the wave length having the same sequence of digits given in the table. This change is shown in the following form.

λ	f in kilocycles	LC	ω
1 to 99.5	2 places to right	4 places to left	2 places to right
100 to 995	1 place to right	2 places to left	1 place to right
10,000 to 99,500	1 place to left	2 places to right	1 place to left

Example:

Find f, LC and ω for wave lengths 20m., 200m., and 20,000m.

Solution:

λ	f in kilo-cycles	LC	ω
2,000	150.0	1.126	942,000
20	15,000.	.0001126	94,200,000
200	1,500.	.01126	9,420,000
20,000	15.00	112.6	94,200

TABLE 14.

MULTIPLIERS FOR CONVERTING ELECTRICAL UNITS.

Inductance.

Unit	h	mh	μh	emu	esu
Henry (h)	1	$1 \cdot 10^{3}$	$1 \cdot 10^{6}$	$1 \cdot 10^{9}$	$1.1 \cdot 10^{-12}$
Millihenry (mh)	$1 \cdot 10^{-3}$	1	$1 \cdot 10^{3}$	$1 \cdot 10^{6}$	$1.1 \cdot 10^{-15}$
Microhenry (μh)	$1 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	1	$1 \cdot 10^{3}$	$1.1 \cdot 10^{-18}$
El. mag. unit (emu)	$1 \cdot 10^{-9}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	1	$1.1 \cdot 10^{-21}$
El. static unit (esu)	$9 \cdot 10^{11}$	$9 \cdot 10^{14}$	$9 \cdot 19^{17}$	$9 \cdot 10^{20}$	1

Capacity.

Unit	f	μf	emμf	$\mu\mu$f	emu	esu
Farad (f)	1	$1 \cdot 10^{6}$	$1 \cdot 10^{9}$	$1 \cdot 10^{12}$	$1 \cdot 10^{-9}$	$9 \cdot 10^{11}$
Microfarad (μf)	$1 \cdot 10^{-6}$	1	$1 \cdot 10^{3}$	$1 \cdot 10^{6}$	$1 \cdot 10^{-15}$	$9 \cdot 10^{5}$
Milli-microfarad (mμf)	$1 \cdot 10^{-9}$	$1 \cdot 10^{-3}$	1	$1 \cdot 10^{3}$	$1 \cdot 10^{-18}$	$9 \cdot 10^{2}$
Micro-microfarad ($\mu\mu$f)	$1 \cdot 10^{-12}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	1	$1 \cdot 10^{-21}$	$9 \cdot 10^{-1}$
El. mag. unit (emu)	$1 \cdot 10^{9}$	$1 \cdot 10^{15}$	$1 \cdot 10^{18}$	$1 \cdot 10^{21}$	1	$9 \cdot 10^{20}$
El. static unit (esu)	$1.1 \cdot 10^{-12}$	$1.1 \cdot 10^{-6}$	$1.1 \cdot 10^{-3}$	1.1	$1 \cdot 10^{-19}$	1

Current.

Unit	ampere	emu	esu
Ampere	1	$1 \cdot 10^{-1}$	$3 \cdot 10^{9}$
El. mag. unit (emu)	10	1	$3 \cdot 10^{10}$
El. static unit (esu)	$3.33 \cdot 10^{-10}$	$3.33 \cdot 10^{-11}$	1

Potential.

Unit	volt	emu	esu
Volt	1	$1 \cdot 10^{8}$	$3.33 \cdot 10^{-3}$
El. mag. unit (emu)	$1 \cdot 10^{-8}$	1	$3.33 \cdot 10^{-11}$
El. static unit (esu)	$3 \cdot 10^{2}$	$3 \cdot 10^{10}$	1

Resistance.

Unit	ohm	emu	esu
Ohm	1	$1 \cdot 10^{9}$	$1.11 \cdot 10^{-12}$
El. mag. unit (emu)	$1 \cdot 10^{-9}$	1	$1.11 \cdot 10^{-21}$
El. static unit (esu)	$9 \cdot 10^{11}$	$9 \cdot 10^{20}$	1

Power.

Unit	watt	emu	esu
Watt	1	$1 \cdot 10^7$	$1 \cdot 10^7$
El. mag. unit (emu)	$1 \cdot 10^{-7}$	1	1
El. static unit (esu)	$1 \cdot 10^{-7}$	1	1

Explanation:

These tables have been arranged so that multiplication is the only operation to be performed in changing a unit from one system to another. The multipliers, on account of their size, have been expressed in powers of 10. It is only necessary to remember that exponents should be added algebraically when multiplying.

To change a unit from one system to another: Enter the vertical column on the left under "Unit" with the unit to be changed. Move to the right on line on which unit is found until in the vertical column headed by the unit to which it is desired to change. The multiplier is found on this line in the column just mentioned.

Example:

Change 2.75 millihenries to microhenries.

Solution:

Enter table in vertical column on left and find millihenry (mh). This is the second unit in column. Move on same horizontal line to right until in column headed "μh". The multiplier is found to be $1 \cdot 10^3$.

Multiplying:

$$2.75 \times 1 \cdot 10^3 = 2.75 \cdot 10^3 = 2750$$

whence $$2.75\text{mh} = 2750\mu\text{h}.$$

Example:

Change 0.0015 μf to esu (cms).

Solution:

$$0.0015\ \mu\text{f} = 1.5 \cdot 10^{-3}\ \mu\text{f}.$$

Proceeding as in the first example, the multiplier is found to be $9 \cdot 10^5$.

Multiplying:

$$1.5 \cdot 10^{-3} \times 9 \cdot 10^5 = 13.5 \cdot 10^2 = 1350$$

whence $$0.0015\mu\text{f} = 1350 \text{ esu (cms)}.$$

Note: The cgs system includes both the electromagnetic and electrostatic units. The el. mag. unit of inductance is the centimeter and the el. static unit of capacity is the centimeter. The el. static unit of inductance and the el. mag. unit of capacity have no names.

TABLE 15.

TABLES FOR CONVERTING U. S. MEASURES.
(1) Customary to Metric.

Linear.

No.	Inches to millimeters.	Feet to meters.	Yards to meters.	Statute miles to kilometers.	Nautical miles to kilometers.
1	25.40	0.3048	0.9144	1.6094	1.8533
2	50.80	0.6096	1.8288	3.2187	3.7065
3	76.20	0.9144	2.7432	4.8280	5.5598
4	101.60	1.2192	3.6576	6.4374	7.5130
5	127.00	1.5240	4.5720	8.0467	9.2663
6	152.40	1.8288	5.4864	9.6561	11.1195
7	177.80	2.1336	6.4008	11.2654	12.9728
8	203.20	2.4384	7.3152	12.8748	15.0260
9	228.60	2.7432	8.2296	14.4841	16.6793

Square.

1	645.2	0.09290	0.836	2.59	3.438

Cubic.

1	1,638.7	0.02832	0.765	— — —	— — —

(2) Metric to Customary.

Linear.

No.	Millimeters to inches	Meters to feet.	Meters to yards.	Kilometers to statute miles.	Kilometers to nautical miles.
1	0.03937	3.2808	1.0936	0.6214	0.5396
2	0.07874	6.5617	2.1872	1.2427	1.0792
3	0.11811	9.8425	3.2808	1.8641	1.6188
4	0.15748	13.1233	4.3744	2.4855	2.1584
5	0.19685	16.4042	5.4681	3.1069	2.6980
6	0.23622	19.6850	6.5617	3.7282	3.2376
7	0.27559	22.9658	7.6553	4.3496	3.7771
8	0.31496	26.2467	8.7489	4.9710	4.3167
9	0.35433	29.5275	9.8425	5.5923	4.8563

Square.

1	0.001551	10.764	1.196	0.3861	0.2912

Cubic.

1	0.0000611	35.314	1.308	— — —	— — —

Explanation:

The above tables, which are adapted from sheets issued by the Bureau of Standards, will aid in converting the customary linear, square and cubic units to their equivalents in the metric system, and vice versa.

The linear units are given for numbers 1 to 9 in order to simplify the conversion. Interpolation is not necessary, as the equivalent can be read directly for each digit of the number to be converted, addition only being necessary, as shown in the following.

Example:

Change 554 feet to meters.

Solution:

$$\begin{array}{r@{\,=\,}l} 500 & 152.4 \\ 50 & 15.24 \\ 4 & 1.2192 \\ \hline 554 & 168.8592 \end{array}$$

whence $554 \text{ ft.} = 168.86 \text{ m},$

The tables also give the multipliers for changing units of area and volume in one system to their equivalents in the other system. As square and cubic measurements are seldom used in radio, multipliers are given only for the unit. It will, therefore, be necessary to multiply the number to be converted by the factor given in the table.

Example:

Change 8 m^2 to square feet.

Solution:

From table $1 \text{ m}^2 = 10.764$ square feet

Hence $8 \text{ m}^2 = 10.764 \times 8 = 86.112$ square feet.

The Metric System.

The fundamental unit of the metric system is the **meter** (unit of length). From this, the **gram** (unit of mass) and the **liter** (unit of capacity) are derived. All other units are decimal subdivisions or multiples of these. These three units are simply related, so that the volume of one kilogram of water is, for practical purposes, equal to one cubic decimeter. The following table shows the metric system.

Prefix	Meaning	Units
milli-	$\frac{1}{1,000}=0.001=10^{-3}$	
centi-	$\frac{1}{100}=0.01=10^{-2}$	Meter (for length)
deci-	$\frac{1}{10}=0.1=10^{-1}$	
unit	$\frac{1}{1}=1.=10^{0}$	Gram (for mass)
deka-	$\frac{10}{1}=10.=10^{1}$	
hecto-	$\frac{100}{1}=100.=10^{2}$	Liter (for capacity)
kilo-	$\frac{1,000}{1}=1,000.=10^{3}$	

The metric terms are formed by combining the words **meter, gram** and **liter** with the six numerical prefixes. These are given below for the units of length.

Linear Units.		
10 millimeters mm	=1 centimeter	cm
10 centimeters	=1 decimeter	dm
10 decimeters	=1 meter	m
10 meters	=1 dekameter	dkm
10 dekameters	=1 hectometer	hm
10 hectometers	=1 kilometer	km

The units of area and volume are the squares and cubes of the linear units. They are written according to the following form:

$$3\ m^2 = 3 \text{ square meters,}$$
$$4\ m^3 = 4 \text{ cube meters.}$$

The metric units in general use are the millimeter, centimeter, meter and kilometer.

TABLE 16.

COMMON LOGARITHMS OF NUMBERS.

(American Practical Navigator.)

No.	0	1	2	3	4	5	6	7	8	9	Av. Diff
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743	
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555	380
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059	350
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301	320
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319	300
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140	280
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789	265
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285	250
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646	235
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885	220
20	30103	30320	30535	30750	30963	31165	31387	31597	31806	32015	212
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044	202
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984	193
23	36173	36361	36549	36736	36922	37107	37291	27475	37658	37840	185
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975	164
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560	158
28	44716	44871	45025	45179	45232	45484	45637	45788	45939	46090	153
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567	148
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996	143
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379	138
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720	134
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020	130
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283	126
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509	122
36	55630	55751	55871	55991	56110	56229	56348	56476	56585	56703	119
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864	116
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995	113
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097	110
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172	107
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221	105
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246	102
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246	100
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225	98
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181	96
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117	93
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034	91
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931	90
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810	88
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672	86
51	70757	70842	70927	71012	71096	71181	71265	71349	71435	71517	84
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346	83
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159	81
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957	79
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741	78
56	74819	74896	74974	75051	75128	75205	75282	75358	75433	75511	77
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268	76
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012	74
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743	73
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462	72
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169	71
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865	70
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550	68
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224	67
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889	66
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543	65
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187	64
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822	63
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448	62
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065	62
71	85126	85187	85248	85309	85370	85431	85491	95552	85612	85673	61
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273	60
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864	59
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448	58

TABLE 16—Continued.

COMMON LOGARITHMS OF NUMBERS.

(American Practical Navigator.)

75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024	57.5
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593	57
77	88649	88705	88762	88818	88874	88930	88936	89042	89098	89154	56
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708	55.5
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255	55
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795	54
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855	53
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376	52
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891	51.5
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399	51
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902	50
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399	50
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890	49
89	94939	93988	95036	95085	95134	95182	95231	95279	95328	95376	48.5
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856	48
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332	47.5
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802	47
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267	46.5
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727	46
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182	45.5
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632	45
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078	44.5
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520	44
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957	44
No.	0	1	2	3	4	5	6	7	8	9	Av. Diff

Explanation:

Calculations made by the use of the above table will be accurate to four places and, by careful interpolation, to five places. The degree of accuracy obtainable is sufficient for practically all radio problems. The **slide rule,** which is generally used for radio calculations, is based on the principle of logarithms and is considered sufficiently accurate.

It will be noted that the logarithms are given to five places, and that the logarithm of a number having three significant figures can be read directly from the table. For larger numbers, use the average difference given in the last column for interpolation.

Example:

Find the logarithm of 3.1416.

Solution:

The log of 3.14 can be found directly and is 0.49693.

Interpolating for the 16

$$
\begin{array}{lr}
\text{Av. diff.} = & 138 \\
 & .16 \\
\hline
\text{Multiplying} & 22.08 \text{ to be added to } 0.49693
\end{array}
$$

$$
\begin{array}{lr}
\text{Log of } 3.14 = & 0.49693 \\
\text{Interpolation} & 22 \\
\hline
\text{Log of } 3.1416 = & 0.49715
\end{array}
$$

Example:

Find the number corresponding to logarithm 3.95493.

Solution:

Characteristic = 3. (for pointing off whole number of 4 places)
Mantissa = 95498 (to be found in the table)
From the table, mantissa lies between the numbers 9010 and 9020

Difference between	95493	(log of 9020)
	95472	(log of 9010)
Difference =	21	
Average difference =	48	

Dividing, $\frac{21}{48} = 0.45$ of difference between 9010 and 9020, or 4.5.

Number = 9014.5, which is correct to .05 of 1%.

Natural logarithms.

The natural logarithms employ $\epsilon = 2.7183$ as their base, while the common logarithms use 10.

When natural logarithms are used in calculations, the fact is usually noted by writing the logarithm in the following form,

$$\log_\epsilon x$$

where x is any natural number.

No table of natural logarithms is given because they can be found more conveniently from the common logarithms.

To change common logarithms to natural logarithms, multiply the former by 2.3026.

Formula: $\log_\epsilon x = 2.3026 \log x$

Example:

Find the natural logarithm of 3.1416.

Solution:

$$\log\ 3.1416 = 0.49715$$
$$\log_\epsilon\ 3.1416 = 2.3026 \times 0.49715$$

whence $\log_\epsilon\ 3.1416 = 1.4473.$

To change natural logarithms to common logarithms, multiply the former by 0.4343.

Formula: $\log\ x = 0.4343 \log_\epsilon x$

Example:

Find the number corresponding to the natural logarithm 0.69315.

Solution:

$$\log\ x = 0.4343 \times 0.69315$$
$$= 0.30103$$

whence $x = 2.0.$

TABLE 17.

CIRCULAR (TRIGONOMETRIC) FUNCTIONS.

(Adapted from Smithsonian Tables.)

Radians	Degrees	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
0.0000	0.0	.0000	∞	1.0000	0.0000	.0000	∞	∞	∞	90.0	1.5708
0.0087	0.5	.0087	7.9408	1.0000	.0000	.0087	7.9409	114.59	1.0580	89.5	1.5621
0.0175	1.0	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	89.0	1.5533
0.0262	1.5	.0262	.4179	.9997	.9999	.0262	.4181	38.188	.5819	88.5	1.5446
0.0349	2.0	.0349	.5428	.9994	.9997	.0349	.5431	28.636	.4569	88.0	1.5359
0.0436	2.5	.0436	.6397	.9990	.9996	.0437	.6401	22.904	.3599	87.5	1.5272
0.0524	3.0	.0523	.7188	.9986	.9994	.0524	.7194	19.081	.2806	87.0	1.5184
0.0611	3.5	.0610	.7857	.9981	.9992	.0612	.7865	16.350	.2135	86.5	1.5097
0.0698	4.0	.0698	.8436	.9976	.9989	.0699	.8446	14.301	.1554	86.0	1.5010
0.0785	4.5	.0785	.8946	.9969	.9987	.0787	.8960	12.706	.1040	85.5	1.4923
0.0873	5.0	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	85.0	1.4835
0.0960	5.5	.0958	.9816	.9954	.9980	.0963	.9836	10.385	.0164	84.5	1.4748
0.1047	6.0	.1045	9.0192	.9945	.9976	.1051	9.0216	9.5144	0.9784	84.0	1.4661
0.1134	6.5	.1132	.0539	.9936	.9972	.1139	.0567	8.7769	.9433	83.5	1.4574
0.1222	7.0	.1219	.0859	.9925	.9968	.1228	.0891	8.1443	.9109	83.0	1.4486
0.1309	7.5	.1305	.1157	.9914	.9963	.1317	.1194	7.5958	.8806	82.5	1.4399
0.1396	8.0	.1392	.1436	.9903	.9958	.1405	.1478	7.1154	.8522	82.0	1.4312
0.1484	8.5	.1478	.1697	.9890	.9852	.1495	.1745	6.7912	.8255	81.5	1.4224
0.1571	9.0	.1564	.1793	.9877	.9946	.1584	.1997	6.3138	.8003	81.0	1.4137
0.1685	9.5	.1650	.2176	.9863	.9940	.1673	.2236	5.9758	.7764	80.5	1.4050
0.1745	10.0	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	80.0	1.3963
0.1833	10.5	.1822	.2606	.9833	.9927	.1853	.2680	5.3955	.7320	79.5	1.3875
0.1920	11.0	.1908	.2806	.9816	.9919	.1944	.2887	5.1446	.7113	79.0	1.3788
0.2007	11.5	.1994	.2997	.9799	.9912	.2035	.3085	4.9152	.6915	78.5	1.3701
0.2094	12.0	.2079	.3179	.9781	.9904	.2126	.3275	4.7046	.6725	78.0	1.3614
0.2182	12.5	.2164	.3353	.9763	.9896	.2217	.3458	4.5107	.6542	77.5	1.3526
0.2269	13.0	.2250	.3521	.9744	.9887	.2309	.3634	4.3315	.6366	77.0	1.3439
0.2356	13.5	.2334	.3682	.9724	.9878	.2401	.3804	4.1653	.6196	76.5	1.3352
0.2443	14.0	.2419	.3837	.9703	.9869	.2493	.3968	4.0108	.6032	76.0	1.3265
0.2531	14.5	.2504	.3986	.9681	.9859	.2586	.4127	3.8667	5873	75.5	1.3177
0.2618	15.0	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.05719	75.0	1.3090
0.2705	15.5	.2672	.4269	.9636	.9839	.2773	.4430	3.6059	.5570	74.5	1.3003
0.2793	16.0	.2756	.4403	.9613	.9828	.2867	.4575	3.4874	.5425	74.0	1.2915
0.2880	16.5	.2840	.4533	.9588	.9817	.2962	.4716	3.3759	.5284	73.5	1.2828
0.2967	17.0	.2924	.4659	.9563	.9806	.3057	.4853	3.2709	.5147	73.0	1.2741
0.3054	17.5	.3007	.4781	.9537	.9794	.3153	.4987	3.1716	.5013	72.5	1.2654
0.3142	18.0	.3090	.4900	.9511	.9782	.3249	.5118	3.0777	.4882	72.0	1.2566
0.3229	18.5	.3173	.5015	.9483	.9770	.3346	.5245	2.9887	.4755	71.5	1.2479
0.3316	19.0	.3256	.5126	.9455	.9757	.3443	.5370	2.9042	.4630	71.0	1.2392
0.3403	19.5	.3338	.5235	.9426	.9743	.3541	.5491	2.8239	.4509	70.5	1.2305
0.3491	20.0	.3420	9.5341	.9397	9.9730	.3640	9.5611	2.7475	0.4389	70.0	1.2217
0.3578	20.5	.3502	.5443	.9367	.9716	.3739	.5727	2.6746	.4273	69.5	1.2130
0.3665	21.0	.3584	.5543	.9336	.9702	.3839	.5842	2.6051	.4158	69.0	1.2043
0.3752	21.5	.3665	.5641	.9304	.9687	.3939	.5954	2.5386	.4046	68.5	1.1956
0.3840	22.0	.3746	.5736	.9272	.9673	.4040	.6064	2.4751	.3936	68.0	1.1868
0.3927	22.5	.3827	.5828	.9239	.9656	.4142	.6172	2.4142	.3828	67.5	1.1781
0.4014	23.0	.3907	.5919	.9205	.9640	.4245	.6279	2.3559	.3721	67.0	1.1694
0.4102	23.5	.3987	.6007	.9171	.9624	.4348	.6383	2.2998	.3617	66.5	1.1606
0.4189	24.0	.4067	.6093	.9135	.9607	.4452	.6486	2.2460	.3514	66.0	1.1519
0.4276	24.5	.4147	.6177	.9100	.9590	.4557	.6587	2.1943	.3413	65.5	1.1432
0.4363	25.0	.4226	9.6259	.9063	9.9573	.4663	9.6687	2.1445	0.3313	65.0	1.1345
0.4451	25.5	.4305	.6340	.9026	.9555	.4770	.6785	2.0965	.3215	64.5	1.1257
0.4538	26.0	.4384	.6418	.8988	.9537	.4877	.6882	2.0503	.3118	64.0	1.1170
0.4625	26.5	.4462	.6495	.8949	.9518	.4986	.6977	2.0057	.3023	63.5	1.1083
0.4712	27.0	.4540	.6570	.8910	.9499	.5095	.7072	1.9626	.2928	63.0	1.0996
0.4800	27.5	.4617	.6644	.8870	.9479	.5206	.7165	1.9210	.2835	62.5	1.0908
0.4887	28.0	.4695	.6716	.8829	.9459	.5317	.7257	1.8807	.2743	62.0	1.0821
0.4974	28.5	.4722	.6787	.8788	.9439	.5430	.7348	1.8418	.2652	61.5	1.0734
0.5061	29.0	.4848	.6856	.8746	.9418	.5543	.7438	1.8040	.2562	61.0	1.0674
0.5149	29.5	.4924	.6923	.8704	.9397	.5658	.7526	1.7675	.2474	60.5	1.0559
		Cosines		Sines		Cotangents		Tangents		Degrees	Radians
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		

TABLE 17—Continued.

CIRCULAR (TRIGONOMETRIC) FUNCTIONS.

(Adapted from Smithsonian Tables.)

Radians	Degrees	Sines		Cosines		Tangent		Cotangents			
		Nat.	Log.	Nat.	Log	Nat.	Log.	Nat.	Log.		
0.5236	30.0	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	60.0	1.0472
0.5323	30.5	.5075	.7055	.8616	.9353	.5890	.7701	1.6977	.2299	59.5	1.0385
0.5411	31.0	.5150	.7118	.8572	.9331	.6009	.7788	1.6643	.2212	59.0	1.0297
0.5498	31.5	.5225	.7181	.8526	.9308	.6128	.7873	1.6319	.2127	58.5	1.0210
0.5585	32.0	.5299	.7242	.8480	.9284	.6249	.7958	1.6003	.2042	58.0	1.0123
0.5672	32.5	.5373	.7302	.8434	.9260	.6371	.8042	1.5697	.1958	57.5	1.0036
0.5760	33.0	.5446	.7361	.8387	.9236	.6494	.8125	1.5399	.1875	57.0	0.9948
0.5847	33.5	.5519	.7419	.8339	.9211	.6619	.8208	1.5108	.1792	56.5	0.9861
0.5934	34.0	.5592	.7476	.8290	.9186	.6745	.8290	1.4826	.1710	56.0	0.9774
0.6021	34.5	.5664	.7531	.8241	.9160	.6873	.8371	1.4550	.1629	55.5	0.9687
0.6109	35.0	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	55.0	0.9599
0.6196	35.5	.5807	.7640	.8141	.9107	.7133	.8533	1.4019	.1467	54.5	0.9512
0.6283	36.0	.5878	.7692	.8090	.9080	.7265	.8613	1.3764	.1387	54.0	0.9425
0.6370	36.5	.5948	.7744	.8039	.9052	.7400	.8692	1.3514	.1308	53.5	0.9338
0.6458	37.0	.6018	.7795	.7986	.9023	.7536	.8771	1.3270	.1229	53.0	0.9250
0.6545	37.5	.6088	.7844	.7934	.8995	.7673	.8850	1.3032	.1150	52.5	0.9163
0.6632	38.0	.6157	.7893	.7880	.8965	.7813	.8928	1.2799	.1072	52.0	0.9076
0.6720	38.5	.6225	.7941	.7826	.8935	.7954	.9006	1.2572	.0994	51.5	0.8988
0.6807	39.0	.6293	.7989	.7771	.8905	.8098	.9084	1.2349	.0916	51.0	0.8901
0.6894	39.5	.6361	.8035	.7716	.8874	.8243	.9161	1.2131	.0839	50.5	0.8814
0.6981	40.0	.6428	9.8081	.7660	9.8843	.8391	9.9238	1.1918	0.0762	50.0	0.8727
0.7069	40.5	.6494	.8125	.7604	.8810	.8541	.9315	1.1708	.0685	49.5	0.8639
0.7156	41.0	.6561	.8169	.7547	.8778	.8693	.9392	1.1504	.0608	49.0	0.8552
0.7243	41.5	.6626	.8213	.7490	.8745	.8847	.9468	1.1303	.0532	48.5	0.8465
0.7330	42.0	.6691	.8255	.7431	.8711	.9004	.9544	1.1106	.0456	48.0	0.8378
0.7418	42.5	.6756	.8297	.7373	.8676	.9163	.9621	1.0913	.0379	47.5	0.8290
0.7505	43.0	.6820	.8338	.7314	.8641	.9325	.9697	1.0724	.0303	47.0	0.8203
0.7592	43.5	.6884	.8378	.7254	.8606	.9490	.9772	1.0538	.0228	46.5	0.8116
0.7579	44.0	.6947	.8418	.7193	.8569	.9657	.9848	1.0355	.0152	46.0	0.8029
0.7767	44.5	.7009	.8457	.7133	.8532	.9827	.9924	1.0176	.0076	45.5	0.7941
0.7854	45.0	.7071	.8495	.7071	.8495	1.0000	0.0000	1.0000	0.0000	45.0	0.7854
		Cosines		Sines		Cotangents		Tangents		Degrees	Radians
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		

TABLE 18.

MATHEMATICAL CONSTANTS.

	Number	Logarithm
$\epsilon = 2.71828$	$\pi = 3.14159$	0.49715
$\epsilon^{-1} = \frac{1}{\epsilon} = 0.36788$	$2\pi = 6.28318$	0.79830
$M = \log_{10}\epsilon = 0.43429$	$4\pi = 12.56636$	1.09911
$(M)^{-1} = \frac{1}{M} = \log_{\epsilon} 10 = 2.30258$	$\pi^2 = 9.86960$	0.99430
$\log_{10} x = M \log_{\epsilon} x$	$\pi^3 = 31.0064$	1.49145
$\log_{\epsilon} x = \frac{1}{M} \log x$	$\frac{1}{\pi} = 0.31831$	9.50285
$\log_{\epsilon} \pi = 1.14473$	$\frac{\pi}{2} = 1.57079$	0.19602
	$\frac{\pi}{4} = 0.78540$	9.89509
$c = 3 \cdot 10^8$ meters/second		
$2\pi c = 1.88496 \cdot 10^9$	$\frac{4}{3}\pi = 4.18879$	0.62209
$(2\pi c)^2 = 3.55306 \cdot 10^{18}$	$\sqrt{\pi} = 1.77245$	0.24857
$\frac{1}{2\pi c} = 5.30515 \cdot 10^{-10}$	$\frac{\sqrt{\pi}}{2} = 0.88623$	9.94754
$\frac{1}{(2\pi c)^2} = 2.81446 \cdot 10^{-21}$	$\frac{\sqrt{\pi}}{4} = 0.44311$	9.96451
	$\frac{1}{\sqrt{\pi}} = 0.56419$	9.75143
	$\frac{2}{\sqrt{\pi}} = 1.12838$	0.05246
	$\sqrt{\frac{\pi}{2}} = 1.25331$	0.09806
	$\sqrt{\frac{2}{\pi}} = 0.79788$	9.90194
	$\sqrt[3]{\pi} = 1.46460$	0.16572

TABLE 19.—VALUES OF X FOR USE IN FORMULA FOR CALCULATING THE INDUCTANCE OF A SINGLE-LAYER COIL OR SOLENOID (p. 341)

Diameter / Length $\frac{2a}{b}$	K	Difference	Diameter / Length $\frac{2a}{b}$	K	Difference	Diameter / Length $\frac{2a}{b}$	K	Difference
0.00	1.0000	−0.0209	2.00	0.5255	−0.0118	7.00	0.2584	−0.0047
.05	.9791	203	2.10	.5137	112	7.20	.2537	45
.10	.9588	197	2.20	.5025	107	7.40	.2491	43
.15	.9391	190	2.30	.4918	102	7.60	.2448	42
.20	.9201	185	2.40	.4816	97	7.80	.2406	40
0.25	0.9016	−0.0178	2.50	0.4719	−0.0093	8.00	0.2366	−0.0094
.30	.8838	173	2.60	.4626	89	8.50	.2272	86
.35	.8665	167	2.70	.4537	85	9.00	.2185	79
.40	.8499	162	2.80	.4452	82	9.50	.2106	73
.45	.8337	156	2.90	.4370	78	10.00	.2033	....
0.50	0.8181	−0.0150	3.00	0.4292	−0.0075	10.0	0.2033	−0.0133
.55	.8031	146	3.10	.4217	72	11.0	.1903	113
.60	.7885	140	3.20	.4145	70	12.0	.1790	98
.65	.7745	136	3.30	.4075	67	13.0	.1692	87
.70	.7609	131	3.40	.4008	64	14.0	.1605	78
0.75	0.7478	−0.0127	3.50	0.3944	−0.0062	15.0	0.1527	−0.0070
.80	.7351	123	3.60	.3882	60	16.0	.1457	63
.85	.7228	118	3.70	.3822	58	17.0	.1394	58
.90	.7110	115	3.80	.3764	56	18.0	.1336	52
.95	.6995	111	3.90	.3708	54	19.0	.1284	48
1.00	0.6884	−0.0107	4.00	0.3654	−0.0052	20.0	0.1236	−0.0085
1.05	.6777	104	4.10	.3602	51	22.0	.1151	73
1.10	.6673	100	4.20	.3551	49	24.0	.1078	63
1.15	.6573	98	4.30	.3502	47	26.0	.1015	56
1.20	.6475	94	4.40	.3455	46	28.0	.0959	49
1.25	0.6381	−0.0091	4.50	0.3409	−0.0045	20.0	0.0910	−0.0102
1.30	.6290	89	4.60	.3364	43	35.0	.0808	80
1.35	.6201	86	4.70	.3321	42	40.0	.0728	64
1.40	.6115	84	4.80	.3279	41	45.0	.0664	53
1.45	.6031	81	4.90	.3238	40	50.0	.0611	43
1.50	0.5950	−0.0079	5.00	0.3198	−0.0076	60.0	0.0528	−0.0061
1.55	.5871	76	5.20	.3122	72	70.0	.0467	48
1.60	.5795	74	5.40	.3050	69	80.0	.0419	38
1.65	.5721	72	5.60	.2981	65	90.0	.0381	31
1.70	.5649	70	5.80	.2916	62	100.0	.0350	....
1.75	0.5579	−0.0068	6.00	0.2854	−0.0059			
1.80	.5511	67	6.20	.2795	56			
1.85	.5444	65	6.40	.2739	54			
1.90	.5379	63	6.60	.2685	52			
1.95	.5316	61	6.80	.2633	49			

TABLE 20.—VALUES OF CORRECTION TERM *A* IN FORMULAS USED IN CALCULATING THE INDUCTANCE OF SINGLE-LAYER COILS OR SOLENOIDS, SINGLE-LAYER SQUARE COILS AND FLAT SQUARE COILS.

$\frac{d}{D}$	A	Difference	$\frac{d}{D}$	Difference	A	$\frac{d}{D}$	A	Difference
1.00	0.557	−0.051	0.40	−0.359	−0.052	0.15	−1.340	−0.069
0.95	.506	54	.38	.411	54	.14	1.409	74
.90	.452	57	.36	.465	57	.13	1.483	80
.85	.394	61	.34	.522	61	.12	1.563	87
.80	.334	65	.32	.583	64	.11	1.650	96
0.75	0.269	−0.069	0.30	−0.647	−0.069	0.10	−1.746	−0.105
.70	.200	74	.28	.716	74	.09	1.851	.118
.65	.126	80	.26	.790	80	.08	1.969	.133
.60	.046	87	.24	.870	87	.07	2.102	.154
.55	− .041	95	.22	.957	96	.06	2.256	.173
0.50	−0.136	−0.041	0.29	−1.053	−0.051	0.05	−2.439	−0.223
.48	.177	43	.19	1.104	54	.04	2.662	.288
.46	.220	44	.18	1.158	57	.03	2.950	.405
.44	.264	47	.17	1.215	61	.02	3.355	.693
.42	.311	48	.16	1.276	64	.01	4.048	....

TABLE 21.—VALUES OF CORRECTION *B* IN FORMULAS USED IN CALCULATING THE INDUCTANCE OF SINGLE-LAYER COILS OR SOLENOIDS, SINGLE-LAYER SQUARE COILS AND FLAT SQUARE COILS.

Number of turns, n	B	Number of turns, n	B
1	0.000	40	0.315
2	.114	45	.317
3	.166	50	.319
4	.197	60	.322
5	.218	70	.324
6	0.233	80	0.326
7	.244	90	.327
8	.253	100	.328
9	.260	150	.331
10	.266	200	.333
15	0.286	300	0.334
20	.297	400	.335
25	.304	500	.336
30	.308	700	.336
35	.312	1000	.336

TABLE 22.—VALUES OF B_s FOR USE IN FORMULA FOR CALCULATING THE INDUCTANCE OF MULTIPLE-LAYER COILS.

$\frac{b}{c}$	B_s	$\frac{b}{c}$	B_s
1	0.0000	16	0.3017
2	.1202	17	.3041
3	.1753	18	.3062
4	.2076	19	.3082
5	.2292	20	.3099
6	0.2446	21	0.3116
7	.2563	22	.3131
8	.2656	23	.3145
9	.2730	24	.3157
10	.2792	25	.3169
11	0.2844	26	0.3180
12	.2888	27	.3190
13	.2927	28	.3200
14	.2961	29	.3209
15	.2991	30	.3218

TABLE 23.—CONSTANTS USED IN THE FORMULAS USED IN CALCULATING THE INDUCTANCE OF FLAT-SPIRAL, DISK OR PANCAKE COILS.

b/c or c/b	y_1	Difference	b/c	y_3	Difference
0	0.5000	0.0253	0	0.597	0.002
0.025	.5253	237	....	....	....
.05	.5490	434	0.05	.599	3
.10	.5924	386	.10	.602	6
0.15	0.6310	0.0342	0.15	0.608	0.007
.20	.6652	301	.20	.615	9
.25	.6953	266	.25	.624	9
.30	.7217	230	.30	.633	10
0.35	0.7447	0.0198	0.35	0.643	0.011
.40	.7645	171	.40	.654	11
.45	.7816	144	.45	.665	12
.50	.7960	121	.50	.677	13
0.55	0.8081	0.0101	0.55	0.690	0.012
.60	.8182	83	.60	.702	13
.65	.8265	66	.65	.715	14
.70	.8331	52	.70	.729	13
0.75	0.8383	0.0039	0.75	0.742	0.014
.80	.8422	29	.80	.756	15
.85	.8451	19	.85	.771	15
.90	.8470	10	.90	.786	15
0.95	0.8480	0.0003	0.95	0.801	0.015
1.00	.8483	....	1.00	.816	....

TABLE 24.—DIELECTRIC CONSTANT (SPECIFIC INDUCTIVE CAPACITY) OF VARIOUS GASES, LIQUIDS AND SOLIDS.

Substance	Temp. °C	Dielectric Constant	Remarks
Gases:			
Air	0	1.000	Pressure, 1 atmos.
	19	1.022	Pressure, 20 atmos.
Carbon dioxide	0	1.001	Pressure, 1 atmos.
	15	1.020	Pressure, 20 atmos.
Vacuum	. .	0.999	. . . . .
Liquids:			
Alcohol:			
Ethyl	20	25.8	Grain
Methyl	20	31.2	Wood
Glycerine	15	56.2	
Oils:			
Castor	11	4.67	
Petroleum	. .	2.13	
Transformer	. .	2.5	
Vaseline	. .	2.17	
Water	. .	81.0	Distilled
Solids:			
Caoutchouc	. .	2.22	
Ebonite	. .	2.5—3.5	
Fibre	. .	5—8	Vulcanized
Glass	Density		
Flint	4.5	9.9	
Flint	2.87	6.6	
Hardcrown	2.48	7.0	
Lead	3.0—3.5	5.4—8.0	
Gutta Percha	. .	3.3—4.9	
Mica	. .	4—8	Indian and Canadian
Paper:	. .	1.5—3.0	Dry
Cable	. .	2.0—2.5	Treated
Telephone	. .	2.0	Treated
Phenolic base			
Molded material	. .	5.0—7.5	Bakelite
Paraffin	. .	2.1 – 2.3	
Porcelain	. .	4.4 approx.	Glazed
Silk	. .	4.6	
Quartz	. .	4.5	
Shellac	. .	3.0 – 3.7	
Wood			
Maple	. .	3.0 – 4.5	Untreated; dry
Oak	. .	3.0 – 6.0	Untreated; dry

Explanation:

The **dielectric constant** K, also named **specific inductive capacity** and **inductivity,** is the ratio of the capacity C_X of a condenser having

any substance other than air as the dielectric to the capacity C_a of the same condenser when air alone is the dielectric. Thus,

$$K = \frac{C_x}{C_a}$$

where K = dielectric constant,
C_X = capacity of given condenser with other than air dielectric,
C_a = capacity of same condenser with only air dielectric.

Example:

The capacity of a certain condenser with air dielectric, measured at a radio frequency, was found to be 0.005518μf. The condenser was then filled with medium government oil (petroleum base), and the measured capacity was increased to 0.01181μf. What was the dielectric constant of the oil?

Solution:

Formula: $$K = \frac{C_x}{C_a}$$

substituting $$= \frac{0.01181}{0.005518} = 2.14$$

whence $$K = 2.14$$

In the above table, air is taken as the standard. Its dielectric constant is, therefore, unity. The dielectric constants of the other substances are factors to be used in determining what the capacity of a given condenser with air dielectric would be if some other substance were used as the dielectric. The formula is

$$C_X = KC_a$$

where the quantities are the same as previously given.

The values of the dielectric constant given in the table are for radio-frequencies. Except in the case of the gas and liquid dielectrics, it is difficult to substitute for air. Solid specimens are tested for dielectric constant by using mercury for the two plates. Unless a full substitution is effected, erratic results will be obtained. Impurities in the dielectric under test will also materially affect the results.

TABLE 25.

Tables for Calculating the Inductance of Single-Layer Coils.

(a) VALUES OF *P*.

Length of Coil in Inches. (Rows: Diameter of Coil in Inches.)

	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	10	11	12	13	14	15	16	17	18	19	20
2.0	.0865	.1035	.147	.186	.217	.348	.280	3.04	.336	.368	.400	.430	.460	.490	.522	.581	.643	.702	.760	.818	.878	.940	1.000	1.060	1.138	1.186
2.5	.107	.145	.188	.226	.285	.328	.363	.403	.443	.483	.522	.563	.603	.643	.686	.768	.855	.935	.995	1.100	1.175	1.260	1.340	1.41	1.49	1.595
3.0	.166	.228	.290	.370	.424	.494	.580	.655	.725	.795	.870	.940	.980	1.062	1.130	1.265	1.390	1.530	1.675	1.820	1.94	2.08	2.22	2.35	2.48	2.64
3.5	.212	.292	.357	.458	.547	.655	.705	.871	.965	1.05	1.14	1.24	1.32	1.38	1.55	1.69	1.88	2.05	2.26	2.43	2.64	2.80	2.98	3.28	3.32	3.54
4.0	.280	.373	.490	.597	.711	.823	.978	1.08	1.21	1.40	1.53	1.66	1.78	1.91	2.04	2.31	2.53	2.77	3.02	3.33	3.55	3.78	4.05	4.30	4.56	4.78
4.5	.315	.438	.567	.700	.830	.968	1.11	1.28	1.41	1.58	1.83	1.97	2.14	2.25	2.42	2.68	3.01	3.29	3.60	3.94	4.22	4.47	4.84	5.15	5.48	5.78
5.0	.375	.523	.670	.840	1.000	1.16	1.35	1.52	1.74	1.88	2.07	2.32	2.62	2.76	2.98	3.34	3.72	4.08	4.45	4.84	5.18	5.58	5.94	6.32	6.84	7.08
5.5	.425	.600	.782	.980	1.160	1.36	1.56	1.76	1.96	2.17	2.46	2.63	2.87	3.20	3.52	3.96	4.40	4.84	5.32	5.77	6.20	6.72	7.05	7.50	8.00	8.55
6.0	.458	.668	.882	1.04	1.32	1.55	1.77	2.02	2.25	2.49	2.75	3.00	3.25	3.56	4.55	4.93	5.10	5.62	6.18	6.68	7.20	7.70	8.25	8.75	9.25	9.90
7.0	.662	.811	1.14	1.43	1.71	2.03	2.35	2.73	3.00	3.32	3.65	3.97	4.33	4.62	5.04	5.80	6.85	7.65	8.38	9.14	9.80	10.6	11.5	12.1	12.8	13.5
8.0	.692	1.02	1.39	1.80	2.16	2.59	2.94	3.38	3.76	4.15	4.62	5.10	5.50	5.90	6.35	7.55	8.35	9.38	10.9	11.8	12.8	13.7	14.7	15.7	16.6	17.5
9.0	.720	1.21	1.72	2.07	2.59	3.05	3.49	4.03	4.58	5.03	5.63	6.11	6.72	7.20	7.79	8.85	10.0	11.3	12.4	14.4	15.8	17.1	18.1	19.4	20.6	21.7
10	.838	1.35	1.90	2.43	2.99	3.57	4.24	4.80	5.40	6.03	6.75	7.35	8.01	8.75	9.38	10.7	12.0	13.6	15.1	16.7	18.3	21.1	22.3	23.8	25.0	26.8
12	1.20	1.63	2.30	2.96	3.73	4.54	5.33	6.12	7.05	7.88	8.80	9.61	10.5	11.4	12.3	14.1	16.1	18.0	19.4	22.0	24.0	25.7	28.4	30.6	33.6	36.3
14	1.35	2.06	2.60	3.68	4.78	5.68	6.78	7.85	9.10	10.3	11.4	12.4	13.8	15.1	16.2	18.8	21.3	23.9	26.6	29.2	32.0	36.4	37.8	40.5	43.5	46.5
16	1.62	2.44	3.20	4.21	5.50	6.80	8.10	9.70	11.0	12.6	14.2	14.8	17.2	18.6	20.3	23.1	26.7	30.0	33.0	36.6	40.2	43.6	47.0	50.1	54.7	58.2
18	1.85	2.76	4.15	4.84	5.99	7.82	9.68	11.5	12.9	14.7	16.6	18.4	20.7	22.4	24.4	28.0	33.4	35.5	40.6	45.0	49.0	53.2	57.5	62.1	66.8	70.0
20	2.54	3.18	4.45	6.03	6.99	8.71	10.8	12.7	14.8	17.4	19.0	21.6	23.8	26.2	28.6	33.6	38.2	43.2	48.2	54.0	58.8	64.1	70.0	75.0	80.0	85.8

(b) VALUES OF K FOR N.

$$K = \left(\frac{N}{25}\right)^2$$

N	K	N	K	N	K	N	K	N	K
1	.0015	15	.360	29	1.33	44	3.10	72	8.30
2	.0064	16	.410	30	1.44	46	3.38	74	8.74
3	.0144	17	.462	31	1.54	48	3.68	76	9.25
4	.0256	18	.519	32	1.64	50	4.00	78	9.72
5	.0400	19	.578	33	1.74	52	4.32	80	10.20
6	.0575	20	.640	34	1.85	54	4.65	82	10.80
7	.0784	21	.705	35	1.96	56	5.00	84	11.25
8	.1024	22	.772	36	2.08	58	5.38	86	11.82
9	.1296	23	.849	37	2.17	60	5.75	88	12.4
10	.160	24	.940	38	2.31	62	6.13	90	12.9
11	.194	25	1.000	39	2.46	64	6.54	92	13.5
12	.230	26	1.080	40	2.56	66	6.95	94	14.2
13	.270	27	1.165	41	2.70	68	7.40	96	14.7
14	.314	28	1.25	42	2.82	70	7.80	98	15.3

(c) VALUES OF N.

N = Number of Turns Per Inch.

B. & S. GAUGE	ENAMEL	S. C. C.	D. C. C.
12		11.4	10.9
14		14.2	13.5
16		17.8	16.9
17		20.0	18.8
18		22.2	20.8
19		25.6	23.2
20	30	27.7	25.0
21	33	30.7	27.3
22	37	34.4	30.3
23	42	37.5	32.6
24	46	41.4	35.5
25	53	45.6	38.6
26	58	50.2	41.8
27	66	54.9	45.0

B. & S. GAUGE	ENAMEL	S. C. C.	D. C. C.
28	73	60.2	48.5
29	82	65.3	51.8
30	91	71.4	55.5
31	104	77.5	59.1
32	116	84.0	62.8
33	138	90.0	66.2
34	145	97.0	69.9
35	161	104	73.5
36	178	117	83.3
37			
38			
39			
40			
42			

Explanation:

It is often desirable to know how much inductance can be obtained by closely winding a single-layer coil, having a certain diameter and length, with various sized wires and insulation. Tables 25 (a), (b) and (c) permit this to be done with a minimum amount of calculation.

Referring to the Tables, the inductance of any single-layer coil can be found by following formula:

$$L = PK$$

where L = inductance in mh,

P = factor for coil size, Table 25 (a),

$K = \left(\frac{N}{25}\right)^2$ = constant for number of turns and size of wire, Tables 25 (b) and (c).

Example:

Calculate the inductance of a coil 5 inches in diameter closely wound with 200 turns of No. 20 B & S gauge, DCC wire.

Solution:

Using Table 25 (c), it is seen that No. 20 DCC wire can be wound 25 turns per inch.

Hence $N=25$

Entering Table 25 (b) with 25 under N, find value of K, opposite 25.

It is $K=1.000$

Since the coil is to have 200 turns, the length of the winding will be

$$\frac{200}{N}=\frac{200}{25}=8$$

or length of coil = 8.

Enter Table 25 (a) with 5 inches (diameter of coil), which is found in column at left, then move to right along this line until under 8. Read the number.

It is $P=2.62$

Formula $L=PK$

substituting $=2.62\times1.000=2.62$

whence $L=2.62$ mh.

SYMBOLS AND ABBREVIATIONS.

B = magnetic flux density; Bg = magnetic induction in air.
C = capacity (electrostatic capacity).
D = electric displacement; electric surface density.
E = effective electromotive force; E_0 maximum emf.
F = force.
H = magnetic field intensity; heat; H_c = coercive force.
I = effective current; I_0 = maximum current; I_r = received current; I_s = current in transmitter antenna.
J = intensity of magnetization.
K = dielectric constant.
L = inductance.
M = mutual inductance.
N = number of conductors or turns.
P = average power.
Q = quantity of electricity.
R = resistance.
S = area.
T = period, or one complete cycle.
V = potential difference of a condenser; V_0 = maximum condenser voltage.
W = average energy.
X = reactance.
Y = admittance.
Z = impedance.

b = breadth or width.
c = velocity of light $= 2.9982 \cdot 10^8$ meters/second.
d = diameter; distance.
e = instantaneous electromotive force.
f = frequency (cycles per second).
g = conductance.
h = height.
i = instantaneous current.
k = coefficient of coupling.
l = length.
m = mass; magnetic pole strength.
n = velocity of rotation (revolutions per second); number of turns of wire.
o = subscript letter for maximum.
p = instantaneous power.
r = distance from a point (radius); subscript letter for receiving.
s = length along a path; subscript letter for transmitting.
t = time.
v = velocity.
w = instantaneous energy.

Φ = magnetic flux.
Ψ = electrostatic flux.
Ω = ohm.
γ = electric conductivity.
δ = logarithmic decrement.
$\epsilon = 2.7183$ (base of Napierian logarithms).
η = efficiency (per cent).
θ = phase angle (degree or radian).

κ = susceptibility (J/H).
λ = wave length in meters; λ_{km} = wave length in kilometers, etc.
μ = permeability (B/H).
π = 3.1416 = ratio of circumference to diameter.
ρ = volume resistivity.
τ = thickness.
φ = angle.
ψ = difference in phase.
$\omega = 2\pi f$ (angular velocity in radians per second); megohm.

$\mathcal{E}$ = electric field indensity (electric gradient).
$\mathfrak{F}$ = magnetomotive force.
$\mathcal{R}$ = reluctance.

SPECIAL SYMBOLS.

h = henry.
mh = millihenry.
μh = microhenry.
f = farad.
μf = microfarad.
mμf = milli-microfarad.
$\mu\mu$f = micro-microfarad.
Ω = ohm.
ω = megohm.
emu = electromagentic unit.
esu = electrostatic unit.
rf = radio frequency.
af = audio frequency.
emf = electromotive force.
mmf = magnetomotive force.
ac = alternating current.
dc = direct current.
rms = root-mean-square.
cgs = centimeter-gram-second (absolute) system.

SIGNS.

$\propto$	is proportional to; varies as.
$=$	is exactly equal to.
$\times$	multiplied by; also for powers of 10, as $1 \cdot 10^3$.
$\div$	divided by; also / and –, as 2/3 and $\frac{2}{3}$.
$+$	plus, sign of addition.
$-$	minus, sign of subtraction.
$\sim$	cycle.
$>$	is greater than.
$\gg$	is much greater than.
$<$	is less than.
$\ll$	is much less than.
∞	infinity.
$\angle$	angle.
$\odot$	circle.

GREEK ALPHABET.

Letters		Names	Letters		Names	Letters		Names
Α	α	Alpha	Ι	ι	Iota	Ρ	ρ	Rho
Β	β	Beta	Κ	κ	Kappa	Σ	σ ς	Sigma
Γ	γ	Gamma	Λ	λ	Lambda	Τ	τ	Tau
Δ	δ	Delta	Μ	μ	Mu	Υ	υ	Upsilon
Ε	ϵ	Epsilon	Ν	ν	Nu	Φ	ϕ	Phi
Ζ	ζ	Zeta	Ξ	ξ	Xi	Χ	χ	Chi
Η	η	Eta	Ο	o	Omicron	Ψ	ψ	Psi
Θ	θ	Theta	Π	π	Pi	Ω	ω	Omega

INDEX.

A

D